# Influence of exchange scattering on superfluid $^3$He states in nematic aerogel

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The superfluid state in bulk liquid $^3$He is realized in the form of A or B phases. Uniaxially anisotropic aerogel (nafen) stabilizes the transition from the normal to the polar superfluid state which on further cooling transitions to the axipolar orbital glass state [Dmitriev, Senin, Soldatov, and Yudin, Phys. Rev. Lett. **115**, 165304 (2015)]. This is the case in nafen aerogel preplated by several atomic layers of $^4$He. When pure liquid $^3$He fills the same nafen aerogel a solid-like layer of $^3$He atoms coats the aerogel structure. The polar state is not formed anymore and a phase transition occurs directly to the axipolar phase [Dmitriev, Soldatov, and Yudin, Phys. Rev. Lett. **120**, 075301 (2018)]. The substitution of $^4$He by $^3$He atoms at the aerogel surface changes the potential and adds the exchange scattering of quasiparticles on the aerogel strands. A calculation shows that both of these effects can decrease the degree of anisotropy of scattering and suppress the polar phase formation. The derived anisotropy of the spin diffusion coefficient in globally anisotropic aerogel is determined by the same parameter which controls the polar state emergence, which allows one to check the effect of anisotropy change for different types of covering.

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## I. INTRODUCTION

The superfluid state of liquid $^3$He is formed by means Cooper pairing with spin and orbital angular momentum equal to 1. In isotropic space the phase transition depending on pressure occurs in either the A or B superfluid phase [1]. Investigation of superfluid phases in high porosity aerogel allows one to study the influence of impurities on superfluidity with nontrivial pairing [2,3]. It has been found that similar to bulk $^3$He two superfluid A-like and B-like phases exist in $^3$He in aerogel [4]. However, both the superfluid fraction and the temperature at which the superfluid is manifested are suppressed from their bulk values [2]. The interesting possibility is opened in globally anisotropic aerogel of lifting the degeneracy between the different superfluid phases with $p$ pairing. It was shown [5] that in the case of easy-axis anisotropy a new superfluid phase of $^3$He, the polar phase, is stabilized below the transition temperature. It was also predicted [5] that on further cooling a second-order transition into a polar-distorted A phase should occur. Indeed, quite recently, the first observation of the polar phase was reported [6]. This phase appears in $^3$He confined in new type of "nematically ordered" aerogel called "nafen" with a nearly parallel arrangement of strands which play the role of ordered impurities. It was shown that in nafen the transition to the superfluid state always occurs to the polar phase and the region of its existence increases with density of strands. In another type of nematically ordered but less dense and much less anisotropic aerogel called "Obninsk aerogel" [6] the superfluid state is always formed in the orbital glassy A-like state.

To avoid a paramagnetic signal from surface solid $^3$He, the nafen samples in the measurements [6] were preplated by $\sim$2.5 $^4$He monolayers. Then a new experiment series was performed with the same samples filled by pure $^3$He [7]. In this case the temperature of the superfluid transition is suppressed more strongly and this effect increases with strand density such that in the most dense nafen the superfluid transition was not detected down to the lowest attained temperatures. The superfluid transition occurs directly to the polar-distorted A phase without the formation of an intermediate region of polar state. The small addition of $^3$He in the surface $^4$He layer, corresponding to 0.1 monolayer, also completely kills superfluidity at 29.3 bar in the most dense nafen, and in the less dense aerogel noticeably suppresses the critical temperature. In this case also the transition occurs directly to the distorted A state. Thus, the situation looks as if the $^3$He covering suppresses anisotropy necessary for the existence of the polar phase.

There was pointed out in Ref. [7]: "The observed phenomena cannot be explained by a change of the scattering specularity because they are observed also at high pressures where the scattering should be diffusive regardless of the presence or absence of solid $^3$He." This statement is based on previous studies (see Refs. 5–9 in the paper [7]) of the degree of specularity of $^3$He quasiparticles scattering on metallic surfaces with different coverings. The corresponding information for liquid $^3$He filling nafen aerogel is absent. But, generally speaking, the substitution of $^4$He by $^3$He atoms at aerogel surface changes the potential and adds the exchange scattering of quasiparticles on the aerogel strands.

I study this problem taking into account both the potential and the exchange scattering of quasiparticles of liquid $^3$He on $^3$He atoms localized at the strands surface. In Sec. II I show that $^3$He covering changes the intensity and the anisotropy of scattering. In Sec. III, the presented derivation of spin diffusion current shows that the anisotropy of the spin diffusion coefficient in globally anisotropic aerogel is expressed through the same parameter that determines the polar state emergence. Thus, being measured, the anisotropy decrease of spin diffusion in nafen filled by pure $^3$He can serve as a direct

 

indication of suppression of the temperature interval of polar state existence.

## II. SUPERFLUID $^3$He IN UNIAXIALLY ANISOTROPIC AEROGEL WITH MAGNETIC SCATTERING

The order parameter of superfluid phases of $^3$He is given [1] by the complex $3 \times 3$ matrix $A_{\alpha i}$, where $\alpha$ and $i$ are the indices enumerating the Cooper pair wave function projections on spin and orbital axes respectively. All the phases with different order parameters $A_{\alpha i}$ have the same critical temperature. The degeneracy is lifted by the fourth-order terms with respect to $A_{\alpha i}$ in the Landau expansion of the free energy density. The most energetically profitable are the B phase with the order parameter $A^B_{\alpha i} = \Delta R_{\alpha i} e^{i\varphi}$, where $R_{\alpha i}$ is a rotation matrix, and (in the high pressure region) the A phase with the order parameter

$$A^A_{\alpha i} = \Delta V_\alpha (m_i + i n_i), \tag{1}$$

where $\mathbf{V}$ is the unit spin vector and $\mathbf{m}$ and $\mathbf{n}$ are the orthogonal unit vectors such that $\mathbf{m} \times \mathbf{n} = \mathbf{l}$ is the unit vector directed along the Cooper pair angular momentum.

The different pairing states of superfluid $^3$He in a random medium with global uniaxial anisotropy can be compared by making use the second-order terms in the Landau free energy density. They consist of an isotropic part, common to all the superfluid phases with $p$ pairing, and the anisotropic part

$$\begin{aligned} F^{(2)} &= F_i^{(2)} + F_a^{(2)} \\ &= \alpha_0 \left( \frac{T}{T_c} - 1 \right) A_{\alpha i} A^\star_{\alpha i} + \eta_{ij} A_{\alpha i} A^\star_{\alpha j}, \end{aligned} \tag{2}$$

where $T_c = T_c(P)$ is the transition temperature in the superfluid state suppressed with respect of transition temperature in the bulk liquid $T_{c0}(P)$. The medium uniaxial anisotropy with anisotropy axis parallel to $\hat{z}$ direction, coincident in our case with the average direction of aerogel strands, is given by the traceless tensor

$$\eta_{ij} = \eta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \tag{3}$$

In the absence of global anisotropy ($\eta = 0$) all $p$-wave phases have the same critical temperature. At positive $\eta > 0$ the polar state with the order parameter of the form

$$A_{\alpha i} = a V_\alpha z_i, \tag{4}$$

where $V_\alpha$ is the unit spin vector, has the lowest energy of anisotropy [8],

$$F_a = -2\eta |a|^2. \tag{5}$$

Hence, it has the highest critical temperature $T_{c1}$ of transition from the normal state. At some lower temperature $T_{c2}$ the polar state passes to the more energetically profitable distorted A state [9] with the order parameter

$$A_{\alpha i} = V_\alpha [a \hat{z}_i + i b (\hat{x}_i \cos\varphi + \hat{y}_i \sin\varphi)] \tag{6}$$

intermediate between the polar state at $b = 0$ and the A state at $a = b$. This state has the Cooper pair angular momentum $\hat{\mathbf{l}} = -\hat{x} \sin\varphi(\mathbf{r}) + \hat{y} \cos\varphi(\mathbf{r})$ lying in the basal plane and locally ordered [$\varphi(\mathbf{r}) = \text{const}$] on scales $L$ exceeding the coherence length $\xi_0$ but smaller than the dipole length $\xi_d$ and randomly distributed on scales larger than $L$. The pure polar state exists in the temperature interval roughly determined by the energy of anisotropy difference between of the polar and the distorted A states [9],

$$T_{c1} - T_{c2} \approx \frac{\eta}{\alpha_0} T_c. \tag{7}$$

Hence, at small $\eta$ parameter the temperature interval of the polar state existence is small and hardly observable.

The quasiparticle interaction with the nafen strands is modeled by the interaction with the randomly distributed impurities including the globally anisotropic potential and the globally anisotropic exchange part,

$$H_{\text{int}} = \sum_i \int d^3 r \psi^\dagger_\alpha(\mathbf{r}) [u(\mathbf{r} - \mathbf{r}_i)\delta_{\alpha\beta} + J(\mathbf{r} - \mathbf{r}_i)\boldsymbol{\sigma}_{\alpha\beta}\mathbf{S}]\psi_\beta(\mathbf{r}), \tag{8}$$

where $\mathbf{S}$ is the spin of the impurity and $\boldsymbol{\sigma}$ are the $^3$He quasiparticles' spin matrices. The exchange scattering in an isotropic aerogel has been considered by Sauls and Sharma [10] and by Baramidze and Kharadze [11]. They have shown that if the scattering amplitude on impurities includes an exchange part then the critical temperature splitting of $A_1$ and $A_2$ transitions under an external field $H$ decreases in comparison with the impurity free case:

$$T_{A_1} - T_{A_2} = (\gamma_0 - \gamma_{\text{imp}}) H.$$

The effect arises due to an interference between scalar and exchange scattering such that

$$\gamma_{\text{imp}} \propto u J$$

is proportional to the product of the corresponding amplitudes. In NMR experiments [6,7] the field is small and this effect is negligible, but one needs to consider an influence of the globally anisotropic scattering on critical temperature.

To find the critical temperature of superfluid transition in globally anisotropic aerogel one must calculate the second-order terms in the Landau free energy density:

$$\begin{aligned} F^{(2)} = \frac{1}{3} \Bigg\{ \frac{1}{g} \delta_{ij}\delta_{\mu\nu} - T \sum_\omega \int \frac{d^3 p}{(2\pi)^3} \hat{p}_i \Gamma^{\mu\nu}_j(\mathbf{p},\omega,) G(\mathbf{p},\omega) \\ \times G(-\mathbf{p},-\omega) \Bigg\} A^\star_{\mu i} A_{\nu j}, \end{aligned} \tag{9}$$

where $g$ is the constant of $p$-wave triplet pairing. Here,

$$G(\mathbf{p},\omega) = \frac{1}{i\omega - \xi_{\mathbf{p}} - \Sigma_{\mathbf{p}}(\omega)} \tag{10}$$

is the normal state quasiparticle Green function and $\Gamma^{\mu\nu}_j(\mathbf{p},\omega)$ is the vertex part. $\xi_{\mathbf{p}} = \varepsilon_{\mathbf{p}} - \mu$ is the quasiparticle's energy counted from the chemical potential, and $\omega = \pi T(2n + 1)$ is the fermion Matsubara frequency. The Planck constant $\hbar$ was everywhere put equal to 1. The self-energy part is given by the equation

$$\Sigma_{\mathbf{p}}(\omega) = \int \frac{d^3 p'}{(2\pi)^3} U^2_{\mathbf{p}-\mathbf{p}'} G(\mathbf{p}',\omega). \tag{11}$$

Here, according to Abrikosov and Gor'kov [12], the "impurity line" $U^2_{\mathbf{p}-\mathbf{p}'}$ arises after averaging over impurity positions and also over the orientation of the spins of all impurity atoms, $\langle S_i S_k\rangle = \frac{1}{3}S(S+1)\delta_{ik}$, where in our particular case $S = 1/2$. Then taking into account $\sigma^i_{\alpha\gamma}\sigma^i_{\gamma\alpha} = \frac{3}{4}$ we obtain

$$U^2_{\mathbf{p}} = n_i\left[u^2_{\mathbf{p}} + \left\langle S_i\sigma^i_{\alpha\gamma}S_k\sigma^k_{\gamma\alpha}\right\rangle J^2_{\mathbf{p}}\right] = n_i\left[u^2_{\mathbf{p}} + \tfrac{1}{4}S(S+1)J^2_{\mathbf{p}}\right], \tag{12}$$

where $n_i$ is impurity concentration and $u(\mathbf{p})$ and $J(\mathbf{p})$ are the Fourier transforms of the amplitudes of potential and exchange scattering from Eq. (8). According to assumption about global anisotropy they depend on the momentum direction such that

$$n_i u^2_{\mathbf{p}} = \frac{1}{2\pi N_0\tau_p}\left\{1 - \delta_p\left[\hat{p}_z^2 - \frac{1}{2}\left(\hat{p}_x^2 + \hat{p}_y^2\right)\right]\right\}, \tag{13}$$

$$n_i J^2_{\mathbf{p}} = \frac{1}{2\pi N_0\tau_{ex}}\left\{1 - \delta_{ex}\left[\hat{p}_z^2 - \frac{1}{2}\left(\hat{p}_x^2 + \hat{p}_y^2\right)\right]\right\}, \tag{14}$$

where $N_0$ is the density of states per one spin projection, $\hat{p}_i$ are the projections of momentum unit vector $\frac{\mathbf{p}}{|\mathbf{p}|}$ on the $i = (x,y,z)$ coordinate axis, $\tau_p$ and $\tau_{ex}$ are the isotropic parts of mean free time of potential and exchange scattering, and $\delta_p$ and $\delta_{ex}$ are the corresponding degrees of anisotropy. The anisotropic part of $U^2_{\mathbf{p}}$ is taken with the sign opposite to that in [5] and chosen such that $\int \frac{d\Omega}{4\pi}U^2_{\mathbf{p}}$ is independent of the anisotropic part of scattering.

So, the self-energy obtained from Eqs. (11)–(14) is

$$\Sigma_{\mathbf{p}}(\omega) = -\frac{i}{2\tau}\left\{1 - \delta\left[\hat{p}_z^2 - \frac{1}{2}\left(\hat{p}_x^2 + \hat{p}_y^2\right)\right]\right\}\operatorname{sign}\omega. \tag{15}$$

Along with the isotropic term it includes a term describing the global uniaxial anisotropy. Each of these terms consists of two independent parts: the potential part and the exchange one determined in the following way:

$$\frac{1}{\tau} = \frac{1}{\tau_p} + \frac{1}{\tau_{ex}}, \quad \frac{\delta}{\tau} = \frac{\delta_p}{\tau_p} + \frac{\delta_{ex}}{\tau_{ex}}. \tag{16}$$

The vertex part must be found from the integral equation

$$\Gamma^{\mu\nu}_j(\mathbf{p},\omega) = \hat{p}_j\delta_{\mu\nu} + n\int\frac{d^3p'}{(2\pi)^3}\left[u^2_{\mathbf{p}-\mathbf{p}'} + \frac{1}{3}S(S+1)(g^\dagger)^\mu_{\alpha\beta} \times \sigma^i_{\lambda\alpha}\sigma^i_{\rho\beta}g^\nu_{\lambda\rho}J^2_{\mathbf{p}-\mathbf{p}'}\right] \times \Gamma^{\mu\nu}_j(\mathbf{p}',\omega)G(\mathbf{p}',\omega)G(-\mathbf{p}',-\omega). \tag{17}$$

It is known [12] that for the case of singlet superconductivity the exchange part of scattering in this equation is given by

$$\tfrac{1}{3}S(S+1)g^t_{\alpha\beta}\sigma^i_{\lambda\alpha}\sigma^i_{\rho\beta}g_{\lambda\rho}J^2_{\mathbf{q}} = -\tfrac{1}{4}S(S+1)J^2_{\mathbf{q}}, \tag{18}$$

where the matrix $\hat{g} = \left(\begin{smallmatrix}0 & 1\\ -1 & 0\end{smallmatrix}\right)$, and the superscript $t$ indicates transposition. As result, there are two different "scattering times" originating from the self-energy and the vertex [12]. The corresponding combination for the triplet pairing is

$$\tfrac{1}{3}S(S+1)(g^\dagger)^\mu_{\alpha\beta}\sigma^i_{\lambda\alpha}\sigma^i_{\rho\beta}g^\nu_{\lambda\rho}J^2_{\mathbf{q}} = \tfrac{1}{4}S(S+1)J^2_{\mathbf{q}}\delta_{\mu\nu}, \tag{19}$$

where $g^\nu_{\lambda\rho} = (-\sigma^z_{\lambda\rho}, i\delta_{\lambda\rho}, \sigma^x_{\lambda\rho})$, such that the scattering times originating from the self-energy and the vertex are equal to each other. Thus, Eq. (17) is

$$\Gamma^{\mu\nu}_j(\mathbf{p},\omega) = \hat{p}_j\delta_{\mu\nu} + \int\frac{d^3p'}{(2\pi)^3}U^2_{\mathbf{p}-\mathbf{p}'}\Gamma^{\mu\nu}_j(\mathbf{p}',\omega)G(\mathbf{p}',\omega) \times G(-\mathbf{p}',-\omega), \tag{20}$$

and its solution has the form

$$\Gamma^{\mu\nu}_j(\omega,\mathbf{p}) = \left\{\hat{p}_j + \Gamma_\omega\left[\hat{p}_z\hat{z}_j - \tfrac{1}{2}(\hat{p}_x\hat{x}_j + \hat{p}_y\hat{y}_j)\right]\right\}\delta_{\mu\nu}, \tag{21}$$

where for $\delta \ll 1$

$$\Gamma_\omega = \frac{\delta}{3\tau\left|\omega + \frac{1}{2\tau}sign\omega\right|} + \mathcal{O}(\delta^2). \tag{22}$$

Substitution of the vertex $\Gamma^{\mu\nu}_j(\omega,\mathbf{p})$ and the Green function $G(\mathbf{p},\omega)$ into Eq. (9) yields

$$F_2 = \alpha A^\star_{\mu i}A_{\mu i} - 2\eta\left[A^\star_{\mu z}A_{\mu i z} - \tfrac{1}{2}(A^\star_{\mu x}A_{\mu x} + A^\star_{\mu y}A_{\mu y})\right], \tag{23}$$

where

$$\alpha = \frac{N_0}{3}\left[\ln\frac{T}{T_{c0}} + \psi\left(\frac{1}{2} + \frac{1}{4\pi T\tau}\right) - \psi\left(\frac{1}{2}\right) - \frac{1}{5}\frac{\delta}{4\pi T\tau}\psi^{(1)}\left(\frac{1}{2} + \frac{1}{4\pi T\tau}\right)\right], \tag{24}$$

$$\eta = \frac{8N_0}{45}\frac{\delta}{4\pi T\tau}\psi^{(1)}\left(\frac{1}{2} + \frac{1}{4\pi T\tau}\right). \tag{25}$$

Here, $\psi(z)$ and $\psi^{(1)}(z)$ are the digamma function and its first derivative.

At $\delta > 0$ the critical temperature of the phase transition to the superfluid state is maximal for the polar phase, Eq. (4), and is determined by the equation

$$\alpha - 2\eta = 0. \tag{26}$$

In the limit of weak scattering $4\pi T\tau_c \gg 1$ the transition to the polar state occurs at

$$T_{c1} = T_{co} - \frac{\pi}{8\tau} + \frac{11\pi}{60\tau}\delta. \tag{27}$$

It is worthwhile to recall that at small degrees of anisotropy the temperature interval of the polar state existence is small and hardly observable.

According to Eq. (16) the degree of global anisotropy $\delta$ is determined by two independent terms originating from the potential and the exchange scattering. The latter can in principle suppress the anisotropy. However, the quasiparticle self-energy and the vertex part due to exchange scattering have the same structure as for pure potential scattering. Hence, the anisotropy suppression can also originate from the potential scattering, which is different for the covering of aerogel stands by a solid $^3$He layer.

The change in anisotropy of scattering for different types of covering must also reveal itself in the changes of spin diffusion anisotropy. In the next section I derive the normal liquid $^3$He spin diffusion current flowing through media filled by randomly distributed impurities with globally anisotropic potential and exchange scattering.

## III. SPIN CURRENT

The spin current in neutral Fermi liquid is calculated [13,14] as the response to the gradient of angle of rotation of the spin space $\boldsymbol{\omega}_i = \nabla_i \boldsymbol{\theta}$,

$$\mathbf{j}_i = -\frac{\delta H}{\delta \boldsymbol{\omega}_i}, \tag{28}$$

where

$$H = \frac{1}{2m}\int d^3r \left(D_i^{\alpha\lambda}\psi_\lambda\right)^\dagger D_i^{\alpha\mu}\psi_\mu + H_{\rm int}, \tag{29}$$

$$D_i^{\alpha\beta} = -i\delta_{\alpha\beta}\nabla_i + \frac{1}{2}\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\omega}_i, \tag{30}$$

and $H_{\rm int}$ includes the Fermi liquid interaction and the interaction with impurities, Eq. (8).

At low temperatures the collisions between the Fermi liquid quasiparticles induce negligibly small correction to the spin diffusion due to the scattering on aerogel strands. On the other hand, we are mainly interested in the spin current anisotropy in the anisotropic media and will ignore the temperature dependence of exchange amplitude of scattering due to the Kondo effect [15]. In this case one can work with the field theory technique for $T = 0$. The response of the gauge field $\boldsymbol{\omega}_i$ is calculated in complete analogy with response to the usual vector potential $A_i$ in the calculation of electric current in an isotropic metal with randomly distributed impurities, performed in [16]. The spin current at finite wave vector $\mathbf{k}$ and external frequency $\omega$ is

$$\mathbf{j}_i(\mathbf{k},\omega) = \frac{i}{4m}Tr\int_{-\infty}^{+\infty}\frac{d\varepsilon}{2\pi}\int\frac{d^3p}{(2\pi)^3}p_i\boldsymbol{\sigma}_{\alpha\beta}(\boldsymbol{\sigma}_{\beta\alpha}\boldsymbol{\omega}_j)\Pi_j - \frac{1}{4}n\boldsymbol{\omega}_i, \tag{31}$$

where $n$ is the number of liquid $^3$He atoms in the unit volume, function $\Pi_j$ is determined by the equation

$$\Pi_j(p,p-k) = G(\mathbf{p},\varepsilon)G(\mathbf{p}-\mathbf{k},\varepsilon-\omega) \times\left[p_j + \int\frac{d^3p'}{(2\pi)^3}U^2(\mathbf{p}-\mathbf{p}')\Pi_j(p',p'-k)\right], \tag{32}$$

$p = (\mathbf{p},\varepsilon)$, $k = (\mathbf{k},\omega)$,

$$G(\mathbf{p},\varepsilon) = \frac{1}{\varepsilon - \xi_{\mathbf{p}} - \Sigma_{\mathbf{p}}(\varepsilon)}, \tag{33}$$

$U^2(\mathbf{p})$ is determined by Eq. (12), and $\Sigma_{\mathbf{p}}(\varepsilon)$ is given by Eq. (15). The vertex correction does not introduce changes in the spin structure of Eq. (31) due to the identity $\sigma^i_{\alpha\lambda}\sigma^p_{\lambda\mu}\sigma^p_{\rho\alpha}\sigma^j_{\mu\rho} = \sigma^i_{\alpha\beta}\sigma^j_{\beta\alpha}$.

At $\mathbf{k} = 0$, $\omega = 0$ the first term in the current expression (31) cancels out the second "diamagnetic" term. We are interested in calculating the current at $\mathbf{k} = 0$, $\omega \neq 0$. In this case,

$$\mathbf{j}_i = \frac{i}{4m}Tr\int_0^{\omega}\frac{d\varepsilon}{2\pi}\int\frac{d^3p}{(2\pi)^3}p_i\boldsymbol{\sigma}_{\alpha\beta}(\boldsymbol{\sigma}_{\beta\alpha}\boldsymbol{\omega}_j)\Pi_j(\mathbf{k}=0), \tag{34}$$

and the solution of Eq. (32) in a linear approximation with respect to $\delta$ and at $\omega\tau \ll 1$ is

$$\Pi_j(\mathbf{k}=0) = G(\mathbf{p},\varepsilon)G(\mathbf{p},\varepsilon-\omega) \times\left\{p_j + \tfrac{2}{3}\delta\left[\hat{p}_z\hat{z}_j - \tfrac{1}{2}(\hat{p}_x\hat{x}_j + \hat{p}_y\hat{y}_j)\right]\right\}. \tag{35}$$

Substituting this into Eq. (34), we obtain

$$\mathbf{j}_i = \tfrac{1}{6}\left\{\delta_{ij} + \tfrac{16}{15}\delta\left[\hat{z}_i\hat{z}_j - \tfrac{1}{2}(\hat{x}_i\hat{x}_j + \hat{y}_i\hat{y}_j)\right]\right\}i\omega\tau N_0 v_F^2\boldsymbol{\omega}_j. \tag{36}$$

Here $v_F$ is the Fermi velocity. Making use of the Larmor theorem

$$\gamma\mathbf{H} = \frac{\partial\boldsymbol{\theta}}{\partial t} = -i\omega\boldsymbol{\theta}, \tag{37}$$

where $\gamma = 2\mu$ is the gyromagnetic ratio and $\mu$ is the magnetic moment of $^3$He atoms, one can rewrite the expression for current as

$$\mathbf{j}_i = -\tfrac{1}{3}\left\{\delta_{ij} + \tfrac{16}{15}\delta\left[\hat{z}_i\hat{z}_j - \tfrac{1}{2}(\hat{x}_i\hat{x}_j + \hat{y}_i\hat{y}_j)\right]\right\}\tau N_0 v_F^2\mu\nabla_j\mathbf{H}. \tag{38}$$

To rewrite the spin current as the magnetic diffusion current one should multiply both sides of this equation by $2\mu$ to obtain

$$\mathbf{j}_i^M = -\tfrac{1}{3}\left\{\delta_{ij} + \tfrac{16}{15}\delta\left[\hat{z}_i\hat{z}_j - \tfrac{1}{2}(\hat{x}_i\hat{x}_j + \hat{y}_i\hat{y}_j)\right]\right\}\tau v_F^2\nabla_j\mathbf{M}, \tag{39}$$

where the Fermi-liquid magnetization is $\mathbf{M} = 2\mu^2 N_0\mathbf{H}$. Thus, the spin diffusion currents along the direction of nafen strands and in the direction perpendicular to them are

$$\mathbf{j}_z^M = -\tfrac{1}{3}\left\{1 + \tfrac{16}{15}\delta\right\}\tau v_7 F^2\nabla_z\mathbf{M}, \tag{40}$$

$$\mathbf{j}_x^M = -\tfrac{1}{3}\left\{1 - \tfrac{8}{15}\delta\right\}\tau v_F^2\nabla_x\mathbf{M}, \tag{41}$$

respectively. One can demonstrate that a similar calculation taking into account the Fermi liquid renormalization adds in these formulas the pre-factor $(1 + F_0^a)(1 + F_1^a/3)$.

Thus, the anisotropy of the spin diffusion coefficient is expressed through the same parameter of anisotropy $\delta$ that determines the temperature interval of the polar state existence.

## IV. CONCLUSION

It was shown that the degree of global anisotropy responsible for polar state stability is determined by two mechanisms, originating from the potential and the exchange scattering. The suppression of anisotropy narrows the temperature interval of

the polar state existence, making it hardly observable. The anisotropy decrease can be controlled by the measurements of spin diffusion because the difference in the spin diffusion coefficients in directions parallel and perpendicular to nafen strands is found to be proportional to the same parameter that determines the polar state emergence.

The authors of the paper [7] have pointed out the dominate role of the exchange scattering in anisotropy suppression (see the citation of Ref. [7] in the Introduction). However, according to the presented results, the exchange interaction yields the quasiparticle self-energy and the vertex part of the same structure as for pure potential scattering. Hence, although the exchange mechanism works only in the case of coating of aerogel strands by a solid $^3$He layer, it is possible that the main role in the anisotropy decrease is played by the change of potential scattering with aerogel strands covered by $^3$He instead of $^4$He. The problem of choosing between the two mechanisms of anisotropy suppression will be addressed in future investigations.

Being mainly interested in the role of anisotropy of exchange scattering, I neglect throughout this paper the possible temperature dependence of the amplitude of exchange scattering due to the Kondo effect. The logarithmic increase of positive ion mobility starting at $T = 50$ mK up to the superfluid transition temperature (see the paper [15] and references therein) means that the exchange scattering has a ferromagnetic character, in agreement with the notion that $^3$He is an almost ferromagnetic Fermi liquid. Thus, the Kondo effect weakens the magnitude of the pair breaking by magnetic scattering.

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