Electromagnon on the surface of a magnetic topological insulator

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We investigate theoretically the electromagnon on the surfaces of the magnetic topological insulator thin films. It is found that when the magnetic asymmetry between the top and bottom surfaces is there, the ferromagnetic resonance is driven by the electric field which is about ten times more efficient compared with that by the magnetic field. The resonant frequency of the electromagnon is also estimated.

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I. INTRODUCTION

Nowadays topological insulator (TI) is one of the central topics in condensed matter physics [1–4]. The two major features are topologically nontrivial bulk band structures and the existence of the gapless surface state whose low-energy effective Hamiltonian is described by the spin-momentum locked Weyl Hamiltonian [2,5,6]. The electromagnetic response owing to the topological bulk band is called topological magnetoelectric effect [1–4,7,8]. It is described by the axion Lagrangian density $\mathcal{L} = \theta(\alpha/4\pi^2)\boldsymbol{E}\cdot\boldsymbol{B}$ with $\alpha = e^2/\hbar c$ the fine structure constant, and $\theta = \pi(0)$ for the topologically nontrivial (trivial) insulator. The associated electromagnetism is described such that a magnetization is induced by an electric field as $4\pi\boldsymbol{M} = \alpha\boldsymbol{E}$ while a magnetic field generates an electric polarization as $4\pi\boldsymbol{P} = \alpha\boldsymbol{B}$. This indicates that the bulk TI exhibits the electromagnetism as multiferroics, which are the materials where magnetism and ferroelectricity coexist [9–11]. It clearly represents that the topological magnetoelectric effect is nothing but the linear magnetoelectric effect which is an inherent property of multiferroics. The multiferroics also have interesting magnetoelectric optical properties such as the emergence of the electromagnon having a natural frequency in the GHz to THz regime and directional dichroism [11]. Likewise the bulk TI, the surface state of the TI also exhibits the physical multifunctionality, e.g., spin-charge transport phenomena, the magnetization dynamics in ferromagnet coupled TI, and optical phenomena intrinsic to the spin-momentum locking [12–24]. We then naturally expect that these rich TI surface properties enable us to realize the multiferroic magnetoelectric optical phenomena.

In this paper we study the magnon of the ferromagnet on the surface of magnetic TI. Especially, we focus on the thin films of magnetic TI, which are now experimentally realized by magnetic doping, for instance, by Cr, V, and Mn ions [25–35], or coating the magnetic insulator (MI) such as EuS to TI (MI/TI heterostructure) [34,36]. The examples of three-dimensional TI materials are Bi_2Se_3, Bi_2Te_3, their compounds $Bi_2(Se_xTe_{1-x})_3$, and Sb_2Te_3.

We demonstrate that when an electromagnetic field (emf) is applied to the magnetic TI and the surface state is in the quantized anomalous Hall state, the magnetization couples not only to the magnetic field but also to the electric field so that the magnetization behaves as the electric polarization. In such circumstance, the ferromagnetic resonance (FMR) due to the electric field is much stronger than that by the magnetic field. This indicates that the magnon as the fluctuation of the electric polarization, i.e., the electromagnon, is induced. Such an electromagnon can be created when the exchange coupling between the magnetization and surface state is large enough and a magnetic anisotropy along the out of surface plane is strong. We then analyze in what conditions does this electromagnon emerge in terms of the helicities of the surface states, magnetization direction, the exchange coupling signs, and the helicity of emf. Here we use the terminology "helicity" for the TI surface states to describe how the spin and momentum are locked. Mathematically, the helicity of the TI surface state is defined by the inner product between momentum vector and the vector product of an unit vector perpendicular to the surface and a spin vector whose components are described by the Pauli matrices. Two TI surface states, top and bottom surface states, have opposite helicities. For emf field we use it to distinguish whether the light is left or right circularly polarized.

We show that the relevant quantities to create the electromagnon are the helicities of the surface states, the magnetization direction, and the emf helicity. The electromagnon emerges when two surfaces have asymmetric magnetization configuration and the emf applied with the appropriate combination of the emf helicity and magnetization direction. Such a situation can be realized by making the system into a semimagnetic thin film or by surface doping using two different types of magnetic ions. Furthermore, we analyze the resonant frequency of the electromagnon.

II. MODEL

We first focus on one of the surfaces, say the top surface in the two-dimensional plane $\boldsymbol{x} = (x, y)$ located at $z = 0$. The surface state is coupled to an applied emf and the magnetization through the exchange coupling. The Hamiltonian of this

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system is

$$\mathcal{H} = v_{\rm F}(\sigma^y \Pi_x - \sigma^x \Pi_y) - \frac{JS}{2} n_a(\boldsymbol{x},t)\sigma^a, \tag{1}$$

where $v_{\rm F}$ is the Fermi velocity of the surface state, σ^a ($a = x,y,z$) is the Pauli matrices describing the spin, $\Pi_j = -i\hbar\partial_j - eA_j$ ($j = x,y$) with $p^j = -i\hbar\partial_j$ the momentum operator, $-e < 0$ is the electron charge, and A_j is the external electromagnetic vector potential, respectively. The summation for the spin index a is taken here. $J = J^* n_0$ is the exchange coupling with J^* being intrinsic to the ferromagnetic materials, and $n_0 = 1/a_{\rm m}^2$ is the averaged two-dimensional density of magnetic ions with $a_{\rm m}$ the separation between the localized spin in the xy plane. The vector n_a ($a = x,y,z$) is a normalized localized spin field with S its spin magnitude. We assume that the ferromagnet has an easy axis parallel to the z direction and treat S_z as a constant. We analyze the fluctuations of the in-plane magnetization $n_{x,y}$ generated by the applied emf.

From Eq. (1) one can define the generalized vector potential as

$$\mathcal{A}_j(\boldsymbol{x},t) = A_j(\boldsymbol{x},t) + a_j(\boldsymbol{x},t) \quad (j = x,y), \tag{2}$$

where we have defined the emergent vector potential

$$a_x(\boldsymbol{x},t) = \frac{JS}{2ev_{\rm F}} n_y(\boldsymbol{x},t), \quad a_y(\boldsymbol{x},t) = -\frac{JS}{2ev_{\rm F}} n_x(\boldsymbol{x},t). \tag{3}$$

When the Fermi energy is inside the gap, $\Delta_{\rm M} = |JSn_z|$, induced by the z component of the magnetization, the system shows the quantized anomalous Hall effect, and the electromagnetic response of the system is characterized by the half-quantized Hall conductance $\sigma_H = \frac{e^2}{2h}\mathrm{sgn}(Jn_z)$ such that the electric current density takes the form

$$J_j(\boldsymbol{x},t) = \epsilon_{jk}\sigma_H(E_k + e_k), \tag{4}$$

where $E_k = -\partial_t A^k$ ($=\partial_t A_k$) and $e_k = -\partial_t a^k$ ($=\partial_t a_k$) are the external and emergent electric fields, respectively. ϵ_{jk} is the antisymmetric tensor with $\epsilon_{xy} = -\epsilon_{yx} = 1$. The above quantized Hall current originates from the Chern-Simons term.

Now let us focus on the second term of the Hall current in Eq. (4) which is described as the response to the emergent electric field. From the relation between the electric polarization and the polarization current $\boldsymbol{P} = \partial \boldsymbol{j}_P/\partial t$, we see that the in-plane magnetization $n_{x,y}$ can be identified with the two-dimensional electric polarization [13]

$$P_j^M(\boldsymbol{x},t) = -\frac{JS\sigma_H}{2ev_{\rm F}} n_j(\boldsymbol{x},t). \tag{5}$$

Thus, Eq. (5) reflects that the surface of the magnetic TI exhibits both the ferromagnetism and the ferroelectricity, i.e., the multiferroicity. Putting the Fermi velocity $v_{\rm F} = 4 \times 10^5$ m/s and $J^* = 145$ meV nm^2 [28], this electric polarization is estimated as $P_j^M \cong 3.51 \times 10^{-30} n_0 S n_j$ C/m when the density of the magnetic ions n_0 is measured in the unit of m^{-2}.

III. FERROMAGNETIC RESONANCE AND ELECTROMAGNON

We now study the FMR on the surface of the magnetic TI. The incident light propagates normal to the surface plane. We apply the left (right) circular polarized light $E_x = E_0 e^{-i\omega t}$, $E_y = +(-)iE_0 e^{-i\omega t}$, or the linear polarized light $E_x = E_x^0 e^{-i\omega t}$, $E_y = E_y^0 e^{-i\omega t}$, with E_0, E_x^0, and E_y^0 as real constants. Hereinafter we abbreviate the left (right) circular polarized light as LCPL (RCPL). We show that the resonance due to the electric field is stronger than that by the magnetic field implying the electromagnon creation. We also study the condition for the emergence of this electromagnon. The resonant frequency of this magnon is also being estimated.

The magnetization in Eq. (5) couples to the applied emf as

$$\begin{aligned} H_{em} &= -\left(P_j^M E_j + \hbar\gamma_e S n_0 n_j B_j\right) \\ &= -n_0 S\left(-\frac{J^*\sigma_H}{2ev_{\rm F}} E_j + \hbar\gamma_e B_j\right) n_j, \end{aligned} \tag{6}$$

where $j = x,y$ and $\gamma_e = 1.76 \times 10^{11}$ rad/T s is the gyromagnetic ratio of the electron. Here we set a g factor of localized spin to be 2 [37]. Let us estimate the coupling strength between the electric field and the magnetization for the case of magnetic doping by using the same values for the exchange coupling J^* and the Fermi velocity $v_{\rm F}$ presented previously. Then the coupling strength between the electric field and a single localized spin is estimated as $1.05 \times 10^{-22} n_0 S$ J/T m^2 where we used the relation $E_x = \tilde{c}B_y$ and $E_y = -\tilde{c}B_x$ with $\tilde{c}$ the speed of light in the TI. It is described as $\tilde{c} = c/\sqrt{\epsilon_{\rm TI}}$, where $c = 3.0 \times 10^8$ m/s is the speed of light in the vacuum and $\epsilon_{\rm TI}$ is the relative dielectric constant of TI, which is around 100 [38]. In contrast, the Zeeman interaction strength in Eq. (6) is about $1.86 \times 10^{-23} n_0 S$ J/T m^2. Thus we see that the coupling strength due to the electric field is much stronger than that owing to the magnetic field. This indicates that when the emf is applied to the surface of the magnetic TI, instead of the ordinary magnon created by the magnetic field, the electromagnon can be dominantly generated due to the nature of the spin-momentum locking. Here we note that in Ref. [39] the magnetic surface gap induced by the out-of plane magnetization has been estimated for Mn-doped TI by the first-principle calculation which is 16 meV. The surface gap reported in Ref. [28] is around 60 meV, and therefore they are comparable implying that the coupling between the electric field and the magnetization n_i exceeding the Zeeman coupling can also be realized for Mn-doped TI.

Next we study the dynamics of the magnetization up to the linear order in the electric field and approximate n_z as $\mathrm{sgn}(n_z)$. The relevant action terms are the Berry phase term, magnetic anisotropic energy term, and the Chern-Simons terms represented by the generalized gauge potential in Eq. (2). It generates the coupling between electric field and magnetization described by the first term in the second line of Eq. (6). The Zeeman coupling term in Eq. (6) is going to be neglected since it is much smaller than the electric-field channel coupling. We assume the transverse emf $\boldsymbol{E}(\boldsymbol{x},z,t) = (\bar{E}_x, \bar{E}_y, 0)e^{i(k_z z - \omega t)}$, $\boldsymbol{B}(\boldsymbol{x},z,t) = (\bar{B}_x, \bar{B}_y, 0)e^{i(k_z z - \omega t)}$ is applied ($\bar{E}_x$, $\bar{E}_y$, $\bar{B}_x$, and $\bar{B}_y$ are complex constants, whereas k_z is the wave number and ω is the dispersion satisfying $\omega = \tilde{c}k_z$). The equation of motion for the magnetization n_j is

$$\begin{aligned} \dot{n}_x + \omega_0 n_y &= -\frac{JeE_y}{8\pi\hbar v_{\rm F} S C_1}\mathrm{sgn}(Jn_z), \\ \dot{n}_y - \omega_0 n_x &= -\frac{JeE_x}{8\pi\hbar v_{\rm F} S C_1}\mathrm{sgn}(Jn_z), \end{aligned} \tag{7}$$

where the dot "·" represents the time derivative and

$$\omega_0 = \frac{K a^2 a_z n_0}{C_1}, \tag{8}$$

$$C_1 = \frac{\hbar n_0}{S}\mathrm{sgn}(n_z) + \frac{J^2}{16\pi\hbar v_{\mathrm{F}}^2}\mathrm{sgn}(J n_z), \tag{9}$$

with K a magnetic anisotropy constant. Equation of motion (7) describes a forced oscillation of the in-plane magnetization n_j where the sum between the Berry phase term and anisotropic energy term play a role of oscillator with a resonant frequency given by Eq. (8), while the Chern-Simons term acts as a time dependent external force in terms of the electric field. By using the physical parameters shown in the previous discussion, we have $(\hbar n_0/S)\mathrm{sgn}(n_z) \simeq (1.29\times10^{-16}/S)\mathrm{sgn}(n_z)$ J s/m^2 and $J^2/16\pi\hbar v_{\mathrm{F}}^2 \simeq 9.50\times10^{-19}$ J s/m^2. Here we have used proximate values of data for lattice constants of tetradymites as $a = 4.2$ Å and $a_z = 30$ Å, where a and a_z are the lattice constants for the in-plane and z directions, respectively [40]. Then the separations for localized spin in the in-plane and z directions are given by $a_{\mathrm{m}} = a\times10^{1/3} \simeq 9.05$ Å, and $a_{\mathrm{m},z} = a_z\times10^{1/3} \simeq 6.46\times10$ Å, respectively. Here we assumed 10% magnetic doping. Thus, the coefficient C_1 in Eq. (9) is approximated as $C_1 \approx (\hbar n_0/S)\mathrm{sgn}(n_z)$, and subsequently, for the frequency (8) we obtain $\omega_0 \approx (K a_{\mathrm{m}}^2 a_{\mathrm{m},z} S/\hbar)\mathrm{sgn}(n_z)$. By assuming the solution for Eq. (7) in a form $n_j = \bar{n}_j e^{-i\omega t}$ ($\bar{n}_j$ is a complex constant), for the top surface we have the solution

$$n_x^{\mathrm{T}} = \frac{C_2^{\mathrm{T}}\left(i\omega\bar{E}_y + \omega_0^{\mathrm{T}}\bar{E}_x\right)}{v_{\mathrm{F}}\left[\left(\omega_0^{\mathrm{T}}\right)^2 - \omega^2\right]}e^{-i\omega t},$$

$$n_y^{\mathrm{T}} = \frac{C_2^{\mathrm{T}}\left(-i\omega\bar{E}_x + \omega_0^{\mathrm{T}}\bar{E}_y\right)}{v_{\mathrm{F}}\left[\omega^2 - \left(\omega_0^{\mathrm{T}}\right)^2\right]}e^{-i\omega t}, \tag{10}$$

where

$$C_2^{\mathrm{T}} = \frac{|J^{\mathrm{T}}|e}{8\pi\hbar^2 n_0^{\mathrm{T}}}. \tag{11}$$

We introduced the superscript T representing the top surface. The physical quantities such as the exchange coupling J^{T}, the averaged two-dimensional density n_0^{T}, and the z-component normalized magnetization n_z^{T} in Eq. (10) are those for the top surface. $\omega_0^{\mathrm{T}} = [K^{\mathrm{T}}(a_{\mathrm{m}}^{\mathrm{T}})^2 a_{\mathrm{m},z}^{\mathrm{T}} S^{\mathrm{T}}/\hbar]\mathrm{sgn}(n_z^{\mathrm{T}})$ is the resonant frequency of the magnetization on the top surface. Assuming $\omega > 0$, when the LCPL (RCPL) is applied the electric-induced FMR is generated only when the out-of-plane magnetization n_z^{T} is positive (negative). On the other hand, the magnetization dynamics for the linear polarized light is obtained by setting $\bar{E}_x$ and $\bar{E}_y$ as real constants. In this case, the electric-induced FMR is generated for both n_z^{T} being positive and negative.

Next, let us estimate the resonant frequency of the electromagnon $f_0 = \omega_0/2\pi$. In Ref. [27] the out-of plane anisotropic magnetic field B_K and the saturation magnetization M_S were measured in the Cr-doped TI thin films as $B_K = 0.9$ T and $M_S = 16\times10^3$ J/T m^3. By using the relation $K = B_K M_S/2$, the magnetic anisotropic constant becomes $K = 7.2\times10^3$ J/m^3, and the resonant frequency is estimated as $f_0 \approx 5.75S\mathrm{sgn}(n_z)\times10$ GHz. We note that we have not included the Zeeman interaction between the spin of TI surface state and the applied magnetic field in the Hamiltonian in Eq. (1). When we include this, we obtain the term $(J\gamma_e S/16\pi v_{\mathrm{F}}^2\tilde{c})\mathrm{sgn}(J n_z)\dot{E}_j$ in the equations of motion (7). In the 100 GHz range, it is about 10^5 times smaller than the term on the right-hand side of Eq. (7). Thus, such Zeeman term is negligible in the dynamics we consider.

In the magnetic TI thin film, the top and bottom surfaces act as a single surface. Thus, we need to discuss whether the FMR is triggered or not by the total magnetization dynamics of top and bottom surfaces, and indeed, this is what we observe in the experiment. To obtain this, we also need to analyze the magnetization dynamics on the bottom surface. It can be easily done by applying the similar argument for the top surface. First, the magnetization for the bottom surface corresponding to Eq. (10) is obtained by the replacement $v_{\mathrm{F}} \to -v_{\mathrm{F}}$ since the helicities of top and bottom surfaces are opposite. Second, the resonant frequency is in the order of 100 GHz and we have $k_z d_z = \omega d_z/\tilde{c} < 10^{-3} \ll 1$. The plane-wave part of the emf on the bottom surface can be approximated as $e^{i(k_z d_z - \omega t)} \approx e^{-i\omega t}$, where we took the thickness of the TI thin film $d_z = 100$ Å. This indicates that the electric field is spatially homogeneous in the magnetic TI thin film. Two surface states penetrate inside the magnetic TI and experience the same electric field. Finally, the magnetization dynamics on the bottom surface corresponding to Eq. (7) is described by replacing the quantities such as magnetic anisotropy energy, the averaged two-dimensional local spin density, the out-of-plane magnetization, and the exchange coupling for the bottom surface ones. The total in-plane magnetization $n_j = n_j^{\mathrm{T}} + n_j^{\mathrm{B}}$ becomes

$$n_x = \left[\frac{C_2^{\mathrm{T}}\left(i\omega\bar{E}_y + \omega_0^{\mathrm{T}}\bar{E}_x\right)}{\left(\omega_0^{\mathrm{T}}\right)^2 - \omega^2} - \frac{C_2^{\mathrm{B}}\left(i\omega\bar{E}_y + \omega_0^{\mathrm{B}}\bar{E}_x\right)}{\left(\omega_0^{\mathrm{B}}\right)^2 - \omega^2}\right]\frac{e^{-i\omega t}}{v_{\mathrm{F}}},$$

$$n_y = \left[\frac{C_2^{\mathrm{T}}\left(-i\omega\bar{E}_x + \omega_0^{\mathrm{T}}\bar{E}_y\right)}{\omega^2 - \left(\omega_0^{\mathrm{T}}\right)^2} - \frac{C_2^{\mathrm{B}}\left(-i\omega\bar{E}_x + \omega_0^{\mathrm{B}}\bar{E}_y\right)}{\omega^2 - \left(\omega_0^{\mathrm{B}}\right)^2}\right]\frac{e^{-i\omega t}}{v_{\mathrm{F}}}, \tag{12}$$

where

$$C_2^{\mathrm{B}} = \frac{|J^{\mathrm{B}}|e}{8\pi\hbar^2 n_0^{\mathrm{B}}}. \tag{13}$$

ω_0^{B} is the resonant frequency for the magnetization at the bottom surface. In the following, we discuss the electric-field induced FMR in the magnetic TI thin film based on Eq. (12) for various magnetization configurations.

(i) *Symmetric surface doping*—We consider the case when the single type of magnetic ions, for instance, Cr, Mn, and V, are doped [25–35]. The magnetic doping is performed symmetrically on the two surfaces. The magnetization is generated such that the exchange coupling, the two-dimensional local spin density, the z component of magnetization, and the resonant frequency of two surfaces are identical. The only difference is the two TI helicities which are opposite, and this leads to the cancellation of the magnetization dynamics between the

two surfaces. Thus, the electric-field induced FMR and the associated electromagnon are not generated.

(ii) *Asymmetric surface doping*—We next consider the case when the magnetic doping is performed asymmetrically on the two surfaces. One way is to dope the magnetic ions only to the single surface, i.e., the semimagnetic thin film configuration [30]. Then one of the surfaces shows the electric-induced FMR and is described by Eq. (10). Another way is to perform the magnetic doping using two types of ions, for instance, doping V ions to the top surface while Cr ions to the bottom surface [35]. In such circumstance, since $C_2^{\rm t} \neq C_2^{\rm b}$ and $\omega_0^{\rm t} \neq \omega_0^{\rm b}$, there is no magnetization cancellation between the top and bottom surfaces, and we will observe the electric-field induced FMR as well as the electromagnon.

(iii) *MI/TI heterojunction*—We also examine the case for MI/TI heterostructure settings. When EuS, EuO, or YIG are used as a ferromagnetic insulator, it seems to be difficult to realize the FMR and the associated electromagnon. This is because the coupling which generates the electromagnon [the first term in Eq. (6)] is realized when the surface state is in the quantized anomalous Hall state with a surface gap larger than the light frequency. When the magnetic anisotropy is along the in-plane direction [34,36], the surface gap is small. Thus, it is hard to observe the electromagnon. Instead of them, using an antiferromagnetic insulator MnSe might be a good choice. It has been investigated by the first-principle calculation that the surface gap about 54 meV can be induced [41], while in Ref. [42] the gap of the TI surface state is 8.5 meV. Both values are comparable with the magnetic surface gap obtained in [28] which is around 60 meV. Although the electronic states and the associated ferromagnetism at the interface between the MI and TI are quite complicated in MI/TI heterostructured systems, we still believe that the electromagnon on the surface TI can also be created for these systems with such choice.

Consequently, in order to generate the electric-field induced FMR and electromagnon, we have to prepare the asymmetric magnetization configurations between two surfaces so as to prevent from the cancellation originating from the opposite TI surface-state helicities. This can be done by either the magnetic doping or coating the MI on one of the surfaces of TI. Once such asymmetric magnetization configurations are set up, the electromagnon emerges with the proper combinations of the magnetization direction and emf helicity.

IV. CONCLUSION

In this paper we have investigated the FMR on the surface of magnetic TI thin film. We have found that when the exchange coupling is large enough around 10 meV, the magnetization behaves as two-dimensional electric polarization and couples to the electric field so that such coupling exceeds the Zeeman coupling. This reflects that the surface of the magnetic TI exhibits the multiferroics. As a result, the FMR can be induced by the electric field more efficiently than the magnetic field and the associated electromagnon as a fluctuation of the two-dimensional electric polarization (in-plane magnetization) is generated.

To analyze the magnetization dynamics in the magnetic TI thin film, we needed to consider the contributions from the two surfaces. From the equation of motion (7) and the solution (12), we have carefully analyzed in what conditions does the electromagnon emerges by focusing on the magnetization configuration, the exchange-coupling sign, the surface-state helicities, and the emf helicity. We demonstrated that to generate the electromagnon we have to create the asymmetry between the top and bottom surfaces. This could be done, for instance, by doping the magnetic ions only on the single surface or using two different kinds of ions. Besides the magnetic doping, this electric-field induced FMR and the associated electromagnon can also be realized by the MI/TI heterostructure with the proper choice of MI. If we could observe that the electric-field induced FMR is inactive in the symmetric configuration while the activation of that in the asymmetric configuration, it will be the strong experimental evidence of the electromagnon emergence. Another way to experimentally verify the electromagnon emergence is to compare the FMR spectrum due to the light with the electric field polarized in the in-plane direction, which propagates into the z direction, and due to the light with the electric field polarized in the z direction propagating parallel to the in-plane. For the former case, the electric-field induced FMR is generated, whereas in the latter case only the magnetic-field induced FMR is realized. This is because the z component of the electric field does not couple to the in-plane magnetization and does not generate the electromagnon. The observation of these two different FMR spectra is another strong evidence of the electromagnon emergence. Finally, we have estimated the resonant frequency of the electromagnon for the 10% Cr doping, which was on the order of 100 GHz. By increasing the magnetic anisotropic energy or the saturation magnetization about one order of magnitude greater than the above case, we may create the electromagnon in the THz regime, implying that we can perform the ultrafast manipulation of the magnetization by the electric field owing to the spin-momentum locking of the surface TI.

We note that in Ref. [14] the Hall-current (or the electric field) induced magnetization switching, i.e., the inverse spin-Galvanic effect, and the associated magnon was pointed out. Furthermore, in Ref. [22] the effect on conductivity due to the electric-field induced magnon has been presented. Although the electric-field induced magnon has been referred to in these articles, in our paper we have found some new insights. First, we have numerically shown that the coupling between the electric field and magnetization mediated by the TI surface state is stronger than that between the magnetic field and magnetization, i.e., the possibility of the electric-field induced FMR. For doing this we have used the recent experimental data and compared with the results due to the first-principle calculation. Second, we then have focused on two surfaces and examined in what conditions in terms of the magnetization configuration and its direction as well as the surface-state and emf-field helicities does the electromagnon emerge on TI surfaces. Our result indicates not only the possibility of the electromagnon emergence but may guide us to explore the hidden electromagnetic and optical properties of surfaces of TI as multiferroics, or those of other two-dimensional multiferroic materials.

Recently, the FMR has been performed in magnetic TI [21,31]. The condition adopted in these studies, however, are

different from ours: The magnetic TI in Ref. [21] consists of permalloy ($Ni_{81}Fe_{19}$) and TIs where the surface states are in the metallic regime. In Ref. [31], the magnetization induced by the Mn doping has an easy axis parallel to the surface-plane alignment. Once the conditions for the magnetization and TI surface states proposed in our paper are applied experimentally, the electromagnon may be observed by the FMR in the near future.

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