Computational investigation of inverse Heusler compounds for spintronics applications

Jianhua Ma,^{1,*} Jiangang He,² Dipanjan Mazumdar,³ Kamaram Munira,⁴ Sahar Keshavarz,^{4,5} Tim Lovorn,^{4,5} C. Wolverton,² Avik W. Ghosh,¹ and William H. Butler^{4,5,†}

¹*Department of Electrical and Computer Engineering, University of Virginia, Charlottesville, Virginia 22904, USA*

²*Department of Materials Science and Engineering, Northwestern University, Evanston, Illinois 60208, USA*

³*Department of Physics, Southern Illinois University, Carbondale, Illinois 62901, USA*

⁴*Center for Materials for Information Technology, University of Alabama, Tuscaloosa, Alabama 35401, USA* ⁵*Department of Physics and Astronomy, University of Alabama, Tuscaloosa, Alabama 35401, USA*

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First-principles calculations of the electronic structure, magnetism, and structural stability of inverse Heusler compounds with the chemical formula *X*2*YZ* are presented and discussed with a goal of identifying compounds of interest for spintronics. Compounds for which the number of electrons per atom for *Y* exceed that for *X* and for which *X* and *Y* are each one of the 3*d* elements, Sc-Zn, and *Z* is one of the group IIIA-VA elements: Al, Ga, In, Si, Ge, Sn, P, As, or Sb were considered. The formation energy per atom of each compound was calculated. By comparing our calculated formation energies to those calculated for phases in the inorganic crystal structure database of observed phases, we estimate that inverse Heuslers with formation energies within 0.052 eV*/*atom of the calculated convex hull are reasonably likely to be synthesizable in equilibrium. The observed trends in the formation energy and relative structural stability as the *X*, *Y ,* and *Z* elements vary are described. In addition to the Slater-Pauling gap after 12 states per formula unit in one of the spin channels, inverse Heusler phases often have gaps after 9 states or 14 states. We describe the origin and occurrence of these gaps. We identify 14 inverse Heusler semiconductors, 51 half-metals, and 50 near-half-metals with negative formation energy. In addition, our calculations predict 4 half-metals and 6 near-half-metals to lie close to the respective convex hull of stable phases, and thus may be experimentally realized under suitable synthesis conditions, resulting in potential candidates for future spintronics applications.

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I. INTRODUCTION

During the past two decades, the field of magnetoelectronics and spintronics has experienced significant development [\[1–3\]](#page-24-0). Half-metals have the potential to play an important role in the continuing development of spintronics because they have a gap in the density of states (DOS) at the Fermi energy for one of the spin channels, but not for the other, creating an opportunity to achieve 100% spin-polarized currents [\[4,5\]](#page-24-0). A half Heusler compound inspired the term "half-metal" when, in 1983, de Groot and collaborators calculated the electronic structure of NiMnSb and recognized its unusual electronic structure [\[6\]](#page-24-0). Since then, many Heusler compounds have been analyzed and have been theoretically predicted to be half-metals or near-half-metals [\[7–12\]](#page-24-0).

Due to the near 100% spin polarization at the Fermi level and the relatively high Curie temperatures shown by some Heusler compounds [\[13–15\]](#page-24-0), half-metallic Heusler compounds have been exploited for devices [\[16\]](#page-24-0), such as electrodes for magnetic tunnel junctions (MTJs) [\[17–22\]](#page-24-0), electrodes for giant magnetoresistive spin valves (GMRs) [\[23–26\]](#page-24-0) and for injection of spins into semiconductors [\[27\]](#page-24-0).

Heusler-like compounds, sometimes called "inverse Heuslers" (IH) which are described below, can have electronic structures similar to those of the full Heusler and half Heusler compounds, in the sense that they may be half-metallic or semiconducting. In addition, the IH compounds can have an electronic structure that has been described as a "spin-gapless semiconductor (SGS)" $[28]$, i.e., a large gap at the Fermi energy for one spin channel and a very small gap at the Fermi energy for the other channel.

In this paper we will discuss our survey of IH compounds. We will concentrate on their bulk properties. For many applications, heterostructures with interfaces will be needed. These interfaces will affect the electronic structure and transport properties, but are outside the scope of this paper.

The family of Heusler compounds has three subfamilies: the full Heuslers, the half Heuslers, and the inverse Heuslers. All three of these subfamilies are based on decorations of atoms and vacancies on an underlying bcc lattice and are most easily visualized by beginning with a bcc lattice represented by the familiar cubic cell shown in Fig. [1\(a\)](#page-1-0) with all *X*-type atoms. If the center atom is made different from the atoms at the corners, the structure becomes the *B*2 structure with composition XY [Fig. [1\(b\)\]](#page-1-0). If, in addition, alternate atoms at the corners in Fig. $1(b)$ are made different, as shown in the larger cell of Fig. $1(c)$, the structure becomes the full Heusler or $L2_1$ structure with composition X_2YZ . The half Heusler structure (composition *XYZ*) can be visualized by removing half of the *X* atoms from Fig. $1(c)$ as shown in Fig. $1(d)$. The inverse Heusler or XA structure (composition X_2YZ) can be

^{*}jm9yq@virginia.edu

[†] wbutler@mint.ua.edu

FIG. 1. Schematic representation of full Heusler*L*21 structure and inverse Heusler *XA* structure. (a) The body-centered cubic (bcc) structure. (b) The *B*2 cubic structure. (c) The $L2_1$ structure consisting of four interpenetrating fcc sublattices. (d) The half Heusler $C1_b$ structure consists of three interpenetrating fcc sublattices. (e) The *XA* structure consists of four interpenetrating fcc sublattices. In the *XA* structure, *X*¹ and *X*² are the same transition-metal element but they have different environments and magnetic moments.

obtained from $L2_1$ by switching one of the *X* atoms in the $L2_1$ structure with a *Y* or *Z* as shown in Fig. 1(e). In all three families, the *X* and *Y* atoms are typically transition metals (at least for the systems that are potential half-metals), while the *Z* atom is typically an *s-p* metal.

We have recently completed a survey of the calculated properties of the three subfamilies of Heusler compounds. The results of these calculations are available in a database [\[29\]](#page-24-0). We have previously reported on our study of the half Heusler compounds [\[30\]](#page-24-0). In this paper, we describe the results of calculations of the properties of IH compounds with particular emphasis on those IH compounds that may be of use for spintronics applications and which may have a reasonable probability of being synthesized.

A universal feature of the electronic structure of Heusler alloys that we consider is the existence of a relatively broad *s* band followed at higher energy by bands that are primarily derived from *d* states (Fig. 2). One particular feature of the electronic structure of the IH compounds is especially interesting. The half Heusler and full Heusler families often have band gaps after three states per atom counting from and including the just mentioned *s* band. We call this type of gap a

FIG. 2. The band structures of $Mn₂CoAl$ in (a) spin-up and (b) spin-down channels are shown. Bands from 1 to 14 are labeled in (a) and (b). The zero of the energy axis corresponds to the Fermi level.

Slater-Pauling gap because Slater and Pauling, but especially Slater, noticed that many bcc-based alloys have approximately three states per atom in the minority-spin channel $[31-33]$. Furthermore, Slater and Koster showed that for a nearestneighbor tight-binding model for a B2 alloy (see Fig. 1) with only *d* states, a gap forms in the center of the *d* band [\[31\]](#page-24-0).

The "Slater-Pauling" state for bcc alloys has been ascribed to a minimum in the electronic DOS near the center of the *s*-*d* band. This minimum becomes an actual gap after three states per atom in one of the spin channels in some Heusler compounds. In contrast, although many IH compounds show Slater-Pauling gaps at three states per atom, some have gaps after a number of states per atom that differs from three.

Thus, IH compounds often have gaps after 9, 12, and 14 states per formula unit in the gapped channel. The gap after 12 states corresponds to the Slater-Pauling gap. The gaps after 9 states and 14 states are additional gaps allowed by the reduced symmetry of the inverse Heusler compared to the full Heusler. Semiconducting states are possible with half Heuslers and full Heuslers, but these typically occur when both spin channels have Slater-Pauling gaps, and both spin channels are identical, implying no magnetic moments on any of the atoms. In contrast, the multiple gaps of the IH compounds allows the possibility of a magnetic semiconductor, e.g., when the minority channel has a gap after 12 filled states and the majority has a gap after 14 filled states. The multiple gap possibilities and the possibility of a magnetic semiconductor are illustrated by the partial band structures of $Mn₂CoAl$ shown in Fig. 2. Here, it can be seen that the down-spin channel has a gap after 12 states per formula unit and the up-spin channel has potential gaps after 9 and after 14 states per formula unit. The effect of reduced symmetry on the potential up-spin gap after 9 states can be seen in the avoided crossing of bands 9 and 10. For some systems, this avoided crossing leaves only a tiny gap. A full explanation of possible band gaps requires an analysis of the band ordering at symmetry points and the symmetry-dependent band compatibility relations.

We note that other authors [\[7,8,12,34\]](#page-24-0) use the term "Slater-Pauling gap" to refer to any gap that can create a half-metal in a Heusler compound. We prefer to reserve the term for gaps after three states per atom. We apologize for any confusion our attempt to refine nomenclature may cause.

The reduced-symmetry-derived gaps after 9 and 14 states are often quite narrow so that a state called a spin gapless semiconductor (SGS), a special type of semiconductor in which the Fermi level falls into a relatively large gap in the minority-spin channel (after 12 states) and within a narrow gap for the majority (after 14 states) [\[34,35\]](#page-24-0). This band structure allows both electrons and holes to be excited simultaneously carrying 100% spin-polarized current, potentially with high mobility. Recently, the existence of some IH compounds has been confirmed by experiment [\[36–38\]](#page-24-0). Additionally, it has been experimentally verified that $Mn₂CoAl$ is an SGS with high Curie temperature [\[28\]](#page-24-0).

Interestingly, the half Heusler $C1_b$ phase has the same space group as the inverse Heusler *XA* phase and is also prone to gaps after 9 and 14 states per formula unit in a spin channel. However, in this case 9 states per formula unit coincide with the Slater-Pauling gap after 3 states/atom.

We anticipate that a large and complete database of consistently calculated properties of IH compounds will allow the testing of hypotheses that may explain the occurrence and size of Slater-Pauling band gaps as well as the gaps that occur in some IH compounds after 9 and 14 states per formula unit.

Although a large number of IH compounds have been analyzed by first-principles calculations [\[12,34,39–42\]](#page-24-0), a comprehensive study of the structural, electronic, and magnetic properties of the IH family is useful because it is not clear how many of the IH half-metals and semiconductors that can be imagined are thermodynamically stable in the *XA* structure. Thus, a systematic study of the structural stability of the IH family can provide guidance for future work.

In Sec. \mathbf{I} we describe the details of our computational approach. Section IIA describes the DFT calculations. In Sec. IIB, we discuss how the equilibrium structures for each compound were determined and the possibility of multiple solutions in energy and magnetic configuration for some specific compounds. Our approach to estimating the stability of the IH compounds is described in Sec. [IIC.](#page-4-0)

Our results are described in Secs. [III](#page-5-0) and [IV.](#page-9-0) Section [III](#page-5-0) describes the formation energies that we calculated for the IH compounds and how these formation energies compare to those of competing phases. Section \overline{II} A describes a study of the calculated formation energies and stability of those IH phases that have been observed experimentally. The relationship between stability and composition is discussed in Sec. [IIIB.](#page-7-0) Section [IIIC](#page-8-0) describes our calculated results for the formation energy and stability of the 405 IH compounds that we studied, including an investigation of the relation between gaps at the Fermi energy and stability. Section [IV](#page-9-0) covers the calculated electronic structure of the IH compounds with particular emphasis on those that form semiconductors and half-metals. In Sec. [IV A,](#page-9-0) IH semiconductors, half-metals, and near-half-metals with negative formation energy are listed and discussed in terms of their electronic structure and structural stability. Section V is a summary of our results and conclusions.

II. COMPUTATIONAL DETAILS AND STRUCTURE DETERMINATION

The IH compound X_2YZ crystallizes in the face-centeredcubic *XA* structure with four formula units per cubic unit cell (Fig. [1\)](#page-1-0). Its space group is no. 216, $F43m$ (the same as the half Heusler). The IH structure can be viewed as four interpenetrating fcc sublattices, occupied by the *X*, *Y* , and *Z* elements, respectively. The *Z* and *Y* elements are located at the $(0, 0, 0)$ and $\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$, respectively, in Wyckoff coordinates, while the *X*₁ and *X*₂ elements are at $(\frac{3}{4}, \frac{3}{4}, \frac{3}{4})$ and $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$, respectively, resulting in two different rock-salt structures $[X_1Y]$ and $[X_2Z]$ as shown in Fig. [1\(e\).](#page-1-0) We use X_1 and X_2 to distinguish these two *X* atoms sitting at the two inequivalent sites in the *XA* structure. Since we also calculated the energy for each compound in the $L2_1$ structure, we also show the $L2_1$ structure in Fig. $1(c)$ for comparison.

In this study, (a) *X* is one of 9 elements (Sc, Ti, V, Cr, Mn, Fe, Co, Ni, or Cu), (b) *Y* is one of 9 elements (Ti, V, Cr, Mn, Fe, Co, Ni, Cu, or Zn), and (c) *Z* is one of 9 elements (Al, Ga, In, Si, Ge, Sn, P, As, or Sb). In summary, we calculated the 405 inverse Heusler compounds for which the valence (or atomic number since *X* and *Y* are both assumed to be 3*d*) of *Y* is larger than that of *X*. For each of these 405 potential IH compounds, we calculated its electronic and magnetic structure, stability against structural distortion, and formation energy. We also compared its formation energy to those of all phases and combinations of phases in the open quantum materials database (OQMD) [\[43,44\]](#page-24-0).

A. Density functional theory calculations

All calculations were performed using density functional theory (DFT) as implemented in the Vienna *ab initio* simulation package (VASP) [\[45\]](#page-24-0) with a plane-wave basis set and projectoraugmented wave (PAW) potentials [\[46\]](#page-24-0). The set of PAW potentials for all elements and the plane-wave energy cutoff of 520 eV for all calculations were both chosen for consistency with the OQMD [\[43,44\]](#page-24-0). The Perdew-Burke-Ernzerhof (PBE) version of the generalized gradient approximation (GGA) to the exchange-correlation functional of DFT was adopted [\[47\]](#page-24-0). The integrations over the irreducible Brillouin zone (IBZ) used the automatic mesh generation scheme within VASP with the mesh parameter (the number of *k* points per A^{-1} along each reciprocal lattice vector) set to 50, which usually generated a $15 \times 15 \times 15$ Γ -centered Monkhorst-Pack grid [\[48\]](#page-24-0), resulting in 288 *k* points in the IBZ. The integrations employed the linear tetrahedron method with Blöchl corrections [\[49\]](#page-24-0). Finally, all the crystal structures are fully relaxed until the energy change in two successive ionic steps is less than 1×10^{-5} eV.

The total magnetic moment of each compound is calculated by integrating the spin-polarized charge density over the calculation cell. The division of the total magnetic moment among the atoms in the cell is somewhat ambiguous in VASP. The magnetic moment for a particular atom presented in this paper is evaluated within a sphere of 1.45 \AA radius centered on the atomic site. It should be noted that the sum of the atomic moments defined this way is not exactly equal to the total cell moment.

B. Determination of the relaxed structure

We explain our procedure for obtaining the relaxed structures in some detail in order to make clear that there are many possible structures (in addition to the *XA* structure) that an *X*2*YZ* compound can assume and that the determination of the ground state can be complicated by the additional degrees of freedom associated with the possible formation of magnetic moments of varying magnitude and relative orientation and by possible distortions of the cell.

We calculated the electronic structure of each compound in the *XA* phase by minimizing the energy with respect to the lattice constant. For each system and each lattice constant, we found it to be important to initiate the calculation with multiple initial configurations of the magnetic moments. We considered four kinds of initial moment configurations: (1) moments on the three transition-metal atoms are parallel; (2) moment on the *Y* site is antiparallel to the moments on the X_1 and X_2 ; (3) moment on the X_1 site is antiparallel with the moments on the X_2 site and Y ; (4) moment on the X_2 site is antiparallel with the moments on the X_1 and Y . After the lattice constant and magnetic configuration that determined the minimum energy for the *XA* structure was determined, we compared this energy to the minimum energy determined in a similar way for the *L*21 structure.

*Competition between XA and L*21. Comparing the calculated energies of the *XA* and *L*21 phases of the 405 compounds that we considered, we found that 277 have lower energy in the *L*21 phase. The remaining 128 have lower energy in the *XA* phase. This result may be compared to the conclusion in Refs. [\[12,40,50–](#page-24-0)[55\]](#page-25-0) that the *XA* structure is energetically preferred to the $L2_1$ structure when the atomic number of the *Y* element is larger than the atomic number of *X*. Although this hypothesis is consistent with some experimental observations [\[37,38,40](#page-24-0)[,56,57\]](#page-25-0), it does not seem to be generally valid because more than two thirds of the systems in our data set (all of which satisfy $N_Y > N_X$) have lower calculated energy as $L2_1$.

Test for tetragonal distortions via ionic relaxation. After determination of the cubic lattice constant and magnetic configuration in our initial survey, all of the 405 compounds in our survey, both $L2_1$ and XA , were tested for stability against tetragonal distortions using an 8-atom tetragonal cell and full ionic relaxation. For both XA and $L2₁$ we considered the structure to be cubic if $|c/a - 1| < 0.01$. All relaxations started from the XA structure or $L2₁$ structure adapted to an 8-atom tetragonal cell.

Of the 277 compounds that preferred the $L2₁$ structure, 136 remained in the $L2_1$ structure while 141 relaxed to a tetragonal " $L2_1$ " structure. We notice that few systems with $X = Sc$, Ti, or V are calculated to be stable as *XA*. None of the 81 considered compounds with $X =$ Sc were found to be XA . 10 relaxed into the tetragonal "*L*21" phase, 63 preferred the *L*21 phase, and the remaining 8 became tetragonal "*XA*" phase. Among the 72 compounds with $X = Ti$, only 4 compounds preferred XA phase, while 37 relaxed into the tetragonal "*L*21" phase, 29 preferred the *L*21 phase, and 2 distorted to be tetragonal "*XA*" phase. Among the 63 compounds with $X = V$, 43 compounds distorted into tetragonal "*L*21" phase, 2 preferred the *L*21 phase, and 2 distorted to be tetragonal "*XA*" phase.

When the 128 compounds that were more stable as *XA* than $L2₁$ were tested for susceptibility to tetragonal distortion, 95 were found to remain in the *XA* structure while 33 relaxed to a tetragonal *XA* phase. In contrast to the half Heuslers which are prone to trigonal distortions [\[30\]](#page-24-0), we find that the full Heuslers and IH compounds tend to retain tetragonal or cubic symmetry during this type of relaxation. The susceptibility of the half

FIG. 3. Calculated total energies of $Mn₂ZnP$ with full-Heusler $L2_1$ structure and inverse-Heusler *XA* structure as a function of the lattice constant *a* in ferrimagnetic, ferromagnetic and nonmagnetic states.

Heuslers to trigonal distortions is likely related to the vacant site in the cell.

If we ignore the competition between XA and $L2₁$ phases, of the 405 potential *XA* compounds that we considered, 141 were unstable with respect to a tetragonal distortion and the remaining 264 compounds have lower energy in *XA* phase.

Competition between magnetic configurations. For some compounds, we found multiple solutions to the DFT equations at the same or similar lattice constants with different magnetic configurations. These can be categorized into six groups: in the $L2_1$ structure, (1) the moments of the *X* atoms (which are identical by assumption for $L2_1$) are parallel to those for *Y* $(L2₁$ ferromagnetic), or (2) the moments of *X* are antiparallel to those for $Y(L2_1$ ferrimagnetic); in the *XA* structure, (3) the moments for *Y* are parallel with those for X_1 but antiparallel with those for X_2 (*XA* ferrimagnetic A1), or (4) the moments for Y are parallel with those for X_2 but antiparallel with those for X_1 (*XA* ferrimagnetic A2), or (5) the moments for X_1 and *X*² are parallel with each other but antiparallel with those for *Y* (*XA* ferrimagnetic), or (6) the moments for X_1 , X_2 , and *Y* are parallel (*XA* ferromagnetic). Magnetic transitions, in both magnitude and orientation as a function of lattice constant, are clearly possible and often observed in our calculations. We did not consider the possibility of noncollinear moments [\[58\]](#page-25-0), and because the calculations did not include spin-orbit coupling, we did not obtain information about the orientation of the moments relative to the crystal axes.

Example: Mn₂ZnP. Mn₂ZnP provides an example of competition between different magnetic states for both the *L*21 and *XA* structures as shown in Fig. 3. In this case, the *L*21 phases have lower energy. In addition to a nonmagnetic phase with a lattice constant of 5.645 Å, two energy minima were found for magnetic phases. These occur at $a = 5.69$ and 5.88 Å. For the solution at $a = 5.69$ Å the total moment per cell is $2.53\mu_B$ while the moments within a sphere of radius 1.45 Å surrounding each atom are 1.32 for Mn, −0*.*01 for Zn, and -0.10 for P. For the solution at $a = 5.88$ Å the compound has a total magnetic moment of $5.37\mu_B$ per unit cell, and the moments within the 1.45- \AA spheres in this case are 2.75 μ _B, $-0.03\mu_B$, and $-0.13\mu_B$ for Mn, Zn, and P, respectively. The solution at $a = 5.88$ Å has higher energy than the solution at 5.69 Å by $\Delta E = 0.230$ eV per formula unit.

There are also multiple *XA* magnetic phases. The two shown are both ferrimagnetic in the sense that the two Mn moments have opposite signs. The phase with larger lattice constant actually has smaller net moment because of the closer approximate cancellation of the Mn moments.

To be clear, the energy comparisons of the preceding paragraph only consider the *XA* and *L*21 phases and their possible tetragonal distortions. Phase stability is discussed more generally in Sec. IIC.

Competition between ordered and disordered phases. For a few systems, we considered the energy difference between ordered and disordered phases. As will be discussed in detail in Sec. [III A,](#page-5-0) some Mn_2CoZ and Mn_2RuZ XA compounds (especially when *Z* is a larger atom, e.g., In, Sn, or Sb,) seem to have disorder in the occupation of the Mn_1 and *Y* sublattice sites. To approximate this disorder in the *XA* Mn₂Co*Z* ($Z =$ In, Sn, and Sb) and $Mn₂RuSn$, special quasirandom structures (SQS) as described by Zunger *et al.* [\[59\]](#page-25-0) were generated. The SQS are designed to best represent the targeted disordered alloy by a periodic system of a given size.

Our SQS comprised 32-atom cells for the case of completely random (50%/50%) occupation of the X_1 and Y sites, and 64-atom cells for the case in which the $X_1(Y)$ site occupation was 75%/25% (25%/75%). The SQS was generated using the Monte Carlo algorithm of Van de Walle *et al.* [\[60\]](#page-25-0) as imple-mented in the alloy theoretic automated toolkit (ATAT) [\[61\]](#page-25-0). The cluster correlations used to define the SQS were specified using a distance-based cutoff of all two-, three-, and four-body clusters with a maximum distance of $5-6$ Å. We did not consider the case with smaller *X/Y* ratios because much larger unit cells would have been required. More detailed results will be discussed in Sec. [III A.](#page-5-0)

C. Calculation of energetic quantities

1. Formation energy

We further investigated the structural stability of the 405 potential IH compounds by calculating their formation energies. The formation energy of an IH compound X_2YZ is defined as

$$
\Delta E_f (X_2 Y Z) = E (X_2 Y Z) - \frac{1}{4} (2\mu_X + \mu_Y + \mu_Z), \quad (1)
$$

where $E(X_2 Y Z)$ is the total energy per atom of the IH compound, and μ_i is the reference chemical potential of element *i*, chosen to be consistent with those used in the OQMD database. Typically, the reference energy is the total energy per atom in the elemental phase (see Refs. [\[43,44\]](#page-24-0) for details). A negative value of ΔE_f indicates that at zero temperature, the compound is more stable than its constituent elements. It is a *necessary but not sufficient* condition for ground-state thermodynamic stability. It does not, for example, guarantee the stability of an inverse Heusler (IH) phase over another competing phase or mixture of phases. For that, we must determine the convex hull.

2. Convex hull distance

A compound can be thermodynamically stable only if it lies *on* the convex hull of formation energies of all phases in the respective chemical space. Every phase on the convex hull has a formation energy lower than any other phase or linear combination of phases in the chemical space at that composition. Thus, any phase on the convex hull is, by definition, thermodynamically stable at 0 K. Conversely, any phase that does not lie on the convex hull is thermodynamically unstable; i.e., there is another phase or combination of phases on the convex hull that is lower in energy.

The distance from the convex hull ΔE_{HD} for a phase with formation energy ΔE_f can be calculated as

$$
\Delta E_{\rm HD} = \Delta E_f - E_{\rm hull},\tag{2}
$$

where E_{hull} is the energy of the convex hull at the composition of the phase. The energy of the convex hull at any composition is given by a linear combination of energies of stable phases. Thus, the determination of *E*hull from a database of formation energies is a linear composition-constrained energy minimization problem [\[62,63\]](#page-25-0), and is available as a lookup feature called "grand canonical linear programming"(GCLP) on the OQMD website [\(http://oqmd.org/analysis/gclp\)](http://oqmd.org/analysis/gclp). Obviously, if the hull distance ΔE_{HD} for a phase on the convex hull is 0, there is no other phase or linear combination of phases lower in energy than the phase at that composition. The distance of the formation energy of a phase from the convex hull is an indicator of its thermodynamic stability and the likelihood of it being fabricated because the greater the distance from the convex hull, the higher is the thermodynamic driving force for transformation into another phase or decomposition into a combination of phases.

We note that the distance of a phase from the calculated convex hull depends on the completeness of the set of phases considered in the construction of the convex hull. Ideally, to calculate the convex hull of a system, *X*-*Y* -*Z*, one would calculate the energies of all possible compounds that can be formed from elements *X*, *Y* , and *Z*. Unfortunately, such a comprehensive study is not currently feasible.

A practical approach is to construct the convex hull using all the currently reported compounds in the *X*-*Y* -*Z* phase space. Here, we have limited our set of considered phases to those in the OQMD, which includes many ternary phases that have been reported in the inorganic crystal structure database (ICSD) [\[64,65\]](#page-25-0), and \sim 500 000 hypothetical compounds based on common crystal structures. Thus, the calculated formation energy of each X_2YZ IH compound is compared against the calculated formation energies of all phases and all linear combinations of phases with total composition $X_2 YZ$ in the OQMD.

A phase that we calculate to be above the convex hull may still be realized experimentally for three reasons: (a) There may be errors in the calculation of the energies of the IH compound or in the reference compounds. These errors may be intrinsic to PBE-DFT or due to insufficient precision in the calculation or inability to distinguish the ground state. We have applied considerable effort to reduce or eliminate the latter two possibilities so far as possible. (b) The compounds are usually synthesized at temperatures well above 0 K. Because entropic effects will differ for different phases, the phase with lowest free energy at the synthesis temperature may be different from the phase with lowest energy at 0 K and this high-temperature phase may persist to low temperature due to slow kinetics at laboratory time scales. (c) Nonequilibrium processing techniques can sometimes be used to fabricate nonequilibrium metastable phases.

Conversely, an IH phase that we calculate to be on the calculated convex hull may nevertheless be difficult or impossible to fabricate because of reasons (a) or (b) or because the database used to generate the convex hull does not contain all phases so that the true convex hull lies below the calculated one.

Finally, we remark that our stability investigations are most directly pertinent to bulk phases. The growth of alloy films is a sophisticated art. Phases that are stable in bulk can be difficult to grow as films. Conversely, clever use of substrates, surface energies, and kinetics can stabilize film structures different from the equilibrium bulk phase. Even for films, however, the hull distances provided here may provide useful guidance concerning the likelihood that a phase or structure may be stabilized.

III. FORMATION ENERGY AND STABILITY

A. Formation energy and convex hull distance

In this section, we systematically investigate the formation energy and thermodynamic stability of the 405 IH compounds considered in this paper. The formation energies at 0 K were evaluated by using Eq. [\(1\)](#page-4-0) and the convex hull distances were calculated using Eq. [\(2\)](#page-4-0). We first explored the relationship between formation energy and convex hull distance for the known synthesized IH compounds, including those collected in the ICSD.

We compiled a list of the reported IH compounds by extracting IH compounds from the ICSD and literature including all elements as potential *X*, *Y* , or *Z* atoms. From this list we removed the compounds with partially occupied sites. Some of the ICSD entries were from DFT calculations. These were included except for those with a higher DFT energy than the corresponding full Heusler which were obviously included in the ICSD by mistake. Finally, we tabulated the formation energy and convex hull distance as calculated in the OQMD.

A total of 48 distinct IH compounds are reported in ICSD. Of these, 36 have been synthesized experimentally. References to six additional synthesized IH compounds, not included in ICSD (Fe₂CoGa, Fe₂CoSi, Fe₂CoGe, Fe₂NiGa, Mn₂NiGa, and Fe₂RuSi) were found in recent literature $[66,67]$. These can be used for cross validation of our method. Although $Mn₂NiSn$ was synthesized by Luo *et al.* [\[68\]](#page-25-0), a more recent study shows that it has atomic disorder on the transition-metal sites [\[67\]](#page-25-0) and our DFT calculation shows it is 103 meV*/*atom above the convex hull.

These data from ICSD and the recent literature are displayed in Fig. 4, from which it can be seen that most of the synthesized IH compounds are within 52 meV*/*atom of the convex hull. The green diamonds represent the IH compounds that have not been synthesized but have been sourced into the ICSD by DFT predictions.

FIG. 4. DFT-calculated formation energy vs convex hull distance of inverse Heuslers reported in ICSD and in recent experimental studies. A hull distance $\Delta E_{HD} = 0$ (on the convex hull) indicates the compound is thermodynamically stable at 0 K. Blue circles and red squares represent experimentally synthesized IH through equilibrium and nonequilibrium process, respectively. Purple triangle is synthesized IH coexisting with other phase. Yellow stars represent synthesized compounds reported in literature but not in ICSD. Green diamonds represent IH compounds predicted using DFT.

Overall, we find most of the experimentally reported IH compounds (38 of 42) have a convex hull distance less than 52 meV*/*atom. Thus, the convex hull distance of the DFTcalculated formation energy at 0 K appears to be a good indicator of the likelihood of synthesis of an IH compound and 52 meV*/*atom is a reasonable empirical threshold separating the compounds likely to be synthesized by equilibrium processing from those unlikely to be so synthesized.

The empirical 52-meV threshold assumed here may be contrasted with the 100-meV threshold that seemed appropriate for the half Heusler compounds [\[30\]](#page-24-0). We speculate that the more open structure of the half Heusler phase is associated with a high density of states for low-frequency phonons which reduces the free energy of the half Heusler phase relative to competing phases at temperatures used for synthesis.

Before proceeding, it is important to note that several Mn₂*YZ* compounds (Mn₂CoSn, Mn₂CoSb, Mn₂CoIn, and Mn₂RuSn) appear to significantly violate our empirical hull distance threshold. They also have vanishing or positive formation energy! A discussion of our rationale for *not* extending our hull distance threshold to include these systems follows.

(a) Mn2CoSn: Our calculations predict that *XA* phase Mn₂CoSn lies well above the convex hull. A mixture of phases, CoSn-Mn is predicted to be lower in energy by $\Delta E_{HD} =$ 0*.*107 eV*/*atom. However, there are reasons to believe that the experimentally reported phase may not be pure *XA*.

The ICSD entry for $Mn₂CoSn$ is based on the work of Liu *et al.* who described synthesis via melt spinning as well as calculations that employed the FLAPW technique [\[40\]](#page-24-0). They found that the nonequilibrium melt-spinning technique generated a material with a "clean" x-ray diffraction (XRD) pattern consistent with the *XA* phase. The measured magnetic

moment of $2.98\mu_B$ per formula unit was consistent with their calculated value of 2.99 and with their calculated DOS, which indicates that this material may be a Slater-Pauling near-half-metal.

Subsequently, Winterlik *et al.* synthesized this compound by the quenching from 1073 K of annealed arc-melted samples [\[38\]](#page-24-0). They obtained similar XRD results to Liu *et al.*, but with a small admixture of a tetragonal $MnSn₂$ phase. They also concluded that the experimental moment was near the Slater-Pauling half-metal value of $3\mu_B$ per formula unit. However, their calculations for XA -phase $Mn₂CoSn$ which also employed FLAPW found, in addition to the near-half-metallic phase found by Liu *et al.*, a different magnetic structure with lower energy.

Our calculations using VASP confirm the Winterlik *et al.* results. The PBE-DFT predicted phase for XA $Mn₂CoSn$ has a magnetic moment around $1.53\mu_B$ rather far from the value of $3\mu_B$ necessary to place the Fermi energy into the Slater-Pauling gap after 12 states per atom in the minority channel. There is an interesting competition between two very similar ferrimagnetic states that plays out in many of the 26- and 27-electron Mn₂Co*Z* systems. The Mn₂ and Co moments are aligned and are partially compensated by the Mn_1 moment. For smaller lattice constants, the compensation yields the correct moment for a half-metal or near-half-metal [i.e., for $Mn_2Co(Al,Ga,Si,Ge)$, but as the lattice constant increases [i.e., for $Mn_2Co(In,Sn)$], the Mn_1 moment becomes too large in magnitude and the partial compensation fails to yield a half-metal.

Winterlik *et al.* conclude from XRD, NMR and Mössbauer studies that their quenched samples of $Mn₂CoSn$ are actually disordered on the MnCo layers. They modeled this disorder using the Korringa-Kohn-Rostoker (KKR) and the coherent potential approximation (CPA) (KKR-CPA) approach and obtained values of the moments in reasonable agreement with magnetometry and x-ray magnetic circular dichroism (XMCD). They argued that their model with Mn and Co randomly occupying the sites on alternate (001) layers separated by ordered Mn-Sn layers agreed with XRD as well as the *XA* model because Mn and Co scatter x rays similarly. This disordered $L2_1B$ structure has both Mn and Co at the $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ and the $(\frac{3}{4}, \frac{3}{4}, \frac{3}{4})$ sites, whereas the *XA* structure has only Co at $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ and only Mn at $(\frac{3}{4}, \frac{3}{4}, \frac{3}{4})$ site.

We investigated the Winterlik *et al.* hypothesis by comparing the energy of a few cubic 16-atom supercells with differing equiatomic occupations of the MnCo layers to the energy of the *XA* occupation. The energies of these different site occupations were all lower than the *XA* energy, and most of them were half-metallic or near-half-metallic. The decrease in energy for the non-*XA* site occupations ranged from 8 to 33 meV*/*atom. These should be considered lower bound estimates of the decrease since we did not optimize the geometries of the model "disordered" structures. We also performed calculations for an SQS designed to mimic a system with disorder on the MnCo layers. Our calculation indicates that the SQS with 25% Mn/Co mixing is 31 meV*/*atom lower in energy than the *XA* phase. The net magnetic moment of this SQS is about $1.28\mu_B$ per f.u., which is lower than that of *XA* phase $(1.53\mu_B)$ due to the atom disordering.

We propose the following hypothesis and rationale for omitting $Mn₂CoSn$ from the estimate for the hull energy distance threshold: During synthesis at high temperature, Mn₂CoSn is thermodynamically stable in a phase with substantial configurational disorder on the MnCo planes. The configurational disorder on these sites may also decrease the free energy of the compound. A simple mean-field estimate of the configurational entropy (*S*) of disorder gives a −*T S* contribution of −32 meV*/*atom at 1073 K. If we add this entropic contribution to the approximately 30-meV*/*atom difference in total energy between the ordered and disordered systems, we can estimate a decrease in the relative free energy of formation by approximately 60 meV*/*atom due to disorder. This would yield a negative free energy and a hull distance less than our empirical 52-meV threshold.

(b) $Mn₂CoSb$: $Mn₂CoSb$ is predicted to lie above the convex hull with a mixture of phases CoMnSb-Mn lower in energy by $\Delta E_{\text{HD}} = 0.133 \text{ eV/atom}$. This phase also entered the ICSD database from the paper of Liu *et al.* who generated their sample by melt spinning and also provided FLAPW calculations that predicted XA $Mn₂CoSb$ to be a half-metal with a Slater-Pauling moment of $4\mu_B$ per formula unit, consistent with their measured value.

Unlike some $Mn₂CoZ$ compounds with smaller *Z* atoms, synthesis of single XA phase $Mn₂CoSb$ is difficult via arc melting and a high-temperature annealing step. Prior x-ray diffraction investigations $[40]$ revealed a multiphase $Mn₂CoSb$ in arc-melt samples. Our synthesis supports this finding [\[69\]](#page-25-0). On the other hand, a melt-spinning treatment of arc-melt Mn₂CoSb ingots is reported to produce a pure cubic structure [\[40](#page-24-0)[,70\]](#page-25-0).

We consider it likely that $Mn₂CoSb$ is similar to $Mn₂CoSn$. A study by Xin et al. found that "disordered" Mn₂CoSb [apparently modeled by replacing the (001) MnCo layers by layers of pure Mn and pure Co alternating with the MnSb layers] was lower in energy than *XA* by 21 meV*/*atom [\[39\]](#page-24-0). Our SQS calculations show that the 50% Mn/Co mixture within one rock-salt lattice is 60 meV*/*atom lower in energy than the *XA* phases. Again, our assertion is that the free-energy reduction due to configurational entropy together with the lower energy of the disordered configurations may be sufficient to stabilize the $L2_1B$ phase as it is rapidly quenched from the melt.

(c) Mn_2Coln : According to Liu *et al.* [\[40\]](#page-24-0), Mn_2Coln appears to be multiphase in samples prepared by arc melting. Apart from the cubic phase identified as inverse Heusler, a second phase described as fcc is also apparent. Liu *et al.* also report FLAPW calculations performed at a much smaller lattice constant (5.8 Å) than the experimentally derived lattice constant (6.14 Å) . These calculations yielded a moment of 1.95μ _B in reasonable agreement with experimental value of 1.90μ _B. A subsequent calculation by Meinert *et al.* found a lattice constant of 6.04 Å and a moment of $1.95 \mu_B$ [\[52\]](#page-24-0).

Our calculations for the *XA* phase yielded a lattice constant of 6.10 Å , much closer to the experimental value and a moment of 1.74μ _B. We also found a surprising and unusual sensitivity of our results to the number of plane waves. We performed SQS calculations which indicated that the disordered phase is essentially degenerate in energy with the *XA* phase. The magnetic moment of SQS is about $1.01\mu_B$ per f.u.

Given the significant amount of second phase in the experimental sample which would affect the composition of the phase identified as *XA*, together with the possibility of disorder on the MnCo layers we tentatively conclude that it is reasonable to discount the large calculated formation energy and hull distance in setting our threshold for likely fabricability of *XA* phases. Additional studies of this putative phase are probably advisable.

(d) Mn_2RuSn : Mn_2RuSn is predicted to lie above the convex hull with a mixture of phases $MnSnRu+Mn+Sn₇Ru₃$ by $\Delta E_{HD} = 0.085$ eV/atom. The first reports of fabrication by Endo *et al.* [\[71\]](#page-25-0), were unable to distinguish between the *XA* and *L*21*B* phases. A later study by Kreiner *et al.* seemed to confidently conclude from powder XRD studies that their sample prepared by alloying through inductive heating had the $L2_1B$ structure [\[67\]](#page-25-0). Our calculation of the energy of a SQS for Mn2RuSn with 25% Mn/Ru mixing gave an energy of the model disordered alloy 18 meV lower than *XA*. The calculated magnetic moment of the SQS is about $0.24\mu_B$ per f.u.. A study by Yang *et al.* using the KKR-CPA approach estimated 11% antisite Mn based on a comparison of the calculated moment to the experimental one [\[72\]](#page-25-0).

We tentatively conclude that the four experimental systems in the ICSD with anomalously large calculated hull distances and formation energies are probably not *XA* phase and exclude them from consideration in determining the threshold for estimating which *XA* phases are likely to be fabricated by equilibrium approaches.We also remind that an additional potential inverse Heusler, Mn₂NiSn, was excluded from our database of experimental *XA* phases, because of experimental evidence of similar disorder to that observed in the Mn₂Co_Z phases.

We recommend additional studies of the competition between XA and $L2_1B$ phases, especially for compounds which have large-*Z* atoms. Such studies may require careful experimental analysis because of the similarities of the XRD patterns of the two phases. Theoretical studies may also be difficult because of the need to accurately calculate the formation energy of the disordered *L*21*B* phase.

We extended the comparison of formation energy and convex hull distance to all the 405 IH compounds considered in this work as shown in Fig. 5. It can be seen from Fig. 5 that the calculated formation energies span a range from −0*.*45 to 0.38 eV*/*atom. This may be compared with a range of −1*.*1 to 0.7 eV/atom that was observed for half Heuslers [\[30\]](#page-24-0). 248 of the IH have negative formation energy, indicating thermal stability against decomposition into the constituent elements at 0 K. The calculated hull distances vary from 0 to nearly 1 eV*/*atom. It is clear from the lack of correlation between formation energy and hull distance in Fig. 5 that the formation energy is not a reliable indicator of stability.

The IH compounds considered in this work that are included in ICSD are indicated by red squares in Fig. 5. These all lie within 0.052 eV/atom of the convex hull except for three compounds mentioned previously $(Mn_2CoSn, Mn_2CoIn, and$ $Mn₂CoSb$) that were synthesized by nonequilibrium processing and are likely $L2_1B$ rather than *XA* phase. Our calculations predict 13 (out of 405) IH compounds to be within 52 meV*/*atom of the convex hull. The "success rate," i.e., the number of predicted potentially stable compounds versus total number of systems investigated is lower for IH (13*/*405) than

FIG. 5. DFT-calculated formation energy vs convex hull distance of all IH considered in this paper. A hull distance $\Delta E_{HD} = 0$ (on the convex hull) indicates the compound is thermodynamically stable at 0 K. Red diamonds and green squares represent IH compounds sourced into ICSD by synthesis and DFT computation, respectively. Blue circles represent the IH compounds calculated in this work.

for half Heusler (50*/*378) [\[30\]](#page-24-0), but this result is affected by the different hull distance thresholds and the different choices of potential compounds.

B. Stability and composition

We systematically investigated the formation energies of the 405 IH compounds in the *XA* phase. We found that many of these potential IH phases actually have lower energy in other phases, e.g., *L*21(136), tetragonal "*L*21" phase (141), or tetragonal *XA*(33). Here, however, since we are interested in gaining insight into the systematics of the stability of the *XA* phase, we compare formation energies as if all 405 compounds remained in the *XA* phase. To better illustrate the relation between structural stability and composition, we arranged the formation energy data according to *X*, *Y* , and *Z* elements, respectively, in Figs. [6–8.](#page-8-0)

Figure [6](#page-8-0) shows the calculated formation energies and hull distances of the 405 calculated X_2YZ compounds arranged by *X* species, then by *Y* , and finally by *Z*. From Fig. [6,](#page-8-0) we can see that Sc, Ti, and V generate more negative formation energy *XA* compounds as *X* elements. In terms of the fraction with negative formation energy, we found $75/81$ with $X = Sc$, 60/72 with $X = Ti$, 25/45 with $X = Mn$, and 20/36 with $X =$ Fe, and $15/18$ with $X = Ni$.

Figure [7](#page-8-0) presents the calculated formation energies and hull distances arranged first by *Y* , then by *X*, and finally by*Z*. From Fig. [7,](#page-8-0) it can be seen that when *Y* is Fe, Co, or Ni many *XA* compounds with negative formation energies result. However, when one considers the fraction of negative formation energy compounds with a given *Y* in our database, the highest are 9*/*9 for $Y = Ti$, 17/18 for $Y = V$, and 24/27 for $Y = Cr$.

When the compounds are ordered by*Z* element as presented in Fig. $\frac{8}{3}$, we can see that the smaller atoms within a given group $(i.e., Z = A1, Si, In)$ tend to have a larger fraction of negative

FIG. 6. DFT formation energies and hull distances for potential inverse Heusler compounds grouped by the element on the *X* site. The numbers near the top (in blue) and center (in brown) of each column denote the number of compounds with negative formation energy ΔE_f and hull distance $\Delta E_{HD} \leq 0.1$ eV/atom, respectively, in the corresponding *Z*-element group. Within a given *X*-element column, the compounds are ordered first by the element on the *Y* site (same order as in Fig. 7) and then by the element on the *Z* site (same order as in Fig. 8), i.e., *Z* varies more rapidly than *Y* .

energy *XA* phases. The large number of $\Delta E_f < 0$ compounds with $Z = A1$ is striking in Fig. 8, but only 14 compounds have negative formation energy for $Z =$ Sb. Although the number of negative formation energy compounds with $Z = AI$ is greatest, the number among them with formation energies less than −0*.*2 eV (13) is less than for compounds with *Z* = Si (25) and $Z = P(24)$.

FIG. 7. DFT formation energies and hull distances for potential inverse Heusler compounds grouped by the element on the *Y* site. The numbers near the top (in blue) and center (in brown) of each column denote the number of compounds with negative formation energy ΔE_f and hull distance $\Delta E_{HD} \leq 0.1$ eV/atom, respectively, in the corresponding *Y* -element group. Within a given *Y* -element column, the compounds are ordered first by the element on the *X* site (same order as in Fig. 6) and then by the element on the *Z* site (same order as in Fig. 8), i.e., *Z* varies more rapidly than *X*.

FIG. 8. DFT formation energies and hull distances for potential inverse Heusler compounds grouped by the element on the *Z* site. The numbers near the top (in blue) and center (in brown) of each column denote the number of compounds with negative formation energy ΔE_f and hull distance $\Delta E_{HD} \leq 0.1$ eV/atom, respectively, in the corresponding *Z*-element group. Within a given *Z*-element column, the compounds are ordered first by the element on the *X* site (same order as in Fig. 6) and then by the element on the *Y* site (same order as in Fig. 7), i.e., *Y* varies more rapidly than *X*.

It is interesting to contrast the trends in formation energy to those observed by considering the OQMD hull distance. Although Sc and Ti tend to generate negative formation energies as *X* atoms in *XA* phase $X_2 Y Z$ compounds, the picture is opposite for the hull distances. Very few of these compounds have a hull distance near the empirical 0.052 eV*/*atom range which is indicative of a potentially synthesizable phase. Using a larger hull distance of 0.1 eV as an (arbitrary) stability threshold, we find no $X = Sc$ phases that meet the criterion and only 2 of $72 X = Ti$ phases. Sc and Ti are "active" elements that form numerous low-energy phases. Hull distances tend to be lower when the *X* atom is Mn or Fe with 22 of 45 $X = Mn$ and 12 of 36 $X =$ Fe meeting the criterion of hull distance less than 0.1 eV*/*atom.

While $9/9 Y = Ti$ and $17/18 Y = VI$ H compounds have negative formation energy, none of them meet the relaxed hull distance threshold of 0.1 eV*/*atom. However, relatively large fractions of $Y = \text{Fe} (10/45)$, $Y = \text{Co} (11/45)$, and $Y =$ Ni (8*/*63) meet the criterion. Thus, there is better correlation between low formation energy and low hull distance when the *XA* compounds are grouped by *Y* element. By both measures, $Y = Ni$, Co, or Fe tend to generate more stable compounds.

The trends in stability with *Z* atom are also somewhat similar when viewed from the perspectives of hull distance and formation energy. Al, Ga, Si, and Ge as *Z* atom are associated with compounds having relatively low hull distances and negative formation energies. One big difference, however, is $Z = P$ which tends to generate numerous *XA* compounds with low formation energy, but none with very low hull distances.

C. Stability and gaps near the Fermi energy

We calculated the electronic structure of each compound and obtained its spin polarization at the Fermi level E_F . The

FIG. 9. The distribution of the 405 potential inverse Heusler compounds with negative ($\Delta E_f < 0$ eV/atom) and positive ($\Delta E_f >$ 0 eV*/*atom) formation energies as a function of spin polarization $P(E_F)$ [given by Eq. (3)]. In the central region, we show the number of inverse Heusler compounds grouped by 10 percentage points of spin polarization. In an additional region to the left, we show the 15 semiconductors, including 14 compounds with a negative formation energy and 1 with a positive formation energy. In the additional region on the right, we show 52 half-metals, including 51 with negative formation energy and 1 with positive formation energy.

spin polarization P at E_F is defined as

$$
P(E_F) = \frac{N_{\uparrow}(E_F) - N_{\downarrow}(E_F)}{N_{\uparrow}(E_F) + N_{\downarrow}(E_F)},
$$
\n(3)

where N_{\uparrow} and N_{\downarrow} are the densities of states for majority (spin-up) and minority (spin-down) electrons, respectively. We define the compounds with 100% spin polarization $[P(E_F) =$ 1] to be half-metals in this paper.

In a study of the half-Heusler compounds, gaps at the Fermi energy in one or both spin channels appeared to be associated with low formation energies and low hull distances [\[30\]](#page-24-0). Figure 9 shows how the number of IH compounds with positive and negative formation energy varies with spin polarization. It can be seen that for semiconductors and half-metals the ratio of the number of negative formation energy compounds to positive formation energy compounds is particularly high.

Figure 10 shows the number of IH compounds with positive and negative hull distances less than and greater than 0.1 eV*/*atom versus spin polarization. In contrast to the case of the half Heuslers, the plot of hull distance versus spin polarization does not present an obvious case for a correlation between gaps near the Fermi energy and stability. As will be discussed in Sec. IV A, this is due to the large number of $X =$ Sc and $X =$ Ti compounds in our database that are semi-conductors or half-metals and although they tend to have negative formation energies, they tend to be less stable than competing compounds. On the other hand, as will be discussed in Sec. [III,](#page-5-0) only 20 of the IH compounds in our database systems met our 0.052-eV*/*atom hull distance stability criterion. Half of these are half-metals or near-half-metals. We consider this to be strong support for the notion that gaps in or near

FIG. 10. The distribution of the 405 potential inverse Heusler compounds that lie near (ΔE_{HD} < 0.1 eV/atom) or far (ΔE_{HD}) 0*.*1 eV*/*atom) from the convex hull, as a function of spin polarization $P(E_F)$ [given by Eq. (3)]. In the central region, the number of inverse Heusler compounds grouped by 10 percentage points of spin polarization is shown. In the additional region on the left are shown 15 semiconductors (of which 2 compounds have E_{HD} < 0.1 eV/atom). On the right are shown the 52 half-metals (of which 9 compounds have E_{HD} < 0.1 eV/atom).

the Fermi energy in one or both spin channels contribute to stability.

IV. ELECTRONIC STRUCTURE

In this section the electronic structures of the IH compounds are discussed with a particular emphasis on gaps in the density of states, particularly those after 9, 12, and 14 states per formula unit.

A. IH semiconductors and half-metallic ferromagnets

In total we found 14 semiconductors, 51 half-metals, and 50 near-half-metals with negative formation energy among the 405 potential IH compounds. We will discuss these materials in this section.

1. Semiconductors

The 14 *XA* semiconductors with negative formation energy that we found are listed in Table [I](#page-10-0) where the DFT-calculated properties (i.e., lattice constant, band gap, gap type, local moments, formation energy, hull distance) are presented. Remarkably, all of these 14 systems are 18 electron *non-Slater-Pauling* semiconductors with gaps after 9 electrons per formula unit (2.25 per atom) per spin channel. Recall that *Slater-Pauling* gaps (at least according to the nomenclature we prefer) occur after 3 electrons per atom per spin.

From Table [I,](#page-10-0) it can be seen that all of these semiconductors have either Sc or Ti as *X* atoms. This is not surprising (given that they all have 18 electrons per formula unit) because the choice of systems that comprise our database (see Sec. [II\)](#page-2-0) implies that if the number of valence electrons on an *X* atom

TABLE I. For each of the 14 18-electron X_2YZ inverse Heusler compounds that are non-Slater-Pauling semiconductors with negative formation energy, we list the calculated lattice constant *a*, band gap in two spin channels E_g^{\uparrow} , E_g^{\downarrow} within DFT, local moments for atoms on the X, Y, and Z sites: $m(X_1)$, $m(X_2)$, $m(Y)$, and $m(Z)$, the formation energy of the compound in the XA and $L2_1$ structures, distance from the convex hull for the *XA* phase ΔE_{HD}^{XA} , previous experimental reports.

| | $\mathfrak a$ | E_g^\uparrow | E_g^{\downarrow} | | $m(X_2)$ | m(Y) | | ΔE_f^{XA} | $\Delta E^{L2_1}_f$ | $\Delta E_{\rm HD}^{XA}$ | |
|----------------------|---------------|----------------|--------------------|----------------|----------------|----------------|----------------|-------------------|---------------------|--------------------------|--|
| X_2YZ | $\check{A})$ | (eV) | (eV) | $m(X_1)$ | (μ_B) | | m(Z) | (eV/atom) | | | |
| Sc ₂ MnP | 6.18 | 0.052 | 0.052 | $\mathbf{0}$ | $\mathbf{0}$ | θ | $\overline{0}$ | -0.286 | -0.471 | 0.631 | |
| Sc ₂ FeSi | 6.255 | 0.347 | 0.347 | $\overline{0}$ | $\mathbf{0}$ | Ω | $\overline{0}$ | -0.292 | -0.416 | 0.311 | |
| Sc ₂ FeGe | 6.33 | 0.356 | 0.356 | $\overline{0}$ | $\mathbf{0}$ | θ | $\overline{0}$ | -0.262 | -0.435 | 0.371 | |
| Sc ₂ FeSn | 6.60 | 0.337 | 0.337 | $\overline{0}$ | $\mathbf{0}$ | θ | $\overline{0}$ | -0.230 | -0.358 | 0.319 | |
| Sc_2CoAl | 6.42 | 0.475 | 0.475 | $\overline{0}$ | $\overline{0}$ | θ | $\overline{0}$ | -0.280 | -0.350 | 0.130 | |
| Sc ₂ CoGa | 6.36 | 0.532 | 0.532 | $\overline{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | 0 | -0.328 | -0.454 | 0.159 | |
| Sc ₂ Coln | 6.61 | 0.470 | 0.470 | $\overline{0}$ | $\mathbf{0}$ | $\overline{0}$ | $\overline{0}$ | -0.205 | -0.281 | 0.200 | |
| Ti ₂ VSb | 6.47 | 0.427 | 0.119 | -1.236 | -0.521 | 1.881 | -0.009 | -0.215 | -0.208 | 0.162 | |
| Ti ₂ CrSi | 6.11 | 0.475 | 0.086 | -1.157 | -0.787 | 2.129 | -0.036 | -0.337 | -0.385 | 0.230 | |
| Ti ₂ CrGe | 6.20 | 0.522 | 0.104 | -1.254 | -0.883 | 2.348 | -0.022 | -0.262 | -0.305 | 0.259 | |
| Ti ₂ CrSn | 6.47 | 0.579 | 0.133 | -1.396 | -1.018 | 2.727 | -0.016 | -0.181 | -0.154 | 0.139 | |
| Ti ₂ MnAl | 6.23 | 0.561 | 0.043 | -1.202 | -1.067 | 2.606 | -0.043 | -0.268 | -0.302 | 0.088 | |
| Ti ₂ MnGa | 6.20 | 0.603 | 0.034 | -1.201 | -1.115 | 2.611 | -0.020 | -0.287 | -0.341 | 0.114 | |
| Ti ₂ MnIn | 6.47 | 0.394 | 0.042 | -1.336 | -1.266 | 3.043 | -0.020 | -0.100 | -0.05 | 0.092 | |

exceeds 4, the total number of electrons per formula unit will exceed 18.

Of the 9 *XA* compounds with $X = Sc$ in our database, 7 are calculated to be nonmagnetic semiconductors. The two that are not semiconductors are $Sc₂MnAs$ and $Sc₂MnSb$. The former is nonmagnetic with a very low DOS pseudogap at the Fermi energy.

The absence of a gap in $Sc₂MnAs$ can be traced to a reordering of the states at Γ . Between Sc₂MnP and Sc₂MnAs, a singly degenerate A_1 state (band 7) switches places with a triply degenerate T_2 state (bands 8, 9, and 10), as shown in Fig. 11. This *A*¹ state (composed of *s* contributions from all four atoms) decreases in energy relative to the *d* states as the lattice expands from $Sc₂MnP$ to $Sc₂MnAs$ to $Sc₂MnSb$. However,

FIG. 11. The band structures of (a) $Sc₂MnP$ and (b) $Sc₂MnAs$ are shown. The singly degenerate A_1 state and triply degenerate T_2 state are labeled in (b). Bands 7, 8, 9, and 10 are labeled in (a) and (b). The zero of the energy axis corresponds to the Fermi level.

Sc₂MnSb becomes magnetic as the lattice is expanded before this reordering occurs so it is predicted to be a magnetic near-half-metal with a tiny moment (see Table [III\)](#page-13-0).

The reason $Sc₂MnSb$ is magnetic while $Sc₂MnP$ and $Sc₂MnAs$ are not is almost certainly due to the larger lattice constant induced by the larger atom (Sb). For lattice constants less than approximately 6.50 Å, $Sc₂MnSb$ is predicted to be a nonmagnetic semiconductor similar to $Sc₂MnP$.

The DOS of the nonmagnetic Sc-based semiconductors in Table I can be understood by referring to Fig. $12(a)$ which shows the site decomposed DOS for Sc_2CoAl . Because Sc is an early transition metal compared to Co, its *d* states must lie higher. Thus, the ordering of the states is Al-*s* (in this case forming a narrow band well below the Fermi level and not shown in the figure.) followed by Al-*p*, followed by Co-*d*, and finally Sc_1-d and Sc_2-d . The gap after 9 electrons per spin channel occurs from the hybridization of the Co-*d* and Sc-*d* states or, more generally, between the *Y* -*d* and *X*-*d* states.

Thus, bonding in these systems appears to have a considerable ionic component with transfer of electrons to the Al and Co atoms from the Sc atoms. It can be seen from Table I that all of the $X = Sc$ 18-electron semiconductors are predicted to be substantially more stable in the *L*21 phase than in the *XA* phase. In addition, the *XA* hull distances all exceed our 0.052-eV threshold for potentially being stable in equilibrium.

Of the 9 18-electron compounds in our database with $X =$ Ti, 7 are predicted to be semiconductors and are included in Table I. One, $Ti₂VP$, is predicted to be a near-half-metal (see Table III) with a tiny moment. The final compound, $Ti₂Vs$, is predicted to be a magnetic semiconductor similar to $Ti₂VSb$, but its formation energy is greater than zero and so is omitted from Table I.

In contrast to the nonmagnetic 18-electron semiconductors with $X = Sc$, the systems with $X = Ti$ develop magnetic moments and have gaps of different size in the majorityand minority-spin channels. The net moment per formula unit is predicted to be zero for the semiconductors, with the Ti

FIG. 12. Density of states curves for the 18-electron semiconductor (a) Sc_2CoAl and (b) Ti_2CrSn . The curve presents the total DOS (black) and the projected DOS contributed by $Sc₁$ (T_{i1}) (red), $Sc₂$ (Ti₂) (orange), Co (Cr) (blue), and Al (Sn) (green). Zero energy corresponds to the Fermi level.

moments having the same sign and being balanced by a larger moment of opposite sign on the *Y* site. If the sign of the moment on the *Y* atom is taken to be positive, then the larger gaps are in the majority channel while the smaller gaps are in the minority channel, becoming slightly negative for $Ti₂VP$.

The larger gap (in the majority channel in Table [I\)](#page-10-0) for the $Ti₂YZ$ magnetic semiconductors is very similar to the gaps in both channels for the Sc_2YZ nonmagnetic semiconductors. The positive moment on the *Y* atom shifts its majority *d* level down while negative moments on the Ti atoms shift their majority d levels up, the larger moment on the $Ti₁$ giving it a slightly larger upward shift. As a consequence, the state ordering in the majority channel is Z -*s*, Z -*p*, Y -*d*, T_{12} -*d*, T_{11} -*d* so that the majority gap after nine states per formula unit arises from hybridization between $Y - d$ and $X_2 - d$ and $X_1 - d$ similar to the case for both channels in the $Sc₂YZ$ 18-electron semiconductors.

The persistence of the very small gaps after nine states per formula unit in the minority channel is surprising and seems to be aided by avoided band crossings along Γ to K and along *L* to *W*. We find these small gaps to be common when the *d* levels of the three transition metals are similar. The avoided crossings and hence the gaps are likely symmetry related since the nonmagnetic compound V_2VGa has several allowed band crossings that are avoided when the symmetry between X_1 , *X*2, and *Y* atoms is broken. These avoided band crossings lead to the small gap after nine states and are quite common when X_1, X_2 , and Y are similar, but not identical in one of the spin channels. For example, simple electron counting ignoring charge transfer and using the calculated moments yields for Ti2MnGa that the number of electrons for the minority channel

on each atom are approximately 2.6 for X_1 and X_2 and 2.2 for *Y* . The corresponding minority gap is tiny, 0.034 eV.

The atom-projected DOS of $Ti₂CrSn$ is presented in Fig. 12. The energy ordering of the atomic orbitals is Sn-*s* (in this case forming a narrow band more than 10 eV below the Fermi level and not shown in the figure.) followed by Sn-*p*, followed in the majority channel by Cr-*d*, and then the closely spaced $Ti₂$ -*d* and $Ti₁$ -*d*. In the minority channel the ordering of the d states is different, Ti_1-d , Ti_2-d , followed by Cr- d . In the majority channel, the hybridization between the lower-lying Cr-*d* and the higher Ti-*d* states produces a relatively large gap (0.579 eV), while in the minority channel, the relatively closely spaced Cr-*d* and Ti-*d* states create a smaller gap (0.133 eV).

Ti2CrSn might have interesting spin-dependent transport and tunneling properties if it can be synthesized. The calculated hull distance for the compound is 0.139 eV*/*atom meaning that the OQMD contains phases or a combination of phases (in this case a mixture of binaries) that is lower in energy than our 0.052-eV*/*atom empirical threshold.

We found no 24-electron Slater-Pauling semiconductors among the *XA* compounds that we investigated. Of the 405 *XA* compounds that we studied, 24 had a valence electron count of 24. None were semiconducting, though 6 were halfmetallic with zero moment or near-half-metallic with a very small moment. The reason for the absence of 24-electron *XA* semiconductors is relatively simple. As we shall discuss in the next section, the Slater-Pauling gap in the *XA* phase requires formation of significant magnetic moments to create a significant difference between the *d*-onsite energies of the two *X* atoms. When moments form, the two spin channels will generally be different and a semiconductor becomes unlikely. The interesting exception to this rule is the 18-electron semiconductors with moments, but zero net moment in which even very small differences between the *d*-onsite energies of the *X* atoms can effectuate a small gap. All of the semiconductors in Table [I](#page-10-0) have hull distances that exceed our stability threshold criterion of 0.052 eV*/*atom.

2. Half-metals and "near"-half-metals

Although a large number of*XA*phase half-metals have been predicted based on theoretical simulations [\[12\]](#page-24-0), we found that most of them do not remain in the *XA* phase after relaxation, or they have lower energies in the *L*21 or the tetragonal "*L*21" phase. However, in order to better understand the*XA* phase, we have listed in Tables [II](#page-12-0) and [III](#page-13-0) all of the half-metals and nearhalf-metals found in our survey of 405 *XA* systems for which the calculated formation energy was negative. Our definition of a "near-half-metal" is given in Sec. [IVB.](#page-15-0)

Both tables are divided into three sections according to whether the gap leading to half-metallicity is located after 9, 12, or 14 states. The tables give N_V , the number of valence electrons per formula unit, *a*, the lattice constant, the total magnetic moment per formula unit as well as the individual moments on each atom evaluated within atom centered spheres of $1.45 - \text{\AA}$ radius.

For almost all of the half-metals and near-half-metals that follow the Slater-Pauling rule (gap after 12 states), the spin moments on the X_1 atoms are antiparallel to those on X_2 atoms. If one uses the local moments given in Table II to

TABLE II. DFT-calculated properties of 51 half-metals in the inverse Heusler phase that have negative formation energy. Successive columns present number of valence electrons per formula unit N_V , calculated lattice constant *a*, total spin moment per f.u. M_{tot} , local moments for atoms on the X_1, X_2, Y , and Z sites: $m(X_1), m(X_2), m(Y)$, and $m(Z)$, formation energy ΔE_f , distance from the convex hull ΔE_{HD} , formation energy in L_2 ₁ phase $\Delta E_f(L_2)$, band gap E_g , gap type, experimental reports of compounds with composition X_2YZ , and experimental reports of corresponding *Y*2*XZ* full Heusler compounds, if any.

| | | \boldsymbol{a} | $M_{\rm tot}$ | $m(X_1)$ | $m(X_2)$ | m(Y) | m(Z) | | $\Delta E_{\rm HD}$ | $\Delta E_f(L2_1)$ | $E_{\it g}$ | Experimental | Y_2XZ |
|----------------------|-------|------------------|-------------------------|----------|-----------|----------|----------|--------------|---------------------|--------------------|-------------|--------------|-------------|
| X_2YZ | N_V | (\AA) | | | (μ_B) | | | ΔE_f | | (eV/atom) | (eV) | reports | reports |
| Sc ₂ VIn | 14 | 6.938 | -4 | -0.072 | -0.509 | -2.675 | 0.041 | -0.013 | 0.276 | -0.100 | 0.504 | | |
| Sc ₂ VSi | 15 | 6.53 | -3 | 0.058 | -0.452 | -2.228 | 0.073 | -0.125 | 0.388 | -0.196 | 0.499 | | |
| Sc ₂ VGe | 15 | 6.61 | -3 | 0.084 | -0.424 | -2.274 | 0.070 | -0.118 | 0.446 | -0.203 | 0.480 | | |
| Sc ₂ VSn | 15 | 6.87 | -3 | 0.144 | -0.327 | -2.388 | 0.058 | -0.157 | 0.323 | -0.210 | 0.518 | | |
| Sc ₂ CrAl | 15 | 6.67 | -3 | 0.295 | -0.064 | -3.133 | 0.082 | -0.039 | 0.225 | -0.096 | 0.684 | | |
| Sc ₂ CrGa | 15 | 6.62 | -3 | 0.284 | -0.050 | -3.121 | 0.067 | -0.065 | 0.283 | -0.158 | 0.684 | | |
| Sc_2CrIn | 15 | 6.86 | -3 | 0.337 | 0.056 | -3.302 | 0.049 | -0.003 | 0.286 | -0.069 | 0.646 | | |
| Sc ₂ CrSi | 16 | 6.43 | -2 | 0.380 | 0.020 | -2.454 | 0.079 | -0.120 | 0.393 | -0.161 | 0.632 | | |
| Sc ₂ CrGe | 16 | 6.52 | -2 | 0.429 | 0.112 | -2.613 | 0.065 | -0.110 | 0.453 | -0.184 | 0.646 | | |
| Sc ₂ CrSn | 16 | 6.79 | -2 | 0.505 | 0.234 | -2.878 | 0.051 | -0.136 | 0.344 | -0.180 | 0.631 | | |
| Sc ₂ MnSi | 17 | 6.37 | -1 | 0.465 | 0.443 | -2.218 | 0.062 | -0.200 | 0.350 | -0.285 | 0.518 | | |
| Sc ₂ MnGe | 17 | 6.461 | -1 | 0.536 | 0.554 | -2.455 | 0.041 | -0.188 | 0.410 | -0.314 | 0.428 | | |
| Sc ₂ MnSn | 17 | 6.75 | -1 | 0.628 | 0.654 | -2.833 | 0.031 | -0.202 | 0.312 | -0.287 | 0.266 | | |
| Ti ₂ VSi | 17 | 6.173 | -1 | 0.575 | -0.028 | -1.476 | 0.007 | -0.316 | 0.217 | -0.379 | 0.219 | | |
| Ti ₂ VGe | 17 | 6.262 | -1 | 0.688 | 0.094 | -1.706 | 0.008 | -0.246 | 0.240 | -0.316 | 0.285 | | |
| Ti ₂ VSn | 17 | 6.523 | -1 | 0.879 | 0.257 | -2.052 | 0.004 | -0.166 | 0.139 | -0.189 | 0.423 | | |
| Sc ₂ CoSi | 19 | 6.28 | $\mathbf{1}$ | 0.426 | 0.173 | 0.220 | 0.053 | -0.268 | 0.406 | -0.506 | 0.67 | | |
| Sc ₂ CoGe | 19 | 6.367 | $\mathbf{1}$ | 0.432 | 0.221 | 0.165 | 0.029 | -0.248 | 0.425 | -0.521 | 0.591 | | |
| Sc ₂ CoSn | 19 | 6.627 | $\mathbf{1}$ | 0.421 | 0.206 | 0.182 | 0.010 | -0.239 | 0.345 | -0.421 | 0.594 | | |
| Ti ₂ MnSi | 19 | 6.024 | $\mathbf{1}$ | 1.055 | 0.652 | -0.920 | 0.043 | -0.394 | 0.231 | -0.409 | 0.461 | | |
| Ti ₂ MnGe | 19 | 6.123 | $\mathbf{1}$ | 1.204 | 0.844 | -1.314 | 0.024 | -0.295 | 0.265 | -0.346 | 0.517 | | |
| Ti ₂ MnSn | 19 | 6.39 | $\mathbf{1}$ | 1.41 | 1.106 | -1.916 | 0.017 | -0.188 | 0.188 | -0.178 | 0.593 | | |
| Ti ₂ FeAl | 19 | 6.12 | $\mathbf{1}$ | 1.013 | 0.709 | -0.964 | 0.031 | -0.256 | 0.187 | -0.279 | 0.536 | | |
| Ti ₂ FeGa | 19 | 6.122 | $\mathbf{1}$ | 1.005 | 0.757 | -0.966 | 0.009 | -0.265 | 0.212 | -0.344 | 0.57 | | |
| Ti ₂ FeIn | 19 | 6.37 | $\mathbf{1}$ | 1.208 | 1.064 | -1.673 | 0.004 | -0.045 | 0.274 | -0.072 | 0.532 | | |
| Ti ₂ FeSi | 20 | 5.99 | $\overline{\mathbf{c}}$ | 1.257 | 0.621 | -0.101 | 0.041 | -0.395 | 0.295 | -0.447 | 0.628 | | |
| Ti ₂ FeGe | 20 | 6.07 | $\overline{\mathbf{c}}$ | 1.282 | 0.677 | -0.186 | 0.011 | -0.297 | 0.307 | -0.394 | 0.622 | | |
| Ti ₂ FeSn | 20 | 6.339 | $\boldsymbol{2}$ | 1.356 | 0.801 | -0.509 | 0.001 | -0.185 | 0.256 | -0.227 | 0.617 | | |
| Ti ₂ CoAl | 20 | 6.13 | \overline{c} | 1.252 | 0.705 | -0.204 | 0.020 | -0.287 | 0.185 | -0.293 | 0.684 | | |
| Ti ₂ CoGa | 20 | 6.11 | \overline{c} | 1.230 | 0.773 | -0.211 | -0.005 | -0.295 | 0.178 | -0.363 | 0.689 | | |
| Ti ₂ CoIn | 20 | 6.351 | $\boldsymbol{2}$ | 1.245 | 0.837 | -0.381 | -0.009 | -0.089 | 0.195 | -0.086 | 0.613 | | |
| V_2CrGe | 20 | 5.94 | $\overline{\mathbf{c}}$ | 1.167 | -0.415 | 1.158 | 0.003 | -0.118 | 0.151 | -0.120 | 0.086 | | |
| V_2MnAl | 20 | 5.922 | $\boldsymbol{2}$ | 1.354 | -0.309 | 0.870 | 0.016 | -0.148 | 0.097 | -0.072 | 0.142 | | $[74]$ |
| V_2MnGa | 20 | 5.924 | $\boldsymbol{2}$ | 1.229 | -0.416 | 1.138 | -0.007 | -0.118 | 0.123 | -0.070 | 0.124 | | $[75]$ |
| Ti ₂ CoSi | 21 | 6.02 | 3 | 1.528 | 0.789 | 0.394 | 0.059 | -0.387 | 0.307 | -0.437 | 0.798 | | |
| Ti ₂ CoGe | 21 | 6.10 | 3 | 1.518 | 0.812 | 0.396 | 0.018 | -0.299 | 0.303 | -0.384 | 0.779 | | |
| Ti ₂ CoSn | 21 | 6.35 | 3 | 1.495 | 0.802 | 0.373 | -0.004 | -0.217 | 0.220 | -0.197 | 0.727 | | |
| Ti ₂ NiAl | 21 | 6.19 | 3 | 1.524 | 0.982 | 0.149 | 0.028 | -0.285 | 0.200 | -0.277 | 0.46 | | |
| Ti ₂ NiGa | 21 | 6.17 | 3 | 1.491 | 1.056 | 0.158 | -0.003 | -0.289 | 0.193 | -0.364 | 0.522 | | |
| Ti ₂ Niln | 21 | 6.4 | 3 | 1.452 | 1.058 | 0.135 | -0.013 | -0.127 | 0.216 | -0.096 | 0.418 | | |
| Cr ₂ MnAl | 22 | 5.831 | -2 | -1.835 | 1.536 | -1.678 | -0.021 | -0.086 | 0.048 | 0.002 | 0.204 | | |
| Cr ₂ NiAl | 25 | 5.7 | $\mathbf{1}$ | -1.239 | 1.828 | 0.452 | -0.057 | -0.071 | 0.236 | 0.148 | 0.142 | | $[76]$ |
| Cr ₂ CoP | 26 | 5.619 | $\overline{\mathbf{c}}$ | -0.794 | 1.912 | 0.854 | -0.012 | -0.22 | 0.217 | -0.159 | 0.48 | | |
| Mn ₂ CoAl | 26 | 5.735 | 2 | -1.617 | 2.701 | 0.958 | -0.061 | -0.27 | 0.036 | -0.141 | 0.38 | XA [28,40] | $[77]$ |
| Mn ₂ FeP | 27 | 5.558 | 3 | -0.452 | 2.632 | 0.733 | 0.024 | -0.352 | 0.096 | -0.292 | 0.252 | | |
| Mn ₂ FeAs | 27 | 5.72 | 3 | -0.732 | 2.772 | 0.885 | 0.022 | -0.071 | 0.086 | 0.014 | 0.337 | | |
| Mn ₂ CoSi | 27 | 5.621 | 3 | -0.548 | 2.670 | 0.849 | -0.012 | -0.365 | 0.018 | -0.177 | 0.513 | | $[78 - 80]$ |
| Mn ₂ CoGe | 27 | 5.75 | 3 | -0.804 | 2.863 | 0.903 | 0.005 | -0.153 | 0.03 | -0.011 | 0.323 | XA [40] | [81] |
| Mn ₂ CoP | 28 | 5.581 | 4 | 0.091 | 2.793 | 1.017 | 0.032 | -0.333 | 0.216 | -0.232 | 0.508 | | |
| Mn ₂ CoAs | 28 | 5.738 | 4 | -0.007 | 2.916 | 1.013 | 0.025 | -0.085 | 0.08 | 0.081 | 0.295 | | |
| Mn ₂ CoSb | 28 | 5.985 | 4 | -0.041 | 3.093 | 0.938 | -0.003 | 0.042 | 0.133 | 0.193 | 0.499 | XA [40,70] | [76, 82] |
| Mn ₂ CuSi | 29 | 5.762 | $\mathbf{1}$ | -1.876 | 2.838 | 0.003 | 0.018 | -0.101 | 0.177 | -0.091 | 0.318 | | |

TABLE III. DFT-calculated properties of 50 near-half-metallic *X*2*YZ* inverse Heusler compounds with negative formation energy. Successive columns present number of valence electrons per formula unit N_V , calculated lattice constant (a_{calc}), total spin moment (M_{tot}) per f.u., local moments for atoms on the X_1 , X_2 , Y , and Z sites: $m(X_1)$, $m(X_2)$, $m(Y)$, and $m(Z)$, formation energy (ΔE_f) , distance from the convex hull ΔE_{HD} , formation energy in *L*2₁ phase $\Delta E_f(L2_1)$, experimental reports of compounds with composition X_2YZ , and experimental reports of corresponding *Y*2*XZ* full Heusler compounds.

| | | $a_{\rm calc}$ | $M_{\rm tot}$ | | | | | ΔE_f | $\Delta E_{\rm HD}$ | $\Delta E_f(L2_1)$ | Experimental | Rep. |
|----------------------|-------|----------------|---------------|----------|----------|----------|----------|--------------|---------------------|--------------------|--------------|---------|
| X_2YZ | N_V | (\AA) | (μ_B) | $m(X_1)$ | $m(X_2)$ | m(Y) | m(Z) | | (eV/atom) | | records | Y_2XZ |
| Sc ₂ VAl | 14 | 6.756 | -3.8779 | -0.077 | -0.508 | -2.588 | 0.040 | -0.033 | 0.231 | -0.134 | | |
| Sc ₂ VGa | 14 | 6.708 | -3.9989 | -0.114 | -0.579 | -2.613 | 0.050 | -0.064 | 0.285 | -0.184 | | |
| Sc ₂ MnAl | 16 | 6.59 | -1.9983 | 0.429 | 0.231 | -2.903 | 0.074 | -0.122 | 0.174 | -0.160 | | |
| Sc ₂ MnGa | 16 | 6.54 | -1.9999 | 0.425 | 0.285 | -2.919 | 0.052 | -0.159 | 0.229 | -0.243 | | |
| Sc ₂ MnIn | 16 | 6.80 | -1.9969 | 0.489 | 0.398 | -3.203 | 0.038 | -0.085 | 0.221 | -0.138 | | |
| Sc ₂ VSb | 16 | 6.78 | -2.0011 | 0.319 | -0.188 | -1.949 | 0.049 | -0.150 | 0.439 | -0.173 | | |
| Ti ₂ VAL | 16 | 6.32 | -1.9633 | 0.380 | -0.579 | -1.541 | 0.016 | -0.120 | 0.153 | -0.179 | | |
| Ti ₂ VGa | 16 | 6.27 | -1.9529 | 0.314 | -0.567 | -1.499 | 0.022 | -0.145 | 0.189 | -0.219 | | |
| Sc ₂ CrP | 17 | 6.303 | -1.0043 | 0.491 | 0.349 | -2.051 | 0.063 | -0.166 | 0.699 | -0.296 | | |
| Sc ₂ CrSb | 17 | 6.74 | -1.0001 | 0.660 | 0.526 | -2.598 | 0.046 | -0.097 | 0.492 | -0.178 | | |
| Sc ₂ FeAl | 17 | 6.47 | -0.9071 | 0.248 | 0.216 | -1.539 | 0.029 | -0.1323 | 0.216 | -0.244 | | |
| Sc ₂ FeGa | 17 | 6.43 | -0.9488 | 0.264 | 0.277 | -1.656 | 0.015 | -0.1735 | 0.267 | -0.345 | | |
| Sc ₂ FeIn | 17 | 6.69 | -0.9714 | 0.344 | 0.412 | -2.042 | 0.013 | -0.069 | 0.290 | -0.189 | | |
| Ti ₂ CrAl | 17 | 6.297 | -0.8021 | 1.032 | 0.775 | -2.723 | 0.019 | -0.1556 | 0.141 | -0.197 | | |
| Ti ₂ CrGa | 17 | 6.266 | -0.8659 | 1.008 | 0.785 | -2.747 | 0.011 | -0.1781 | 0.166 | -0.245 | | |
| Ti ₂ CrIn | 17 | 6.51 | -0.8538 | 1.141 | 0.939 | -3.078 | 0.005 | -0.002 | 0.149 | 0.022 | | |
| Sc ₂ MnSb | 18 | 6.685 | 0.0047 | -0.656 | -0.712 | 1.984 | 0.018 | -0.1449 | 0.444 | -0.296 | | |
| Ti ₂ VP | 18 | 6.06 | 0.0033 | -0.941 | -0.365 | 1.371 | -0.013 | -0.4397 | 0.442 | -0.495 | | |
| Sc ₂ NiAl | 19 | 6.502 | 0.9649 | 0.413 | 0.217 | 0.120 | 0.031 | -0.2413 | 0.249 | -0.425 | | |
| Sc ₂ NiGa | 19 | 6.453 | 0.9738 | 0.423 | 0.272 | 0.081 | 0.015 | -0.2817 | 0.278 | -0.544 | | |
| Sc ₂ NiIn | 19 | 6.69 | 0.9839 | 0.414 | 0.271 | 0.078 | 0.006 | -0.1993 | 0.309 | -0.375 | | |
| Ti ₂ CrAs | 19 | 6.11 | 0.8077 | 1.015 | 0.535 | -0.941 | 0.023 | -0.2035 | 0.444 | -0.255 | | |
| Ti ₂ CrSb | 19 | 6.38 | 0.965 | 1.301 | 0.775 | -1.441 | 0.019 | -0.111 | 0.279 | -0.084 | | |
| Sc ₂ CoSb | 20 | 6.64 | 1.9999 | 0.656 | 0.299 | 0.664 | 0.055 | -0.138 | 0.528 | -0.394 | | |
| Sc ₂ NiSi | 20 | 6.38 | 1.8015 | 0.689 | 0.459 | 0.220 | 0.093 | -0.203 | 0.527 | -0.591 | | |
| Sc ₂ NiGe | 20 | 6.46 | 1.8123 | 0.691 | 0.517 | 0.184 | 0.050 | -0.200 | 0.540 | -0.621 | | |
| Sc ₂ NiSn | 20 | 6.72 | 1.8438 | 0.690 | 0.523 | 0.164 | 0.019 | -0.228 | 0.430 | -0.516 | | |
| Ti ₂ MnSb | 20 | 6.323 | 1.9891 | 1.318 | 0.564 | -0.231 | 0.009 | -0.1596 | 0.277 | -0.181 | | |
| Ti ₂ FeAs | 21 | 6.07 | 2.9395 | 1.424 | 0.469 | 0.780 | 0.036 | -0.2374 | 0.475 | -0.391 | | |
| V_2 FeGa | 21 | 5.91 | 2.8487 | 1.803 | -0.231 | 1.200 | -0.001 | -0.146 | 0.091 | -0.047 | | $[76]$ |
| Ti ₂ NiSn | 22 | 6.427 | 3.8602 | 1.604 | 1.307 | 0.451 | 0.001 | -0.189 | 0.294 | -0.236 | | |
| Ti ₂ FeSb | 21 | 6.31 | 2.9885 | 1.449 | 0.441 | 0.802 | 0.009 | -0.1583 | 0.319 | -0.209 | | |
| V_2 FeAl | 21 | 5.914 | 2.911 | 1.862 | -0.197 | 1.132 | 0.026 | -0.191 | 0.088 | -0.023 | | $[83]$ |
| V_2MnGe | 21 | 5.92 | -2.8136 | -1.627 | 0.516 | -1.610 | 0.005 | -0.193 | 0.115 | -0.110 | | |
| V_2MnSn | 21 | 6.20 | -2.9604 | -1.690 | 0.944 | -2.148 | 0.014 | -0.002 | 0.188 | 0.179 | | |
| V_2 FeGe | 22 | 5.891 | -1.9779 | -1.415 | 0.434 | -0.944 | -0.004 | -0.165 | 0.146 | -0.129 | | |
| Cr ₂ MnGa | 22 | 5.832 | -1.993 | -1.880 | 1.614 | -1.716 | -0.003 | -0.004 | 0.049 | 0.038 | | |
| Cr ₂ MnGe | 23 | 5.8 | -0.9919 | -1.478 | 1.610 | -1.147 | 0.010 | -0.007 | 0.122 | -0.008 | | |
| V_2CoSi | 23 | 5.78 | -0.997 | -0.754 | 0.182 | -0.374 | -0.029 | -0.291 | 0.199 | -0.193 | | |
| Cr ₂ CoAl | 24 | 5.79 | 0.0095 | 1.892 | -1.698 | -0.307 | 0.086 | -0.08 | 0.227 | 0.126 | | $[84]$ |
| Cr ₂ CoSi | 25 | 5.657 | 1.0086 | -0.949 | 1.440 | 0.547 | 0.053 | -0.2187 | 0.139 | -0.022 | | |
| Cr ₂ CoGe | 25 | 5.792 | 1.0225 | -1.593 | 2.034 | 0.619 | 0.043 | -0.0171 | 0.109 | 0.155 | | |
| Mn ₂ FeAl | 25 | 5.75 | 1.0009 | -1.856 | 2.714 | 0.142 | -0.025 | -0.158 | 0.008 | -0.086 | | $[75]$ |
| Mn ₂ FeGa | 25 | 5.79 | 1.0386 | -2.185 | 2.888 | 0.315 | -0.004 | -0.075 | 0.018 | -0.038 | | |
| Cr ₂ NiSi | 26 | 5.705 | 1.8915 | -0.857 | 2.130 | 0.573 | -0.023 | -0.195 | 0.201 | -0.035 | | |
| Cr ₂ NiGe | 26 | 5.82 | 1.9414 | -1.132 | 2.434 | 0.573 | -0.008 | -0.012 | 0.176 | 0.124 | | |
| Mn ₂ CoGa | 26 | 5.76 | 2.0045 | -1.820 | 2.879 | 0.955 | -0.030 | -0.167 | 0 | -0.087 | XA[40] | $[85]$ |
| Mn ₂ FeSi | 26 | 5.6006 | 2.0077 | -0.798 | 2.407 | 0.371 | -0.015 | -0.362 | 0 | -0.259 | | $[76]$ |
| Mn ₂ FeGe | 26 | 5.72 | 2.0135 | -1.243 | 2.667 | 0.556 | -0.004 | -0.14 | 0 | -0.035 | | |
| Mn ₂ CuP | 30 | 5.716 | 1.9946 | -1.242 | 3.042 | 0.057 | 0.083 | -0.048 | 0.399 | -0.091 | | |

FIG. 13. Calculated total magnetic moment per unit cell as a function of the total valence electron number per unit cell for 254*XA* compounds with negative formation energies. The orange dashed line represents $|M_{\text{tot}}| = N_V - 18$, the blue dashed line represents the Slater-Pauling rule $|M_{tot}| = N_V - 24$, and the light green dashed line represents $|M_{tot}| = N_V - 28$. The compounds from Table [II](#page-12-0) are listed in the boxes. All of these compounds follow the rules precisely. We used different colors to label different *X*-based inverse Heusler compounds. We used diamond, cubic, circle, and triangle symbols to label ferromagnets/ferrimagnets, half-metals, metals, and semiconductors, respectively.

estimate the number of majority and minority *s*-*d* electrons on the transition-metal sites one finds that for the Slater-Pauling half-metals with minority gap, there are approximately four minority electrons on the X_1 and *Y* sites and approximately two on the X_2 sites. This implies that the transition-metal nearest neighbors always differ in number of electrons by approximately 2. The implied contrast in the positions of the *d* states of these neighbors leads to the hybridization gap in the minority channel. Since the average of 4 and 2 is 3, the gap should fall after 3 states per atom [\[73\]](#page-25-0).

The tables also give the formation energy ΔE_f and the hull distance calculated within the OQMD, ΔE_{HD} (see Sec. IV A 4) as well as the formation energy of the $L2_1$ phase, $E_f(L2_1)$. The final three columns show the calculated energy gap, references to experimental reports of the *XA* phase and references to experimental reports of the $L2_1$ phase for Y_2XZ . The presence of these phases indicates that it may be difficult to distinguish a mixture of $X_2 YZ$ in the *XA* phase and $Y_2 XZ$ in the $L2_1$ phase. Since they have the same XRD reflection peaks, it is difficult to distinguish the composition from the XRD patterns, which may lead to a wrong conclusion about the composition of the sample.

3. Moment vs electron count

Figure 13 shows the total spin magnetic moment per formula unit (M_{tot}) plotted versus the total valence electron number per formula unit (N_V) for 254 *XA* compounds with negative formation energy. In this plot, the half-metallic phases fall along one of the lines, $M_{\text{tot}} = |N_V - 18|$, $M_{\text{tot}} = |N_V - 18|$ 24|, or $M_{\text{tot}} = |N_V - 28|$ depending on whether the system is placing the Fermi energy in a gap after 9, 12, or 14 states, respectively. The half-metals are listed in Table [II](#page-12-0) along with their calculated properties. Table III is a similar list of the near-half-metals.

The half-metals and near-half-metals found along the $M_{\text{tot}} = |N_V - 18|$ line with a gap after 9 states per formula unit in one of the spin channels primarily have $X = Sc$ or Ti, while those found along the $M_{\text{tot}} = |N_V - 24|$ lines generally have $X = Cr$ or Mn. Systems with $X = V$ also fall mainly along the $M_{\text{tot}} = |N_V - 18|$ line, but there is one near-half-metal (V_2CoSi) that uses the gap after 12 states. We find only one half-metal ($Mn₂CuSi$) and one near-half-metal ($Mn₂CuP$) along the $M_{\text{tot}} = |N_V - 28|$ line. The systems that appear to be on that line at $N_V = 30$ and 31 have pseudogaps rather than gaps near the Fermi energy.

Some systems appear on or near two lines. Systems with $N_V = 21$ can in principle have gaps after both 9 states and 12 states. Ti₂CoSi is a half-metal with the Fermi energy in the minority gap after 9 states, but the Fermi energy also falls in a pseudogap at 12 states in majority. The near-half-metals $Ti₂Fe$ (P,As,Sb) all have gaps after 9 states in minority and after 12 states in majority, however, the moment is not large enough to place the Fermi energy in the gaps. In each case it falls just above the minority gap and just below the majority gap by an equal number of states. Similarly, for $N_V = 26$ one can have systems on both the $M_{\text{tot}} = |N_V - 24|$ and the $M_{\text{tot}} =$ $|N_V - 28|$ lines. Thus, Mn₂CoAl has a minority gap after 12 states and a pseudogap at 14 majority states.

Although we predicted that $Mn₂CoSb$ would have a positive formation energy (0.042 eV), we found references indicating that it can be fabricated in the *XA* phase [\[40](#page-24-0)[,70\]](#page-25-0) using melt spinning. For this reason, $Mn₂CoSb$ was included in Fig. 13 and Table [II.](#page-12-0) As discussed in Sec. [III A,](#page-5-0) we speculate that the experimental samples may be $L2_1B$ phase rather than *XA*.

4. Hull distances

Using our calculated *XA* formation energies, and the formation energies of all the other phases in the OQMD, we calculated the hull distance ΔE_{HD} for the negative formation energy half-metallic inverse Heusler compounds listed in Table [II](#page-12-0) and the near-half-metals listed in Table [III.](#page-13-0) Our calculations predict all the Sc-, Ti-, and V-based half-metals lie significantly above the convex hull and have a ΔE_{HD} that exceeds our empirical criterion of $\Delta E_{HD} \leqslant 0.052$ eV/atom for having a reasonable probability of synthesizability. We predict four half-metals (Cr_2MnAl , Mn_2CoAl , Mn_2CoSi , Mn_2CoGe) to lie close enough to the convex hull to meet our criterion. Two of these compounds Mn_2CoAl [\[28,40\]](#page-24-0) and Mn_2CoGe [\[40\]](#page-24-0) have been experimentally observed by x-ray diffraction (XRD) measurements to exist in the *XA* phase. We also found six near-half-metals (Cr_2MnGa , Mn_2FeA1 , Mn_2FeGa , Mn_2FeSi , Mn₂FeGe, Mn₂CoGa) that meet our criterion for stability, three of these (Mn_2FeGa , Mn_2FeSi , Mn_2FeGe) have a calculated hull distance of zero, meaning that no phase or combination of phases in the OQMD had lower energy.

B. Electronic structure of each IH compound

In this section we will briefly discuss each of the 405 *XA* compounds discussing gaps in their DOS, and whether they are half-metals or near-half-metals. The designation of halfmetal is binary. If the Fermi energy lies in a gap for one of the spin channels, but not for the other, then it is a half-metal. There may be other important quantitative questions such as the distances between the Fermi energy and the band edges, but the calculated band structure either corresponds to a half-metal or it does not. It should be noted that the determination of whether a given inverse Heusler has a gap is not always trivial if the gap is small because the Brillouin zone is sampled at a finite number of points. We used both the DOS calculated using the linear tetrahedron method and an examination of the bands along symmetry directions to decide difficult cases.

In contrast to the binary choice between half-metal or not, the designation of a "near-half-metal" is more qualitative. There are a couple of ways a calculated DOS could be "almost" half-metallic: (a) the Fermi energy could fall at an energy for which the DOS for one of the spin channels is very small, i.e., a pseudogap, (b) the DOS could have a gap in one of the spin channels with the Fermi energy lying slightly outside the gap. If we choose criterion (a), we must choose a threshold for "very small," and if we choose (b), we must choose a threshold for "slightly." Criterion (a) is difficult to implement because there are an infinite number of ways the DOS can vary near a pseudogap, e.g., a pseudogap may be wide and only fairly small or it might be narrow but extremely small or some combination. Another issue with criterion (a) is that a low density of states is often associated with a highly dispersive band that leads to a large conductivity. For these reasons, we choose criterion (b), i.e., we require that there be an actual gap and that the Fermi energy fall "near" this gap. The (arbitrary) criterion we choose for "near" is 0.2 electron states per formula unit. Other criteria could be used, e.g., the distance from the Fermi energy to the nearest band edge. Our choice of using the number of states has the advantage of being easily related to the magnetic moment. It has the disadvantage that occasionally a system with a highly dispersive band above or below the Fermi energy with a very small density of states can lead to a situation in which we label a system near-half-metallic when the Fermi energy is as much as 0.3 eV into the band.

For each of the systems, we note the gaps in the DOS, typically after 9, 12, or 14 states per formula unit per spin channel and whether the Fermi energy falls in one or more of the gaps or close enough to the gap (0.2 states) to be designated a near-half-metal. Simple state and electron counting dictates that to make a half-metal by taking advantage of a gap after *Ns* states when there are *N* electrons per formula unit, the magnetic moment per formula unit must be $N - 2N_s$.

13-electron systems. Our data set contained 3 systems with 13 valence electrons, $Sc₂Ti(AI, Ga,In)$. All three were magnetic metals with small gaps after 9 states per spin channel in both channels (except for $Sc₂$ TiIn minority). However, the moments were not large enough (a total moment of $5\mu_B$ per formula unit would have been needed) to move the majority Fermi energy into the gap.

14-electron systems. Our data set contained 6 systems with 14 valence electrons, $Sc_2Ti(Si,Ge,Sn)$ and $Sc_2V(Al,Ga,In)$. These systems require a moment of $4\mu_B$ to place the Fermi energy in a majority gap after 9 electrons per spin channel.

 $Sc₂Ti(Si,Ge,Sn)$: $Sc₂TiSi$ and $Sc₂TiGe$ are predicted to be nonmagnetic metals and to have small gaps after 9 states in both spin channels. $Sc₂TiSn$ is predicted to be magnetic and to have a majority gap after 9 states, but its moment closer to $3\mu_B$ than $4\mu_B$.

 $Sc₂V(AI,Ga,In)$: These all have relatively large gaps after 9 states in the majority channel. The position of the Fermi energy rises relative to the gap as the atomic size of the *Z* atom increases the lattice constant, facilitating the formation of large moments on V and even small moments on Sc. For $Sc₂VAL$ and $Sc₂VGa$, the moment is not quite large enough to place the Fermi energy in the gap making them near-half-metals (Table [III\)](#page-13-0). However, the lattice constant of $Sc₂VIn$ *is* large enough to support a moment large enough to move the Fermi energy into the gap so it is a half-metal (Table [II\)](#page-12-0).

15-electron systems. Our data set contained 9 systems with 15 valence electrons, $Sc₂Ti(P,As, Sb)$, $Sc₂V(Si, Ge, Sn)$, and Sc₂Cr(Al,Ga,In). A moment of $3\mu_B$ is needed to place the Fermi energy in a majority gap after 9 states.

 $Sc₂Ti(P, As, Sb)$: These systems do not have gaps after 9 states per atom in either channel because of a reordering of the states at Γ . A singly degenerate state, usually band 10 at Γ , interchanges with a triply degenerate T_2 state so that bands 8, 9, and 10 are associated with the triply degenerate T_2 state, precluding a gap.

 $Sc_2V(Si,Ge,Sn)$: These systems have majority gaps after 9 states. The Fermi energy is in the gap and they are classified as half-metals as described in Table [II.](#page-12-0)

 $Sc_2Cr(AI,Ga,In)$: These systems all have large gaps after 9 states in majority and small gaps or pseudogaps near 9 states in minority. The Fermi energy falls in the majority gap for all three so they are classified as half-metals and described in Table [II.](#page-12-0)

16-electron systems. Our data set contained 12 systems with 16 valence electrons per formula unit, $Sc_2V(P,As,Sb)$, $Sc_2Cr(Si,Ge,Sn)$, $Sc_2Mn(Al,Ga,In)$, and $Ti_2V(Al,Ga,In)$. For 16 valence electrons per formula unit and a gap after 9 states in the majority-spin channel, a moment of $2\mu_B$ is needed to place the Fermi energy in the gap to make a half-metal.

 $Sc_2V(P,As,Sb)$: Sc_2VP and Sc_2VAs have a region with an extremely low but nonzero DOS at the Fermi energy. These pseudogaps can be be traced to a band reordering similar to that mentioned previously in which a singly degenerate state

at Γ drops below a triply degenerate state so that the triply degenerate state connects to bands 8, 9, and 10, thus precluding a gap. In the case of $Sc₂VSb$, the singly degenerate state is a few hundredths of an eV above the triply degenerate state so it is a near-half-metal with a tiny gap.

 $Sc_2Cr(Si,Ge,Sn)$: These systems have large majority gaps after 9 states. They generated half-metals as described in Table [II.](#page-12-0)

 $Sc₂Mn(AI, Ga,In)$: The three systems with $X = Sc$ and $Y =$ Mn have large majority and tiny minority gaps after 9 states. Their moments are very slightly less than $2\mu_B$ so they generated near-half-metals as described in Table [III](#page-13-0) by placing the Fermi energy at the valence edge of the majority gap.

 $Ti₂V(AI,Ga,In)$: These systems have tiny majority and minority gaps after 9 states per spin channel. All take advantage of the majority gap to generate near-half-metals as shown in Table [III.](#page-13-0) Ti₂VIn is not included in the table because it has a positive formation energy.

17-electron systems. Our data set contained 15 systems that had 17 electrons per formula unit $Sc_2Cr(P,As,Sb)$, $Sc₂Mn(Si,Ge,Sn)$, $Sc₂Fe(Al,Ga,In)$, $Ti₂V(Si,Ge,Sn)$, and $Ti₂Cr(AI, Ga,In)$. Systems with 17 electrons per formula unit can generate half-metals by taking advantage of a gap after 9 states in majority if their magnetic moment per formula unit is $1\mu_B$.

 $Sc_2Cr(P, As, Sb)$: Sc_2CrP and Sc_2CrAs both have majority pseudogaps at the Fermi energy. The gap after 9 electrons per formula unit is converted into a pseudogap by a band reordering at Γ similar to that occurring for $Sc_2V(P,As)$ and $Sc₂Ti(P, As, Sb)$. Sc₂CrSb has a "normal" band ordering at Γ and is a near-half-metal similar to $Sc₂VSb$, but with a much larger gap.

 $Sc₂Mn(Si,Ge,Sn)$: These systems have majority gaps after 9 states and are all half-metals as described in Table [II.](#page-12-0) In addition to the majority gap, $Sc₂MnSi$ has a gap after 9 states in minority. A pseudogap near 9 states in minority for $Sc₂MnSi$ can also be identified.

 $Sc₂Fe(Al,Ga,In)$: These systems have gaps after 9 states in both majority and minority channels. All three take advantage of the majority gap to generate near-half-metals as described in Table [III.](#page-13-0) Sc₂FeAl also has a minority pseudogap near 12 states and a majority pseudogap near 14 states.

 $Ti₂V(Si,Ge,Sn)$: These all have gaps after 9 states in both majority and minority with the majority gap being larger. All three compounds utilize the majority gap to form half-metals as described in Table [II.](#page-12-0)

 $Ti₂Cr(AI, Ga,In)$: These have gaps after 9 states in both spin channels. They are all near-half-metals (Table [III\)](#page-13-0), having moments just large enough to satisfy our criterion. The large moment on the Cr atom opens a large gap in the majority channel (hybridization between Cr and Ti-*d* and with the Cr-*d* onsite energy well below that of the Ti atoms). The gap in minority is smaller and is dependent on the difference between the two Ti potentials and avoided band crossings.

18-electron systems. Our data set contained 18 systems with 18 valence electrons. These were described in Sec. [IV A 1.](#page-9-0)

19-electron systems. Our data set contained 21 systems with 19 valence electrons per formula unit $Sc_2Fe(P,As,Sb)$, $Sc_2Co(Si,Ge,Sn),$ $Sc_2Ni(Al,Ga,In),$ $Ti_2Cr(P,As,Sb),$ $Ti₂Mn(Si,Ge,Sn)$, $Ti₂Fe(Al,Ga,In)$, and $V₂Cr(Al,Ga,In)$.

Systems with 19 valence electrons need a moment of $1\mu_B$ to place the Fermi energy in a gap after 9 states in the minority channel.

 $Sc_2Fe(P,As, Sb)$: Sc_2FeP is nonmagnetic with gaps after 9 states in both spin channels. $Sc₂FeAs$ is also nonmagnetic, but the gaps after 9 states are converted to pseudogaps by the inversion of singly and triply degenerate states at Γ similar to Sc_2CrP and Sc_2CrAs . The larger lattice constant of Sc_2FeSb allows it to develop a moment. There are sizable gaps after 9 states in both the minority and majority channels, but the moment is not quite large enough $(0.79\mu_B)$ to meet our criterion for calling it a near-half-metal.

 $Sc_2Co(Si,Ge,Sn)$: All three of these are half-metals with sizable gaps after 9 states in both spin channels (Table [III\)](#page-13-0). The Fermi energy falls near the top of the gap in all three cases, but especially for Ge.

 $Sc₂Ni(Al,Ga,In)$: All three have sizable gaps after 9 states in both spin channels and moments as given in Table [III](#page-13-0) that qualify them as near-half-metals.

 $Ti₂Cr(P, As, Sb)$: All three compounds have gaps in the minority channel after 9 states, but only $Ti₂CrAs$ and $Ti₂CrSb$ have sufficiently large moments to be classified as near-halfmetals. (Ti₂CrAs only barely made the cutoff.) Ti₂CrSb also has a tiny gap after 9 states in majority.

 $Ti₂Mn(Si,Ge,Sn)$: All three of these compounds have sizable gaps after 9 states in minority and much smaller gaps in majority. They also have pseudogaps near 12 states in minority. For all three, the Fermi energy falls in the gap so they are are included in Table [III.](#page-13-0)

 $Ti₂Fe(AI, Ga,In)$: All three of these compounds have gaps after 9 states in both channels and pseudogaps in both channels after 12 states. The gaps after 9 states are much smaller in majority than in minority. The Fermi energy for each of the three falls in the minority gap creating three half-metals as described in Table [II.](#page-12-0)

 $V_2Cr(AI,Ga,In)$: All three of these systems have gaps after 9 states in both spin channels. In all cases, however, the magnetic moment is not large enough to meet our criterion for a nearhalf-metal. The formation energy is small and negative for V_2 CrAl and V_2 CrGa. It is positive for V_2 CrIn.

20-electron systems. Our data set contained 24 compounds with 20 electrons per formula unit $Sc_2Co(P, As, Sb)$, $Sc_2Ni(Si,$ Ge, Sn), $Sc_2Cu(Al, Ga, In)$, $Ti_2Mn(P, As, Sb)$, $Ti_2Fe(Si, Ge,$ Sn), $Ti_2Co(Al,Ga,As)$, $V_2Cr(Si, Ge, Sn)$, and $V_2Mn(Al, Ga,$ In).

 $Sc_2Co(P,As,Sb)$: Sc_2CoP and Sc_2CoAs both have gaps after 9 states per formula unit in both spin channels, but in both cases the net magnetic moment is much too small to pull the Fermi energy into one of the gaps. $Sc₂CoSb$, on the other hand, due to its larger lattice constant, supports a moment sufficiently large (1.999 μ_B) that the Fermi energy falls at the top edge of the minority gap. We classify it as a near-half-metal (Table [III\)](#page-13-0).

 $Sc₂Ni(Si,Ge,Sn)$: The compounds $Sc₂Ni(Si,Ge,Sn)$ all have gaps after 9 electrons in both spin channels. However, the total moment $(1.80\mu_B, 1.81\mu_B, \text{ and } 1.84\mu_B, \text{ respectively})$ does not reach the value $(2.00\mu_B)$ needed to place the Fermi energy in the gap. Even when the lattice is expanded, the magnetic moment of these $Sc_2Ni(Si,Ge,Sn)$ compounds hardly increases because the *d* bands of Ni are filled so that Ni only supports a very small moment in these materials. Nevertheless, these compounds meet (barely) our threshold to be called near-half-metals (Table [III\)](#page-13-0).

 $Sc_2Cu(Al,Ga,In)$: The systems $Sc_2Cu(Al,Ga,In)$ all have pseudogaps after 9 states in both spin channels.

 $Ti₂Mn(P, As, Sb)$: All three of these compounds have gaps after 9 electrons in both spin channels. However, only $Ti₂MnSb$ (due to its larger lattice constant) has a moment $(1.989\mu_B)$ close enough to $2\mu_B$ to be classified a "near-half-metal." Ti₂MnAs (1.7967 μ_B) barely misses the cutoff to be included in Table [III.](#page-13-0)

 $Ti₂Fe(Si, Ge, Sn)$: The compounds $Ti₂Fe(Si, Ge, Sn)$ all have gaps in both channels after 9 states and pseudogaps after 12 states. The Fermi energy falls within the gap in all three cases so they are predicted to be half-metals as described in Table [II.](#page-12-0)

 $Ti₂Co(AI, Ga, As)$: The compounds $Ti₂Co(AI, Ga, In)$ have large minority gaps after 9 states and somewhat smaller gaps after 9 states in majority. The Fermi energy falls within the minority gap in all three cases so they are are predicted to be half-metals as described in Table [II.](#page-12-0)

 $V_2Cr(Si,Ge,Sn): V_2CrSi$ has small gaps after 9 states in both spin channels, but is not magnetic. V_2 CrGe is magnetic with a moment of $2\mu_B$. The Fermi energy falls very close to the top of the gap, but V_2 CrGe is predicted to be a half-metal as described in Table [II.](#page-12-0) V_2CrSn has a gap after 9 states in the majority channel which is not useful for generating a half-metal. There is no gap in minority.

 $V_2Mn(Al, Ga, In): V_2MnAl$ has a tiny majority gap and a sizable minority gap after 9 states. It also has pseudogaps after 12 states in majority and 14 states in minority. It takes advantage of the minority gap after 9 states to become a halfmetal. The DOS for V_2MnGa is similar. It is also a half-metal. Both compounds are included in Table [II.](#page-12-0) V_2 MnIn has a gap after 9 states in the majority, but only a pseudogap after 9 states in minority. Its large moment, in excess of $3\mu_B$, places the Fermi energy well below this pseudogap.

21-electron systems. Our database contains 27 systems with 21 valence electrons $Sc₂Ni(P,As,Sb)$, $Sc₂Cu(Si,Ge,Sn)$, $Sc_2Zn(Al,Ga,In),$ $Ti_2Fe(P,As,Sb),$ $Ti_2Co(Si,Ge,Sn),$ $Ti₂Ni(Al,Ga,In),$ $V₂Cr(P,As,Sb),$ $V₂Mn(Si,Ge,Sn),$ V2Fe(Al,Ga,In). Systems with 21 valence electrons are interesting because they can, in principle, place the Fermi energy in a gap after 9 electrons or a gap after 12 electrons or both. If a majority gap present after 12 states is present as well as a minority gap after 9 states, a moment of $3\mu_B$ would yield a magnetic semiconductor.

 $Sc₂Ni(P,As, Sb)$: $Sc₂NiP$ has tiny gaps in both spin channels after 9 states while $Sc₂NiAs$ has pseudogaps after 9 states in both spin channels. Both have very small moments. $Sc₂NiSb$ has gaps after 9 states in both spin channels, but its moment $(2.15\mu_B)$ is too small to place the Fermi energy in or near the gap.

Sc2Cu(Si,Ge,Sn): All three of these compounds have gaps after 9 states in minority and much smaller gaps in majority. However, their moments are too small to place the Fermi energy in the minority gap.

Sc2Zn(Al,Ga,In): These three nonmagnetic compounds only have pseudogaps after 9 states in both spin channels because of a highly dispersive band 10 which drops below band 9 along Γ to *X*.

None of the 21-electron systems with $X = Sc$ generate halfmetals or near-half-metals because a moment of $3\mu_B$ would be necessary to take advantage of a gap after 9 states per spin per formula unit or to take advantage of a gap after 12 states per spin per formula unit. The compounds are not able to do this because Sc is difficult to polarize and Ni, Cu, and Zn cannot support a large moment because their *d* bands are filled or (in the case of Ni) nearly filled.

 $Ti₂Fe(P, As, Sb)$: These three compounds have large gaps after 9 states in the minority channel and smaller gaps in the majority channel after 9 states. T₁₂FeP and T₁₂FeAs have pseudogaps after 12 states in the majority which becomes a very small real gap for $Ti₂FeSb$. The moments for the three are $2.72\mu_B$, $2.94\mu_B$, and $2.99\mu_B$, respectively. Ti₂FeAs and Ti2FeSb meet our criterion for near-half-metallicity. If the moment for Ti₂FeSb were very slightly larger, we would predict it to be a magnetic semiconductor with a large minority gap and a tiny majority gap, i.e., a 9*/*12 magnetic semiconductor. The present prediction is for a magnetic semimetal that has a pocket of down-spin electrons along Γ to K and a Γ -centered pocket of up-spin holes.

 $Ti₂Co(Si,Ge,Sn)$: These three compounds all have gaps in both spin channels after 9 states, with the minority gap being larger. Ti₂CoSi also has a majority pseudogap after 12 states. For all three, the Fermi energy falls within the minority gap giving each a moment of $3\mu_B$ and making them half-metals (Table II). For Ti₂CoSi the current at zero K would be carried by up-spin holes at Γ and up-spin electrons at *X*.

 $Ti₂Ni(Al, Ga, In)$: These three compounds all have gaps after 9 states in both spin channels. The Fermi energy falls well inside the minority gaps yielding half-metals with moments of $3\mu_B$ in each case (Table [II\)](#page-12-0).

 $V_2Cr(P, As, Sb): V_2CrP$ has gaps after 9 states in both spin channels, but is nonmagnetic. V_2 CrP has a gap in the minority channel after 9 states and a pseudogap after 9 states in the majority channel. Its moment is too small $(1.85\mu_B)$ for it to be classified as a near-half-metal. V_2CrSb has a pseudogap after 9 states in the majority channel and a sizable gap after 12 states also in majority channel. Its moment is large enough $(2.89 \mu_B)$ that we would classify it as a near-half-metal. However, its formation energy is positive so it is not included in Table [III.](#page-13-0)

 $V_2Mn(Si,Ge,Sn): V_2MnSi$ has pseudogaps after 9 states in both spin channels and is nonmagnetic. V₂MnGe has gaps after 9 states in both spin channels and a pseudogap after 12 states in majority. Its moment of $2.81\mu_B$ qualifies it (barely) for inclusion with the near-half-metals in Table [III.](#page-13-0) V_2MnSn has majority gaps after 9 and 12 states and a pseudogap after 9 states in minority. Its moment of $2.96\mu_B$ qualifies it for inclusion in Table [III](#page-13-0) as a Slater-Pauling near-half-metal.

 $V_2Fe(Al,Ga,In)$: V_2FeAl has gaps after 9 and 14 states in minority and after 12 states in majority. Its moment of $2.91\mu_B$ qualifies it as a "double near-half-metal" since its Fermi energy is near gaps after 9 states in minority and 12 states in majority. V_2 FeGa has a gap after 9 states in minority and a pseudogap after 12 states in majority. Its moment of $2.85\mu_B$ meets the threshold for inclusion in Table [III](#page-13-0) as a near-half-metal. V_2Fefn has gaps in both spin channels after 9 states and a pseudogap after 12 states in majority. Its moment is large enough for inclusion for inclusion in Table [III,](#page-13-0) but its formation energy is positive.

22-electron systems. Our database contains 27 systems with 22 electrons $Sc_2Cu(P, As, Sb)$, $Sc_2Zn(Si, Ge, Sn)$, $Ti₂Co(P,As,Sb)$, $Ti₂Ni(Si,Ge,Sn)$, $Ti₂Cu(Al,Ga,In)$, $V_2Mn(P,As,Sb)$, $V_2Fe(Si,Ge,Sn)$, $V_2Co(Al,Ga,As)$, and $Cr₂Mn(AI,Ga,In)$. These systems would need a moment of 4μ _B to place the Fermi energy into a gap after 9 states in the minority channel or a moment of $2\mu_B$ to place the Fermi energy into a gap after 12 states in the majority channel.

 $Sc_2Cu(P, As, Sb)$: The systems with $X = Sc$ and $Y = Cu$ generate only small moments (0.40, 0.62, and $(0.79)\mu_B$ for $Z =$ P, As, and Sb, respectively. They have gaps in both channels after 9 states and no gaps after 12 states.

 $Sc_2Zn(Si,Ge,Sn)$: The systems with $X = Sc$ and $Y = Zn$ also generate moments that are too small $(0.55\mu_B, 0.92\mu_B,$ and 0.93μ _B for Si, Ge, and Sn respectively). These systems have pseudogaps after 9 states in both spin channels, but no gaps after 12 states.

 $Ti₂Co(P, As, Sb)$: These three systems have gaps after 9 states in both spin channels, but their moments (2.20, 2.38, and $(2.73)\mu_B$ are much smaller than the $4\mu_B$ needed to place the Fermi energy in the minority gap.

 $Ti₂Ni(Si,Ge,Sn)$: These three systems have gaps after 9 states in both spin channels. Their moments are larger than those of the Ti₂Co(P,As,Sb) systems $(3.47\mu_B, 3.69\mu_B,$ and $3.86\mu_B$) for Si, Ge, and Sn, respectively. Thus, Ti₂NiSn meets our threshold for inclusion in Table [III](#page-13-0) as a near-half-metal.

 $Ti₂Cu(Al,Ga,In)$: Ti₂CuAl has gaps after 9 states in both spin channels, but its moment is too low $(3.72\mu_B)$ to meet our criterion for near-half-metal status. Ti₂CuGa and Ti₂CuIn have gaps after 9 states in majority, but only pseduogaps after 9 states in minority.

The 22-electron systems with $X = Ti$ and $Y = Co$, Ni, or Cu tend to have gaps after 9 states per spin channel per formula unit in both spin channels. They do not have gaps after 12 states per spin channel, presumably because insufficient contrast can be generated between the *d*-onsite energies of the two Ti atoms. In most cases, the moments are too small to place the Fermi energy in the minority gap after 9 states. The moments are largely on the Ti atoms and the moments tend to increase with the atomic number of the *Y* atom, increasing from Co to Ni to Cu, and with the size of the *Z* atom.

 $V_2Mn(P,As,Sb)$: V_2MnP has gaps after 9 states in both spin channels but a very small moment $(0.59\mu_B)$ so it cannot take advantage of the minority gap to make a half-metal. V2MnAs also has gaps after 9 states in both channels and a pseudogap after 12 states in the majority. Its moment of $1.97\mu_B$ places the Fermi energy close to the majority pseudogap after 12 states, but the absence of a real gap precludes its designation as a near-half-metal. V_2MnSb has a majority gap after 9 states, a minority pseudogap after 9 states and a majority gap after 12 states. Its moment of $1.9987\mu_B$ places the Fermi energy just below the majority gap after 12 states. Only its positive formation energy precludes its inclusion in Table [III.](#page-13-0)

 $V_2Fe(Si,Ge,Sn)$: All three compounds have gaps after 9 states in both spin channels. V₂FeGe also has a majority gap after 12 states and V_2 FeSn has a pseudogap after 12 states in the majority channel. The moment of V_2 FeGe (1.98 μ_B) places its Fermi energy quite close to the majority gap so it is a near-half-metal as described in Table [III.](#page-13-0)

 $V_2Co(AI, Ga, As)$: All three of these compounds have gaps in both channels after 9 states with the minority gap being much larger than the small majority gap. All three also have pseudogaps after 12 states in the majority. The pseudogap for V_2 CoAl is particularly pronounced. The moment is slightly less than $2\mu_B$ for all three compounds, but according to our definition, they are not near-half-metals.

 $Cr₂Mn(AI, Ga,In): Cr₂MnA1$ and $Cr₂MnGa$ both have gaps after 12 states in the majority channel. $Cr₂MnIn$ only has a pseudogap there. $Cr₂MnAl$ is a half-metal as described in Table [II](#page-12-0) and Cr_2MnGa is a near-half-metal as described in Table [III.](#page-13-0) All three of these compounds are predicted to have ferrimagnetic states in which the moments of $Cr₁$ and Mn are aligned, and the moment of $Cr₂$ is opposite.

23-electron systems. Our database contains 27 systems with 23 electrons $Sc_2Zn(P,As,Sb)$, $Ti_2Ni(P,As,Sb)$, $Ti₂Cu(Si,Ge,Sn)$, $Ti₂Zn(Al,Ga,In)$, $V₂Fe(P,As,Sb)$, $V_2Co(Si,Ge,Sn)$, $V_2Ni(Al,Ga,In)$, $Cr_2Mn(Si,Ge,Sn)$, and $Cr₂Fe(Al,Ga,In)$. These compounds can take advantage of a gap after 9 states in minority to make a half-metal, but a moment of $5\mu_B$ would be needed. Alternatively, they could take advantage of a gap after 12 states in majority which would only require a moment of $1\mu_B$. A few of these systems have pseudogaps after 14 states. In principle, one could have a 23-electron inverse Heusler magnetic semiconductor with a moment of $5\mu_B$ that uses gaps after 9 states in minority and 14 states in majority; however, such solutions did not appear in our database.

 $Sc₂Zn(P,As,Sb)$: The three systems $Sc₂Zn(P,As,Sb)$ have pseudogaps after 9 states in both channels. They are all nonmagnetic and have no gaps after 12 sates.

 $Ti₂Ni(P,As,Sb)$: These compounds all have gaps after 9 states in both spin channels, but their moments are much too small to make half-metals or near-half-metals.

 $Ti₂Cu(Si,Ge,Sn)$: The three systems $Ti₂Cu(Si,Ge,Sn)$ have gaps in both channels after 9 and pseudogaps in majority after 14 states. Their moments $(3.00, 3.16, \text{ and } 3.33)\mu_B$, however, are much less than the $5\mu_B$ that would be needed to place the Fermi energy in the minority gap.

 $Ti₂Zn(AI, Ga,In)$: These compounds all have pseudogaps near 9 states per formula unit in the minority channel but no gaps at 9 or 12 states nor is the Fermi energy near the pseudogap. $Ti₂ZnAl$ and $Ti₂ZnGa$ also have majority gaps after 9 states and $Ti₂ZnAl$ also has a pseudogap after 14 states in majority.

 $V_2Fe(P, As, Sb)$: These compounds have gaps in both spin channels after 9 states. V_2 FeP and V_2 FeAs have pseudogaps in the majority channel after 12 states. V_2 FeSb, however, does have a majority gap after 12 states and the magnetic moment $(0.9996\mu_B)$ places the Fermi energy at the lower edge of this gap. V_2 FeSb is not included in Table [III](#page-13-0) because it has a positive formation energy.

 $V_2Co(Si,Ge,Sn)$: These compounds have gaps after 9 states in both channels, but only pseudogaps after 12 states except for $V₂CoSi$ which has a tiny gap after 12 states. The Fermi energy falls just below this gap (moment $= 0.997 \mu_B$). (See Table [III.](#page-13-0))

 $V_2Ni(Al,Ga,In)$: These compounds have sizable gaps after 9 states in both spin channels, but the moments are small ($\approx 1 \mu_B$) and there are no gaps after 12 states in the majority channel, hence, no half-metals or near-half-metals.

 $Cr₂Mn(Si,Ge,Sn): Cr₂MnSi$ has a gap after 9 states in the majority channel and a pseudogap after 9 states in minority. Cr2MnGe has a pseudogap after 9 states in majority and a sizable gap after 12 states in majority. Its moment of $0.99 \mu_B$ allows it to use this gap to become a near-half-metal (Table [III\)](#page-13-0). $Cr₂MnSn$ also has a majority gap after 12 states and a moment of approximately 1.00μ _B so it is a near-half-metal, but it is not included in Table [III](#page-13-0) because its formation energy is positive.

 $Cr_2Fe(AI,Ga,In)$: Cr_2FeAI has minority gaps after 9 states and 14 states, and a majority pseudogap near 12 states. Its moment of approximately 1.00μ _B places the Fermi energy into this pseudogap. $Cr₂FeGa$ has a minority pseudogap after 9 states and a majority gap after 12 states. Its moment of $0.98\mu_B$ makes it a near-half-metal, but it is not included in Table [III](#page-13-0) because its formation energy is positive. Cr_2FeGa has a majority pseudogap near 12 states.

24-electron systems. Our database contained 24 systems with 24 valence electrons $Ti₂Cu(P,As,Sb)$, $Ti₂Zn(Si,Ge,Sn)$, $V_2Co(P,As,Sb)$, $V_2Ni(Si,Ge,Sn)$, $V_2Cu(Al,Ga,In)$, $Cr_2(P,As,Sb)$, $Cr_2Fe(Si,Ge,Sn)$, $Cr_2Co(Al,Ga,In)$. Based on electron count alone, these systems would offer the opportunity for nonmagnetic semiconductors or zero net moment half-metals arising from Slater-Pauling gaps after 12 states per formula unit in one or both spin channels. In practice, we find that only 4 of the 24 systems display Slater-Pauling gaps (and only in one spin channel). In all four of these systems the Fermi energy falls at the edge of the gap but just outside so that they would be classified as near-half-metals. Three of these four near-half-metals have positive formation energy.

The gaps after 9 states, on the other hand, seem to be more robust. Only $Cr₂MnSb$ failed to show a gap or a pseudogap after 9 states in at least one of the spin channels. Most systems showed gaps after 9 states in both spin channels. However, none of the systems were able to generate a moment of $6\mu_B$ that would have been necessary to use one of the gaps after 9 states to generate a half-metal.

 $Ti₂Cu(P, As, Sb)$: These systems showed gaps in both channels after 9 states, but their moments were much too small to place the Fermi energy near the minority gap.

 $Ti₂Zn(Si,Ge,Sn)$: These systems showed pseudogaps in both channels near 9 states per formula unit. The gaps were converted to pseudogaps by a highly dispersive band 10 along Γ to *X* dropping down below the maximum of bands 7–9 at Γ .

The 9 systems in our database with $X = V$ all showed gaps after 9 states in both spin channels. In all cases, the moments were quite small so that the Fermi energy could not fall into these gaps. Six of the 9 systems $V_2Co(P,As,Sb)$ and V2Ni(Si,Ge,Sn) have pseudogaps around 12 states per atom in at least one spin channel.

 $V_2Co(P, As, Sb)$: These systems show gaps after 9 states and pseudogaps after 12 states in both spin channels. The moments are too small (V_2CoSb) or zero precluding half-metals.

 $V_2Ni(Si,Ge,Sn): V_2NiSi$ has gaps in both spin channels after 9 states and pseudogaps after both spin channels after 12 states. V_2 NiGe has gaps in both spin channels after 9 states and a marginal pseudogap after 12 states in majority. V_2NiSn has gaps after 9 states in both spin channels and pseudogaps after 12 states in minority and after 14 states in majority. The lack of real gaps after 12 states and their very small moments preclude half-metals.

 $V_2Cu(Al,Ga,In)$: These compounds have gaps after 9 states in both spin channels and minority gaps after 14 states. V_2CuIn also has a majority gap after 14 states while V_2 CuAl and $V₂CuGa$ have pseudogaps in majority near 14 states. Their small moments and the absence of gaps after 12 states preclude half-metals.

 $Cr₂Mn(P,As,Sb)$: In the minority channel, $Cr₂MnP$ has a gap after 9 states and a pseudogap near 12 states. It also has a pseudogap near 9 states in majority. It has a small moment placing the Fermi energy in the minority pseudogap near 12 states. $Cr₂MnAs$ has minority gaps after 9 and 12 states. The very small moment $(0.005\mu_B)$ places the Fermi energy just below the gap at 12 states. Thus, $Cr₂MnAs$ is a near-half-metal, but it is not included in Table [III](#page-13-0) because its calculated formation energy is positive. $Cr₂MnSb$ has a minority gap after 12 states. Its tiny moment $(0.0002\mu_B)$ makes it a near-half-metal, but it is not included in Table [III](#page-13-0) because its formation energy is positive.

 $Cr_2Fe(Si,Ge,Sn)$: Cr_2FeSi is nonmagnetic and has gaps after 9 states in both channels. $Cr₂FeGe$ has gaps in both spin channels after 9 states and a minority gap after 12 states. Its small moment makes it a near-half-metal, but we do not list it in Table [III](#page-13-0) because of its positive formation energy. $Cr₂FeSn$ has a majority gap after 9 states and a minority gap after 12 states. Its small moment places the Fermi energy somewhat below this gap yielding a near-half-metal. It is not listed in Table [III](#page-13-0) because of its positive formation energy.

 $Cr_2Co(A1,Ga,In)$: These systems all have gaps after nine states in the majority channel and all have gaps (Al,Ga) or pseudogaps (In) in the minority channel after 12 states. All three have tiny moments. Cr_2CoAl and Cr_2CoGa are near-halfmetals, but only the former has a negative formation energy. Cr2CoAl is the only Slater-Pauling 24-electron near-half-metal with negative formation energy in Table [III.](#page-13-0)

25-electron systems. Ti₂Zn(P,As,Sb), V₂Ni(P,As,Sb), Cu(Si,Ge,Sn) V₂Zn(Al,Ga,In), Cr₂Fe(P,As,Sb), $V_2Cu(Si,Ge,Sn)$ $V_2Zn(Al,Ga,In),$ $Cr_2Fe(P,As,Sb),$ $Cr_2Co(Si,Ge,Sn)$, $Cr_2Ni(Al,Ga,As)$, $Mn_2Fe(Al,Ga,As)$. Many of the 25-electron inverse Heusler systems show gaps after 9 states per formula in one or both spin channels. None of these systems, however, are able to take advantage of these gaps to make a half-metal or near-half-metal by moving the Fermi energy to the gap because that would require a relatively large moment ($7\mu_B$ per formula unit). This large moment would be difficult to produce in systems whose transition-metal elements come from the early or late part of the transition-metal series, e.g., $X = Ti$ or V and $Y =$ Ni, Cu, or Zn. The systems that could produce such large moments, e.g., $X = Cr$ or Mn and $Y = Fe$ or Co seem to prefer smaller net moments closer to $1\mu_B$ which allows them to take advantage of gaps and pseudogaps after 12 states per formula unit. Several systems show gaps after 14 states, but none of these have the correct moment $(3\mu_B)$ to make a half-metal.

 $Ti₂Zn(P, As, Sb)$: All three compounds have gaps after 9 states in both spin channels, but their moments are too small to move the Fermi energy near these gaps.

 $V_2Ni(P,As,Sb)$: All three compounds have gaps after 9 states in both spin channels, but their moments are too small to move the Fermi energy near these gaps.

 $V_2Cu(Si,Ge,Sn)$: All three compounds have gaps after 9 states in both spin channels. V_2CuSn also has gaps after 14 electrons in both spin channels. In all three cases, the moments are too small to move the Fermi energy near the minority gap after 9 states or the majority gap after 14 states.

 $V_2Zn(Al,Ga,Sb)$: V_2ZnAl has a gap after 14 states in majority and a pseudogap after 14 states in minority. V_2ZnGa has a pseudogap after 14 states in majority and a gap after 14 states in minority. V_2ZnIn has a gap in both spin channels after 14 states. Their small moments preclude half-metals or near-half-metals.

 $Cr₂Fe(P, As, Sb): Cr₂FeP has gaps in both spin channels after$ 9 states and a pseudogap in minority after 12 states. $Cr₂FeAs$ has gaps in both spin channels after 9 states and a sizable gap in minority after 12 states. $Cr₂FeSb$ has a gap after 9 states in majority and a pseudogap after 9 states in minority. It also has a gap after 12 states in minority. All three have moments of approximately 1.01μ _B which places the Fermi energy near the gap or pseudogap at 12 states. Cr_2FeAs and Cr_2FeSb are near-half-metals, but are not included in Table [III](#page-13-0) because of their positive formation energies.

 $Cr_2Co(Si,Ge,Sn): Cr_2CoSi$ has gaps in both spin channels after 9 states and a minority gap after 12. $Cr₂CoGe$ has a majority gap after 9 states and a minority gap after 12 states. Both of these systems are predicted to be near-half-metals as described in Table [III.](#page-13-0) Cr_2CoSn has a majority gap after 9 states and a pseudogap in minority near 12 states.

 $Cr_2Ni(Al,Ga,In)$: Cr_2NiAl has gaps after 9 and 14 states in majority and after 12 states in minority. It uses the gap after 12 states in minority to generate a half-metal as described in Table [II.](#page-12-0) Cr_2NiGa has gaps after 9 states in majority and after 14 states in minority. It also has a pseudogap near 12 states in minority. $Cr₂NiIn$ has a gap after 9 states in majority and a pseudogap near 12 states in minority.

 $Mn2Fe(Al,Ga,In)$: $Mn₂FeA1$ and $Mn₂FeGa$ both have gaps after 9 states in majority and after 12 states in minority. Both place the Fermi energy slightly below the minority gap to gen-erate near-half-metals as described in Table [III.](#page-13-0) In $Mn₂FeIn$, the minority gap after 12 states becomes a pseudogap. It also has minority gaps after 9 states and after 14 states.

26-electron systems. V₂Cu(P,As,Sb), V₂Zn(Si,GeSn), $Cr_2Co(P, As, Sb)$, $Cr_2Ni(Si, Ge, Sn)$, $Cr_2Cu(Al, Ga, In)$, $Mn₂Fe(Si, GeSn), Mn₂Co(Al, Ga, As).$ Many of the 26-electron systems show gaps after 9 states, some show gaps after 12 states, and a few show gaps after 14 states. None of these systems can generate the $8\mu_B$ moment needed to generate a half-metal using the gap after 9 states. Several, however, can generate the moment of $2\mu_B$ needed to generate a half-metal or near-half-metal by using a minority gap after 12 states. A moment of $2\mu_B$ would also be consistent with a half-metal taking advantage of a majority gap after 14 states. In principle, a majority gap after 14 states and a minority gap after 12 combined with a moment of $2\mu_B$ could lead to a magnetic semiconductor. $Mn₂CoAl comes close to this situation.$

 $V_2Cu(P, As, Sb)$ have gaps in both spin channels after 9 states and are nonmagnetic. V_2CuSb also has pseudogaps in both spin channels after 14 states.

 $V_2Zn(Si,Ge,Sn)$ have only pseudogaps near 9 states and are also nonmagnetic. None of the 26-electron systems with $X =$ V are predicted to be half-metals or near-half-metals.

 $Cr_2Co(P, As, Sb)$ have majority gaps after 9 states and minority gaps after 12 states. Cr_2CoP also has a small minority gap after 9 states that becomes a pseudogap for Cr_2CoAs and Cr₂CoSb. Cr₂CoP is a half-metal with a moment of $2\mu_B$. Cr_2CoAs and Cr_2CoSb are near-half-metals with moments of $2.0047\mu_B$ and $2.0016\mu_B$, respectively, however they are omitted from Table [III](#page-13-0) because of their positive formation energy.

 $Cr₂Ni(Si,Ge,Sn)$ are remarkable for the large gaps after 9 states in the majority channel. Cr_2NiSi and Cr_2NiGe also have smaller gaps after 9 states in the minority channel. $Cr₂NiSi$ and $Cr₂NiGe$ also have gaps after 12 states in minority. The Fermi energy falls just above these gaps after 12 states giving moments of $1.89\mu_B$ and $1.94\mu_B$, respectively. Cr₂NiGe also has a minority gap after 14 states. $Cr₂NiSn$ has a single gap after 9 states in the majority channel and a relatively small moment ($0.83\mu_B$). Cr₂NiSi and Cr₂NiGe satisfy our criteria for near-half-metals and are listed in Table [III.](#page-13-0)

 $Cr_2Cu(A1,Ga,In)$ all show gaps after 9 states in both spin channels. In addition, Cr_2CuAl has gaps after 14 states in both channels while Cr_2CuGa has gaps after 14 states in the minority channel. For all cases, however, the net moments are too small to place the Fermi energy near any of the gaps. These three systems are ferrimagnets with relatively large antialigned moments on the two Cr atoms.

 $Mn₂Fe(Si,Ge,Sn)$ all have minority gaps after 12 states. Mn2FeSi and Mn2FeSn also have pseudogaps after 9 states in majority while Mn₂FeGe has a gap after 9 states in minority. The moments $(2.008\mu_B, 2.016\mu_B, \text{ and } 1.999\mu_B)$ are the right size to make all three of these systems near-half-metals (Table [III\)](#page-13-0); however, the formation of $Mn₂FeSn$ is positive.

 $Mn₂Co(Al,Ga,In)$ tend to have minority gaps (Al,Ga) or pseudogaps (In) after 12 states and their net moments tend to be large enough to take advantage of these gaps to make near-half-metals. Thus, $Mn₂CoAl$ is a half-metal (Table [II\)](#page-12-0) and $Mn₂CoGa$ is a near-half-metal (Table [III\)](#page-13-0).

In addition to the gap in the minority channel after 12 states that $Mn₂CoAl$ utilizes to make itself a half-metal, it also has a slightly negative gap in the majority channel, i.e., a very slight overlap between bands 14 and 15. The maximum of band 14 at the Γ point is only slightly higher than the minimum of band 15 at the *X* point. This situation has led some to refer this system as a "spin-gapless semiconductor" [\[34,](#page-24-0)[86\]](#page-25-0). In our calculation and also that of Ouardi *et al.* [\[28\]](#page-24-0), it is a semimetal rather than a semiconductor.

27-electron systems. The 21 systems in our database comprise $V_2Zn(P,As,Sb)$, $Cr_2Ni(P,As,Sb)$, $Cr_2Cu(Si,Ge,Sn)$, $Cr_2Zn(Al,Ga,In)$, $Mn_2Fe(P,As,Sb)$, $Mn_2Co(Si,Ge,Sn)$, $Mn₂Ni(Al,Ga,As)$. A system with 27 electrons could take advantage of a minority gap after 12 states to make a half-metal with a moment of $3\mu_B$ or a majority gap after 14 states to to make a half-metal with a moment of $1\mu_B$.

 $V_2Zn(P,As,Sb)$: All three have gaps after 9 states in both spin channels. V_2ZnP is nonmagnetic while V_2ZnAs and $V₂ZnSb$ have relatively small moments on the V sites with ferrimagnetic alignment. Because their moments are small and there are no gaps after 12 or 14 states, these systems are very far from being half-metals.

 $Cr₂Ni(P,As, Sb)$: These three systems all generate large gaps after 9 states in the majority channel, and gaps after 12 states and after 14 states in the minority channel. The gaps after 14 states cannot be used to generate a half-metal because they are in the minority channel. The gaps after 12 states could be used to make half-metals or near-half-metals but the net moments are too small $(2.03\mu_B, 1.69\mu_B, \text{ and } 1.29\mu_B, \text{ respectively})$ for the Fermi energy to be near the gap.

 $Cr_2Cu(Si,Ge,Sn)$: These three systems generate gaps after 9 states in both channels and gaps (Si and Ge) or pseudogaps (Sn) after 14 states in the minority channel. There are no gaps after 12 states. The moments are too small to take advantage of the gaps after 9 states and the gaps after 14 states are in the wrong channel to be useful for making half-metals.

 $Cr₂Zn(AI,Ga,In)$: These systems only generate gaps after 14 states; Cr_2ZnAl in both channels, Cr_2ZnGa in the minority, and for $Cr₂ZnIn$ there are no gaps. The one opportunity for making a half-metal is the majority channel of $Cr₂ZnAl$, but the net moment is somewhat smaller $(0.706\mu_B)$ than the $1.0\mu_B$ needed to make a half-metal. Pseudogaps near 9 states can be identified in both majority and minority of all three.

Mn₂Fe(P,As,Sb): These systems all generate gaps after 12 states in the minority channel and the moments have the correct value $(3\mu_B)$ to generate half-metals (Table [II\)](#page-12-0). The predicted half-metal Mn_2FeSb is omitted from Table [II](#page-12-0) because it is predicted to have positive formation energy. These systems are also predicted to have pseudogaps around 9 states in both majority and minority. $Mn₂FeP$ has a small gap after 9 states in minority.

Mn2Co(Si,Ge,Sn): These systems all generate gaps after 12 states in the minority channel. For $Mn₂CoSi$ and $Mn₂CoGe$ the moment is large enough $(3\mu_B)$ to generate a half-metal (Table [II\)](#page-12-0). The moment of $Mn₂CoSn$ is too small to generate a gap. There are also gaps after 9 states in majority for $Mn₂CoSi$ and Mn_2CoGe . This gap becomes a pseudogap for Mn_2CoSn .

Mn2Ni(Al,Ga,As): No gaps are predicted for these three systems. $Mn₂NiAl$ and $Mn₂NiGa$ have pseudogaps around 14 states.

28-electron systems. The 18 systems with 28 valence electrons in our data set comprise $Cr_2Cu(P, As, Sb)$, $Cr_2Zn(Si,Ge,Sn)$, $Mn_2Co(P,As,Sb)$, $Mn_2Ni(Si,Ge,Sb)$, $Mn_2Cu(Al,Ga,In)$, and $Fe_2Co(Al,Ga,In)$. In principle, these systems could take advantage of gaps after 14 electrons to make zero moment half-metals or semiconductors, however, we found no examples of such an electronic structure in a system with negative formation energy. They can also take advantage of a minority gap after 12 states to make half-metals with moments of $4\mu_B$ per formula unit. We found three examples of this type of electronic structure.

Two systems $Cr_2Cu(P, As, Sb)$: These systems all have sizable gaps in majority after 9 states and smaller gaps in minority after 9 states. Cr_2CuP has a pseudogap after 14 states in minority and Cr_2CuSb has a minority pseudogap after 14 states. The moments are small.

 $Cr₂Zn(Si,Ge,Sn)$: These systems have pseudogaps near 9 states per formula unit in both spin channels. $Cr₂ZnSi$ and $Cr₂ZnGe$ have gaps after 14 electrons. $Cr₂ZnSi$ has gaps in both channels so that it is predicted to be a ferrimagnetic semiconductor with zero net moment and two different gap widths. The smaller gap is a direct gap at the *L* point and is on the order of 0.02 eV so that *XA* phase Cr_2ZnSi could be called a "spin gapless semiconductor." $Cr₂ZnGe$ only has a gap after 14 states in one spin channel. Its net moment is almost zero $(0.0004\mu_B)$ so that it is predicted to be a near-half-metal. Unfortunately, both of these interesting systems are predicted to have positive formation energy and hull distances in excess of 0.3 eV*/*atom.

 $Mn_2Co(P, As, Sb)$: Mn_2CoP and Mn_2CoAs have gaps in both channels after 9 states. All three systems have gaps after 12 states in minority. All three have the requisite moment $(4\mu_B)$ to generate half-metals and are listed in Table [II.](#page-12-0) We include details about the calculated $Mn₂CoSb XA$ phase in Table [II](#page-12-0) even though its calculated formation energy is positive because of the controversy over the experimentally observed phase as discussed in Sec. [III A.](#page-5-0)

Mn2Ni(Si,Ge,Sb): These systems all have minority gaps after 12 states. Mn_2NiSi and Mn_2NiGe also have majority pseudogaps near 9 states. The moments on these systems are all too small to take advantage of the minority gap after 12 states.

 $Mn₂Cu(Al,Ga,In)$: One can identify pseudogaps near 9 states in majority in $Mn₂CuAl$ and $Mn₂CuGa$ along with a tiny gap after 9 states in minority in $Mn₂CuGa$. One can also identify pseudogaps near 14 states in both majority and minority in $Mn₂CuAl.$

 $Fe₂Co(Al, Ga, In)$: These systems have minority gaps after 9 states. Pseudogaps near 12 minority states can also be identified above the Fermi energy in all three systems. Even if there were a minority gap after 12 states, the moments in these systems would be too large to take advantage of it.

29-electron systems. Our database contained 18 systems with 29 valence electrons per formula unit: $Cr_2Zn(P,As,Sb)$, Mn2Ni(P,As,Sb), Mn2Cu(Si,Ge,Sn), Mn2Zn(Al,Ga,In), $Fe₂Co(Si,Ge,Sn)$, and $Fe₂Ni(Al,Ga,In)$. In principle, a system with 29 electrons per formula unit could utilize a minority gap after 9, 12, or 14 states together with moments of $9\mu_B$, $5\mu_B$, or $1\mu_B$, respectively, to make a half-metal. In practice, we only find one 29 electron half-metal, $Mn₂CuSi$, with a moment of $1\mu_B$.

 $Cr₂Zn(P, As, Sb)$: These systems all showed gaps after 9 states and 15 states in the minority channel. $Cr₂ZnP$ also has a gap after 9 states in the majority channel and a pseudogap after 14 states in minority. $Cr₂ZnAs$ also has a pseudogap near 9 states and a gap after 14 states in majority. In principle, the gap after 14 states in the majority could be utilized to make a half-metal, but the moment is too small $(0.47 \mu_B)$. Cr₂ZnSb has in addition to the minority gaps after 9 and 15 states, a pseudogap near 9 states in majority. The gap after 15 states observed in the minority channel of $Cr_2Zn(P,As,Sb)$ is unusual and can be traced to a singly degenerate state at Γ that is usually band 15 or band 17 dropping below a triply degenerate state.

 $Mn_2Ni(P,As,Sb)$: These three systems have gaps (Mn_2NiP) or pseudogaps $[Mn_2Ni(As,Sb)]$ near 12 states in the minority channel, but the moments are all much less than the $5\mu_B$ that would be necessary to make a half-metal. In addition, pseudogaps near 9 states in majority and 14 states in minority can be identified for $Mn₂NiP$.

 $Mn₂Cu(Si,Ge,Sn)$: All have gaps after 9 states in both the majority and minority channels. $Mn₂CuSi$ also has a minority pseudogap near 12 states and a minority gap after 14 states. The system uses the minority gap after 14 states to make a half-metal (Table [II\)](#page-12-0).

Mn₂Zn(Al,Ga,In): All have small moments and no gaps. Pseudogaps in both channels near 9 states can be identified.

Fe₂Co(Si,Ge,Sn): All have moments near $5\mu_B$. However, instead of gaps after 12 states in minority there are pseudogaps. All three have gaps after 9 states in minority. Fe₂CoSi also has a pseudogap near 9 states in majority.

Fe₂Ni(Al,Ga,In): Also, all have moments near $5\mu_B$, but the only gaps are after 9 states in the minority channel. $Fe₂NiAl$ has a minority pseudogap near 12 states and majority pseudogaps near 14 and 16 states. Fe $_2$ NiGa has majority pseudogaps near 9 states and 16 states.

30-electron systems. The 15 systems in our database with 30 electrons are $Mn_2Cu(P,As,Sb)$, $Mn_2Zn(Si,Ge,Sn)$, $Fe₂Co(P, As, Sb)$, $Fe₂Ni(Si, Ge, Sn)$, and $Fe₂Cu(Al, Ga, In)$. In principle, gaps after 14, 12, or 9 states could be used to make half-metals with moments of $2\mu_B$, $6\mu_B$, or $12\mu_B$, respectively. In practice, we find one negative-formation-energy near-halfmetal with a moment near $2\mu_B$.

 $Mn₂Cu(P, As, Sb)$: These all have gaps after 9 states in both majority and minority. $Mn₂CuP$ also has a gap after 14 states in the minority channel. The moment is nearly $2\mu_B$ which makes it a near-half-metal.

 $Mn₂Zn(Si,Ge,Sn)$: These systems have pseudogaps in both spin channels near 9 states. Mn₂ZnSi also has a gap after 14 electrons in the minority channel and a spin moment of exactly $2\mu_B$ which makes it a half-metal. Unfortunately, its calculated formation energy is positive. $Mn₂ZnGe$ and $Mn₂ZnSn$ do not have gaps in the DOS at the experimental lattice constant. If, however, their lattices are significantly contracted, the gap after 14 states reappears and the Fermi energy falls into the gap.

 $Fe₂Co(P,As,Sb)$: These all have gaps after 9 electrons in the minority channel and pseudogaps near 12 states also in the minority channel. The moments are large enough $(5.76\mu_B,$ $5.95\mu_B$, $5.93\mu_B$, respectively) to place the Fermi energy in the vicinity of the 12-state pseudogap, but because there is not really a gap, these are not near-half-metals.

 $Fe₂Ni(Si,Ge,Sn)$: These all have gaps after 9 states in the minority. Fe₂NiSi and Fe₂NiGe also have majority gaps after 9 states. Fe₂NiSi also has a minority gap after 12 states while Fe₂NiGe has a minority pseudogap after 12 states. The moments are relatively large $(4.78\mu_B, 5.09\mu_B, 5.23\mu_B,$ respectively) but not large enough $(6\mu_B)$ would have been required) to make them half-metals or near-half-metals.

 $Fe₂Cu(Al,Ga,In)$: These all have relatively large minority gaps after 9 states and small majority gaps after 14 states $(Fe₂CuA₁$ and Fe₂CuGa) or after 16 states (Fe₂CuIn). Fe₂CuIn also has a pseudogap after 14 states in the majority. The majority gaps after 14 states cannot be used to make a halfmetal and although the majority gap after 16 could be used, the moment is too large, $4.84\mu_B$ rather than $2\mu_B$.

31-electron systems. The 15 systems in our database with 31 electrons comprise Mn₂Zn(P,As,Sb), Fe₂Ni(P,As,Sb), $Fe₂Cu(Si,Ge,Sn)$, $Fe₂Zn(Al,Ga,In)$, and $Co₂Ni(Al,Ga,In)$. Compounds with 31 electrons could, in principle, take advantage of a minority gap after 14, 12, or 9 states with a moment of $3\mu_B$, $7\mu_B$, or $13\mu_B$, respectively. In addition, we find one example of a gap after 15 states which would allow a half-metal with a moment of $1\mu_B$. In practice, we find no 31-electron half-metals or near-half-metals with negative formation energy.

Mn₂Zn(P,As,Sb): All three systems have gaps after 9 states in minority. Mn₂ZnP also has a gap after 9 states in majority while $Mn₂ZnAs$ has a pseudogap near 9 states in majority. $Mn₂ZnP$ has a pseudogap near 14 states in minority. $Mn₂ZnAs$ has a tiny minority gap after 15 states and a moment $(1.001\mu_B)$ that places the Fermi energy very near this gap. Unfortunately, its formation energy is positive. $Mn₂ZnSb$ has a similar electronic structure and moment, but the tiny gap in $Mn₂ZnAs$ is replaced by a pseudogap in $Mn₂ZnSb$.

 $Fe₂Ni(P,As,Sb)$: These all have minority gaps after 9 states. $Fe₂NiP$ also has a minority gap after 12 states while $Fe₂NiAs$ and Fe2NiSb have pseudogaps after 12 states. The moments are too small to take advantage of the gap or pseudogaps after 12 states.

Fe2Cu(Si,Ge,Sn): These all have gaps after 9 states in both spin channels, with the minority gap being larger than the majority gap. In addition, $Fe₂CuSi$ and $Fe₂CuSn$ have narrow gaps after 14 states in the majority. None of these gaps are near the Fermi energy.

 $Fe₂Zn(AI, Ga,In)$: These systems have gaps (Al) or pseudogaps (Ga,In) at 14 states in minority. The moments are too large to place the Fermi energy near these gaps or pseudogaps.

 $Co₂Ni(Al,Ga,In)$: These systems have numerous gaps and pseudogaps. All three have narrow, deep, pseudogaps near 14 states in minority and have moments $(2.99, 3.02, 3.13)\mu_B$ that place the Fermi energy in the pseudogaps. They are not "nearhalf-metals" according to our definition. In addition, all three have minority pseudogaps near 12 states. They all also have gaps after 9 states in minority. $Co₂NiAl$ has a gap after 14 states in majority. $Co₂NiGa$ has a gap after 9 states in majority and pseudogaps near 14 and 16 states in majority.

32-electron systems. The 12 systems with 32 electrons in our database include $Fe₂Cu(P,As,Sb)$, $Fe₂Zn(Si,Ge,Sn)$, $Co₂Ni(Si,Ge,Sn)$, and $Co₂Cu(Al,Ga,In)$.

 $Fe₂Cu(P,As,Sb)$: These systems all have gaps after 9 states in both spin channels and pseudogaps after 14 states in the minority channel. Their moments are all greater than $4\mu_B$, and the moment of Fe₂CuP is $4.018\mu_B$, quite close to the value of 4 needed to generate a half-metal. Unfortunately, the Fermi energy is on the edge of a pseudogap rather than a gap.

Fe2Zn(Si,Ge,Sn): The 31-electron Fe2Zn*Z XA* compounds have pseudogaps rather than gaps near 9 states in both spin channels, but similar to the 31-electron $Fe₂CuZ$ systems, they have pseudogaps near 14 states in the minority channel. The pseudogap near 14 states in $Fe₂ZnSi$ is remarkable in that the DOS plots appear to show a gap after 14 states. Its moment $(3.982\mu_B)$ is close enough to 4 that we would classify Fe₂ZnSi as a near-half-metal and include it in Table [III](#page-13-0) because it also has a negative formation energy. However, bands 14 and 15 cross between *X* and *W*, spoiling the gap.

 $Co₂Ni(Si,Ge,Sn)$: These all have gaps after nine states in both spin channels and pseudogaps near 12 states in minority. Their moments are too small $(2.51\mu_B, 2.56\mu_B, 2.70\mu_B)$ to place the Fermi energy near the pseudogaps. None are halfmetals or near-half-metals.

 $Co₂Cu(AI, Ga,In)$: These all have gaps after 9 states in the minority channel. None are half-metals or near-halfmetals.

33-electron systems. The 12 systems with 33 electrons in our database include $Fe₂Zn(P,As,Sb)$, Co₂Ni(P,As,Sb), $Co₂Cu(Si,Ge,Sn)$, and $Co₂Zn(Al,Ga,In)$. These systems would need a moment of $5\mu_B$ to utilize a gap after 14 states to make a half-metal. They have neither the gaps nor the moment.

 $Fe₂Zn(P, As, Sb)$: These systems have either gaps (P, As) or pseudogaps after 9 states in both spin channels.

 $Co₂Ni(P,As,Sb)$: These systems all have gaps after 9 states in both spin channels. Pseudogaps near 12 states in minority can also be identified for all three systems.

Co2Cu(Si,Ge,Sn): These systems also have gaps after 9 states in both channels. Two (Si,Ge) have pseudogaps after 12 states in both spin channels.

Co2Zn(Al,Ga,In): No gaps or convincing pseudogaps were observed in these systems.

34-electron systems. The 9 systems in our database with 34 electrons include $Co₂Cu(P,As,Sb)$, $Co₂Zn(Si,Ge,Sn)$, and $Ni₂Cu(Al,Ga,In).$

 $Co₂Cu(P, As, Sb)$: These systems have gaps after 9 states in both channels. $Co₂CuP$ also has a majority gap after 12 states which becomes a pseudogap for the other two compounds. The moments are not nearly large enough to place the Fermi energy near the gaps or pseudogaps.

 $Co₂Zn(Si,Ge,Sn)$: These systems all have pseudogaps near 9 states in both spin channels and also a pseudogap near 12 states in the minority channel.

 $Ni₂Cu(A1,Ga,In)$: These systems are nonmagnetic and have no gaps. Ni2CuAl and Ni2CuGa have pseudogaps near 12 states in both channels. Ni₂CuGa and Ni₂CuIn have pseudogaps after 9 states in both channels.

35-electron systems. The 9 systems in our database with 35 electrons include $Co₂Zn(P,As,Sb)$, $Ni₂Cu(Si,Ge,Sn)$, and $Ni₂Zn(Al,Ga,In)$. A system with 35 electrons would need a moment of $7\mu_B$ to take advantage of a gap after 14 states in minority. Such moments would be unlikely in these systems because the *d* bands are nearly filled for Co and especially for Ni.

 $Co₂Zn(P, As, Sb)$: These systems are magnetic with moments relatively small moments $(2.4\mu_B, 2.5\mu_B, \text{ and } 2.7\mu_B)$. $Co₂ZnP$ and $Co₂ZnAs$ have gaps after 9 states in both spin channels while $Co₂ZnSb$ has pseudogaps near 9 states in both spin channels. Pseudogaps near 14 states can be identified in all three systems.

Ni2Cu(Si,Ge,Sn): These are all nonmagnetic and have gaps after 9 states in each of the identical spin channels. Pseudogaps near 12 states can be identified in all three as well as near 14 states in $Ni₂CuSn$.

36-electron systems. The 6 systems in our database with 36 electrons include Ni2Cu(P,As,Sb) and Ni2Zn(Si,Ge,Sn). None of these systems are magnetic.

 $Ni₂Cu(P, As, Sb)$: These have gaps after 9 states and pseudogaps near 12 states and 14 states.

Ni2Zn(Si,Ge,Sn): These have pseudogaps near 14 states.

37-electron systems. The 6 systems in our database with 37 electrons are $Ni₂Zn(P,As,Sb)$ and $Cu₂Zn(Al,Ga,In)$. None of these 6 systems have a magnetic moment and therefore cannot be half-metals.

 $Ni₂Zn(P,As,Sb)$: $Ni₂ZnP$ and $Ni₂ZnAs$ have gaps after 9 states. Ni2ZnSb has a pseudogap near 9 states. Pseudogaps after 14 states can be identified for all three.

 $Cu₂Zn(AI, Ga,In)$: These systems have no gaps.

38-electron systems. The three systems in our database with 38 electrons are $Cu₂Zn(Si,Ge,Sn)$. They have neither moments nor gaps.

39-electron systems. The three systems in our database with 39 electrons are $Cu₂Zn(P, As, Sb)$. They have neither moments nor gaps.

V. CONCLUSION

We have performed extensive first-principles calculations on 405 inverse Heusler compounds X_2YZ , where $X = Sc$, Ti, V, Cr, Mn, Fe, Co, Ni, or Cu; *Y* = Ti, V, Cr, Mn, Fe, Co, Ni, Cu, or Zn; and $Z = A1$, Ga, In, Si, Ge, Sn, P, As, or Sb with a goal of identifying materials of interest for spintronic applications. We identified 14 semiconductors with negative formation energy. These are listed in Table [I.](#page-10-0) We identified many half-metals and near-half-metals. Those half-metals and near-half-metals that we identified and that have negative formation energy are listed in Tables [II](#page-12-0) and [III,](#page-13-0) respectively.

We have identified four half-metals and six near-half-metals that meet a criterion for stability that we estimated based on a study of the hull distances of known *XA* phases. Seven of these ten predicted phases have not (to our knowledge) been observed experimentally.

In establishing our stability criterion, we analyzed the experimental and theoretical data for several reported *XA* phases $(Mn_2Coln, Mn_2CoSb, Mn_2CoSn, Mn_2RuSn)$ and tentatively concluded that they are not *XA* but are more likely a disordered phase such as $L2_1B$. We recommend more extensive studies of these materials.

The 10 half-metallic or near-half-metallic phases, that we identified as reasonably likely to be stable, are all Slater-Pauling phases, i.e., they have a gap in the density of states after three states per atom in one of the two spin channels, and the Fermi energy is in or near this gap. Many half-metallic and near-half-metallic phases based on a gap after 9 states per formula unit (2.25 states per atom) were identified, but we estimate that other phases or combination of phases are likely to be more stable in equilibrium.

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