

Solitonic thermal transport in a current-biased long Josephson junctionClaudio Guarcello,^{1,*} Paolo Solinas,² Alessandro Braggio,¹ and Francesco Giazotto¹¹*NEST, Istituto Nanoscienze-CNR and Scuola Normale Superiore, Piazza San Silvestro 12, I-56127 Pisa, Italy*²*SPIN-CNR, Via Dodecaneso 33, I-16146 Genova, Italy*

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We investigate the coherent energy and thermal transport in a temperature-biased long Josephson tunnel junction, when a Josephson vortex, i.e., a soliton, steadily drifts driven by an electric bias current. We demonstrate that thermal transport through the junction can be controlled by the bias current, since it determines the steady-state velocity of the drifting soliton. We study the effects on thermal transport of the damping affecting the soliton dynamics. In fact, a soliton locally influences the power flowing through the junction and can cause the variation of the temperature of the device. When the soliton speed increases approaching its limiting value, i.e., the Swihart velocity, we demonstrate that the soliton-induced thermal effects significantly modify. Finally, we discuss how the appropriate material selection of the superconductors forming the junction is essential, since short quasiparticle relaxation times are required to observe fast thermal effects.

DOI: [10.1103/PhysRevB.98.104501](https://doi.org/10.1103/PhysRevB.98.104501)**I. INTRODUCTION**

Long Josephson junctions (LJJs) are physical systems often used as a framework to explore nonlinear dynamics [1,2]. Nonetheless, coherent thermal transport in this context was explored only recently [3–5]. In fact, as a temperature gradient is imposed across the device, namely, as the electrodes forming the junction reside at different temperatures, a heat current depending on the configurations of Josephson vortices, i.e., solitons, flows through the device [4,5]. The phase-dependent heat current was recently explored theoretically and experimentally in both Josephson junctions (JJs) [6,7] and superconducting quantum-interference devices (SQUIDs) [8,9]. This phenomenon is the core of the emerging field of phase-coherent caloritronics [10–12], from which fascinating devices, such as heat diodes [13], thermal transistors [14], solid-state memories [15,16], microwave refrigerators [17], thermal engines [18], thermal routers [4,19], and heat amplifier [20], were conceived. Recently, it was demonstrated theoretically that a static soliton in a temperature-biased long tunnel junction induces a localized warming in one of the electrodes of the device, according to which the application as a fast solitonic thermal router was suggested [4]. The scenario changes if we consider a time-dependent external magnetic field, since more solitons can be excited along the device and the soliton configuration reflects on the temperature profile of the junction, so that every magnetically excited soliton induces a well-localized temperature peak [5].

In this paper we discuss how an electric bias current affects thermal transport when a soliton is steadily drifting along the system as driven by the bias current. We demonstrate that the phase-dependent components of the heat current depend on the soliton speed, the latter being also a function of the bias current. Therefore, the dissipationless bias current can

be used as a knob to locally modify thermal transport across the device. Additionally, we study how the damping affecting the soliton dynamics influences thermal transport. In fact, in the case of a low value of the damping parameter [21], we demonstrate that the energy and thermal transport profiles along the junction induced by the soliton significantly change, as the bias current increases.

In the following, we will make two realistic approximations, namely, we write the phase solution representing an electrically driven soliton in a closed simplified form [22], and we consider a soliton traveling at the steady drift velocity [23].

The paper is organized as follows. In Sec. II, we examine how an electric biasing current generally affects energy transport in a temperature-biased JJ. In Sec. III, we focus on a soliton moving under the influence of a bias current in a LJJ. In Sec. IV, we discuss the behavior of heat currents through the junction as a function of the bias current in the adiabatic limit. In Sec. V, we discuss the adiabatic limit and its implication on material selection in order to find appropriate thermal response time scales. In Sec. VI, conclusions are drawn.

II. ENERGY TRANSPORT

In a current-biased LJJ, see Fig. 1(a), the phase difference $\varphi(x, t)$ along the junction in the presence of a soliton can be approximatively expressed as the sum of a dynamic contribution $\phi(x, t)$, and a static contribution $\sin^{-1} i_b$, that is [22]

$$\varphi(x, t) \simeq \phi(x, t) + \sin^{-1} i_b, \quad (1)$$

where the phase shift induced by the bias current $i_b = I_{\text{bias}}/I_c$ (with I_c being the critical current of the device) is simply added to the soliton solution ϕ . This dissipationless current can be directly delivered by a current source, or it can coincide with the current circulating in a magnetically driven superconducting ring containing the junction.

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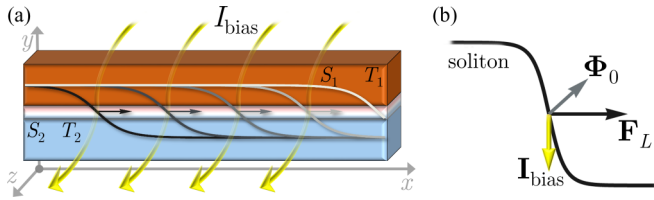


FIG. 1. (a) A superconductor-insulator-superconductor (SIS) temperature-biased rectangular long Josephson junction (LJJ) driven by an electrical bias current I_{bias} . The temperature T_i of the electrode S_i is also indicated. A soliton drifting due to the bias current is shown. (b) Lorentz force due to a bias current on a soliton. Indeed, a bias current flowing through the junction acts on the soliton with a Lorentz force, $\mathbf{F}_L \propto \mathbf{I}_{\text{bias}} \times \Phi_0$, with the direction of Φ_0 depending on the polarity of the soliton [see Eq. (10)].

In a washboardlike picture [21], the term $\sin^{-1} i_b$ in Eq. (1) represents the shift of the potential minimum, within which the phase profile $\phi(x, t)$ lies. This shift is induced by the tilting of the potential imposed by the external bias current.

Energy transport in a temperature-biased JJ can be written as [4,24–27]

$$P_{\text{tot}}(T_1, T_2, \varphi) = P_{\text{qp}}(T_1, T_2) - \cos \varphi P_{\text{qp-pair}}(T_1, T_2, V) + \sin \varphi P_{\text{pair}}(T_1, T_2, V), \quad (2)$$

where $V(x, t) = \frac{\Phi_0}{2\pi} \frac{\partial \varphi(x, t)}{\partial t}$ is the local voltage drop ($\Phi_0 = h/2e \simeq 2 \times 10^{-15}$ Wb is the magnetic flux quantum, with e and h being the electron charge and the Planck constant, respectively) and T_i is the temperature of the electrode S_i . In Eq. (2), P_{qp} , $P_{\text{qp-pair}}$, and P_{pair} represent the quasiparticle and the “anomalous” contributions to thermal current density flowing through the junction [10,24,26,28–30] (see Appendix A). In fact, P_{qp} is the heat flux density carried by quasiparticles and represents an incoherent flow of energy through the junction from the hot to the cold electrode. Instead, the “anomalous” terms P_{pair} and $P_{\text{qp-pair}}$ determine the phase-dependent part of the heat current originating from the energy-carrying tunneling processes involving Cooper pairs and recombination/destruction of Cooper pairs on both sides of the junction. We remark that P_{pair} linearly depends on the voltage drop (see Appendix A), so that $P_{\text{pair}} \rightarrow 0$ when $V \rightarrow 0$ [26].

According to Eq. (1), the phase-dependent terms in Eq. (2) become

$$\begin{aligned} \cos \varphi P_{\text{qp-pair}} &= \cos(\phi + \sin^{-1} i_b) P_{\text{qp-pair}} \\ &= (\sqrt{1 - i_b^2} \cos \phi - i_b \sin \phi) P_{\text{qp-pair}}, \end{aligned} \quad (3)$$

and

$$\begin{aligned} \sin \varphi P_{\text{pair}} &= \sin(\phi + \sin^{-1} i_b) P_{\text{pair}} \\ &= (\sqrt{1 - i_b^2} \sin \phi + i_b \cos \phi) P_{\text{pair}}. \end{aligned} \quad (4)$$

Accordingly, P_{tot} can be recast by defining two phase-dependent terms,

$$P_{\text{tot}}(T_1, T_2, \varphi) = P_{\text{qp}}(T_1, T_2, \varphi) + P_{\phi}^0(T_1, T_2) + P_{\phi}^1(T_1, T_2), \quad (5)$$

which depends on the bias current according to

$$P_{\phi}^0(T_1, T_2) = (-\cos \phi P_{\text{qp-pair}} + \sin \phi P_{\text{pair}}) \sqrt{1 - i_b^2}, \quad (6)$$

$$P_{\phi}^1(T_1, T_2) = (\sin \phi P_{\text{qp-pair}} + \cos \phi P_{\text{pair}}) i_b. \quad (7)$$

Notably, for $i_b = 0$ the usual energy transport across a tunnel junction is recovered,

$$P_{\text{tot}}(T_1, T_2, \varphi) = P_{\text{qp}}(T_1, T_2, \varphi) - \cos \phi P_{\text{qp-pair}}(T_1, T_2, \varphi) + \sin \phi P_{\text{pair}}(T_1, T_2, \varphi), \quad (8)$$

instead, in the limit of $i_b \rightarrow 1$, Eq. (2) becomes

$$P_{\text{tot}}(T_1, T_2, \varphi) \rightarrow P_{\text{qp}}(T_1, T_2, \varphi) + \sin \phi P_{\text{qp-pair}}(T_1, T_2, \varphi) + \cos \phi P_{\text{pair}}(T_1, T_2, \varphi). \quad (9)$$

Interestingly, by increasing the bias current the role of $\sin \phi$ and $\cos \phi$ tends to swap. Moreover, we observe that by inverting the flowing direction of the bias current only the sign of P_{ϕ}^1 changes [see Eq. (6)], whereas P_{ϕ}^0 is invariant with respect to changes of sign of the bias current.

The behavior of P_{tot} in the absence of solitons, namely, $\phi = 0$ in Eq. (1) so that $\varphi = \sin^{-1}(i_b)$, as a function of the normalized bias current i_b is shown in Fig. 2, for $T_1 = 7$ K and $T_2 = 4.2$ K. We observe that the bias current causes the power P_{tot} to monotonically increase.

Finally, we note that in the time-independent case an equilibrium dissipationless current does not generate any Joule heating terms contributing to the energy exchange.

III. SOLITON DYNAMICS IN AN ELECTRICALLY BIASED LJJ

Although LJJs were first measured more than 40 years ago [31,32], they are still the subject of both theoretical [33–41] and experimental [42–50] studies, also because they are the ideal solid-state environment to investigate the properties of soliton [1,2]. These excitations give rise to step structures in the I-V characteristic of LJJs, microwave radiation emission, and they carry a quantum of magnetic flux Φ_0 , induced by a supercurrent loop surrounding it, with the local magnetic field perpendicularly oriented with respect to the junction length [23]. For this reason, solitons in the context of LJJs are usually referred to as fluxons or Joseph-

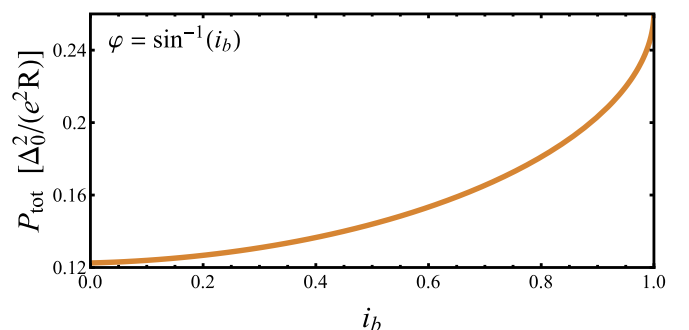


FIG. 2. P_{tot} [in units of $\Delta_0^2/(e^2 R)$] as a function of the normalized bias current i_b , in the absence of soliton, namely, $\phi = 0$ in Eq. (1) so that $\varphi = \sin^{-1}(i_b)$, for $T_1 = 7$ K and $T_2 = 4.2$ K.

son vortices. Solitons in LJJs can be easily generated by an external magnetic field [4,5]. Alternatively, in an annular geometry [51], i.e., a “closed” LJJ folded back into itself in which solitons move with no interactions with edges, fluxons can be excited at will [52,53], permitting highly controllable soliton dynamics.

We consider a current-biased long Josephson tunnel junction, with normal-state resistance R and specific capacitance C_s , connecting two superconducting leads, S_1 and S_2 residing at temperatures T_1 and T_2 [see Fig. 1(a)]. We assume leads made by the same superconductor, so that $T_{c1} = T_{c2} = T_c$ and $\Delta_0 = 1.764k_B T_c$ is the zero-temperature superconducting gap.

A bias current flowing through the junction acts on the soliton with a Lorentz force, $\mathbf{F}_L \propto \mathbf{I}_{\text{bias}} \times \mathbf{\Phi}_0$ [with the direction of $\mathbf{\Phi}_0$ depending on the polarity of the soliton; see Eq. (10)]; see Fig. 1(b). So, in the presence of an external bias current a soliton shifts along the junction.

The phase solution representing a soliton moving with velocity u along a LJJ, in the presence of a bias current I_{bias} , can be written approximatively as [22]

$$\varphi_s(x, t) \simeq 4 \arctan\{\exp[\sigma \xi(x, t)]\} + \sin^{-1}(i_b), \quad (10)$$

where

$$\xi(x, t) = \frac{x - x_0 - ut}{\lambda_J \sqrt{1 - \left(\frac{u}{\bar{c}}\right)^2}} = \frac{\tilde{x} - \tilde{x}_0 - \tilde{u}t}{\sqrt{1 - \tilde{u}^2}}, \quad (11)$$

and $\sigma = \pm 1$ is the polarity of the soliton. Here, we used the normalized units $\tilde{x} = x/\lambda_J$, $\tilde{t} = \omega_p t$, and $\tilde{u} = u/\bar{c}$, with $\omega_p = \sqrt{\frac{2\pi}{\Phi_0} \frac{J_c}{C_s}}$ and $\lambda_J = \sqrt{\frac{\Phi_0}{2\pi\mu_0} \frac{1}{t_d J_c}}$ being the plasma frequency and the Josephson penetration depth, respectively. Moreover, μ_0 is the vacuum permeability, $t_d = \lambda_{L,1} + \lambda_{L,2} + d$ is the effective magnetic thickness (where $\lambda_{L,i}$ is the London penetration depth of the i th superconductor and d is the insulating thickness), and $\bar{c} = \omega_p \lambda_J$ is the Swihart velocity, namely, the limiting soliton velocity in the junction [21]. The Swihart velocity of typical high-quality superconductor-insulator-superconductor (SIS) junctions is $\bar{c} \sim 10^6 - 10^7$ m/s. The velocity-dependent factor in Eq. (11) represents the relativistic contraction of the soliton when its velocity approaches the maximum speed, i.e., $\tilde{u} \rightarrow 1$ [23]. This is the consequence of Lorentz invariance of the unperturbed sine-Gordon equation describing the electrostatics of a LJJ [21]. The width of the soliton is

$$W_s = \lambda_J \sqrt{1 - \left(\frac{u}{\bar{c}}\right)^2}, \quad (12)$$

so that the faster the soliton, the narrower it is.

According to the perturbation approach [23], in the presence of an external bias current the steady-state drifting velocity of the soliton, in units of \bar{c} , reads [21,23]

$$\tilde{u}_d(i_b) = \frac{1}{\sqrt{1 + \left(\frac{4\alpha}{\pi i_b}\right)^2}}, \quad (13)$$

where $\alpha = 1/(\omega_p R C_s)$ is the damping parameter, namely, the parameter quantifying the dissipation in the system [21]. This is the velocity at which the power input from the bias current

is equal to the power loss due to damping affecting the soliton dynamics [23].

A moving soliton locally induces a voltage drop,

$$V(x, t) = \frac{\hbar}{2e} \frac{\partial \varphi_s}{\partial t} = \frac{\hbar}{e} \frac{\text{sech}[\xi(x, t)]}{\sqrt{1 - \tilde{u}^2}} \tilde{u} \omega_p, \quad (14)$$

and generates a magnetic field (in units of $\frac{2\pi\mu_0 t_d}{\Phi_0}$),

$$H_{\text{in}}(x, t) = \frac{\partial \varphi_s}{\partial x} = \frac{2 \text{sech}[\xi(x, t)]}{\lambda_J \sqrt{1 - \tilde{u}^2}}. \quad (15)$$

From Eq. (13) one obtains $\frac{\tilde{u}_d}{\sqrt{1 - \tilde{u}_d^2}} = \frac{\pi i_b}{4\alpha}$, so that Eq. (14) for a steadily drifting soliton, i.e., with $\tilde{u} = \tilde{u}_d$, becomes

$$V_d(x, t) = \frac{\hbar}{e} \text{sech}[\xi(x, t)] \frac{\pi i_b}{4\alpha} \omega_p. \quad (16)$$

In order to estimate the modifications to energy and thermal transport, we analyze in the next sections the profile of the exchanged power along the junction as a function of various parameters. In particular, we will investigate the steady dynamics of the soliton profile under a constant bias current.

IV. THERMAL TRANSPORT

In this paragraph we wish to investigate the consequences on energy and thermal transport across the junction in the presence of a steadily drifting soliton under the effect of a current biasing.

We expect that the energy flowing through the system will produce evidences, such as a potential modification of the temperature of the junction. Anyway, exploring the thermal behavior of the junction one has to distinguish between dissipative and reactive contributions in Eq. (2). In fact, with the aim to determine the temperature profile, the total thermal power density to take into account has to contain only the dissipative contributions, namely,

$$P_{\text{tot}}^*(T_1, T_2, \varphi) = P_{\text{qp}}(T_1, T_2) - \cos \varphi P_{\text{qp-pair}}(T_1, T_2, V), \quad (17)$$

since the term P_{pair} is purely reactive [26,27].

Furthermore, the time evolution of the temperatures can be obtained by solving self-consistently both the sine-Gordon equation for the phase dynamics [21] and the heat balance equation for each electrode [5]. Conversely, in the following, in the place of solving numerically these equations, we will exploit the simple closed expressions of both the solitonic phase solution and the stationary speed of a soliton [see Eqs. (10) and (13)], respectively, to directly gain insight on energy and thermal transport across the system in the adiabatic regime (see Sec. V A). The solution obtained in this manner well approximates the full solution, since the two equations governing the evolution are weakly coupled. Finally, we observe that the characteristic time scales of thermal processes may differ from the typical evolution time scale of solitons, and strongly depend on the specific characteristics of the junction (see Sec. V B).

The investigation of the direct effect on the temperature profile is beyond the scope of this work, and we have motivated reason to think that focusing on single-soliton effects

is not the appropriate manner to observe experimentally the reported phenomenology. Anyway, for didactic purposes we discuss the single soliton case as the key element for more complex dynamics.

So, in the next section, we are going to discuss the energy and thermal transport profiles at fixed times as a function of the bias current, in both high and low damping cases.

A. Results

In this section we discuss the impact of a bias current on the power flowing through a temperature-biased LJJ, as a soliton is set in. With the aim to only explore how energy and heat transport is affected by i_b , we assume to can work in the adiabatic limit [26], and we use Eq. (2) to calculate the heat current flowing through the junction. The range of validity of the adiabatic limit approximation will be thoroughly discussed in Sec. V A.

We consider a soliton defined by Eq. (10) which moves with a steady velocity $\tilde{u}_d(i_b)$, see Eq. (13), along a junction with length (in units of λ_J) $L = 40$. Specifically, we investigate thermal transport in the presence of a soliton started from the point $\tilde{x}_0 = L/4$ and traveling along the junction for a time $\tilde{t} = 30$. Clearly, the higher the bias current, the faster the soliton and then the farther it arrives in the time \tilde{t} . We analyze heat transport as a function of the position of the soliton along the junction, for two values of the damping parameter, namely, $\alpha = 3$ and $\alpha = 0.3$, at a few values of the bias current; see Figs. 3 and 4, respectively. Hereafter, we set the values $\omega_p = 1$ THz, $T_1 = 7$ K, $T_2 = 4.2$ K, and $T_c = 9.2$ K (i.e., a Nb-based junction).

In the following, we will first discuss the high damping case, since in this regime we can safely use the adiabatic approximation [26] to study the transport across the JJ, and then we make a comparison with the low damping case.

High damping case. In Fig. 3 we show the phase profile [Fig. 3(a)], the voltage drop [Fig. 3(b)], the local magnetic field [Fig. 3(c)], energy transport [Fig. 3(d)], and heat transport [Fig. 3(e)] in the high damping case.

The solitonic phase evolutions, at different values of i_b , are shown in Fig. 3(a). By increasing the bias current, the soliton, namely, the 2π step in the phase, moves faster, so that at a fixed time \tilde{t} it moves rightwards, and becomes sharper. In Fig. 3(b), we show the corresponding voltage drop distributions by varying i_b . In the right vertical axes of this panel the normalized voltage values eV/Δ_0 are shown. The voltage drop along the junction is peaked in the center of the soliton; see Eq. (14). Furthermore, by increasing the bias current, we observe the voltage peak to become higher and narrower, since the soliton speeds up and shrinks. In Fig. 3(c) we show the local magnetic field, $H_{in}(x, t) = \partial\varphi/\partial x$, which instead keeps roughly the same amplitude in spite of the width and is only slightly changed by increasing i_b , since $\tilde{u} \ll 1$; see Eq. (15).

Both energy and heat transport P_{tot} and P_{tot}^* [see Eqs. (2) and (17), respectively, for $T_1 = 7$ K and $T_2 = 4.2$ K] are shown in Figs. 3(d) and 3(e). By increasing the bias current, we expect the phase dependence of the energy exchanged P_{tot} to change its profile shape. Specifically, for i_b close to zero one obtains $V \rightarrow 0$ according to Eq. (16), so that the

term P_{pair} is vanishingly small [26]. In this case, the $-\cos\phi$ term dominates P_{tot} [see Eq. (8)], which is positive and single peaked in correspondence of the soliton. We essentially already investigate the temperature evolution in this regime in Ref. [4]. Conversely, by increasing i_b , both the $P_{qp-pair}$ and P_{pair} contributions are affected by the average phase shifting, $\sin^{-1}(i_b)$, and by the generation of a voltage V . As a result, the sine dependence of P_{tot} tends to emerge. Moreover, since the reactive contribution P_{pair} is quite small in this case, we observe that the heat power P_{tot}^* does not differ too much from P_{tot} [see Fig. 3(d)]. Finally, we speculate that, in the high damping case [see Fig. 3(e) for $\alpha = 3$], the deformation of P_{tot}^* may induce a local heating of the electrode S_2 [and concurrently a local cooling of S_1 ; see Fig. 1(a)], which depends on i_b . We expect to see in this regime a temperature profile different than that one in the case previously reported [4].

Low damping case. The scenario changes by reducing the damping parameter (see Fig. 4 for $\alpha = 0.3$). In fact, by decreasing α , the velocity of the soliton, for a given bias current, increases, as well as the distance it covers in the time \tilde{t} . This is why the curves shown in Fig. 4 tend to overlap less than those in Fig. 3. Moreover, the lower α , the higher the maximum value of the voltage drop, according to Eq. (16) [see Fig. 4(b)]. We observe also the substantial contraction of the soliton by increasing i_b [see Fig. 4(c)], which results also in a stronger increase of the intensity in the magnetic field peak. Concerning the energy exchange, we observe that at high values of the voltage drop, the term P_{pair} in Eq. (2) can become more effective with respect to the terms P_{qp} and $P_{qp-pair}$, as it has been discussed in Ref. [26]. This behavior becomes stronger for low α values, since in this case the soliton speed, as well as the local voltage drop, is higher. In fact, we observe that the total power P_{tot} flowing through the system behaves quite differently by reducing α ; see Fig. 4(d) for $\alpha = 0.3$. First, the profile of P_{tot} is single peaked for low bias currents, but it is distorted when i_b is increased. For such a small α , if $i_b \rightarrow 1$ the voltage V significantly enhances and P_{pair} becomes greater than $P_{qp-pair}$, so that the $+\cos\phi$ term in Eq. (9) dominates P_{tot} . In this case, we observe a negative peak of P_{tot} [see Fig. 4(d)], so that the soliton could even induce a localized change of sign in the total exchange of energy between S_1 and S_2 . Notably, the intensity of this peak can be intensified by reducing the damping, since $\tilde{u}_d \rightarrow 1$ only if $\alpha \rightarrow 0$. Unfortunately, this negative peak is mainly due to the reactive contribution P_{pair} , so it affects less the dissipative heat power P_{tot}^* , which appears however highly distorted when the bias current increases [see Fig. 4(e)]. Finally, we observe that the peaks shown in Figs. 4(d) and 4(e) stem from the alignment of the singularities of the BCS DOSs in the superconductors [14,21,26,54,55].

Beyond these well-localized thermal effects induced by the soliton, we observe that the background value of P_{tot}^* , namely, the heat current flowing far from the soliton, tends to increase with i_b [see Figs. 4(d) and 3(d)]. This means that the mean temperature of the electrode will globally enhance by increasing the bias current, according to what we discussed in Fig. 2. This behavior can be understood by considering that an increase of i_b causes an overall slight increase of the phase; see Figs. 3(a) and 4(a). This means that, by increasing i_b , the

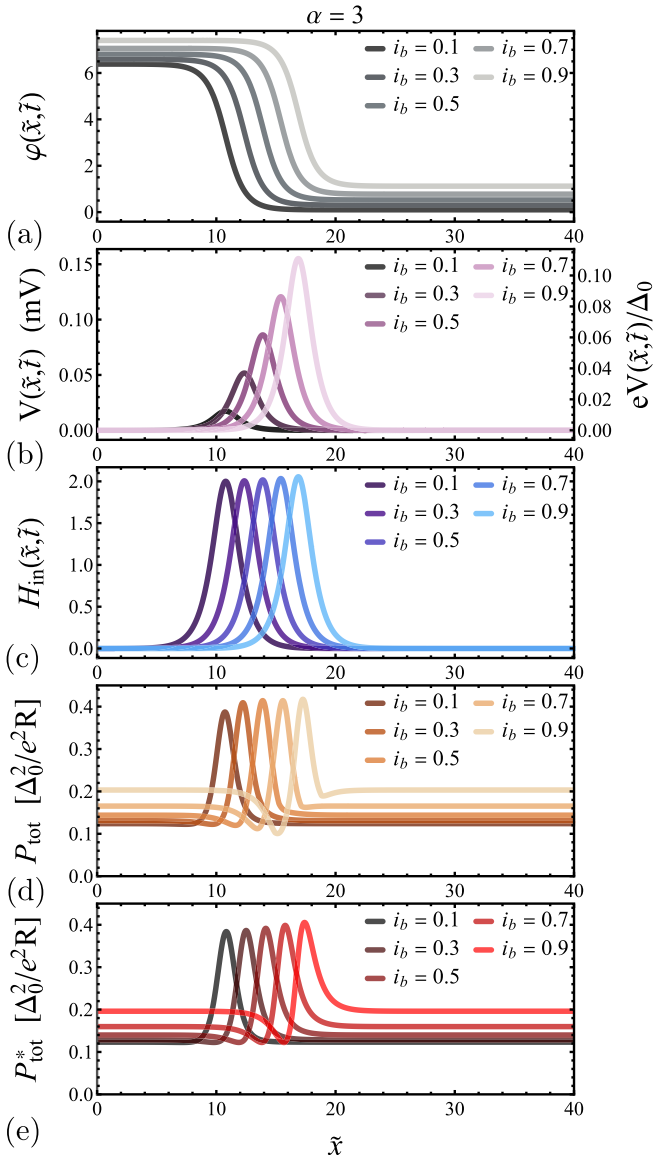


FIG. 3. Phase evolution (a), voltage drop (b), space derivative of the phase (c), energy transport (d), and heat transport (e) for a soliton moving in a LJJ for $\alpha = 3$, at a few values of the bias current i_b . The other parameters are $L = 40$, $\tilde{x}_0 = L/4$, $\tilde{t} = 30$, $\omega_p = 1$ THz, $T_1 = 7$ K, and $T_2 = 4.2$ K. The heat currents are in units of $\Delta_0^2/(e^2 R)$.

contribution of the term $\cos \varphi P_{\text{qp-pair}}$, which opposes P_{qp} in Eq. (2), tends to reduce, resulting in an increase of P_{tot}^* . So the previously discussed modulation of the P_{tot}^* profile can be eventually detected as a temperature smaller than the average temperature.

As discussed so far, the distortion of P_{tot} induced by the bias current is stronger for low values of the damping parameter [c.f., Figs. 3(d) and 4(d)], since this case gives faster solitons, higher voltage drops, and therefore an increase of the effectiveness of the P_{pair} term. Anyway, the reliability of our argument is based on working in the adiabatic regime [26]. Therefore, the results discussed so far for high bias currents and low damping should be taken with a grain of salt. In the next section, we will discuss the range of validity of the adiabatic limit as a function of the bias current.

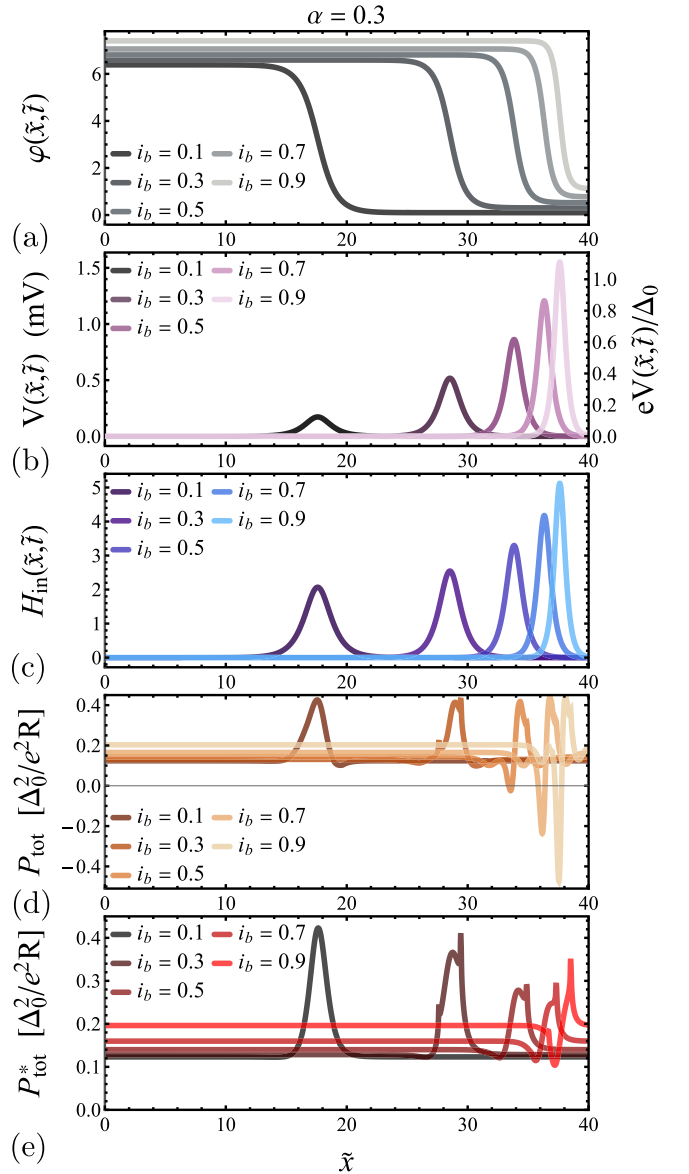


FIG. 4. Phase evolution (a), voltage drop (b), space derivative of the phase (c), energy transport (d), and heat transport (e) for a soliton moving in a LJJ for $\alpha = 0.3$, at a few values of the bias current i_b . The other parameters are $L = 40$, $\tilde{x}_0 = L/4$, $\tilde{t} = 30$, $\omega_p = 1$ THz, $T_1 = 7$ K, and $T_2 = 4.2$ K. The heat currents are in units of $\Delta_0^2/(e^2 R)$.

V. VALIDITY REGIMES

In the previous section we have discussed how heat transport is affected by a bias driven soliton. Those calculations are based on the validity of the adiabatic approximation. Hereafter, we wish to discuss how in a real system this regime can be safely realized. Finally, we will discuss how the thermal relaxation time scales imply a careful material selection in order to hopefully observe some consequence of the discussed phenomenology.

A. The adiabatic regime

Here, we estimate the range of bias current values according to which the adiabatic regime, and therefore the heat

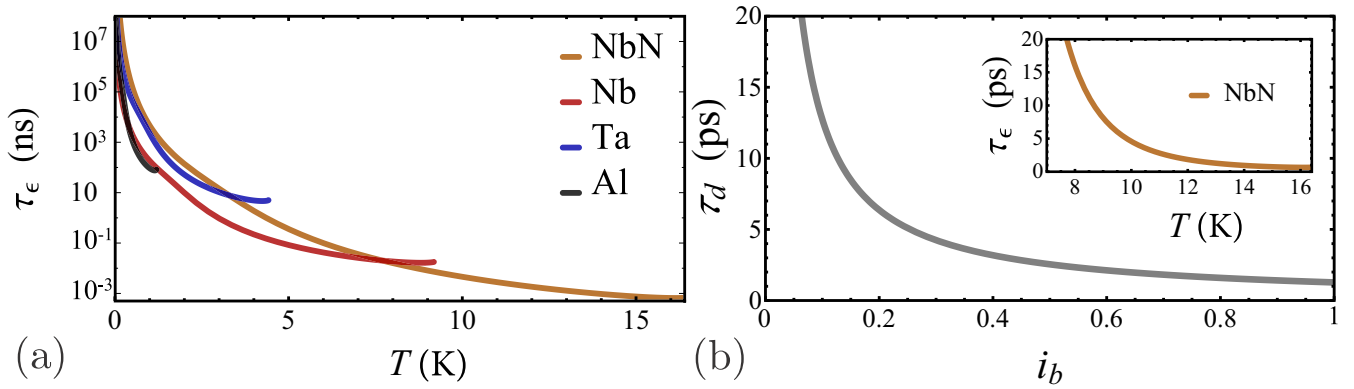


FIG. 5. (a) Quasiparticle relaxation time τ_ϵ as a function of the temperature calculated for several superconductors, specifically, Al ($T_c = 1.2$ K and $\tau_0 = 687$ ns [56]), Ta ($T_c = 4.43$ K and $\tau_0 = 49$ ns [56]), Nb ($T_c = 9.2$ K and $\tau_0 = 0.149$ ns [57]), and NbN ($T_c = 16.4$ K and $\tau_0 = 0.5T[\text{K}]^{-1.6}$ ns [58]). (b) Time τ_d that a drifting soliton needs to span a length W_s as a function of the bias current, for $\alpha = 1$ and $\omega_p = 1$ THz. (Inset) τ_ϵ as a function of the temperature for an NbN junction.

current formulation given by Golubev *et al.* [26], holds, in the case of a soliton drifting in a LJJ.

The adiabatic regime persists for

$$eV \ll \text{Min}\{k_B T_1, k_B T_2, \Delta_1(T_1), \Delta_2(T_2)\}. \quad (18)$$

Hereafter, for simplicity, we assume $T_1 \sim T_2 \sim T$ and $\Delta_1 \sim \Delta_2 \sim \Delta$, since we wish to only give a simple estimation. In particular, we will compute the upper threshold bias current $i_{b,\text{th}}$, well below which the adiabaticity is safely satisfied, i.e., $i_b \ll i_{b,\text{th}}$. Indeed, we expect that increasing i_b the soliton moves faster, enhancing the voltage drop, bringing the system out from the adiabatic condition Eq. (18). We take into account the maximum voltage V_{max} along the junction, namely, the voltage drop in the center of the soliton, $x_m = x_0 + vt$ [see Figs. 3(b) and 4(b)]. Since $\text{sech}[\xi(x_m, t)] = 1$, for a drifting soliton $eV_{d,\text{max}}$ reads

$$eV_{d,\text{max}} = \hbar \frac{\pi i_b}{4\alpha} \omega_p. \quad (19)$$

Therefore, in order to satisfy Eq. (18), for $T \leq T^*$, with T^* being the temperature at which $k_B T^* = \Delta(T^*)$, one can estimate $i_{b,\text{th}}(T)$ from the relation $eV_{d,\text{max}} = k_B T$, so that

$$i_{b,\text{th}}(T) = \frac{4k_B\alpha}{\pi\hbar\omega_p} T = \eta_\alpha T \quad (\text{for } T \leq T^*). \quad (20)$$

Instead, for $T > T^*$, $i_{b,\text{th}}(T)$ goes to zero according to $eV_{d,\text{max}} = \Delta(T)$.

For $\alpha = 1$, $\omega_p = 1$ THz, and $T_{c1} = T_{c2} = 9.2$ K, namely a Nb-based junction, we obtain $T^* \simeq 8.31$ K and $\eta_\alpha \simeq 0.17$ K $^{-1}$, so that, for instance, $i_{b,\text{th}} \simeq 0.71$ at $T = 4.2$ K.

In conclusion, at a given temperature, the adiabatic condition is satisfied if $i_b \ll i_{b,\text{th}}(T)$, in which case we can advantageously use the usual Golubev's formulation [26] to calculate the heat current flowing through the junction. We observe that, at a fixed temperature, the range of i_b values assuring the adiabatic regime can be enlarged by increasing η_α , namely, by increasing the damping parameter, since the soliton slows down and the voltage accordingly reduces, and/or by decreasing

the plasma frequency, since the voltage reduces according to Eq. (14).

B. Characteristics time scales

To eventually measure a localized heating induced by a soliton moving along the junction, the system needs “enough” time to locally adjust its temperature. In fact, although the soliton locally affects the thermal flux, the superconductor takes some time to thermally respond. Then, we can minimally assume that, to effectively induce a local temperature variation, the soliton dynamics should be slower than the time scales dictated by the thermalization processes in the system. The characteristic thermalization time can be estimated as the quasiparticle relaxation time τ_ϵ in the superconductor, which is given by [57]

$$\tau_\epsilon^{-1} = \tau_s^{-1} + \tau_r^{-1}. \quad (21)$$

In the above equation τ_s and τ_r represent, respectively, the scattering and recombination lifetimes, defined according to the well-known model of quasiparticle energy relaxation developed by Kaplan *et al.* [57]. The time τ_s concerns scattering processes involving emission and absorption of a phonon, whereas τ_r is related to the recombination of two quasiparticles to form a pair, with the emission of a phonon [57].

In Fig. 5(a), the quasiparticle relaxation time as a function of the temperature for several superconductors, specifically, Al ($T_c = 1.2$ K and $\tau_0 = 687$ ns [56]), Ta ($T_c = 4.43$ K and $\tau_0 = 49$ ns [56]), Nb ($T_c = 9.2$ K and $\tau_0 = 0.149$ ns [57]), and NbN ($T_c = 16.4$ K and $\tau_0 = 0.5T[\text{K}]^{-1.6}$ ns [58]), is shown. As expected, the quasiparticle relaxation time reduces by increasing the temperature and strongly depends on the material. Specifically, the higher the critical temperature, the lower the τ_ϵ values that can be achieved. Indeed, the thermalization time of a NbN junction is of the order of picoseconds at high temperatures.

Finally, in order to estimate the soliton visibility in a temperature profile, we can assume that the temperature is locally affected by a moving soliton, if the latter stays in the

same place for a time long enough to permit the temperature of the superconductor to locally adjust. Therefore, the reported phenomenology could be appreciable in temperature if the time τ_d that the drifting soliton needs to span the characteristic soliton width W_s [see Eq. (12)] is higher than τ_ϵ , namely, $\tau_d \gtrsim \tau_\epsilon$. The time τ_d can be estimated as

$$\tau_d = \frac{W_s}{u_d} = \frac{\lambda_J}{\bar{c}} \frac{\sqrt{1 - \tilde{u}_d^2}}{\tilde{u}_d} = \frac{1}{\omega_p} \frac{4\alpha}{\pi i_b}. \quad (22)$$

In Fig. 5(b), the behavior of τ_d as a function of the bias current is shown, for $\alpha = 1$ and $\omega_p = 1$ THz. We observe that the condition $\tau_d \gtrsim \tau_\epsilon$ can be effectively fulfilled for a NbN junction, in the limit of low bias currents and high temperature regimes [see the inset of Fig. 5(b)]. Markedly, by using superconductors with a higher critical temperature, the thermalization time τ_ϵ further reduces. Moreover, the time τ_d linearly grows with both α and the inverse of the plasma frequency ω_p^{-1} . In these cases, the localized temperature change induced by a fast moving soliton could be potentially observed.

Finally, with the aim to effectively observe a soliton-induced temperature variation, we suggest that it is convenient to investigate the temperature of the hot electrode of the junction, in order to increase the response time of the superconductor according to Fig. 5.

VI. CONCLUSIONS

In summary, we have investigated theoretically the phase-coherent heat current flowing through a long Josephson tunnel junction in the presence of a soliton driven by a stationary bias current. The latter acts as a force on the soliton, whose steady drift velocity can be written in a closed simple form [23]. We analyzed the distribution of heat currents along the junction by varying the bias current and the damping parameter, as a temperature gradient is imposed across the device. We observed that the bias current significantly affects heat transport; this effect depends on the value of the damping parameter α , since the smaller α , the faster the soliton moves. In fact, although in the case of a slow soliton a localized heating could be observed in the cold electrode, the heat current profile through the junction significantly modifies when the soliton moves with a speed approaching its limit value, namely, the Swihart velocity.

Furthermore, we discussed the range of values of bias current well below which the adiabatic limit [26] is assured. Here, we observed that for intermediate values of α the discussed phenomenology could produce observable thermal effects. Moreover, we compared the characteristic time scales of thermalization processes for several superconductors and solitonic dynamics, to establish the limiting regimes to eventually appreciate temperature variations locally induced by soliton-sustained thermal transport. Finally, we observe that the adiabatic limit approximation [26], and therefore the reliability of the approach developed in this work, could be not so strictly valid in the limits of low damping and high bias current, namely, as the soliton speed significantly grows.

We expect to see stronger effects of the discussed phenomenology for JJs in the flux-flow regime, namely, when solitons are continuously generated by an external magnetic field and shifted by the current along the junction. In this case, the competitive action between moving solitons, their reflection at a border, and their superimposition will increase additively the discussed effects. We reserve the detailed analysis for future research.

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APPENDIX A: THERMAL POWERS

In the adiabatic regime, the contributions to energy transport in a temperature-biased JJ read [26]

$$P_{\text{qp}}(T_1, T_2, V) = \frac{1}{e^2 R} \int_{-\infty}^{\infty} \mathcal{N}_1(\epsilon - eV, T_1) \mathcal{N}_2(\epsilon, T_2) (\epsilon - eV) [f(\epsilon - eV, T_1) - f(\epsilon, T_2)] d\epsilon, \quad (A1)$$

$$P_{\text{qp-pair}}(T_1, T_2, V) = -\frac{1}{e^2 R} \int_{-\infty}^{\infty} \mathcal{N}_1(\epsilon - eV, T_1) \mathcal{N}_2(\epsilon, T_2) \frac{\Delta_1(T_1) \Delta_2(T_2)}{\epsilon} [f(\epsilon - eV, T_1) - f(\epsilon, T_2)] d\epsilon, \quad (A2)$$

$$P_{\text{pair}}(T_1, T_2, V) = \frac{eV}{2\pi e^2 R} \iint_{-\infty}^{\infty} d\epsilon_1 d\epsilon_2 \frac{\Delta_1(T_1) \Delta_2(T_2)}{E_2} \left[\frac{1 - f(E_1, T_1) - f(E_2, T_2)}{(E_1 + E_2)^2 - e^2 V^2} + \frac{f(E_1, T_1) - f(E_2, T_2)}{(E_1 - E_2)^2 - e^2 V^2} \right], \quad (A3)$$

where $E_j = \sqrt{\epsilon_j^2 + \Delta_j(T_j)^2}$, $f(E, T) = 1/(1 + e^{E/k_B T})$ is the Fermi distribution function, $\mathcal{N}_j(\epsilon, T) =$

$|\text{Re}[\frac{\epsilon + i\gamma_j}{\sqrt{(\epsilon + i\gamma_j)^2 - \Delta_j(T)^2}}]|$ is the reduced superconducting density of state, with $\Delta_j(T_j)$ and γ_j being the BCS energy gap and

the Dynes broadening parameter [59] of the j th electrode, respectively.

These equations derive from processes involving both Cooper pairs and quasiparticles in tunneling through a JJ predicted by Maki and Griffin [28]. In fact, P_{qp} is the heat power density carried by quasiparticle tunneling, namely, it is an incoherent flow of energy through the junction from the hot to the cold electrode [10,28]. Instead, the “anomalous” terms P_{pair} and $P_{qp-pair}$ determine the phase-dependent part of heat transport originating from the energy-carrying tunneling pro-

cesses involving Cooper pairs and recombination/destruction of Cooper pairs on both sides of the junction.

We note that P_{pair} , in the low current regime is vanishingly small with respect to both P_{qp} and $P_{qp-pair}$ contributions, and it can be, in principle, neglected. In fact, since this term depends linearly on the time derivative of the phase, it could be effective only when the phase rapidly changes. Anyway, we stress that Eq. (A3) is a purely reactive contribution [26,27], so that in writing a thermal balance equation [5] we have to properly neglect it.

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