Charge and spin conductivity of a two-dimensional electron gas with a random Rashba interaction

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We calculate the transport relaxation time $\tau_{\rm tr}$ and spin transport relaxation time $\tau_{s,{\rm tr}}$ for a two-dimensional electron gas with spatially fluctuating Rashba spin-orbit interaction. These relaxation times determine the electrical and spin conductivity of the two-dimensional system, respectively. It is shown that the transport relaxation time $\tau_{\rm tr}$ is a nonmonotonic function of electron energy ε, whereas the spin transport relaxation time $\tau_{s,{\rm tr}}$ decreases with increasing ε, similarly to the conventional electron relaxation time τ that characterizes the decay of an electron state corresponding to certain values of the momentum and spin. Such a behavior of the relaxation times leads to unusual temperature dependence of the electrical and spin conductivity.

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I. INTRODUCTION

It is well known that Rashba spin-orbit interaction in two-dimensional (2D) electron systems appears due to asymmetry in confining potentials on both sides of the corresponding heterostructure [1–5]. Such an interaction leads to various spin-orbit related effects, like spin Hall effect, current-induced spin polarization, spin-orbit torque, and others [6–12]. These phenomena have been extensively studied in recent years. However, the Rashba spin-orbit coupling should disappear in two-dimensional electron systems that exhibit symmetry with respect to reflection in the 2D plane. In other words, the corresponding coupling constant [2] vanishes then by symmetry reasons. An example of such a system is a symmetric semiconductor quantum well.

However, even though the symmetry precludes the presence of a uniform Rashba interaction, such a symmetry does not exclude the existence of spatially fluctuating Rashba field with the corresponding mean value equal to zero. It was already shown in detail how the spatially fluctuating Rashba field can appear due to a deviation from homogeneity of the doping impurity distribution in the vicinity of a semiconductor quantum well [13–15]. The main characteristics of the random Rashba coupling in such systems have been studied theoretically in Refs. [13–17]. Moreover, recent experiments on scanning tunneling spectroscopy of InSb surfaces produced a pattern of the Rashba coupling with the $\sim$1 nm spatial resolution, which revealed strong randomness of this coupling [18].

It was also demonstrated that the fluctuating Rashba field can induce a variety of experimentally observable effects. For example, the spin Hall conductivity in a 2D system with homogeneous Rashba interaction vanishes in the presence of spin-independent disorder [7,8,12], but it is robust to scattering on impurities in the presence of spatially fluctuating Rashba field [19,20]. This nonzero value of spin-Hall conductivity not only agrees with the analysis based on the SU(2) symmetry of the spin-orbit coupling [21] and detailed numerical calculations [22], but also can be considered as a mechanism of the spin-charge conversion in 2D systems [23]. Furthermore, the fluctuating spin-orbit interaction is responsible for spin relaxation [14,15,24]. For instance, it is possible that the electron spin relaxation in a free standing graphene is related to the fluctuating Rashba field arising from rippled graphene sheet [15,25], random impurity-induced spin-orbit coupling [26], or strong effects of the randomness introduced by the corrugation [27]. In addition, the random Rashba fields play an important role in transport properties of the edge states in topological insulators [28–32], and also can be crucially important in systems with very strong spin-orbit coupling [33].

In this paper we consider the effect of spatially fluctuating Rashba field on the charge and spin conductivity of a 2D electron gas. We assume that the mechanism of electron scattering from the Rashba field is dominant for both momentum and spin relaxation of electrons, which may happen at very low density of impurities and defects. On the other hand, the contribution of any other scattering mechanism can be taken into account effectively by assuming a certain relaxation time (e.g., due to impurities and defects), and then by using the Matthiessen rule to add the rates related to different relaxation mechanisms.

In Sec. II we describe the model Hamiltonian assumed to describe the system with spatially fluctuating Rashba field and also introduce the Hamiltonian describing interaction of the system with external electromagnetic field. Relaxation time is calculated in Sec. III, whereas the vertex function is derived in Sec. IV. Electrical conductivity is calculated and discussed

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in Sec. V. In turn, in Sec. VI we calculate the spin current and spin conductivity. Final conclusions are in Sec. VII.

II. MODEL

To describe the 2D electron system with fluctuating Rashba field we use the following Hamiltonian:

$$\hat{H} = \hat{H}_0 + \hat{H}^{(\mathrm{so})}, \tag{1}$$

where the first term corresponds to the kinetic energy of 2D electrons with parabolic energy spectrum,

$$\hat{H}_0 = -\frac{\hbar^2\nabla^2}{2m^*}, \tag{2}$$

while the term $\hat{H}^{(\mathrm{so})}$ stands for the random Rashba spin-orbit coupling,

$$\hat{H}^{(\mathrm{so})} = -\frac{\mathrm{i}}{2}\,\sigma_x\{\nabla_y, \lambda(\mathbf{r})\} + \frac{\mathrm{i}}{2}\,\sigma_y\{\nabla_x, \lambda(\mathbf{r})\}. \tag{3}$$

Here, σ_x and σ_y are the Pauli matrices acting in the spin space, whereas $\lambda(\mathbf{r})$ is the Rashba parameter that varies randomly in the 2D space. We assume that the average value of this parameter vanishes, $\langle\lambda(\mathbf{r})\rangle = 0$, so that the random field is characterized by the correlator $\langle\lambda(\mathbf{r})\,\lambda(\mathbf{r}')\rangle$ [15,24]. The matrix elements of the Rashba spin-orbit interaction (3) in the basis of the eigenfunctions of Hamiltonian $\hat{H}_0$ are

$$\hat{H}^{(\mathrm{so})}_{\mathbf{k}\mathbf{k}'} = \frac{\lambda_{\mathbf{k}\mathbf{k}'}}{2}[\sigma_x(k_y + k'_y) - \sigma_y(k_x + k'_x)]. \tag{4}$$

Now we assume that the system is in an external electromagnetic field described by a vector potential $\mathbf{A}(t) = \mathbf{A}_0\, e^{-\mathrm{i}\omega t}$. To find the Hamiltonian which describes interaction of the system under consideration with the electromagnetic field, we make the replacement: $\mathbf{k} \to \mathbf{k} - e\mathbf{A}/\hbar c$. Accordingly, the corresponding spin-orbit dependent part of the interaction with electromagnetic field can be written as

$$\hat{H}^{(\mathrm{so})-A}_{\mathbf{k}\mathbf{k}'} = -\frac{e\lambda_{\mathbf{k}\mathbf{k}'}}{c}(\sigma_x A_y - \sigma_y A_x). \tag{5}$$

When taking into account also the interaction of free electrons with the electromagnetic field, the total Hamiltonian describing coupling of the system to the electromagnetic field can be written in the following form:

$$\hat{H}^{(A)}_{\mathbf{k}\mathbf{k}'} = -\frac{e\hbar\mathbf{k}\cdot\mathbf{A}}{m^*c}\,\delta_{\mathbf{k}\mathbf{k}'} + \frac{e^2A^2}{2m^*c^2}\,\delta_{\mathbf{k}\mathbf{k}'} - \frac{e\lambda_{\mathbf{k}\mathbf{k}'}}{\hbar c}(\sigma_x A_y - \sigma_y A_x). \tag{6}$$

Here, the first and second terms are the usual kinetic and diamagnetic contributions. The third term, in turn, takes into account the coupling mediated by the spin-orbit interaction and corresponds to the anomalous spin-dependent velocity in the form of the commutator $\mathrm{i}[\hat{H}^{(\mathrm{so})},\mathbf{r}]/\hbar$.

Without loss of generality, we assume that the vector potential $\mathbf{A}$ is along the x axis, and calculate the current flowing along this axis. The corresponding matrix elements of the charge current operator can be then written in the form

$$\hat{j}_{x,\mathbf{k}\mathbf{k}'} = -c\,\frac{\partial\hat{H}^{A}_{\mathbf{k}\mathbf{k}'}}{\partial A_x} = \frac{e\hbar}{m^*}\left(k_x - \frac{eA_x}{\hbar c}\right)\delta_{\mathbf{k}\mathbf{k}'} - e\lambda_{\mathbf{k}\mathbf{k}'}\sigma_y. \tag{7}$$

To calculate the electric current flowing in the system we will use the standard Kubo formalism and Green function technique in the loop approximation, with a renormalized vertex function [34,35]. To avoid issues related to electron localization, all the calculations will be performed assuming that scattering from the random Rashba field is weak. This scattering gives rise to a slow relaxation of electron states described by certain momentum and spin. The other effect due to fluctuating Rashba field is related to its correction to the current vertex, similar to the impurity-induced correction [34]. This leads to the substitution of the bare current vertex $\hat{j}_{x,\mathbf{k}\mathbf{k}'}$ by its renormalized counterpart $J_{x,\mathbf{k}\mathbf{k}'}$. Note that the vertex correction does not vanish in the limit of very weak scattering by the fluctuating field since the relative correction to the bare vertex is of the order of unity [34].

Assuming the weak scattering regime, we restrict ourselves to the first term on the right-hand side of Eq. (7). Correspondingly, we do not take into account the last (anomalous velocity-related) term in Eq. (6) related to the spin-orbit induced interaction with electromagnetic field. The above-mentioned terms lead to a negligibly small correction to the calculated conductivity. Indeed, the main contribution to the conductivity is of the order of $(e^2/\hbar)\,(\varepsilon_F\tau_{\mathrm{tr}}/\hbar)$, where ε_F is the Fermi energy and τ_{tr} is the transport relaxation time, whereas the correction related to the anomalous velocity is $\sim e^2/\hbar$. Below we use the units with $\hbar \equiv 1$, $k_B \equiv 1$ and restore $\hbar$ and k_B in the numerical calculations.

Before calculating the electrical current and the corresponding conductivity, we need to find the relaxation time and the vertex function. These will be derived in the following two sections.

III. RELAXATION TIMES

Now we calculate the relaxation time due to scattering on the fluctuating spin-orbit Rashba field. Since the fluctuations are assumed to be small, they can be considered in terms of the perturbation theory. The Green function for electrons in 2D electron gas with disorder can be written in the following general form:

$$G^{R,A}_{\varepsilon,\mathbf{k}} = \frac{1}{\varepsilon - \varepsilon_k \pm \mathrm{i}/2\tau}, \tag{8}$$

where $\varepsilon_k = k^2/2m^*$, the energy ε is measured from the bottom of the electron energy band, and $1/2\tau$ is the relaxation rate due to scattering from the fluctuations of the spin-orbit field. We assume that there are no other scattering centers in the system (like impurities or defects) which would lead to decay of the electron state with the momentum $\mathbf{k}$.

In the Born approximation, the self energy due to scattering from fluctuating spin-orbit field can be calculated from the

formula,

$$\Sigma^R(\varepsilon,k) = -\frac{i\pi}{4}\sum_{\mathbf{k}'} |\lambda_{\mathbf{k}\mathbf{k}'}|^2 (k^2 + k'^2 + 2\mathbf{k}\cdot\mathbf{k}')\,\delta(\varepsilon - \varepsilon_{k'}). \tag{9}$$

In the following we assume the disorder correlator in the form [15,24],

$$\langle|\lambda_{\mathbf{k}\mathbf{k}'}|^2\rangle = 2\pi\langle\lambda^2\rangle R^2 e^{-|\mathbf{k}-\mathbf{k}'|R}, \tag{10}$$

where R is the correlation radius of the spatial fluctuations, while the parameter $\langle\lambda^2\rangle$ characterizes amplitude of these fluctuations. Then, upon averaging over static disorder we obtain from Eq. (9) the following expression for the relaxation rate, $1/\tau = 2\mathrm{Im}\,\Sigma^R(\varepsilon_k,k)$:

$$\frac{1}{\tau} = \frac{1}{\tau_0} R^2k^2 \int_0^\pi d\varphi\, e^{-2Rk|\sin(\varphi/2)|}(1+\cos\varphi), \tag{11}$$

where we introduced a constant τ_0 for the time scale,

$$\frac{1}{\tau_0} \equiv \langle\lambda^2\rangle m^*. \tag{12}$$

The parameter τ_0 has the physical meaning of a characteristic spin rotation time of a particle with momentum $m^*\langle\lambda^2\rangle^{1/2}$ in a constant spin-orbit field $\langle\lambda^2\rangle^{1/2}$.

Dependence of the relaxation time τ on the electron energy, as determined by Eq. (11), is presented in Fig. 1(a). This figure shows that τ is divergent for $\varepsilon \to 0$ and decreases with increasing ε. The divergence is a consequence of the k dependence in Eq. (11), where electrons with large wavelengths, $2\pi/k \gg R$, do not *see* the short-range spin-orbit fluctuations. The prefactor in Eq. (11) includes k^2 and thus goes to zero for $\varepsilon \to 0$.

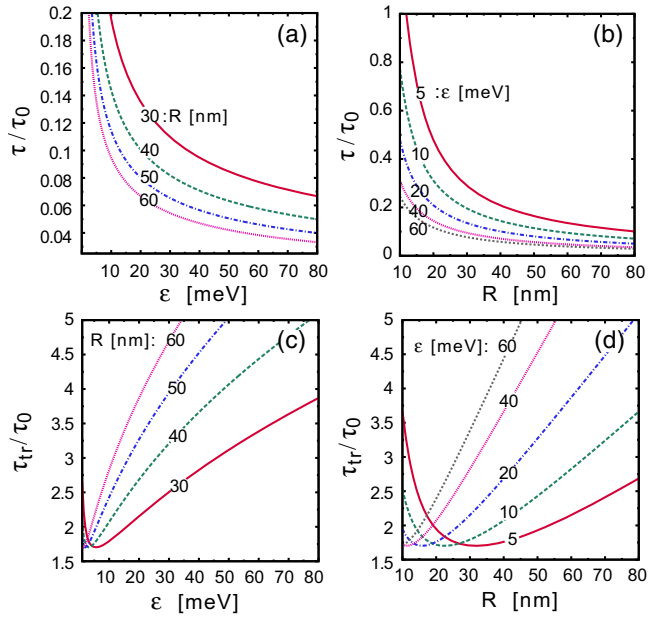

FIG. 1. The relaxation time τ/τ_0 [panels (a),(b)] and transport relaxation time τ_{tr}/τ_0 [panels (c),(d)] (both normalized to τ_0), shown as a function of energy for different values of the disorder correlation length R [panels (a) and (c)] and as a function of R for different values of ε [panels (b) and (d)]. Other parameters: electron effective mass $m^* = 0.03m_0$.

However, in real systems there are always some additional scattering processes, like for example scattering on neutral point defects, which remove the divergence at small energy. In turn, behavior of the relaxation time for $\varepsilon > 0$ is a consequence of the interplay of increase in the prefactor and decrease in the exponential term under the integral. As a result, the relaxation time τ as a function of kR behaves as $\sim 1/(kR)^2$ at $kR \ll 1$ and as $\sim 1/kR$ at $kR \gg 1$. This behavior corresponds to Fig. 1(a) and to Fig. 1(b), where the R dependence of τ is presented. In addition, it should be noted that the time τ in our model becomes simultaneously also the spin relaxation time because neither spin nor momentum are conserved in scattering from the Rashba field.

IV. CURRENT VERTEX

To calculate the electrical conductivity in the Kubo formalism we need the current vertex renormalized by the fluctuating spin-orbit Rashba field. As already mentioned above, we take the bare (unrenormalized) current vertex in the form

$$\hat{j}_x(\mathbf{k}) = \frac{ek_x}{m^*}. \tag{13}$$

Then, the ladder equation for the renormalized current vertex $J_x(\varepsilon,\varepsilon',\mathbf{k})$ takes the form

$$\begin{aligned} J_x(\varepsilon,\varepsilon',\mathbf{k}) = \hat{j}_x(\mathbf{k}) + \frac{1}{4}\sum_{\mathbf{k}'} & J_x(\varepsilon,\varepsilon',\mathbf{k}')|\lambda_{\mathbf{k}\mathbf{k}'}|^2 \\ & \times[\sigma_x(k_y+k'_y) - \sigma_y(k_x+k'_x)]\, G^A_{\varepsilon,\mathbf{k}'} \\ & \times[\sigma_x(k_y+k'_y) - \sigma_y(k_x+k'_x)]\, G^R_{\varepsilon',\mathbf{k}'}. \end{aligned} \tag{14}$$

For brevity of notation, we will omit below ε and ε' in $J_x(\varepsilon,\varepsilon',\mathbf{k})$. Then, using Eq. (8) for the Green's function, we obtain from Eq. (14) the following equation for $J_x(\mathbf{k})$:

$$\begin{aligned} J_x(\mathbf{k}) = \frac{ek_x}{m^*} + \frac{i\pi}{4(\omega + i/\tau)}\sum_{\mathbf{k}'} & J_x(\mathbf{k}')|\lambda_{\mathbf{k}\mathbf{k}'}|^2 \\ & \times(k^2 + k'^2 + 2\mathbf{k}\cdot\mathbf{k}')\,[\delta(\varepsilon-\varepsilon_{k'}) + \delta(\varepsilon'-\varepsilon_{k'})], \end{aligned} \tag{15}$$

where $\omega = \varepsilon' - \varepsilon$.

The detailed calculations, which include disorder averaging and the limit of $\omega \to 0$ (see the Appendix A) lead to the vertex function in the form

$$J_x(\varepsilon,\varepsilon;\mathbf{k}) = \frac{ek_x}{m^*}\frac{\tau_{tr}}{\tau}, \tag{16}$$

where $\varepsilon = k^2/2m^*$ and the transport relaxation time τ_{tr} is given by the formula

$$\frac{1}{\tau_{tr}} = \frac{1}{\tau_0} R^2k^2 \int_0^\pi d\varphi\, e^{-2Rk|\sin(\varphi/2)|}\,\sin^2\varphi, \tag{17}$$

with τ_0 defined by Eq. (12). Variation of the transport relaxation time τ_{tr} with the energy and correlation radius R is shown in Figs. 1(c) and 1(d). In the limit of small energy and small R, behavior of τ_{tr} is similar to that for τ since both these quantities show the $1/(kR)^2$ divergence. In the opposite limit $kR \gg 1$, the contribution from scattering angles $\sim 1/kR$ dominates in the relaxation rate $1/\tau$. However, this small-angle scattering only weakly contributes to the $1/\tau_{tr}$ rate. As a result, τ_{tr} behaves as $\sim kR$ in the $kR \gg 1$ limit, which leads to the increase in τ_{tr}

upon reaching the minima, as clearly seen in Figs. 1(c) and 1(d) [note, such minima are absent in Figs. 1(a) and 1(b)].

V. ELECTRICAL CONDUCTIVITY

Having found the relevant relaxation rates and the vertex function, one can calculate the electrical conductivity. Using the Matsubara technique for finite-temperature Green functions, with discrete frequencies $i\omega_m = 2im\pi T$, where m is an integer number and T is the temperature, we get the following expression for the charge current [34,35]:

$$j_x(i\omega_m) = -\frac{eA_xT}{m^*c}\sum_{n\mathbf{k}} J_x(\mathbf{k};i\varepsilon_n,i\varepsilon_n+i\omega_m) \times G_{\mathbf{k}}(i\varepsilon_n+i\omega_m)\,k_x\sigma_0\,G_{\mathbf{k}}(i\varepsilon_n). \quad (18)$$

Upon analytical continuation to real frequencies, $i\omega_m \to \omega$, one obtains

$$j_x(\omega) = -\frac{eE_0}{2\pi\omega m^*}\,\mathrm{Tr}\int\frac{d^2\mathbf{k}}{(2\pi)^2}\,k_x\int_{-\infty}^{\infty}d\varepsilon[f(\varepsilon+\omega)-f(\varepsilon)] \times J_x(\mathbf{k},\varepsilon,\varepsilon+\omega)\,G^R_{\mathbf{k}}(\varepsilon+\omega)\,G^A_{\mathbf{k}}(\varepsilon), \quad (19)$$

where E_0 is the electric field and the Fermi-Dirac distribution $f(\varepsilon)$ is given by $1/[\exp((\varepsilon-\mu)/T)+1]$, with the chemical potential μ. Then, using Eq. (16) for the vertex function $J_x(\mathbf{k},\varepsilon,\varepsilon)$, one finds from Eq. (19) the following formula for the static ($\omega \to 0$) charge current:

$$j_x = -\frac{e^2E_0}{4\pi^2m^*}\int_0^{\infty}d\varepsilon f'(\varepsilon)\,k^2\,\tau_{\mathrm{tr}}. \quad (20)$$

From this formula follows that the corresponding electrical conductivity σ is determined by the transport relaxation time τ_{tr} as

$$\sigma = -\frac{e^2}{2\pi^2}\int_0^{\infty}d\varepsilon\,\varepsilon f'(\varepsilon)\,\tau_{\mathrm{tr}}(\varepsilon)\,. \quad (21)$$

Numerical results for the electrical conductivity, obtained from Eq. (21) with the transport time given by Eq. (17), are presented in Fig. 2 for indicated parameters describing the system. The increase in conductivity with increasing temperature and chemical potential results from increasing contribution of electrons with higher momentum and from the modification of the total electron concentration. All the dependencies presented in Fig. 2 are related to the dependence of $\tau_{\mathrm{tr}}(\varepsilon)$ on the product kR, see Eq. (17).

VI. SPIN CURRENT AND SPIN CONDUCTIVITY

To complete our considerations we now analyze the spin current flowing in the system. Within the formalism used in this paper, the spin current appears due to a minimal coupling to the spin gauge field. In the first order, this coupling is defined as [21,37–40]:

$$\hat{H}^{(\mathbf{A}_s)}_{\mathbf{k}} = -\hat{j}^{\alpha}_i A^{\alpha}_i, \quad (22)$$

where A^s_j is the adiabatic gauge vector potential related to a spin electric field,

$$E^{\alpha}_j(t) \equiv E^{\alpha}_{j0}\,e^{-i\omega t} = -\frac{1}{c}\frac{\partial A^{\alpha}_j}{\partial t}, \quad (23)$$

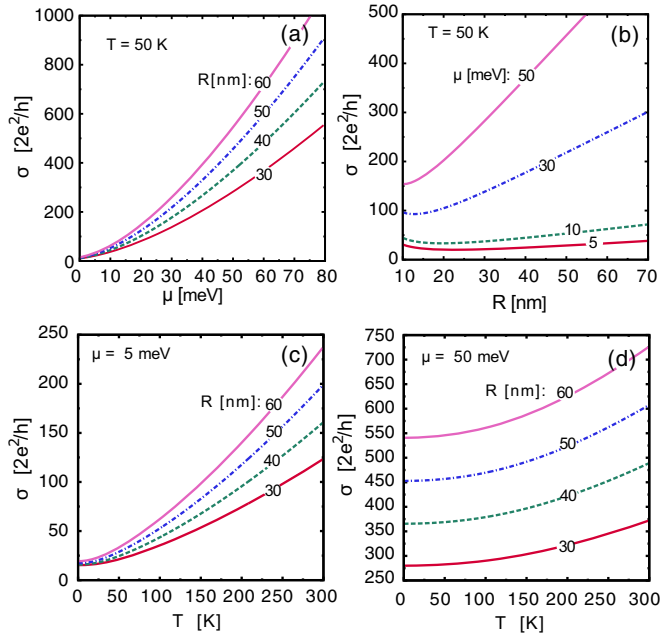

FIG. 2. Electric conductivity as a function of chemical potential μ (a), correlation length R (b), and temperature T [(c), (d)]. Other parameters: electron effective mass [36] $m^* = 0.03m_0$, $\sqrt{\langle\lambda^2\rangle} = 0.15$ eVÅ, and the corresponding $\tau_0 = 0.22$ ns, as follows from Eq. (12).

and $\hat{j}^{\alpha}_i = \hbar[\hat{v}_i,\sigma_{\alpha}]_+/4$ is the spin current operator. For the system under consideration and for transport of the z component of spin polarization along the axis x we get:

$$\hat{j}^z_x = \frac{k_x\sigma_z}{2m^*}. \quad (24)$$

Note that the above spin current operator is based on the unperturbed part of the Hamiltonian [Eq. (2)], which does not contain spin-orbit coupling. Since σ_{α} commutes with $\hat{H}_0$, the definition of spin current is equivalent to a more general one proposed by Niu *et al.* [41,42] (for general overview see also Refs. [43,44]).

Using the Matsubara technique one arrives at the following formula for the spin current:

$$j^z_x(i\omega_m) = -\frac{A^z_xT}{2m^*c}\sum_{n\mathbf{k}} J^z_x(\mathbf{k};i\varepsilon_n,i\varepsilon_n+i\omega_m) \times G_{\mathbf{k}}(i\varepsilon_n+i\omega_m)\,k_x\sigma_z\,G_{\mathbf{k}}(i\varepsilon_n), \quad (25)$$

where $\varepsilon_n = (2n+1)\pi T$. Then, and upon analytical continuation, $i\omega_m \to \omega$, one can write

$$j^z_x(\omega) = -\frac{E^z_{0x}}{4\pi\omega m^*}\,\mathrm{Tr}\int\frac{d^2\mathbf{k}}{(2\pi)^2}\int_{-\infty}^{\infty}d\varepsilon[f(\varepsilon+\omega)-f(\varepsilon)] \times J^z_x(\mathbf{k},\varepsilon,\varepsilon+\omega)\,G^R_{\mathbf{k}}(\varepsilon+\omega)\,k_x\sigma_z\,G^A_{\mathbf{k}}(\varepsilon), \quad (26)$$

where we introduced the spin electric field according to the relation between $A^z_x(\omega)$ and $E^z_x(\omega)$.

To calculate the spin current from Eq. (26), we need to know the vertex function $J^z_x(\mathbf{k})$. The corresponding equation for the

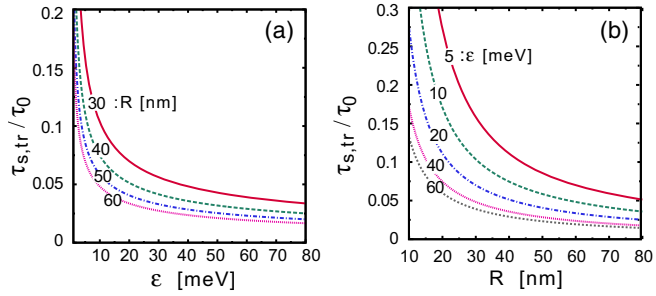

FIG. 3. The normalized spin transport relaxation time, $\tau_{s,\mathrm{tr}}/\tau_0$, as a function of energy ϵ for different values of the radius R (a) and as a function of R for different values of energy (b). Other parameters: electron effective mass $m^* = 0.03m_0$.

spin current vertex reads

$$\mathbf{J}^z(\mathbf{k}) = \frac{\mathbf{k}\sigma_z}{2m^*} + \frac{\mathrm{i}\pi}{4(\omega + \mathrm{i}/\tau)} \sum_{\mathbf{k}'} \mathbf{J}^z(\mathbf{k}')|\lambda_{\mathbf{k}\mathbf{k}'}|^2 \times (k^2 + k'^2 + 2\mathbf{k}\cdot\mathbf{k}')\,[\delta(\varepsilon - \varepsilon_{k'}) + \delta(\varepsilon' - \varepsilon_{k'})]. \quad (27)$$

Solving this equation we find the following simple formula for the spin current:

$$J_x^z = \frac{k_x \sigma_z}{2m^*} \frac{\tau_{s,\mathrm{tr}}}{\tau}, \quad (28)$$

where the spin transport relaxation time $\tau_{s,\mathrm{tr}}$ is given by the formula

$$\frac{1}{\tau_{s,\mathrm{tr}}} = \frac{1}{\tau_0} R^2 k^2 \int_0^\pi d\varphi\, e^{-2Rk|\sin(\varphi/2)|}(1 + \cos\varphi)^2. \quad (29)$$

Dependence of the transport spin relaxation time on the electron energy and correlation radius is presented in Fig. 3. Note that in contrast to τ_{tr} presented in Figs. 1(c) and 1(d), $\tau_{s,\mathrm{tr}}$ does not increase at large kR and behaves similarly as the relaxation time τ presented in Figs. 1(a) and 1(b). This difference is due to the fact that small-angle scattering is essential for relaxation of spin current while it is not essential for the relaxation of charge current. As one can note when comparing Fig. 3 and Fig. 1, the transport spin relaxation time is smaller than τ because it accounts for both effects of electron scattering and spin relaxation.

Similarly to Eq. (21), we obtain the spin conductivity

$$\sigma_s = -\frac{1}{8\pi^2} \int_0^\infty d\varepsilon\, \varepsilon f'(\varepsilon)\, \tau_{s,\mathrm{tr}}(\varepsilon)\,. \quad (30)$$

The numerical results for the spin conductivity, obtained from Eq. (30) with the spin transport time given by Eq. (29), are presented in Fig. 4. These results can be accounted for in a similar way as the results for electrical conductivity in Fig. 2.

VII. CONCLUSIONS

In this paper we presented theoretical results on transport relaxation times τ_{tr} and $\tau_{s,\mathrm{tr}}$, responsible for the charge and spin conductivity in a two-dimensional system, where the main mechanism of electron scattering is related to fluctuations of the Rashba spin-orbit interaction. Since our objective was to describe and explain the role of randomness of Rashba field, we considered an *extreme* situation, where the mean

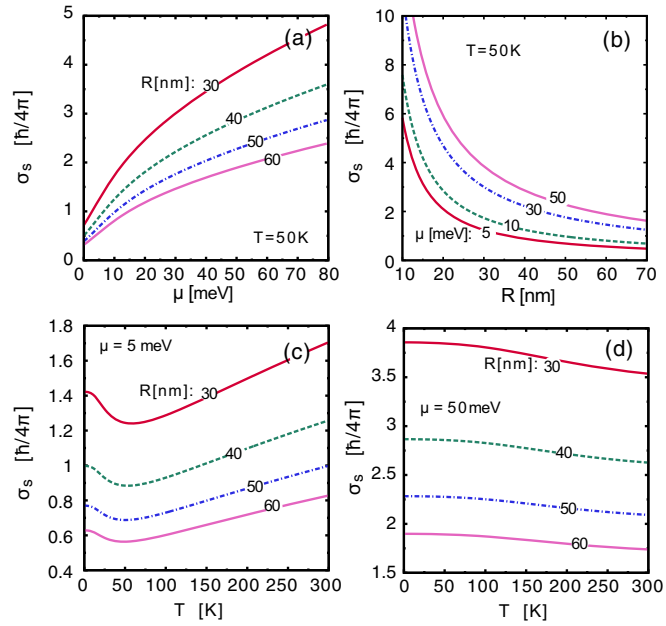

FIG. 4. Spin conductivity as a function of the chemical potential μ (a), correlation length R (b), and temperature T [(c), (d)] for indicated parameters R, T, and μ. Other parameters: electron effective mass $m^* = 0.03m_0$, $\sqrt{\langle\lambda^2\rangle} = 0.15$ eV Å.

value of the Rashba coupling is zero. Thus, we have not considered the role of any uniform or periodically modulated in space spin-orbital fields [44–46]. The mechanism of electron scattering which is discussed here can be important in a general case of $\langle\lambda(\mathbf{r})\rangle \neq 0$, provided that the spin-orbit coupling in the system is sufficiently strong. It can be also related to defects in strongly spin-orbit coupled compounds such as transition-metal chalcogenides or impurities at the surfaces or interfaces with a strong spin-orbit coupling.

The obtained transport relaxation times for the charge and spin current can be essentially different from the times describing the electron momentum and spin relaxation. The observed energy dependence of the transport time leads to nontrivial temperature dependence of the conductivity which may increase with increasing temperature at a constant electron density.

We also note that the spin transport relaxation time considered in this paper characterizes the decay rate of spin current flowing in the system. It takes into account both spin relaxation and momentum relaxation during the motion of electrons in the system with randomly fluctuating in space Rashba spin-orbit coupling. We should also note that the spin relaxation mechanism proposed in this paper is different from the relaxation mechanism proposed by Dyakonov and Perel [47].

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APPENDIX: CALCULATION OF THE VERTEX FUNCTION

Here we present some details concerning calculation of the vertex function in the static limit $\omega \to 0$ (which is of our interest), see Eqs. (14) and (15). Since $J_x \sim k_x$, Eq. (15) can be presented as an equation for the vector vertex $\mathbf{J}(\mathbf{k})$

$$\mathbf{J}(\mathbf{k}) = \frac{e\mathbf{k}}{m^*} + \frac{i\pi}{4(\omega + i/\tau)} \sum_{\mathbf{k}'} \mathbf{J}(\mathbf{k}')|\lambda_{\mathbf{k}\mathbf{k}'}|^2 \times (k^2 + k'^2 + 2\mathbf{k}\cdot\mathbf{k}')\,[\delta(\varepsilon - \varepsilon_{k'}) + \delta(\varepsilon' - \varepsilon_{k'})], \quad \text{(A1)}$$

and we can write $\mathbf{J}(\mathbf{k})$ as

$$\mathbf{J}(\mathbf{k}) = \frac{e\mathbf{k}}{m^*}\, g(k), \quad \text{(A2)}$$

where the scalar function $g(k)$ depends on the module of the vector $\mathbf{k}$ only. Then, from (A1) and (A2) we obtain in the limit $k \to k'$ the following equation for $g(k)$:

$$\mathbf{k}\, g(k) = \mathbf{k} + \frac{i\pi g(k)}{2(\omega + i/\tau)} \sum_{\mathbf{k}'} |\lambda_{\mathbf{k}\mathbf{k}'}|^2 (k^2 + k'^2 + 2\mathbf{k}\cdot\mathbf{k}') \times \mathbf{k}'\, \delta(\varepsilon - \varepsilon_{k'}). \quad \text{(A3)}$$

The right-hand side of Eq. (A3) should be proportional to vector $\mathbf{k}$. As a result, we find $g(k)$ from the following equation:

$$g(k) = 1 + \frac{1}{\tau_0} R^2 k^2 \tau\, g(k) \times \int_0^\pi d\varphi\, e^{-2Rk|\sin(\varphi/2)|} \cos^2(\varphi/2)\, \cos\varphi. \quad \text{(A4)}$$

Finally, (A4) yields

$$g(k) = \left(1 - \frac{\tau}{\tau_0} R^2 k^2 p(k)\right)^{-1}, \quad \text{(A5)}$$

where we introduced the notation,

$$p(k) = \int_0^\pi d\varphi\, e^{-2Rk|\sin(\varphi/2)|} (1 + \cos\varphi) \cos\varphi. \quad \text{(A6)}$$

Using the expression (11) for $1/\tau$ we can write $g(k)$ in the following simple form

$$g(k) = \tau_{\rm tr}/\tau, \quad \text{(A7)}$$

where the transport relaxation time $\tau_{\rm tr}$ is determined as:

$$\frac{1}{\tau_{\rm tr}} = \frac{1}{\tau_0} R^2 k^2 \int_0^\pi d\varphi\, e^{-2Rk|\sin(\varphi/2)|} \sin^2\varphi. \quad \text{(A8)}$$

Finally, the vertex function at $\varepsilon = k^2/2m^*$ has the form:

$$J_x(\mathbf{k};\varepsilon,\varepsilon) = \frac{ek_x}{m^*} \frac{\tau_{\rm tr}}{\tau}. \quad \text{(A9)}$$

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