Anomalous temperature dependence of the relaxation rate measured by muon spin rotation in α-$YbAl_{0.986}Fe_{0.014}B_4$

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Recently, it was reported by MacLaughlin *et al.* in Phys. Rev. B **93**, 214421 (2016) that α-$YbAl_{0.986}Fe_{0.014}B_4$ exhibits an anomalous temperature dependence in the relaxation rate $1/T_1$ of μSR, and stressed that such temperature dependence cannot be understood by the scenario based on the quantum critical valence transition (QCVT) while this compound exhibits a series of the non-Fermi-liquid behaviors explained by the theory of the QCVT. In this paper, we point out that the anomalous temperature dependence in $1/T_1$ can be understood semiquantitatively by assuming that the attraction of a screening cloud of conduction electrons about the μ^+ induces a local magnetic moment arising from a $4f$ hole on the Yb ion, giving rise to the Kondo effect between heavy quasiparticles.

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I. INTRODUCTION

β-$YbAlB_4$ exhibits unconventional non-Fermi-liquid properties in the low-temperature (T) region $T < 10$ K not only at ambient condition [1] but under a range of pressures [2]. The critical exponents of a series of physical quantities were shown to follow those given by a theory of quantum critical valence transition (QCVT) [3–5] which also explains the unconventional non-Fermi-liquid properties of other systems exhibiting the same critical exponents that were observed in $YbCu_{5-x}Al_x$ ($x = 3.5$) [6,7], $YbRh_2Si_2$ [8,9], and quasicrystal compound $Yb_{15}Al_{34}Au_{51}$ [10] in a range of pressures including ambient pressure and quasicrystal approximant $Yb_{14}Al_{35}Au_{51}$ under pressure $P \simeq 1.8$ GPa [11]. Recently, the T/B scaling behavior observed in β-$YbAlB_4$ [12] and $Yb_{14}Al_{35}Au_{51}$ [11] have also been shown to be explained from the theory of the QCVT [13]. On the other hand, a sister compound α-$YbAlB_4$ follows properties of the conventional heavy-electron metal with an intermediate valence of Yb [14]. However, α-$YbAl_{1-x}Fe_xB_4$ ($x = 0.014$) exhibits the same criticality as β-$YbAlB_4$ and the drastic change in valence of Yb [15]. This strongly suggests that the scenario of QCVT is valid. Nevertheless, it was recently reported in Ref. [16] that α-$YbAl_{0.986}Fe_{0.014}B_4$ exhibits an anomalous T dependence in the relaxation rate $1/T_1$ measured by μSR (muon spin rotation), which cannot be simply understood by the scenario based on the QCVT.

Figure 1 shows the T dependence in the muon relaxation rate $1/T_1$ of α-$YbAl_{1-x}Fe_xB_4$ ($x = 0.014$) at ambient pressure [16]. The behavior at $T < 0.05$ K is consistent with the prediction by the theory of QCVT, i.e., $1/T_1T \propto T^{-\zeta}$ with weakly T-dependent exponent ζ ($0.5 < \zeta < 0.7$) [3–5]. On the other hand, the behavior $1/T_1T \propto T^{-1.4}$ (at $T > 0.1$ K), is entirely different, which seems to have led the authors of Ref. [16] to skepticism toward the QCVT scenario. The purpose of this paper is to give a possible explanation to this puzzling behavior by taking account of an influence of μ^+ on the electronic state around it. Namely, μ^+ attracts conduction electrons which lowers the crystalline electric field (CEF) levels of $4f$ holes for the Yb ions neighboring it, inducing the local magnetic moment which should give rise to the Kondo effect that enhances the relaxation rate $1/T_1$ of μ^+ through the conduction electrons around the Yb ion.

II. PHYSICAL PICTURE

Figure 2(a) shows a schematic picture of distribution of conduction electrons modified by the existence of μ^+. Namely, the positive charge of μ^+ attracts conduction electrons so that the density of conduction electrons should increase neighboring μ^+. Figure 2(b) shows a snapshot of the lowest CEF energy levels ε_fs of $4f$ holes in Yb ions around μ^+, which are modified by the increase in conduction electrons density, and the distribution of $4f$ holes in the valence fluctuating situation. A crucial point is that the $4f$-hole's energy level around μ^+ is decreased by the repulsive Coulomb interaction suffered from the excess conduction electrons, so that there arises a localized spin of $4f$ holes at Yb sites adjacent to μ^+. This effect can be rephrased on the hole picture for conduction electrons. Namely, μ^+ decreases the density of conduction holes around it, making the energy level $\varepsilon_f = \bar{\varepsilon}_f + U_{fc}n_c$ of f holes decrease there [3–5], where n_c is the number of conduction electron holes at the Yb site and $\bar{\varepsilon}_f$ is the level of f holes without the μ^+.

This induced local moment of the $4f$ hole at the Yb site would cause the impurity Kondo effect between quasiparticles (consisting of $4f$-hole lattice and conduction electron band through the renormalized hybridization V^*) around there, giving excess spin fluctuations of quasiparticles which in turn

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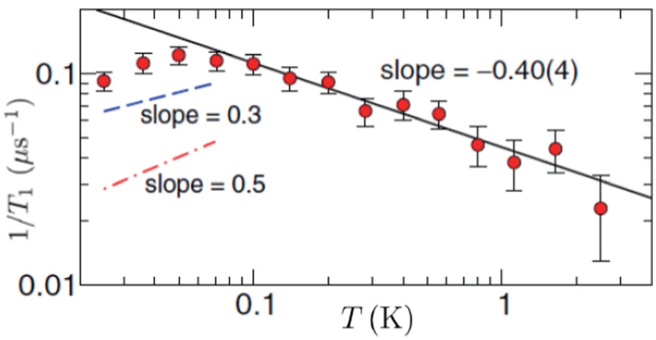

FIG. 1. Temperature dependence (in the common logarithmic scale) of the relaxation rate $1/T_1$ (in the common logarithmic scale) measured by μSR experiment [16].

should give an excess relaxation of the μ^+ spin through the hyperfine coupling between them. The exact position where the μ^+ stops in the crystal is not known, as there was no statement about this in Ref. [16]. However, the result of the anomalous exponent for the temperature dependence in the relaxation rate will not be altered because the effect is not sensitive to a position of μ^+ so long as it stops in the crystal.

III. FORMULATION FOR RELAXATION RATE

For simplicity, hereafter, we treat the problem as the single impurity Kondo effect between the local moment at the Yb site (at the origin of space coordinate) and quasiparticles. The spin relaxation of μ^+ stopped at $\mathbf{r}$ is given by the process of the Feynman diagram shown in Fig. 3 [17], where $J_{\rm qf}$ is the bare exchange interaction between spins of quasiparticles and the localized $4f$ hole, and $\chi_\perp^{\rm local}(\omega)$ is the dynamical transverse spin susceptibility of the localized $4f$ hole at the Yb site. Note that $J_{\rm qf}$ is proportional to the square of the renormalized hybridization V^* between the localized $4f$ hole and the quasiparticles so that it is proportional to the mass renormalization amplitude z. Therefore, the dimensionless coupling constant $J_{\rm qf}N_{\rm F}^*$, $N_{\rm F}^* \propto z^{-1}$ being the renormalized density of states at the Fermi level of quasiparticles, is not subject to the effect of mass renormalization of the quasiparticles. The explicit form of $\chi_\perp^{\rm local}(\omega)$ is the retarded function of $\chi_\perp^{\rm local}(i\omega_m) \equiv \int_0^{1/T} d\tau e^{i\omega_m\tau}\langle S_{\rm f}^+(\tau)S_{\rm f}^-(0)\rangle$, where $S_{\rm f}^\pm$ are the spin-flip operators of the localized f electron and $\langle\dots\rangle$ represents the thermal average.

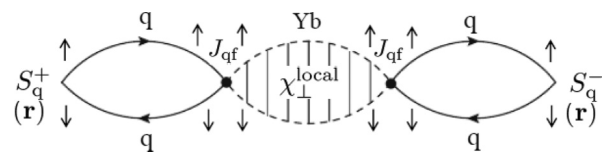

FIG. 3. Feynman diagram giving the relaxation rate $1/T_1T$ of the muon (μ^+) through the hyperfine coupling with spin of quasiparticles at $\mathbf{r}$ (around the μ^+) which is influenced by the local moment at Yb site induced by the effect of existence of μ^+ particle itself.

The nonlocal dynamical transverse spin susceptibility $\chi_\perp^{\rm qp}(\mathbf{r},\omega)$ of quasiparticles, appearing at both sides of $\chi_\perp^{\rm local}$ in Fig. 3, is defined by the retarded function of

$$\chi_\perp^{\rm qp}(\mathbf{r},i\omega_m) \equiv T\sum_{\epsilon_n} G_\uparrow^{\rm qp}(\mathbf{r},i\epsilon_n+i\omega_m)G_\downarrow^{\rm qp}(\mathbf{0},i\epsilon_n), \quad (1)$$

where $G_\sigma^{\rm qp}(\mathbf{r},i\epsilon_n)$ is the Matsubara Green function of the quasiparticles with the spin σ (=↑ or ↓). In the Fermi degenerate region ($T \ll D^*$, with D^* being the effective Fermi energy or temperature of quasiparticles), the explicit form of the retarded function (obtained by the analytic continuation $i\omega_m \to \omega + i\delta$) is given by

$$\chi_\perp^{\rm qp}(\mathbf{r},\omega) = N_{\rm F}^*\frac{2k_{\rm F}^3}{\pi}R(k_{\rm F}r) + i\pi\omega N_{\rm F}^{*2}\frac{[\sin(2k_{\rm F}r)]^2}{(k_{\rm F}r)^2}, \quad (2)$$

where $R(k_{\rm F}r)$ at $k_{\rm F}r \gtrsim 1$ represents the Friedel oscillations appearing in the Ruderman-Kittel-Kasuya-Yosida interaction [18–20] and is defined as

$$R(k_{\rm F}r) \equiv -\frac{\cos(2k_{\rm F}r)}{(2k_{\rm F}r)^3} + \frac{\sin(2k_{\rm F}r)}{(2k_{\rm F}r)^4}, \quad (3)$$

while it approaches a dimensionless constant of the order of $\mathcal{O}(q_{\rm B}a)$, with $q_{\rm B}$ being the wave number of the Brillouin zone and a being the lattice constant, in the limit $k_{\rm F}r \ll 1$.

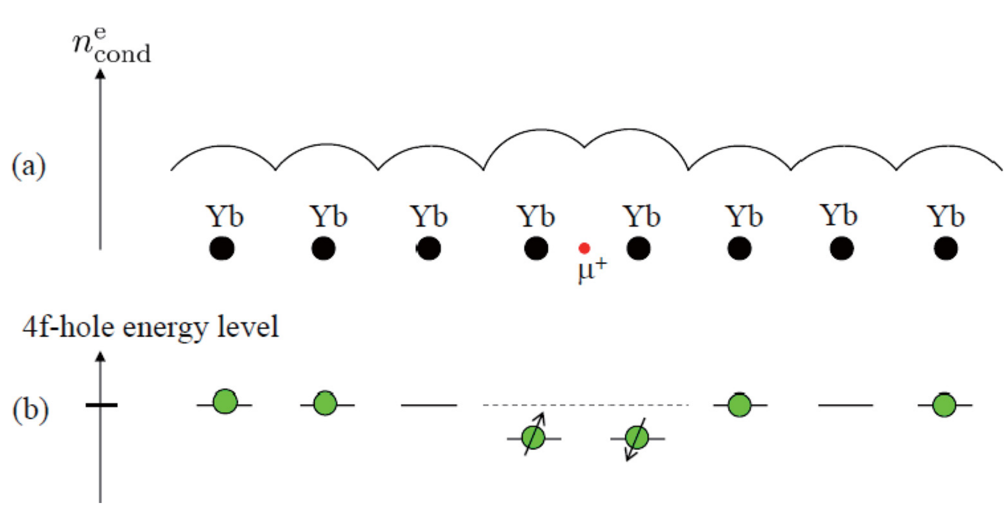

FIG. 2. (a) Schematic picture on electron distribution of conduction electrons attracted by μ^+. (b) Snapshot of the lowest CEF energy level of $4f$ hole at Yb site adjacent to μ^+.

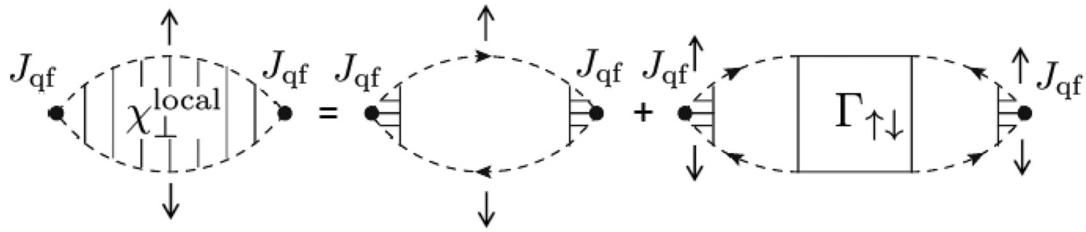

FIG. 4. Feynman diagram giving the renormalization of the local spin susceptibility $\chi_\perp^{local}$ in Fig. 3. Triangles with a dot are the renormalized exchange interaction, a square $\Gamma_{\uparrow\downarrow}$ is the vertex correction causing the Kondo-Yosida singlet formation [21,22], and dashed lines with arrows are the Matsubara Green function of pseudofermion representing the localized $4f$ hole [23].

In deriving Eq. (2), we have assumed the free dispersion for the quasiparticles band. Since the expression [Eq. (2)] is essentially T independent, the crucial T dependence arises from that of $\chi_\perp^{local}(\omega)$ which is given by the Feynman diagram shown in Fig. 4.

The triangle and square $\Gamma_{\uparrow\downarrow}$ in Fig. 4 are renormalized exchange interaction and vertex correction expressing the effect of Kondo-Yosida singlet formation [21,22], respectively. The explicit form of the triangle is given by Fig. 5 up to processes of the two-loop order, and is known to increase by the Kondo renormalization effect [24], which is an origin of the anomalous relaxation rate.

Concluding this section, we note that there also exists the relaxation process through the direct magnetic dipolar coupling between the localized spin of the $4f$ hole on the Yb ion and μ^+ other than the process shown in Fig. 3. However, this process gives only the T independent contribution in $1/T_1$ reflecting the existence of the local moment of the $4f$ hole at the Yb site, so that it is masked by the anomalous contribution through the renormalization effect of J_{qf} shown in Fig. 5 in the high-temperature region $T \gtrsim T_K$ as discussed in the next section. In the low-temperature region $T \lesssim T_K$, it gives the conventional temperature dependence in the relaxation rate $1/T_1$ as expected from the Korringa-Shiba relation at $T \ll T_K$ [25], which is also masked by the contribution given by the theory of QCVT [3].

IV. ANOMALOUS RELAXATION

The temperature dependence of the relaxation rate $(1/T_1T)_{1st}^{local}$ arising from fluctuations of the local moment at Yb site, corresponding to the first term of the right-hand side (rhs) in Fig. 4, is given by

$$\left(\frac{1}{T_1T}\right)_{1st}^{local} = \tilde{A}_{hf}^2[J_{qf}(T/D^*)]^2\left\{\frac{\mathrm{Im}\chi_0^{local}(\omega)}{\omega}[\chi_\perp^{qp}(\mathbf{r},0)]^2 + 2\frac{\mathrm{Im}\chi_\perp^{qp}(\mathbf{r},\omega)}{\omega}\chi_\perp^{qp}(\mathbf{r},0)\chi_0^{local}(0)\right\}, \quad (4)$$

where $\tilde{A}_{hf}$ is the effective hyperfine coupling constant between μ^+ and quasiparticles, and $J_{qf}(T/D^*)$ is the renormalized exchange interaction given by the processes as shown in Fig. 5, and has the strong T dependence characteristic to the Kondo effect [26]. The effective hyperfine coupling $\tilde{A}_{hf}$ is considered to arise mainly from the magnetic dipolar coupling between the muon and quasiparticles around it, while the direct hyperfine coupling between μ^+ and electrons is quite small compared to that for usual nuclei used for NMR measurements.

Here, we are adopting a scheme of the renormalization group (RG) approach in which the effects of intermediate states of quasiparticles with higher energies than the temperature T are absorbed into the renormalized exchange interaction $J_{qf}(T/D^*)$ *à la* the *poorman*'s scaling approach [24,27]. The

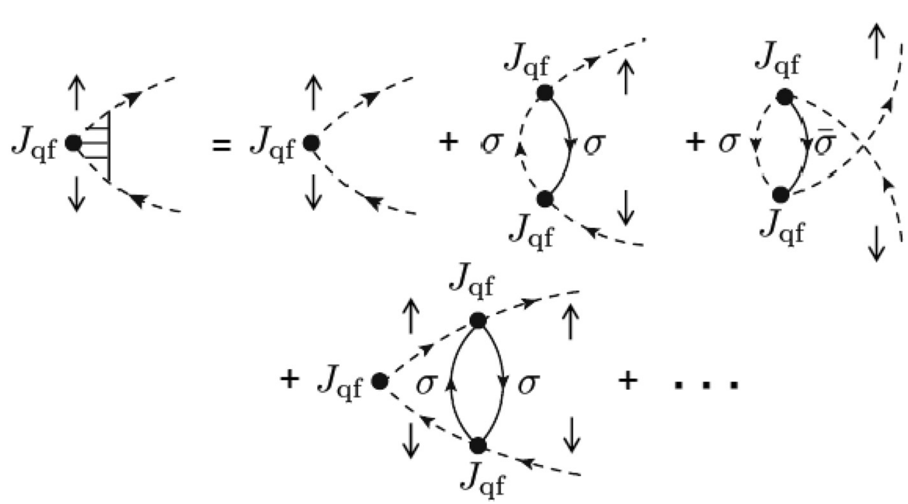

FIG. 5. Feynman diagram giving the renormalization of the exchange interaction J_{qf} corresponding to the spin-flip process shown in Figs. 3 and 4. The labels σ and $\bar{\sigma}$ imply that the summation with respect to σ (↑ or ↓) and $\bar{\sigma}$ (↓ or ↑) is taken.

dynamical susceptibility $\chi_0^{\rm local}(\omega)$ of the local moment in Eq. (2) represents that without the vertex correction and its imaginary part is proportional to ω/T in the limit $\omega \to 0$. Thus,

$$\left(\frac{1}{T_1 T}\right)_{\rm 1st}^{\rm local} \propto \frac{[J_{\rm qf}(T/D^*)]^2}{T}. \tag{5}$$

On the other hand, the contribution from the second term of the rhs in Fig. 4 becomes important in the region $T \lesssim T_{\rm K}^*$, with $T_{\rm K}^*$ being the Kondo temperature given by the one-loop order calculation [24], and works to suppress the relaxation rate through the Kondo-Yosida singlet formation [21,22], leading to the Korringa-Shiba relation of the local Fermi liquid at $T \ll T_{\rm K}^*$ [25]. Therefore, in the region $T \lesssim T_{\rm K}^*$, the relaxation rate $(1/T_1T)^{\rm local}$ due to the fluctuations of the local moment at the Yb site follows the relation

$$\left(\frac{1}{T_1 T}\right)^{\rm local} \propto {\rm const.} \tag{6}$$

Namely, this contribution would be masked by that from the QCVT, $(1/T_1T)^{\rm QCVT} \propto T^{-\zeta}$ [3–5], which dominates over the contribution $(1/T_1T)^{\rm local}$ [Eq. (6)] in the low-temperature region $T \lesssim T_{\rm K}^*$.

As a result, the relaxation rate $(1/T_1T)_{\rm 1st}^{\rm local}$ [Eq. (4)] dominates in the region $T \gtrsim T_{\rm K}^*$, and the exponent α, giving an extra temperature dependence in the relaxation rate $(1/T_1T)_{\rm 1st}^{\rm local}$, arises from that of the renormalized exchange interaction between quasiparticles and the localized $4f$ hole, $J_{\rm qf}(x)$, with x being $x \equiv T/D^*$. Namely, $\alpha(x)$ in the region $T \gtrsim T_{\rm K}^*$ is defined by

$$[J_{\rm qf}(x)]^2 \equiv J_0^2\, x^{-\alpha(x)}, \tag{7}$$

where J_0 is the bare exchange interaction of $J_{\rm qf}$ at $T_{\rm K}^* \ll T \lesssim D^*$. Then, according to the relation Eq. (4), the relaxation rate $(1/T_1T)_{\rm 1st}^{\rm local}$ is expressed as

$$\left(\frac{1}{T_1 T}\right)_{\rm 1st}^{\rm local} \propto T^{-[1+\alpha(T/D^*)]}. \tag{8}$$

With the use of Eq. (7), the exponent $\alpha(x)$ is in turn given by

$$\alpha(x) = -2\frac{\ln[J_{\rm qf}(x)/J_0]}{\ln x}. \tag{9}$$

V. TEMPERATURE DEPENDENCE OF ANOMALOUS EXPONENT

The temperature dependence of the exponent $\alpha(T/D^*)$ is derived by solving the two-loop RG evolution equation for the renormalized exchange interaction $J_{\rm qf}(x)$.

The RG evolution equation for $y \equiv J_{\rm qf}N_{\rm F}^*$ on the two-loop order is given by [28,29]

$$\frac{dy}{dt} = -y^2 + y^3, \tag{10}$$

where $t \equiv \ln x$. This differential equation has a formal solution as

$$\ln\frac{y(y_0-1)}{y_0(y-1)} - \frac{1}{y} + \frac{1}{y_0} = -t, \tag{11}$$

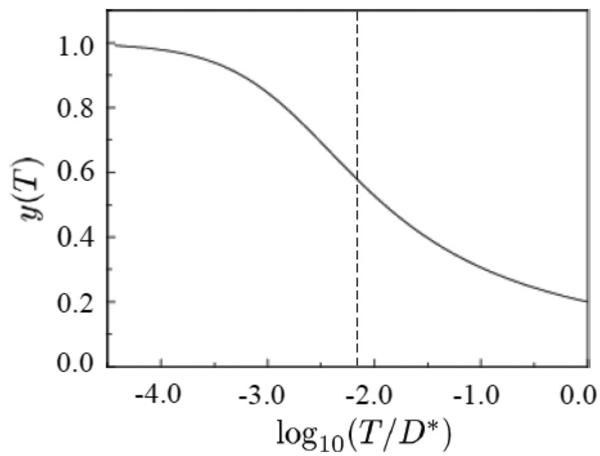

FIG. 6. Temperature dependence of the renormalized exchange interaction $y(T)$ as a function of $\log_{10}(T/D^*)$ on the basis of the RG of the two-loop order for the bare interaction $y_0 = 0.2$. The dashed line corresponds to the Kondo temperature $T_{\rm K}^* \equiv D^* e^{-1/y_0} \simeq 0.67 \times 10^{-2}\, D^*$ given by the RG calculation of one-loop order.

where $y_0 \equiv y(0) = J_{\rm qf}(0)N_{\rm F}^*$. The numerical relation between y and t [$= \ln(T/D^*)$] is easily obtained, as shown in Fig. 6, e.g., in the case of $y_0 = 0.2$.

On the other hand, the RG evolution equation on the one-loop order is simplified and is given by

$$\frac{dy}{dt} = -y^2, \tag{12}$$

which was derived by Anderson on the idea of the poorman's scaling [24]. The solution of Eq. (12) is explicitly given by

$$y = \frac{y_0}{1+y_0 t} = \frac{y_0}{1+y_0 \ln(T/D^*)}, \tag{13}$$

where the explicit T dependence is shown in the second equality. The Kondo temperature $T_{\rm K}^*$ is defined by the condition that the renormalized exchange interaction $y(T)$ diverges: i.e., $T_{\rm K}^* = D^* e^{-1/y_0}$ or $D^* \exp[-1/J_{\rm qf}(0)N_{\rm F}^*]$ although the divergence of $y(T)$ at $T = T_{\rm K}$ is an artifact of insufficient approximation scheme. Nevertheless, it offers the characteristic temperature below which the Kondo-Yosida singlet state begins to be stabilized. Then, the exponent $\alpha(x)$ [Eq. (9)] is given by

$$\alpha(x) = 2\,\frac{\ln[1 + y_0 \ln x]}{\ln x}. \tag{14}$$

Therefore, in the region $T \simeq D^* \gg T_{\rm K}^*$ (or $0 < -\ln x \ll 1$), the exponent $\alpha(x)$ becomes T independent and is given by

$$\alpha(x) = 2y_0 = 2J_{\rm qf}(0)N_{\rm F}^*. \tag{15}$$

However, on the two-loop order, the exponent $\alpha(x)$ has a weak T dependence. With the use of the numerical solution of Eq. (11) with an initial condition $y_0 = 0.2$, the temperature dependence in the exponent $\alpha(T/D^*)$ [Eq. (9)] is given as Fig. 7. It is remarked that the anomalous exponent $\alpha(T/D^*)$ is almost T independent in the region $T \gtrsim T_{\rm K}^* \simeq 0.67 \times 10^{-2}\, D^*$ and is located within the error bar of experiments reported in Ref. [16]. This in turn implies that the bare coupling exchange interaction $J_{\rm qf}(0)$ between quasiparticles and localized $4f$ hole takes the value $J_{\rm qf}(0)N_{\rm F}^* = 0.2$ which is a rather difficult

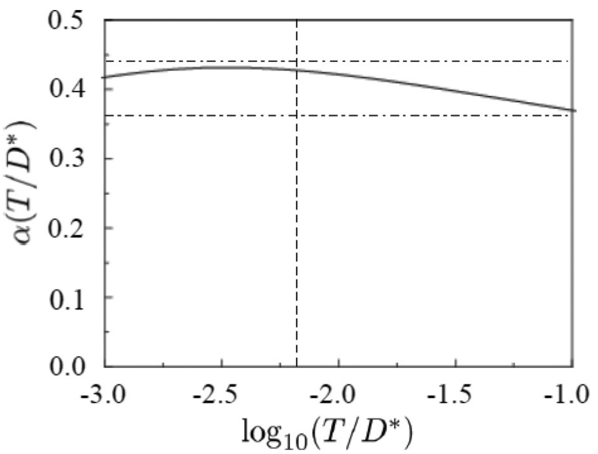

FIG. 7. Temperature dependence of the exponent $\alpha(T/D^*)$ [Eq. (9)] as a function of $\log_{10}(T/D^*)$ on the basis of the two-loop RG equation with the initial condition $y_0 = 0.2$. The dashed line corresponds to the Kondo temperature $T_K^* \equiv D^* e^{-1/y_0} \simeq 0.67 \times 10^{-2}\, D^*$ given by the RG calculation of the one-loop order. The chain lines are lower and upper boundaries corresponding to the error bar in the experiment reported in Ref. [16].

physical quantity to estimate theoretically [30]. Furthermore, the effective Fermi temperature D^* of the quasiparticles should be $D^* \simeq 30$ K considering that the Kondo temperature is estimated as $T_K^* \simeq 0.2$ K where the T dependence of $1/T_1$ is expected to deviate from the scaling behavior $1/T_1 \propto T^{-0.40\pm0.04}$, as shown in Fig. 1. It should be mentioned, however, that $D^* \simeq 30$ K is far smaller than the characteristic temperature $T^* = 200$ K estimated from the T dependence of the Sommerfeld coefficient C/T in Ref. [1].

On the other hand, it turns out that the effective Fermi temperature $D^* \simeq 30$ K is not a ridiculous possibility, if we compare the T dependence of the specific heat of α-$YbAlB_4$ with that of a typical heavy fermion system $CeCu_6$ [31], in which C/T begins to increase from $T_K \simeq 3.5$ K, which is identified with the effective Fermi temperature, and to reach in the low-temperature $\lim_{T\to 0} C/T \simeq 1.6 \times 10^3$ mJ/K^2 mol(Ce). In the case of α-$YbAlB_4$, C/T begins to increase from $\tilde{T} \simeq 30$ K and to reach $\lim_{T\to 0} C/T \simeq 1.3 \times 10^2$ mJ/K^2 mol(Yb) [14]. Therefore, if this $\tilde{T} \simeq 30$ K is identified with the effective Fermi temperature $D^* \simeq 30$ K, the behaviors of the specific heat in both systems are approximately related by changing the temperature scale by about ten times. It should be also remarked that $CeCu_6$ is located near the QCVT point which is approached under the pressure and the magnetic field [5,32–34] as in the case of α-$YbAl_{1-x}Fe_xB_4$ ($x = 0$) [14].

With these reservations, the above result on the exponent $\alpha(T/D^*)$ has offered a possible key concept to resolve the puzzle on the anomalous T dependence in the relaxation rate $1/T_1$ measured by μSR experiment [16].

VI. SUMMARY AND SUPPLEMENTAL DISCUSSIONS

On the basis of the physical picture that the μ^+ stopped at the interstitial in the crystal greatly influences the electronic state around it, it has been predicted that there arises the anomalous temperature dependence of the μSR relaxation rate $1/T_1$ observed in α-$YbAl_{0.986}Fe_{0.014}B_4$ [16]. Namely, the conduction electrons attracted by μ^+ induce the local moment of the $4f$ hole on the Yb ion nearby and the Kondo effect is caused between the local moment and heavy quasiparticles, resulting in the excess contribution to the relaxation rate as $(1/T_1)^{\rm local} \propto T^{-\alpha(T)}$ in the high-temperature region $T \gtrsim T_K^*$. While the exponent $\alpha(T)$ depends on the exchange interaction $J_{\rm qf}$ between the quasiparticles and the local moment of the $4f$ hole and is weakly T dependent, it is possible to choose a reasonable set of parameters, $J_{\rm qf}N_F^* = 0.2$ and $D^* = 30$ K (corresponding to $T_K^* = 0.2$ K), to reproduce the observed value $\alpha = 0.40 \pm 0.04$, as shown in Fig. 7. On the other hand, in the low-temperature region $T \lesssim T_K^*$, the local moment forms the Kondo-Yosida singlet state with quasiparticles so that the local Fermi-liquid behavior is recovered, i.e., $(1/T_1T)^{\rm local} \propto$ const. However, this contribution is buried by the contribution due to the QCVT, $(1/T_1T)^{\rm QCVT} \propto T^{-\zeta}$ $(0.5 < \zeta < 0.7)$, which is really observed at $T < 0.05$ K $\ll T_K^* = 0.2$ K [16].

Although we have discussed the case of zero magnetic field in the present paper, the effect of magnetic field is considered to be also crucial for anomalies of the relaxation rate because the quantum criticality of valence transition is considerably influenced by the magnetic field as discussed in Ref. [35]. Indeed, the magnetic field dependence of the relaxation rate in α-$YbAl_{0.986}Fe_{0.014}B_4$ has some structure at $H \sim 4$ Oe [16], which might have some relevance to magnetic field effect mentioned above. However, detailed analyses are left for future study.

Finally, we have put aside the issue of the possibility of forming a muonium because it seems to be excluded in bulk metallic systems as discussed in Refs. [36,37]. This is because the screening effect on the Coulomb attractive potential from μ^+ works to inhibit the existence of the bound electronic state (i.e., muonium). Note that the screening length λ_s estimated by the Thomas-Fermi formula [38], which is valid also in the heavy fermion system because the charge susceptibility is essentially unrenormalized [39], is given by

$$\lambda_s = \sqrt{\frac{E_F}{6\pi n e^2}} = \frac{\pi}{2}\left(\frac{3}{\pi}\right)^{1/6}\sqrt{\frac{n^{-1/3}}{a_B}}\, a_B, \qquad (16)$$

where E_F is the Fermi energy of free electron, n is the carrier number density, and a_B is the Bohr radius. If we adopt $n^{-1/3} = 4$ Å assuming that each Yb ion supplies one mobile electron [1,14], the screening length is estimated as $\lambda_s \simeq 2.3$ Å. Therefore, the screening is far from perfect at the Yb site so that a finite fraction of conduction electrons can be accumulated at the Yb site giving rise to the local moment of the $4f$ hole at the Yb site because the system is at criticality of the valence transition of the Yb ion, which justifies a physical picture as shown in Fig. 2.

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[1] S. Nakatsuji, K. Kuga, Y. Machida, T. Tayama, T. Sakakibara, Y. Karaki, H. Ishimoto, S. Yonezawa, Y. Maeno, E. Pearson, G. G. Lonzarich, L. Balicas, H. Lee, and Z. Fisk, Nat. Phys. **4**, 603 (2008).
[2] T. Tomita, K. Kuga, Y. Uwatoko, P. Coleman, and S. Nakatsuji, Science **349**, 506 (2015).
[3] S. Watanabe and K. Miyake, Phys. Rev. Lett. **105**, 186403 (2010).
[4] S. Watanabe and K. Miyake, J. Phys.: Condens. Matter **24**, 294208 (2012).
[5] K. Miyake and S. Watanabe, Philos. Mag. B **97**, 3495 (2017).
[6] E. Bauer, R. Hauser, L. Keller, P. Fischer, O. Trovarelli, J. G. Sereni, J. J. Rieger, and G. R. Stewart, Phys. Rev. B **56**, 711 (1997).
[7] C. Seuring, K. Heuser, E.-W. Scheidt, T. Schreiner, E. Bauer, and G. R. Stewart, Phys. B (Amsterdam, Neth.) **281**, 374 (2000).
[8] O. Trovarelli, C. Geibel, S. Mederle, C. Langhammer, F. M. Grosche, P. Gegenwart, M. Lang, G. Sparn, and F. Steglich, Phys. Rev. Lett. **85**, 626 (2000).
[9] K. Ishida, K. Okamoto, Y. Kawasaki, Y. Kitaoka, O. Trovarelli, C. Geibel, and F. Steglich, Phys. Rev. Lett. **89**, 107202 (2002).
[10] K. Deguchi, S. Matsukawa, N. K. Sato, T. Hattori, K. Ishida, H. Takakura, and T. Ishimasa, Nat. Mater. **11**, 1013 (2012); N. K. Sato (private communication).
[11] S. Matsukawa, K. Deguchi, K. Imura, T. Ishimasa, and N. K. Sato, J. Phys. Soc. Jpn. **85**, 063706 (2016).
[12] Y. Matsumoto, S. Nakatsuji, K. Kuga, Y. Karaki, N. Horie, Y. Shimura, T. Sakakibara, A. H. Nevidomskyy, and P. Coleman, Science **331**, 316 (2011).
[13] S. Watanabe and K. Miyake, J. Phys. Soc. Jpn. **83**, 103708 (2014).
[14] Y. Matsumoto, K. Kuga, T. Tomita, Y. Karaki, and S. Nakatsuji, Phys. Rev. B **84**, 125126 (2011).
[15] K. Kuga, Y. Matsumoto, M. Okawa, S. Suzuki, T. Tomita, K. Sone, Y. Shimura, T. Sakakibara, D. Nishio-Hamane, Y. Karaki, Y. Takata, M. Matsunami, R. Eguchi, M. Taguchi, A. Chainani, S. Shin, K. Tamasaku, Y. Nishino, M. Yabashi, T. Ishikawa, and S. Nakatsuji, Sci. Adv. **4**, eaao3547 (2018).
[16] D. E. MacLaughlin, K. Kuga, L. Shu, O. O. Bernal, P. C. Ho, S. Nakatsuji, K. Huang, Z. F. Ding, C. Tan, and J. Zhang, Phys. Rev. B **93**, 214421 (2016).
[17] T. Moriya, J. Phys. Soc. Jpn. **18**, 516 (1963).
[18] M. A. Ruderman and C. Kittel, Phys. Rev. B **96**, 99 (1954).
[19] T. Kasuya, Prog. Theor. Phys. **16**, 45 (1956).
[20] K. Yosida, Phys. Rev. **106**, 893 (1957).
[21] K. Yosida, Phys. Rev. **147**, 223 (1966); Prog. Theor. Phys. **36**, 875 (1966).
[22] A. Yoshimori and K. Yosida, Prog. Theor. Phys. **39**, 1413 (1968).
[23] A. A. Abrikosov, Physics **2**, 5 (1965).
[24] P. W. Anderson, J. Phys. C **3**, 2436 (1970).
[25] H. Shiba, Prog. Theor. Phys. **54**, 967 (1975).
[26] The expression [Eq. (4)] may be regarded as the microscopic manifestation of phenomenological formulation given in H. Alloul, Phys. Rev. Lett. **35**, 460 (1975). Therefore, this expression can be also applied to the analysis of the anomalous T dependence of $1/T_1$ (at $T \gtrsim T_K$) of ^{63}Cu nuclei near the Fe impurity in Cu-Fe dilute alloy reported in H. Alloul, J. Physique Lett. **37**, 205 (1976) for example.
[27] S. Tanikawa, H. Matsuura, and K. Miyake, J. Phys. Soc. Jpn. **78**, 034707 (2009).
[28] M. Fowler and A. Zawadowski, Solid State Commun. **9**, 471 (1971).
[29] A. A. Abrikosov and A. A. Migdal, J. Low Temp. Phys. **3**, 519 (1970).
[30] However, the appearance of the local moment of the $4f$ hole and the existence of a small energy scale T_K^* ($\ll D^*$) itself have been verified by the density-matrix renormalization-group calculations for the one-dimensional model of the extended periodic Anderson model with the negative potential for the conduction hole at Yb site, just as in the case where the phase diagram of the QCVT and the development of correlation of the nearest-neighbor-site Cooper pairing were discussed in S. Watanabe, M. Imada, and K. Miyake, J. Phys. Soc. Jpn. **75**, 043710 (2006). The details of the calculations will be published elsewhere.
[31] K. Satoh, T. Fujita, Y. Maeno, Y. Ōnuki, and T. Komatsubara, J. Phys. Soc. Jpn. **58**, 1012 (1989).
[32] S. Raymond and D. Jaccard, J. Low Temp. Phys. **120**, 107 (2000).
[33] Y. Hirose, J. Sakaguchi, M. Ohya, M. Matsushita, F. Honda, R. Settai, and Y. Ōnuki, J. Phys. Soc. Jpn. **81**, SB009 (2012).
[34] K. Miyake, J. Phys.: Condens. Matter **19**, 125201 (2007).
[35] S. Watanabe, A. Tsuruta, K. Miyake, and J. Flouquet, Phys. Rev. Lett. **100**, 236401 (2008).
[36] F. J. Rogers, H. C. Graboske, Jr., and D. J. Harwood, Phys. Rev. A **1**, 1577 (1970).
[37] S. F. Mott, Proc. R. Soc. London, Ser. A **382**, 1 (1982).
[38] See, for example, J. M. Ziman, *Principles of The Theory of Solids*, 2nd ed. (Cambridge University Press, London, 1972), Sects. 5.2 and 4.3.
[39] C. M. Varma, Phys. Rev. Lett. **55**, 2723 (1985).