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Higher-dimensional Sachdev-Ye-Kitaev non-Fermi liquids at Lifshitz transitions

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We address the key open problem of a higher-dimensional generalization of the Sachdev-Ye-Kitaev (SYK) model. We construct a model on a lattice of SYK dots with nonrandom intersite hopping. The crucial feature of the resulting band dispersion is the presence of a Lifshitz point where two bands touch with a tunable power-law divergent density of states (DOS). For a certain regime of the power-law exponent, we obtain a class of interaction-dominated non-Fermi-liquid (NFL) states, which exhibits exciting features such as a zero-temperature scaling symmetry, an emergent (approximate) time reparameterization invariance, a power-law entropy-temperature relationship, and a fermion dimension that depends continuously on the DOS exponent. Notably, we further demonstrate that these NFL states are fast scramblers with a Lyapunov exponent $\lambda_L \propto T$, although they do not saturate the upper bound of chaos, rendering them truly unique.

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I. INTRODUCTION

Description of quantum many-body systems lacking quasiparticles [1] is a longstanding challenge at the forefront of physics. A solvable example in zero dimension is provided by the SYK model [2,3], which has attracted much attention due to its intriguing connections to quantum gravity in AdS_2 [3–7] and intertwined questions of thermalization and information scrambling. The exciting possibility of generalizing this model to higher dimensions to address questions relating to transport without quasiparticles as well as possible duals to higher-dimensional gravity, has lead to a number of interesting extensions [8–15] of the SYK model.

Such generalizations to higher dimensions typically use a lattice of SYK dots, connected via either random interdot interaction and/or hopping [8–11,15] or through uniform hopping leading to a translationally invariant system [16,17]. The problem has been attempted to be addressed from field theoretic perspectives [18,19] as well. These have been used for extraction of transport quantities and diagnostics of many-body quantum chaos, such as the butterfly velocity [8], in strongly interacting lattice models, demonstrating their connection, e.g., with the phenomena like heavy-Fermi liquids [11] and many-body localization [9]. However, such extensions have met with only partial success towards a higher-dimensional lattice generalization. In particular, the ensuing low-energy behavior turns out to be either qualitatively similar to that of the zero-dimensional SYK model at the leading order [8], or results in a low-temperature phase where interaction becomes irrelevant [11,16] at low energies. Berkooz *et al.* [12] have obtained an interaction-dominated fixed point distinct from the zero-dimensional SYK using a phenomenological 'filter function' construct whose microscopic underpinnings are unclear. Evidently, the higher-dimensional extension of the SYK model remains as an interesting open problem. Here we propose and extensively study a microscopically motivated, translationally invariant lattice model, exhibiting a class of interaction-dominated non-Fermi-liquid (NFL) fixed points, derived from the SYK model.

Our model, illustrated in Fig. 1, is comprised of SYK dots with random intradot q-fermion interactions [7], at the sites of a d-dimensional lattice, connected with *uniform* interdot hoppings. The dispersion arising from the hoppings is designed to produce two particle-hole symmetric bands, which touch each other at zero momentum ($\mathbf{k} = \mathbf{0}$), as in a Lifshitz transition [20], with low-energy dispersion $\varepsilon_\pm(\mathbf{k}) \propto \pm|\mathbf{k}|^p$ ($p \geqslant d$). The resultant particle-hole symmetric density of states (DOS) has a power-law divergence, namely $g(\varepsilon) \sim |\varepsilon|^{-\gamma}$ with $0 \leqslant \gamma = 1 - d/p < 1$. Such DOS singularity, e.g., may arise at topologically protected Fermi points with additional symmetries [21]. This singularity in DOS is what ultimately enables the amalgamation of lattice and SYK physics, thereby producing the family of new fixed points mentioned earlier. The remarkable feature of this model is that by tuning γ a variety of fermionic phases can be realized. For values of γ less than a critical value $\gamma_c = (2q-3)/(2q-1)$, we get a Fermi liquid like phase with perturbative interaction effects, while on the other hand, when $\gamma \approx 1$ the system behaves like the parent SYK model. The most interesting physics occurs for $\gamma_c < \gamma < 1$, where the emergent NFL phases correspond to a new set of fixed points with a continuously tunable fermion dimension $\Delta = (1+\gamma)/2(1-\gamma+2\gamma q)$. As a result, the NFL phases have properties tunable via γ and distinct from that of the parent SYK model. Unlike the pure SYK phase, the NFL phases have zero ground-state entropy [$S(T=0)$], with $S(T)$ varying as a power law in temperature T with a γ dependent exponent. Moreover, the onset of quantum chaos in the model is governed by a Lyapunov exponent $\lambda_L \propto T$, which, however, does not saturate the chaos bound of $\lambda_L = 2\pi T$ [3,22]. As $\gamma \to 1$, the residual zero-temperature entropy of the parent SYK is recovered and $\lambda_L \to 2\pi T$.

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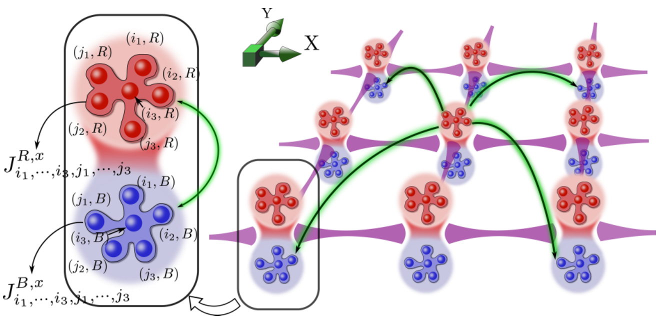

FIG. 1. Model: A d-dimensional hypercubic lattice of SYK dots of two colors—R (red) and B (blue), each with N flavors (indexed by i,j). The flavors within a dot interact via q-body color-preserving interactions (with strength $J^{R,x}$, $J^{B,x}$). Translationally invariant hoppings (green arrows) preserve the flavor but flip color. Figure shown for $d=2$ and $q=3$.

The truly higher-dimensional nature of the model is manifested in the nontrivial dynamical scaling exponent z of the fermions. The perturbative fixed point ($\gamma < \gamma_c$) retains $z = p$ of the noninteracting model. Whereas, for $\gamma > \gamma_c$, interaction changes to $z = p/(2(2q-1)\Delta - 1)$, not unlike a proposed quantum gravity theory [23] which exploits Lifshitz points. Our work, therefore, offers a solution to the much sought after higher-dimensional generalization of the SYK model and provides a framework for addressing problems mentioned in the introduction.

II. MODEL

The Hamiltonian for our lattice model (Fig. 1) is given by

$$\mathscr{H} = -\sum_{\mathbf{x},\mathbf{x}',\alpha,\alpha',i} t_{\alpha\alpha'}(\mathbf{x}-\mathbf{x}')c^{\dagger}_{i\alpha\mathbf{x}}c_{i\alpha'\mathbf{x}'} - \mu\sum_{\mathbf{x},i\alpha} c^{\dagger}_{i\alpha\mathbf{x}}c_{i\alpha\mathbf{x}} + \sum_{\mathbf{x},\alpha}\sum_{\substack{i_1,\ldots,i_q,\\ j_1,\ldots,j_q}} J^{\alpha,\mathbf{x}}_{i_1,\cdots,i_q;j_1,\ldots,j_q} c^{\dagger}_{i_q\alpha\mathbf{x}}\cdots c^{\dagger}_{i_1\alpha\mathbf{x}}c_{j_1\alpha\mathbf{x}}\cdots c_{j_q\alpha\mathbf{x}}, \tag{1}$$

where $i,j = 1,\cdots,N$ denote SYK flavor at point $\mathbf{x}$ and $\alpha,\alpha' = R$ (red), B (blue) index the two colors of fermions with $c(c^{\dagger})$ being the annihilation(creation) operator. The intradot complex random all-to-all q-body SYK couplings $J^{\alpha,\mathbf{x}}_{i_1,\cdots,i_q;j_1,\cdots,j_q}$ are completely local and scatter fermions with same color (see Fig. 1). The Js are identically distributed, independent, Gaussian random variables with a variance of $J^2/qN^{2q-1}(q!)^2$. Hopping from one lattice point to another, $t_{\alpha\beta}(\mathbf{x}-\mathbf{x}')$, flips the color index and conserves the SYK flavor. By tuning the magnitude and range of these hoppings, a low-energy dispersion $\varepsilon_{\pm}(\mathbf{k})$ of the form

$$\varepsilon_{\pm}(\mathbf{k}) \propto \pm|\mathbf{k}|^p \tag{2}$$

can be generated. We construct the dispersion to be particle-hole symmetric about zero energy, and choose p to obtain power-law DOS of the form $g(\varepsilon) \sim |\varepsilon|^{-\gamma}$ where $\gamma = 1 - d/p$ can be tuned via p and d. The exponent γ satisfies $0 < \gamma < 1$ for integrability of the DOS but is otherwise arbitrary. Such dispersions with a low-energy form given by Eq. (2) can be 'designed' for a d-dimensional lattice (see the Supplemental Material [24], S1 for details), e.g., a lattice dispersion in $d=1$ corresponding to Eq. (2) is $\varepsilon_{\pm}(k) \propto \pm|\sin(k/2)|^p$, with k in units of inverse lattice spacing. The approximate low-energy form for the single-particle DOS is $g(\varepsilon) = g_0|\varepsilon|^{-\gamma}\Theta(\Lambda - |\varepsilon|)$, where Θ is the Heaviside step function and $\Lambda > 0$ is an energy cutoff that plays the role of the bandwidth. The constant $g_0 = (1-\gamma)/(2\Lambda^{1-\gamma})$ normalizes the integrated DOS to unity. This low-energy form is sufficient to study the low-temperature properties of our model [see Eq. (1)] for $\Lambda, J \gg T$.

III. SADDLE-POINT EQUATIONS

The model of Eq. (1) is exactly solvable at the level of saddle point in the limit $N \to \infty$, i.e., when there are a large number of SYK flavors. To derive the saddle-point equations [5,8] we disorder average over the random SYK couplings using replicas and obtain an effective action within a replica-diagonal ansatz in terms of a large-N collective field $G_{\alpha\mathbf{x}}(\tau_1,\tau_2) = (1/N)\sum_i \langle c^{\dagger}_{i\alpha\mathbf{x}}(\tau_2)c_{i\alpha\mathbf{x}}(\tau_1)\rangle$, and its conjugate $\Sigma_{\alpha\mathbf{x}}(\tau_1,\tau_2)$, where $\tau_{1,2}$ denote imaginary time (see Ref. [24], S2). Further, retaining color symmetry and lattice translational invariance for the saddle point results in the action (per site, per flavor)

$$\mathcal{S} = -\int d\varepsilon g(\varepsilon)\mathrm{Tr}\ln[(\partial_{\tau_1} + \varepsilon)\delta(\tau_1 - \tau_2) + \Sigma(\tau_1,\tau_2)] - \int d\tau_{1,2}\bigg[(-1)^q\frac{J^2}{2q}G^q(\tau_1,\tau_2)G^q(\tau_2,\tau_1) + \Sigma(\tau_2,\tau_1)G(\tau_1,\tau_2)\bigg], \tag{3}$$

where Tr denotes matrix trace over τ index and $\int d\tau_{1,2} = \int d\tau_1 d\tau_2$. This leads to the self-consistent equations

$$G(\mathrm{i}\omega_n) = \int_{-\Lambda}^{\Lambda} d\varepsilon g(\varepsilon)[\mathrm{i}\omega_n - \varepsilon - \Sigma(\mathrm{i}\omega_n)]^{-1} \tag{4a}$$

$$\Sigma(\tau) = (-1)^{q+1}J^2G^q(\tau)G^{q-1}(-\tau), \tag{4b}$$

where $\omega_n = (2n+1)\pi T$ is the fermionic Matsubara frequency, $\mathrm{i} = \sqrt{-1}$, and we have assumed time translation invariance, $G(\tau_1,\tau_2) = G(\tau_1 - \tau_2)$. At the saddle point, $G(\tau)$ is the on-site fermion Green's function and $\Sigma(\tau)$ is the self-energy, which is completely local in this model. The Green's function of the fermion with momentum $\mathbf{k}$ is given by

$$G_{\pm}(\mathbf{k},\mathrm{i}\omega_n) = (\mathrm{i}\omega_n - \varepsilon_{\pm}(\mathbf{k}) - \Sigma(\mathrm{i}\omega_n))^{-1}. \tag{5}$$

IV. ZERO-TEMPERATURE SOLUTIONS

The low-energy solution of Eqs. (4) can be obtained analytically at $T = 0$. At low temperatures, for ω, $\Sigma(\omega) \ll \Lambda$, we expand the integral for $G(\mathrm{i}\omega_n \to \omega + \mathrm{i}0^+)$ in Eq. (4a) in powers of $(\omega - \Sigma(\omega))/\Lambda$ to get

$$G(\omega) \approx g_0\frac{\pi(1 - e^{\mathrm{i}\gamma\pi})}{\sin(\pi\gamma)(\omega - \Sigma(\omega))^{\gamma}}. \tag{6}$$

This leads to two possible fixed-point solutions: (1) an interaction-dominated fixed point for $\omega \ll \Sigma(\omega)$ as $\omega \to 0$

and (2) the original lattice-dominated fixed point for $\omega \gg \Sigma(\omega)$, essentially a 'Fermi liquid,' where interaction becomes irrelevant for $\omega \to 0$.

In the first case, Eq. (6) becomes

$$G(\omega) = g_0(1 - e^{i\gamma\pi})\pi \csc(\pi\gamma)\Sigma(\omega)^{-\gamma}. \tag{7}$$

At $T = 0$, solution of Eqs. (4b) and (7) is obtained by taking a power-law ansatz for $G(\omega)$, as in the conventional SYK model [2,5], leading to

$$G(\omega) = Ce^{-i\theta}\omega^{2\Delta-1} \tag{8a}$$

$$\Sigma(\omega) = \frac{J^2(C\Gamma(2\Delta)\sin\theta)^{2q-1}}{\Gamma(2\Delta_\Sigma)\sin(\pi\Delta_\Sigma)\pi^{2q-3}}e^{-i\pi\Delta_\Sigma}\omega^{2\Delta_\Sigma-1}. \tag{8b}$$

The expression for $\Sigma(\omega)$ uses the particle-hole symmetric value of $\theta = \pi\Delta$ (see below). The fermion scaling dimension Δ and the prefactor C are determined to be

$$\Delta = \frac{1+\gamma}{2(1-\gamma+2q\gamma)}$$

$$C = \left[\frac{g_0\pi^{2\gamma(q-1)+1}}{J^{2\gamma}\cos(\gamma\pi/2)}\left(\frac{\Gamma(2\Delta_\Sigma)\sin(\pi\Delta_\Sigma)}{(\Gamma(2\Delta)\sin(\pi\Delta))^{2q-1}}\right)^{\gamma}\right]^{\frac{2\Delta}{1+\gamma}}, \tag{9}$$

where $\Delta_\Sigma = (2q-1)\Delta$, and $\Gamma(x)$ is the gamma function. In the Supplemental Material (Ref. [24], S3), we show that the low-energy saddle-point equations, and the constraint $\mathrm{Im}G(\omega) < 0$, completely fix the spectral asymmetry parameter θ to $\pi\Delta$, allowing only particle-hole symmetric scaling solutions at the interacting fixed point. This is in contrast with the usual SYK model where θ can be tuned by filling [5].

The fact that the fermion dimension Δ is determined both by the lattice DOS via γ and by SYK interactions through q, indicates that the fixed point is indeed a 'truly' higher-dimensional analog of the zero-dimensional SYK phase, but yet distinct from it. In fact as $\gamma \to 1$, $\Delta \to 1/2q$ and the fermion dimension of the zero-dimensional SYK model is recovered. However, unlike the SYK model which has an asymptotically *exact* infrared time reparametrization symmetry under $\tau \to f(\tau)$, Eq. (4b) together with Eq. (7) are invariant only under time translation and scaling transformations, $\tau \to a\tau + b; a,b$ being constants. One would expect time reparametrization symmetry to be restored as the zero-dimensional SYK-like fixed point at $\gamma \to 1$ is approached.

Equation (9) also implies that, in principle, Δ can be changed continuously starting from $1/2q$ to $1/2$, as γ is tuned from 1 to 0. However, as evident from Eq. (8b), the assumption $\omega \ll \Sigma(\omega)$ is only self consistent as long as $2\Delta_\Sigma - 1 \leqslant 1$, i.e.,

$$1 \geqslant \gamma \geqslant \frac{2q-3}{2q-1} \equiv \gamma_c(q). \tag{10}$$

Therefore, below a q-dependent critical value $\gamma_c(q)$ the scaling solution (8a) ceases to exist. This brings us to the saddle-point solution for the perturbative fixed point for $\omega \gg \Sigma(\omega)$, henceforth referred to as lattice-Fermi liquid (LFL), where Eq. (6) reduces to

$$G(\omega) \approx g_0(1 - e^{i\gamma\pi})\pi \csc(\pi\gamma)(\omega)^{-\gamma}. \tag{11}$$

It can be shown that at this fixed point $\Sigma(\omega) \sim (J^2/\Lambda)(\omega/\Lambda)^{(1-\gamma)(2q-1)-1} \ll \omega$ for $\gamma < \gamma_c$, i.e., interaction is irrelevant and its effect is only perturbative in J for $\omega \to 0$.

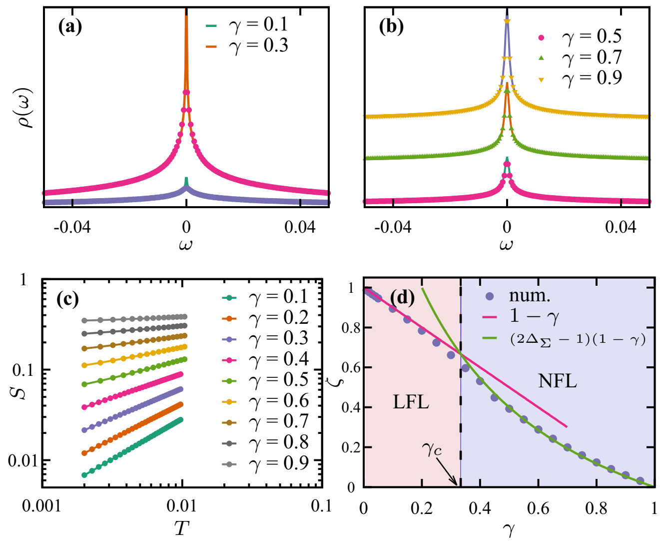

FIG. 2. Numerical results ($q = 2$, $J = \Lambda = 1$): (a) Numerical spectral function (points) at $T = 0.0005$ with power-law ($|\omega|^{-\alpha}$) fits (lines); $\alpha = 0.097 \pm 0.0003, 0.28 \pm 0.0006$ for $\gamma = 0.1, 0.3$, respectively. (b) Numerical spectral function (points) at $T = 0.0005$ for $\gamma = 0.5, 0.7, 0.9$ compared with analytical Eq. (12) (*no fitting parameters*, lines). (c) Entropy (S) vs temperature (T) shown in log-log scale. (d) S-T exponent ζ (points) vs γ compared with theory (lines) in the LFL ($\gamma < \gamma_c$) and NFL ($\gamma > \gamma_c$) [Eq. (13)] regimes [here $\gamma_c = 1/3$, Eq. (10)].

The dominant term in the Green's function above is determined only by the singularity in the single-particle DOS and is temperature independent. At finite temperatures there are small corrections of $O(J^2)$. We use this fact in the next section to numerically verify the existence of the LFL.

In gist, the system undergoes a quantum phase transition at $\gamma = \gamma_c(q)$ upon increasing γ from 0 to 1. For $\gamma < \gamma_c$, we have a LFL, while for $\gamma > \gamma_c$ we get a line of interaction dominated NFLs. This is also indicated by the dynamical exponent, deduced from Eq. (5), that changes from $z = p$ (when $\gamma < \gamma_c$) to $z = p/(2\Delta_\Sigma - 1)$ (when $\gamma > \gamma_c$) across γ_c.

V. FINITE-TEMPERATURE NUMERICS

To gain insight about the finite-temperature properties of the model in Eq. (1), we solve equations (4) numerically at nonzero T for $\Lambda = J = 1$ and $q = 2$ (see Ref. [24], S4 for details). First we calculate the spectral function $\rho(\omega) = -(1/\pi)\mathrm{Im}G(\omega)$ for various $\gamma < \gamma_c(q)$ as shown in Fig. 2(a). In this regime G is given by Eq. (11) and we expect the spectral functions to exhibit the power-law singularity in the DOS with small finite-temperature corrections. Therefore we fit the numerically obtained $\rho(\omega)$ with a power law $|\omega|^{-\alpha}$ and find $\alpha \approx \gamma$ [see Fig. 2(a)], demonstrating the existence of the expected LFL phase.

The numerical verification of the interacting fixed point for $1 > \gamma > \gamma_c(q)$ is less straightforward as, unlike the usual SYK model, our system does not possess asymptotically exact reparameterization invariance in the infrared. Still, expecting time reparametrization invariance to be approximately present when γ is close to 1, we derive a finite-T expression for the spectral function, by the mapping $\tau = (\beta/\pi)\tan(\sigma\pi/\beta)$ [5], to get (see Ref. [24], S5),

$$\rho(\omega) = \frac{C\sin(\pi\Delta)\cosh(\beta\omega/2)}{\pi^2(2\pi/\beta)^{1-2\Delta}}\Gamma(\Delta_-)\Gamma(\Delta_+), \qquad (12)$$

where $\beta = T^{-1}$, and $\Delta_\pm = \Delta \pm i\beta\omega/2\pi$. This is compared with the $\rho(\omega)$ obtained numerically, anticipating only a qualitative match. Surprisingly, excellent quantitative agreement is found *without* any fitting parameters [see Fig. 2(b)]. Moreover, Eq. (12) accurately matches the numerical result even for values of γ far away from 1 [see Fig. 2(b)]. This not only confirms the existence of NFL fixed points [see Eq. (8a)] but also points to an *approximate* emergent reparametrization symmetry at these fixed points.

In order to verify this result further, we estimate finite-temperature entropy $S(T)$ using $\rho(\omega)$ from Eq. (12) and a similar finite-temperature form for $\Sigma(\omega)$, both of which satisfy a scaling relation, e.g., $\rho(\omega) \sim (T/J)^{2\Delta-1} f(\omega/T)$. We obtain the entropy via $S = -\partial F/\partial T$, where the free energy F is obtained by evaluating the action of Eq. (3) with the scaling form for $\rho(\omega)$ and $\Sigma(\omega)$ (see Ref. [24], S6, and also Ref. [25]). We find that the low-temperature entropy vanishes with a power law as $T \to 0$, i.e., $S \sim T^\zeta$, where the exponent

$$\zeta = (2\Delta_\Sigma - 1)(1-\gamma) \qquad (13)$$

varies between $2/(2q-1)$ and 0 for $\gamma_c \leqslant \gamma < 1$. The usual SYK result is recovered for $\gamma \to 1$ (see Ref. [24], S6, Eq. (S6.13)).

To test the prediction $S \sim T^\zeta$, we extract ζ from the numerical entropy-temperature relationship [Fig. 2(c)]. The remarkable agreement of Eq. (13) with the numerical values confirms the presence of an approximate reparametrization symmetry for $\gamma > \gamma_c$ [see Fig. 2(d)]. For $\gamma < \gamma_c$, the expected result for the LFL is $S \sim T^{1-\gamma}$, i.e., $\zeta = 1-\gamma$, and this again is recovered in the numerics [Fig. 2(d)]. The notable aspect of Fig. 2(d) is the abrupt change of the entropy exponent ζ at γ_c, providing a vital confirmation of the underlying quantum phase transition from LFL to NFL.

VI. CHAOS AND THERMALIZATION

While the NFL obtained in the regime $\gamma_c < \gamma < 1$ is distinct from the usual SYK model, it is particularly interesting to explore this distinction further in terms of quantum chaos or information scrambling that gives an early-time diagnostic of thermalization [26]. The SYK model is known to be the most efficient scrambler like a black hole [3], namely the Lyapunov exponent λ_L, characterizing decay of a typical out-of-time-ordered (OTO) correlator, e.g., $\langle c_i^\dagger(t)c_j^\dagger(0)c_i(t)c_j(0)\rangle \simeq f_0 - (f_1/N)e^{\lambda_L t} + \mathcal{O}(N^{-2})$, saturates the upper bound, $\lambda_L = 2\pi T$, imposed by quantum mechanics [22,27]. A natural question is then to ask, whether our new NFL states behave similarly or differently. To this end, we generalize the OTO correlator for our two-band lattice system as $(1/N^2)\sum_{ij}\langle c_{i\alpha x}^\dagger(t)c_{j\beta x'}^\dagger(0)c_{i\alpha x}(t)c_{j\beta x'}(0)\rangle$, $(1/N^2)\sum_{ij}\langle c_{i\alpha x}(t)c_{j\beta x'}^\dagger(0)c_{i\alpha x}^\dagger(t)c_{j\beta x'}(0)\rangle$. We obtain the Lyapunov exponent self-consistently by diagonalizing the relevant retarded kernel as in earlier works [3,7,27] (see Ref. [24], S7). We find that the time evolution of the OTO correlators is governed by two lattice-momentum ($\mathbf{q}$) dependent modes, an intraband mode and an interband one. At $\mathbf{q} = \mathbf{0}$ the lattice dispersion enters into the expressions of OTO correlator for both the modes only through the overall DOS $g(\varepsilon)$ (see Ref. [24], S7, Eq. (S7.24)). Also at $\mathbf{q} = \mathbf{0}$ the intraband mode has the larger Lyapunov exponent ($\lambda_L^{(M)}$) among the two and therefore dominates the onset of chaos.

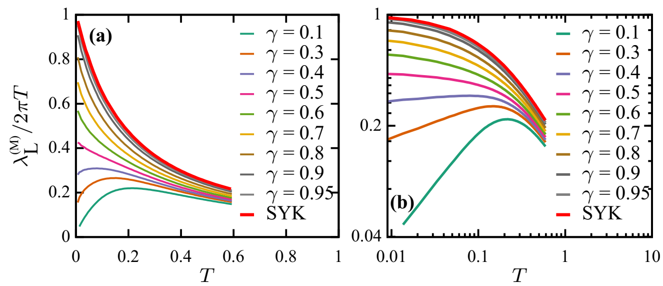

FIG. 3. Zero-mode Lyapunov exponent ($\lambda_L^{(M)}$) for $q = 2$, $J = \Lambda = 1$. (a) $\lambda_L^{(M)}/2\pi T$ vs temperature (T) for $\gamma = 0.1 - 0.95$, bold red line is for zero-dim SYK. (b) Same plot in log-log scale, showing the change from a chaotic(NFL) to a nonchaotic (LFL) fixed point.

We numerically calculate $\lambda_L^{(M)}$ for $q = 2$ as a function of T for values of γ on both sides of the quantum critical point $\gamma_c(q=2) = 1/3$ as show in Fig. 3. $\lambda_L^{(M)}/2\pi T$ becomes identical to that of the zero-dimensional SYK model [bold red line in Fig. 3(a)] as $\gamma \to 1$. In particular $\lambda_L^{(M)}/2\pi T \to 1$ as $T \to 0$, thereby saturating the chaos bound.

For $\gamma_c < \gamma < 1$, our numerical results indicate in the limit $T \to 0$, $\lambda_L^{(M)}/2\pi T \to \alpha$, where $0 < \alpha < 1$. This distinct behavior from the original SYK model implies that the NFL fixed points here *do not* saturate the chaos bound although they are still very efficient scramblers with Lyapunov exponent $\propto T$ at low temperature. Interestingly similar behavior has been reported [28] in systems involving fermions coupled to a gauge field. Around the neighborhood of γ_c for $\gamma = (0.3\text{-}0.5)$, α starts to turn around and tend towards zero [Fig. 3(b)], signifying a change in the chaotic behavior of the system. Finally, for $\gamma < \gamma_c$, $\lambda_L^{(M)}/2\pi T \to 0$ as $T \to 0$ indicating the presence of a slow scrambling phase similar to a Fermi liquid.

VII. DISCUSSION

We have developed a model that achieves a higher-dimensional generalization of the SYK model. The class of NFL phases discovered here should provide a platform to study transport properties in strongly interacting quasiparticleless lattice systems, with nonrandom hopping amplitudes. This allows us to go beyond purely diffusive transport [8–11] and study the interplay of fermion dispersion and interaction in NFL phases, in a manner not possible prior to this work. To this end, it would be interesting to explore connection between transport and scrambling in our model. This work also has relevance towards the study of interaction effects near Lifshitz transitions—particularly interesting from the perspective of transitions between band insulating topological phases that are generically separated by Lifshitz points. Finally, it would be interesting to understand how the approximate time reparametrization symmetry emerges here and what its implications are for the gravitational dual.

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