Many-body effects in transport through a quantum-dot cavity system

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We theoretically describe electric transport through an optically active quantum dot embedded in a single-mode cavity, and coupled to source-drain particle reservoirs. The populations of various many-body configurations (e.g., excitons, trions, biexciton) and the photon-number occupancies are calculated from a master equation which is derived in the basis of dressed states. These take into account both the Coulomb and the light-matter interaction. The former is essential in the description of the transport, while for the latter we identify situations in which it can be neglected in the expression of tunneling rates. The fermionic nature of the particle reservoirs plays an important role in the argument. The master equation is numerically solved for the s-shell many-body configurations of disk-shaped quantum dots. If the cavity is tuned to the biexciton-exciton transition, the most efficient optical processes take place in a three-level Λ system. The alternative exciton–ground-state route is inhibited as nonresonant due to the biexciton binding energy. The steady-state current is analyzed as a function of the photon frequency and the coupling to the leads. An unexpected feature appears in its dependence on the cavity loss rate, which turns out to be nonmonotonic.

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I. INTRODUCTION

The radiative recombination of optically pumped or electrically injected electron-hole pairs in self-assembled quantum dots (QDs) is a certain route to future quantum technologies. The optical driving is suitable for generation of entangled photons via biexciton cascade [1,2], whereas the coupling of the sample to source and drain contacts conduces to polaritonic electroluminescence [3]. More promising applications in quantum lasers and spintronics are expected if the QDs are embedded in microcavities where photons are more efficiently collected and directed [4]. A strongly coupled cavity-mode two-level QD emitter in high-Q cavities has been realized more than a decade ago [5]. Nowadays, the QD exciton levels can be electrically tuned to the optical mode of a photonic crystal membrane [6] and large values of the cooperativity parameter can be obtained [7]. Recently, Somaschi *et al.* [8] reported high photon indistinguishability for InAs dots embedded in a biased micropillar. The strong exciton-photon coupling was shown to induce cavity-mediated coupling between two QDs [9], and Pagliano *et al.* designed an ultrafast photonic crystal cavity diode [10].

The tunneling of electrons and holes in cavity QDs has been on one hand considered as a mere practical tool to supply biexciton or exciton states from contacts or wetting layers in view of lasing [11], two-photon turnstile operations [12], or single-photon pumping [13]. On the other hand, the contact barriers built within a p-i-n structure are carefully controlled in photocurrent experiments [14] and spin-polarized charging operations [15,16] using QD photodiodes.

A lot of theoretical work has been done on photon statistics and correlations in microcavities hosting self-assembled QDs whose exciton transition frequency matches one of the cavity modes. Surprisingly, calculations of input and output currents through a QD-cavity system are quite rare. Djuric *et al.* [17] addressed the problem of spin current in a QD-cavity system coupled to a single reservoir. Calculations of ground-state electroluminescence in the strong light-matter coupling regime and an extensive discussion on the role of the chemical potential of the contacts have been recently presented [18] within an open system approach.

In this work, we explore the interplay of transport and electron-hole pair recombination in a biased quantum-dot photodiode embedded in a single-mode microcavity. The analysis of the transport properties in such a system turns out to be worthwhile, for several reasons: (i) The current passing through the QD is a measurable quantity and provides relevant insight on the transient and steady-state populations of various excitonic states; such hints cannot be traced back from photon statistics. (ii) Just as the electrical injection rates and QD level structure strongly affect the emitted light or the efficiency of the biexciton cascade, the transport properties depend on the optical recombination, cavity losses, and nonradiative exciton losses. Such a discussion is interesting in itself and to our best knowledge has not been done yet. (iii) Last but not least, electrical injection into the valence and conduction states is usually spin degenerate so one cannot expect to fully describe the transport properties within two-level models borrowed from quantum optics.

The formal tool which describes the dynamics of QD-cavity systems and conveniently incorporates various dissipation and recombination processes is the Lindblad form of the master equation (ME). Benson and Yamamoto [19] initiated theoretical studies of a single QD laser within a noninteracting two-level emitter model which describes a single bright exciton selected by suitably polarized light. Later on, Perea *et al.* [20] calculated the first- and second-order coherence functions for a similar system in which excitons are injected from reservoirs via phonon-assisted processes. Steady-state quantities describing a two-level QD laser can be derived directly as a continued fraction within the dissipative Jaynes-Cummings model [21].

The ME approach was also employed to study of biexciton cascade and entangled photon pair in a QD-cavity supporting two modes of different polarizations [22]. A time-dependent formalism has been implemented to study conduction band transient transport in single [23] or double [24] lateral QDs embedded in cavities.

Since optically active QDs are genuine many-body systems, a description of their lasing and transport properties must go beyond the exactly solvable two-level Jaynes-Cummings (JC) model. The photon-probability cluster expansion method proposed by Richter *et al.* [25] allows one to treat few-level interacting QDs strongly coupled to cavity modes. Florian *et al.* [26] also developed an equation-of-motion technique for a four-level QD coupled to both fermionic and bosonic reservoirs and mostly discussed the population dynamics.

Here, we focus on the steady-state transport properties of a QD-cavity system by writing a dressed-states picture of the master equation describing the dynamics of the system. The paper is organized as follows. In Sec. II we present the open system approach to our problem and derive a Markovian master equation for the reduced density matrix of the QD-cavity system in terms of dressed states. It is also shown that if the chemical potential of the contacts are tuned away from the energy required to change the many-body configurations of the QD by single-electron tunneling, one recovers a master equation written entirely in terms of "free" states (i.e., the states of the QD cavity in the absence of the matter-photon coupling). Analytical results for the s-shell states of a disk-shaped QD are presented in the Appendix. Our approach consistently incorporates many-body effects originating from both intradot Coulomb interaction and electron-hole-photon coupling. Some of these effects are illustrated by numerical results in Sec. III. We find that the double occupancy of the s-shell states favors a Λ-type dynamics of the system at the biexciton resonant frequency. Also, the photon-carrier correlations induce a non-monotonic behavior of the steady-state current as function of cavity losses. The conclusions are left for Sec. IV.

II. FORMALISM

A. Many-body states and dressed states

We consider an optically active QD embedded in a p-i-n structure which is also placed in a microcavity supporting a single radiation mode. In the absence of exciton-photon coupling, the "free" dot-cavity Hamiltonian reads as

$$H_S^{(0)} = H_D + \hbar\omega_+ a_{\sigma_+}^\dagger a_{\sigma_+} + \hbar\omega_- a_{\sigma_-}^\dagger a_{\sigma_-}, \qquad (1)$$

where $a_{\sigma_\pm}^\dagger$ is the photon creation operator associated to $\sigma_\pm$ polarizations with frequencies $\omega_\pm$ and H_D describes the Coulomb interacting quantum dot. Its many-body (MB) configurations $\{\nu\}$ and the associated energies E_ν are defined by $H_D|\nu\rangle = E_\nu|\nu\rangle$. Then, by using the Fock states $|n_+,n_-\rangle$ described by the number of photons for each polarization one can write $H_S^{(0)}|\nu,n_+,n_-\rangle = \mathcal{E}^{(0)}_{\nu,n_+,n_-}|\nu,n_+,n_-\rangle$ where

$$\mathcal{E}^{(0)}_{\nu,n_+,n_-} = E_\nu + \hbar(\omega_+ n_+ + \omega_- n_-). \qquad (2)$$

In this work, we present numerical calculations for disk-shaped quantum dots whose valence band states are heavy-hole (HH) like. Then, the corresponding JC, light-matter interaction Hamiltonian in the rotating-wave approximation is given as follows (H.c. denotes the Hermitian conjugate):

$$H_{\text{el-ph}} = -i\hbar\left(g_+ c_\downarrow^\dagger b_\Uparrow^\dagger a_{\sigma_+} - g_- c_\uparrow^\dagger b_\Downarrow^\dagger a_{\sigma_-}\right) + \text{H.c.}, \qquad (3)$$

where $g_\pm$ are the optical coupling constants, and the carrier operators have obvious notations. For instance, $c_\downarrow^\dagger$ creates a spin-down electron on the conduction band (CB) single-particle states of the quantum dot and $b_\Uparrow^\dagger$ creates a heavy hole (HH) with total angular momentum $J_z = \frac{3}{2}$ in the valence band (VB). If we limit the discussion to the s shell in both bands, there is no need for another single-state index aside from the spin. The Hamiltonian (3) can be easily generalized if one needs to include further state indices as well as Luttinger spinors with both light-hole (LH) and HH states.

The coupled QD-cavity system is generally described by dressed states $|\varphi_p\rangle$ and by their energies $\mathcal{E}_p$:

$$H_S|\varphi_p\rangle = \left(H_S^{(0)} + H_{\text{el-ph}}\right)|\varphi_p\rangle = \mathcal{E}_p|\varphi_p\rangle, \qquad (4)$$

where φ_p are linear combinations of free states

$$|\varphi_p\rangle = \sum_{\nu,n_+,n_-} A^{(p)}_{\nu,n_+,n_-}|\nu,n_+,n_-\rangle. \qquad (5)$$

Note that p is actually a multi-index collecting relevant quantum numbers of the QD+cavity system. The coefficients $A^{(p)}_{\nu,n_+,n_-}$ must be calculated by analytical or numerical diagonalization.

It is easy to check that for a Hamiltonian describing QD with rotational symmetry the total spin $F_z(\nu) = J_z(\nu) + S_z(\nu) + M(n_+,n_-)$ of its free-state components is conserved, where the total photon spin is introduced as $M(n_+,n_-) = n_+(\nu) - n_-(\nu)$. Another conserved quantity is the net charge (in elementary charge units) $q(\nu) = n_h(\nu) - n_e(\nu)$, where n_e (n_h) is the number of electrons (holes) in the conduction and valence QD levels. Finally, the optical selection rules lead to the conservation of $n_{\uparrow-} = n_{e\uparrow} + n_-$ and $n_{\downarrow+} = n_{e\downarrow} + n_+$ [27].

Then, the many-body configurations $\{|\nu,n_+,n_-\rangle\}$ which are optically coupled can be organized in invariant orthogonal subspaces specified by such quantum numbers. In other words, the Hamiltonian H_S becomes block diagonal. If one considers only the s-shell states, the largest blocks are four dimensional and contain the neutral states, namely, the ground state, the biexciton, and the two bright excitons with quantum numbers $F_z, q = 0, n_{\uparrow-}$, and $n_{\downarrow+}$. For practical calculations, one has to truncate both the photon numbers n_+,n_- and the many-body configurations such that the dimension and the number of such subspaces stay finite.

For further use, let us introduce below the notation for the 16 many-body configurations derived from the s-shell single-particle conduction and valence band states of an interacting quantum dot. Then, the dot Hamiltonian H_D reads as

$$H_D = \sum_\sigma \epsilon_e \hat{n}_{e\sigma} + \sum_{\sigma'} \epsilon_h \hat{n}_{h\sigma'} - \sum_{\sigma,\sigma'} V_{eh}\hat{n}_{e\sigma}\hat{n}_{h\sigma'} + V_{ee}\hat{n}_{e\uparrow}\hat{n}_{e\downarrow} + V_{hh}\hat{n}_{h\Uparrow}\hat{n}_{h\Downarrow}, \qquad (6)$$

where we introduced the electron/hole single-particle energies ϵ_e/ϵ_h. The electron-number operators are written in terms of creation and annihilation operators for electrons, $\hat{n}_{e\sigma} = c_\sigma^\dagger c_\sigma$ and $\hat{n}_{h\sigma} = b_\sigma^\dagger b_\sigma$. V_{ee} and V_{hh} stand for the intraband interaction while V_{eh} denotes the interband (electron-hole) interaction.

It is convenient to rely on the occupation-number basis associated to spin-up and -down states. We label the valence states by their hole occupation. The ground state $|G\rangle$ corresponds to empty CB and full VB (the electron-hole vacuum). The two bright excitons are $|X_\uparrow\rangle = |\uparrow;\Downarrow\rangle$ and $|X_\downarrow\rangle = |\downarrow;\Uparrow\rangle$, whereas the dark excitons are denoted by small letters $|x_\downarrow\rangle = |\downarrow;\Downarrow\rangle$ and $|x_\uparrow\rangle = |\uparrow;\Uparrow\rangle$. The pairs of positively/negatively charged trion states are $|X_\uparrow^+\rangle = |\uparrow;\Uparrow\Downarrow\rangle$, $|X_\downarrow^+\rangle = |\downarrow;\Uparrow\Downarrow\rangle$ and $|X_\Downarrow^-\rangle = |\uparrow\downarrow;\Downarrow\rangle$ and $|X_\Uparrow^-\rangle = |\uparrow\downarrow;\Uparrow\rangle$. Note that the positive (negative) trions are indexed by their extra electron (hole) spin. Next, we introduce the single-hole states $|h_\Uparrow\rangle = |\Uparrow\rangle$, $|h_\Downarrow\rangle = |\Downarrow\rangle$ and one-electron (no hole) states $|e_\uparrow\rangle = |\uparrow\rangle, |e_\downarrow\rangle = |\downarrow\rangle$. Finally, the biexciton and two electron/hole states are denoted by $|XX\rangle = |\uparrow\downarrow;\Uparrow\Downarrow\rangle$, $|2e\rangle = |\uparrow\downarrow\rangle$, and $|2h\rangle = |\Uparrow\Downarrow\rangle$. We emphasize that with the Coulomb interaction expressed in terms of occupation numbers, there is no configuration mixing, but only energy renormalizations.

B. Transport setting for the QD-cavity system

The interaction with the environment of the QD-photon system is described by the coupling to several reservoirs. Our main interest is in QD electric excitation. The carrier source and sink are described as noninteracting fermionic particle reservoirs provided by contacts to the n and p regions, respectively. Considering alternative processes, instead of tunneling, as carrier capture mechanisms (e.g., Auger processes [28]) would change neither the structure of the master equation nor the necessity to describe it in the dressed-states basis.

In the presence of the Coulomb and JC interactions, the corresponding Lindblad-type dissipative terms in the evolution of the density operator require a more careful consideration.

In describing the flow of the current, we follow the literature and use below the conduction-valence terminology rather than the electron-hole language of the previous subsection. They are, of course, equivalent.

Excited states in the QD are populated electrically by injecting electrons from the n region and by depleting the valence states to the p region (equivalently, by injecting holes). Thus, the conduction (valence) band states of the dot are coupled at the initial time $t = 0$ to a single-particle reservoir denoted by c (v). This model has been used in other theoretical studies [29,30]. The corresponding tunneling Hamiltonian reads as

$$H_T = \sum_\alpha \sum_{k,\sigma} \sum_{i\in\mathrm{QD}} \left(V_{i,k\sigma}^\alpha c_i^\dagger c_{k\alpha\sigma} + \mathrm{H.c.}\right), \tag{7}$$

where i denotes a single-particle conduction or valence state within the QD, (k,σ) stand for the momentum and spin of electron in the reservoir $\alpha = c,v$, and $V_{i,k\sigma}^\alpha$ is the tunneling strength. In this work, we use $V_{i,k\sigma}^\alpha$ as phenomenological parameters. We assume that the tunneling processes conserve the spin such that $V_{i,k\sigma}^\alpha$ vanishes unless one has $S_z(i) = \sigma$, being $S_z(i)$ the electron spin of the state i within the QD. Moreover, we assume for simplicity that $V_{i,k\sigma}^\alpha$ does not depend on k and σ. Our open QD-cavity system is therefore described by the total Hamiltonian

$$H = H_S + H_R + H_B + H_T + H_{S\text{-}B}, \tag{8}$$

where $H_R = \sum_\alpha \sum_{k\sigma} \varepsilon_k c_{k\alpha\sigma}^\dagger c_{k\alpha\sigma}$ describes the two particle reservoirs feeding the QD and ε_k is the energy of an incident electron with momentum k. We denote by μ_c and μ_v the equilibrium chemical potentials of the reservoirs. H_B and $H_{S\text{-}B}$ describe bosonic reservoirs (e.g., some leaky modes) and their coupling to the QD-cavity system. They are responsible for the cavity losses and for the nonradiative exciton recombination

Now, we follow the standard procedure for open systems (see, e.g., [31,32]) and derive the master equation for the reduced density operator $\rho(t)$ of the QD-cavity system. Within the Born-Markov approximation the master equation reads as

$$\dot{\rho}(t) = -\frac{i}{\hbar}[H_S,\rho(t)] - \mathcal{L}_\mathrm{R}[\rho(t)] - \mathcal{L}_\kappa[\rho(t)] - \mathcal{L}_\gamma[\rho(t)], \tag{9}$$

where the contribution of the reservoirs is given by

$$\mathcal{L}_\mathrm{R}[\rho(t)] = \frac{1}{\hbar^2}\int_0^\infty ds\, \mathrm{Tr}_R\{[H_T,[\tilde{H}_T(-s),\rho(t)\rho_R]]\}. \tag{10}$$

Here, we introduced the partial trace with respect to the reservoirs Tr_R, their equilibrium statistical operator ρ_R, and the interaction picture with respect to the decoupled Hamiltonian $H_S + H_R + H_B$ reads as $\tilde{H}_T(t) = e^{\frac{i}{\hbar}tH_S}e^{\frac{i}{\hbar}tH_R}H_T e^{-\frac{i}{\hbar}tH_S}e^{-\frac{i}{\hbar}tH_R}$. Since we are interested in the electrically injected carriers, an (external) optical pumping is not considered. Similar expression can be derived for the dissipative terms $\mathcal{L}_\kappa$ and $\mathcal{L}_\gamma$ associated to cavity losses (κ denotes the decay rate) and to nonradiative processes (with rate γ). For now, let us concentrate on $\mathcal{L}_\mathrm{R}$.

Obviously, one has to bring Eq. (10) to a more explicit form. This is trivial for the H_R exponential whereas for the system Hamiltonian H_S can be done rather easily by expanding the time evolution in terms of its eigenbasis, performing the trace over the degrees of freedom of the reservoirs and the time integral by using the principal-value formula. Then, the dissipative contribution of the reservoirs in the dressed-states basis can be written in the compact form $\mathcal{L}_\mathrm{R}[\rho] = \sum_{\alpha\sigma}\mathcal{L}_{\alpha,\sigma}[\rho]$ where the lead and spin-dependent components are found as

$$\mathcal{L}_{\alpha,\sigma}[\rho(t)] = \frac{\pi}{\hbar}\sum_{p,p',r,r'}\left\{\left[T_{rr'}^{\alpha,\sigma}|\varphi_r\rangle\langle\varphi_{r'}|,\overline{f}_\alpha(\mathcal{E}_{p'}-\mathcal{E}_p)\overline{T}_{p'p}^{\alpha,\sigma}|\varphi_p\rangle\langle\varphi_{p'}|\rho(t)\right] + \left[\overline{T}_{r'r}^{\alpha,\sigma}|\varphi_r\rangle\langle\varphi_{r'}|, f_\alpha(\mathcal{E}_p-\mathcal{E}_{p'})T_{pp'}^{\alpha,\sigma}|\varphi_p\rangle\langle\varphi_{p'}|\rho(t)\right] + \mathrm{H.c.}\right\}. \tag{11}$$

Here, f_α is the Fermi function of the reservoir α which appears from the partial trace over the leads' degrees of freedom, that is, $\mathrm{Tr}_R\{c_{k\alpha\sigma}^\dagger c_{k'\alpha'\sigma'}\rho_R\} = \delta_{\sigma\sigma'}\delta_{\alpha\alpha'}\delta(k-k')f_\alpha(\varepsilon_k)$. We also introduced the shorthand notation $\overline{f}_\alpha = 1 - f_\alpha$. $T_{pp'}^{\alpha,\sigma}$ are the matrix elements of the "jump" operators between two dressed states of the full Hamiltonian

$$T_{pp'}^{\alpha,\sigma} = \sqrt{D_\alpha} \sum_{i\in \mathrm{QD}} V_i^{\alpha,\sigma} \langle \varphi_p | c_i^\dagger | \varphi_{p'} \rangle, \tag{12}$$

and D_α is the density of states (DOS) in the lead α whose weak dependence on energy in the window of interest is assumed here for simplicity. The principal-value terms from the integral over k induce a Lamb shift on the eigenvalues of H_S and were neglected [33]. Obviously, $T_{pp'}^{\alpha,\sigma}$ collects all addition processes of an electron with spin σ on the single-particle states i from the band α. By looking at the dressed-states structure [see also Eq. (5)] we observe that $T_{pp'}^{\alpha,\sigma}$ and its complex conjugate $\overline{T}_{p'p}^{\alpha,\sigma}$ map a subspace made by dressed states with net charge $q(p)$ to its "nearest-neighbor" charge $q(p')$ since the sequential tunneling imposes the condition $|q(p) - q(p')| = 1$.

Let us stress that in the literature (see, e.g., [19,20,26]) it is quite common to write the dissipative Lindblad part of the evolution in terms of single-particle "jump" operators c_i or $c_i^\dagger$, as if all such jumps would require the same energy compensation from the reservoirs. This noninteracting assumption contrasts with the unitary part $[H_S,\rho]$, which contains through H_S the full Coulomb and QD-photon interactions. In fact, the energies involved in each transition depend on the number of photons and "spectator" carriers, which do not participate in the jump themselves. In other words, the proper description of the dissipative processes is done in terms of dressed states and their energies, as in Eq. (11). The situation becomes quite complex, but possible simplifications may occur, as in the cases discussed below.

The master equation (9) can be solved numerically either with respect to the dressed-states basis $\{|\varphi_p\rangle\}$ or in the free basis $\{|\nu,n_+,n_-\rangle\}$, the latter being more convenient when discussing the dynamics and transport in terms of populations and coherences.

Now, it is obvious that it is the presence of the dressed-states energy differences $\mathcal{E}_p - \mathcal{E}_{p'}$ that prevents us from changing in Eq. (11) the decomposition of unity as $\sum_p |\varphi_p\rangle\langle\varphi_p| = \sum_{\nu,n_+,n_-} |\nu,n_+,n_-\rangle\langle\nu,n_+,n_-|$, i.e., expressing the result in terms of states free of JC interaction. The conversion is exact for the term $|\varphi_r\rangle T_{rr'}^{\alpha,\sigma}\langle\varphi_{r'}|$ and its Hermitian conjugate in Eq. (11) since the summations over r,r' do not involve dressed-states energies. Below we discuss that the same holds under certain conditions for the p,p'-dependent factors too.

First, one has to observe that any energy difference $\mathcal{E}_p - \mathcal{E}_{p'}$ equals the tunneling gap (i.e., the charging energy) $W_{\nu\nu'} = E_\nu - E_{\nu'}$ between two QD many-body configurations plus certain JC shifts $w_{pp'}$ which depend on the matter-photon coupling g_c and on photon numbers n_+,n_-. In the Appendix we derive such relations for the s-shell states [see, e.g., Eqs. (A6) and (A7)]. Second, $\mathcal{L}_{\alpha,\sigma}$ depends on this energy difference via the Fermi function, and thus the position of its argument relative to the chemical potential is essential.

Now, assume that for any pair of dressed states φ_p, $\varphi_{p'}$ whose charges satisfy $|q(p) - q(p')| = 1$ one can write

$$f_\alpha(\mathcal{E}_p - \mathcal{E}_{p'}) = f_\alpha(W_{\nu\nu'}), \tag{13}$$

where $|\nu,n_+,n_-\rangle$ and $|\nu',n_+,n_-\rangle$ are JC-free components of φ_p and $\varphi_{p'}$ which are coupled either by creation or annihilation operators on the dot (e.g., $c_i^\dagger|\nu'\rangle = |\nu\rangle$). Then, by using the decomposition of unity on the subspaces with charge $q(p)$ and $q' = q(p')$ one can prove the identity (see Appendix)

$$\begin{aligned}\sum_{\substack{p,p'\\ q-q'=1}} & f_\alpha(\mathcal{E}_p - \mathcal{E}_{p'}) T_{pp'}^{\alpha,\sigma} |\varphi_p\rangle\langle\varphi_{p'}| \\ &= \sum_{\nu,\nu'}\sum_{n_+,n_-} f_\alpha(W_{\nu\nu'}) T_{\nu\nu'}^{\alpha,\sigma} |\nu,n_+,n_-\rangle\langle\nu',n_+,n_-| \\ &= \sum_{\nu,\nu'} f_\alpha(W_{\nu\nu'}) T_{\nu\nu'}^{\alpha,\sigma} |\nu\rangle\langle\nu'|, \end{aligned}\tag{14}$$

where we introduced jump operators associated to a pair of many-body configurations of the QD only, $T_{\nu\nu'}^{\alpha,\sigma} = \sqrt{D_\alpha}\sum_i V_i^{\alpha,\sigma}\langle\nu|c_i^\dagger|\nu'\rangle$. Note that Eq. (14) is much simpler and now allows one to rewrite $\mathcal{L}_{\alpha,\sigma}$ in Eq. (11) in terms of free states only. The action on the photon degrees of freedom is reduced to the unit operator. While this form is the starting point of many theoretical and numerical calculations, its derivation from the dressed-states picture of the master equation has not been previously discussed.

The condition (13) emphasizes that the dressed-states-dependent shifts $w_{pp'} = \mathcal{E}_p - \mathcal{E}_{p'} - W_{\nu\nu'}$ can be neglected as arguments of the Fermi function. We find that [see Eqs. (A6) and (A7)] for the photon numbers considered here, these shifts are less than 1 meV and then at small temperatures Eq. (13) is fulfilled if μ_α lies at least a few meV above or below $W_{\nu\nu'} + w_{pp'}$. Note also that the condition covers the very particular case in which $f_c = \overline{f}_v = 1$ for *all* pairs of dressed states involved in tunneling processes. On the other hand, it should be stressed that the condition does not hold in the nearly resonant regime when one has $\mu_\alpha \approx W_{\nu\nu'}$. In particular, if $W_{\nu\nu'} < \mu_\alpha < W_{\nu\nu'} + w_{pp'}$ one has $f_\alpha(W_{\nu\nu'}) = 1$ while $f_\alpha(W_{\nu\nu'} + w_{pp'}) = 0$, provided $w_{pp'} > 0$. Nevertheless, the electrical injection regime naturally implies that the chemical potentials are set away from the energies of the many-body configuration one wants to feed, so the condition (13) holds. In these circumstances, the influence of the matter-photon interaction on the carrier transport is negligible. On the contrary, the Coulomb MB effects are important and are correctly incorporated into the picture.

The dissipative terms $\mathcal{L}_\kappa$ and $\mathcal{L}_\gamma$ from Eq. (9) can be also expressed in the free-states picture. In this case, the trace over the bath degrees of freedom generates terms containing the Bose function $n_B(\mathcal{E}_p - \mathcal{E}_{p'})$. The main point here is that $\mathcal{E}_p - \mathcal{E}_{p'} \sim \hbar\omega_c \gg k_B T$, so that the Bose function involved is negligible. Only the $n_B + 1$ downward term survives and one easily recovers the usual Lindblad form

$$\mathcal{L}_\kappa = \frac{\kappa}{2}\sum_{s=\sigma_+,\sigma_-}(a_s^\dagger a_s\rho + \rho a_s^\dagger a_s - 2a_s\rho a_s^\dagger). \tag{15}$$

Similarly, the term associated to nonradiative exciton losses reads as

$$\mathcal{L}_\gamma = \frac{\gamma}{2}(c_\downarrow^\dagger b_\Uparrow^\dagger b_\Uparrow c_\downarrow \rho + \rho c_\downarrow^\dagger b_\Uparrow^\dagger b_\Uparrow c_\downarrow - 2b_\Uparrow c_\downarrow \rho c_\downarrow^\dagger b_\Uparrow^\dagger) + \frac{\gamma}{2}(c_\uparrow^\dagger b_\Downarrow^\dagger b_\Downarrow c_\uparrow \rho + \rho c_\uparrow^\dagger b_\Downarrow^\dagger b_\Downarrow c_\uparrow - 2b_\Downarrow c_\downarrow \rho c_\uparrow^\dagger b_\Downarrow^\dagger). \tag{16}$$

The total charge on the QD can be calculated as $\mathcal{Q}_S = \sum_\nu \sum_{n_+,n_-} \langle \nu,n_+,n_-|Q_S\rho(t)|\nu,n_+,n_-\rangle$. The total charge operator $Q_S = e(N_c + N_v)$ where N_c (N_v) are the occupation-number operators of the conduction (valence) QD levels (e.g., $N_c = \sum_\sigma \hat{n}_{e\sigma}$) and $e < 0$ denotes the electron charge. By convention, the current in the left contact J_c is positive if electrons flow from the contact into the QD, while the current in the right contact J_v is positive if electron leaves the valence energy levels of the QD. Using the continuity equation we identify the currents in the two reservoirs

$$\dot{\mathcal{Q}}_S = \mathrm{Tr}_S\{Q_S\dot{\rho}(t)\} = J_c(t) - J_v(t). \tag{17}$$

By straightforward calculations, one can easily check that $\mathrm{Tr}_S\{Q_S\mathcal{L}_\kappa[\rho(t)]\} = \mathrm{Tr}_S\{Q_S\mathcal{L}_\gamma[\rho(t)]\} = 0$. Also, $\mathrm{Tr}_S\{Q_S[H_{\text{el-ph}},\rho(t)]\} = \mathrm{Tr}_S\{[Q_S,H_{\text{el-ph}}]\rho(t)\} = 0$ since the photon-matter Hamiltonian conserves the total charge. The time-dependent currents then read as

$$J_\alpha(t) = e\sum_\sigma \mathrm{Tr}_S\{\hat{N}_\alpha \mathcal{L}_{\alpha,\sigma}[\rho(t)]\}. \tag{18}$$

In the steady state one must have $J_c = J_v := J_S$. The average photon number $\mathcal{N} = \mathcal{N}_{\sigma_+} + \mathcal{N}_{\sigma_-}$ is calculated from

$$\mathcal{N}_{\sigma_\pm} = \mathrm{Tr}_S\{a_{\sigma_\pm}^\dagger a_{\sigma_\pm}\rho(t)\}. \tag{19}$$

Other useful quantities are the population of N-photon states $P_N(t)$ and the population P_ν of a given many-body configuration ν:

$$P_N(t) = \sum_{\substack{\nu,n_+,n_- \\ n_++n_-=N}} \langle \nu,n_+,n_-|\rho(t)|\nu,n_+,n_-\rangle, \tag{20}$$

$$P_\nu(t) = \sum_{n_+,n_-} \langle \nu,n_+,n_-|\rho(t)|\nu,n_+,n_-\rangle. \tag{21}$$

Last but not least, the free-states picture of $\mathcal{L}_{\alpha,\sigma}$ and Eq. (18) allow us to write the time-dependent currents in terms of relevant populations. For our s-shell 16 configurations we find the incoming current J_c^{in} and the outgoing current J_c^{out}:

$$J_c^{\text{in}}(t) = \frac{e\Gamma_c}{\hbar}\sum_{\{\nu,\nu'\}_{\text{in}}} P_\nu(t) f_c(W_{\nu'\nu}), \tag{22}$$

$$J_v^{\text{out}}(t) = \frac{e\Gamma_v}{\hbar}\sum_{\{\nu,\nu'\}_{\text{out}}} P_\nu(t) \overline{f}_v(W_{\nu\nu'}), \tag{23}$$

where we introduced the tunneling coefficients $\Gamma_\alpha = \pi V_\alpha^2 D_\alpha$ and $\{\nu,\nu'\}_{\text{in}}$ ($\{\nu,\nu'\}_{\text{out}}$) denotes all pairs of many-body configurations connected by tunneling-in (-out) processes. For example, the incoming current collects contributions from $W_{X_\uparrow^+,2h}$, $W_{X_\uparrow,h_\Downarrow}$, $W_{x_\uparrow,h_\Uparrow}$ etc. Note that the dark excitons contribute to the transport so they cannot be disregarded in transport calculations. In contrast, states with full CB (i.e., the biexciton, the negative trions, and $|2e\rangle$) do not contribute to the incoming current since tunneling-in processes are no longer allowed. Therefore, the populations of these states do not appear in the expression of J_c^{in}. Similar states can be identified for the outgoing current J_v^{out}.

Let us note that even if the Jaynes-Cummings shifts appearing in the dissipative terms of the master equation are negligible one cannot disentangle the effects of the intradot interaction and exciton-photon coupling. On the one hand, the optical resonant frequencies are fully determined by the Coulomb many-body effects and on the other hand the JC coupling from the unitary term $[H_S,\rho]$ leads to generation/recombination processes which inhibit/activate transport channels.

III. RESULTS

In this section, we restrict our calculation to the spin-degenerate purely heavy-hole (HH) s-shell states which are known to accurately describe the low-energy states of disk-shaped QDs. The numerical results were obtained for a InAs quantum dot. We calculate the single-particle states from the $k \cdot p$ theory [34] using the Luttinger parameters $\gamma_1 = 11.01$, $\gamma_2 = 4.18$, and $\gamma_3 = 4.84$. We considered a quantum dot of radius $R = 15$ nm and height $W = 5$ nm. The cavity supports a single mode, thus, $\omega_+ = \omega_- = \omega_c$ and $g_+ = g_- = g_c$. Also, we assume equal tunneling rates $\Gamma_c = \Gamma_v = \Gamma$.

The energies of the dot E_ν can be written analytically in terms of the single-particle energies and the Coulomb matrix elements. We used the following Coulomb interaction parameters: $V_{hh} = 18.75$ meV, $V_{eh} = 18.5$ meV, $V_{ee} = 15.25$ meV. The differences between the intraband and interband interaction constants generally follow from the stronger confinement of the holes. In the absence of strain, the electron-hole exchange interaction for the s-shell states vanishes due to radial symmetry [35]. Therefore, the bright and dark exciton states are degenerate and one has $E_{X_\uparrow} = E_{X_\downarrow} = E_{x_\uparrow} = E_{x_\downarrow}$ [36]. The same degeneracy holds for the pairs of positive and negative trion states.

Since the cavity mode supports both σ_+ and σ_- polarizations, the bright excitons $|X_\uparrow,n_+,n_- - 1\rangle$ and $|X_\downarrow,n_+ - 1,n_-\rangle$ are simultaneously coupled to the ground state $|G,n_+,n_-\rangle$ and biexciton state $|XX,n_+ - 1,n_- - 1\rangle$. If the biexciton binding energy is negligible, the ground-state-exciton and exciton-biexciton resonant frequencies are equal. Then, at resonance the corresponding dressed states are full mixtures of these four states. However, in real systems the mixing of the free states depends on the binding energy $E_b = 2V_{eh} - V_{ee} - V_{hh}$. For our parameters, the resulting binding energy $E_b = 3$ meV. We recall that such values of the binding energy were reported by Narvaez *et al.* [37] for lens-shaped quantum dots.

Similarly, the matter-photon interaction mixes the positive trion states and one-hole states on the one hand (the charge of the corresponding subspaces being $q = 1$) and the negative trions and one-electron states (i.e., one electron in the CB and no holes in the VB) on the other hand ($q = -1$). One therefore gets two pairs of dressed states (see Appendix). The resonant recombination processes correspond to four frequencies $\hbar\omega_{XX} = E_{XX} - E_X$, $\hbar\omega_X = E_X - E_G$, and $\omega_{X^\pm}$.

We first diagonalize numerically the QD-cavity Hamiltonian H_S. The weights $|A_{\nu,n_+,n_-}|^2$ associated to three dressed states from two neutral subspaces with different photon

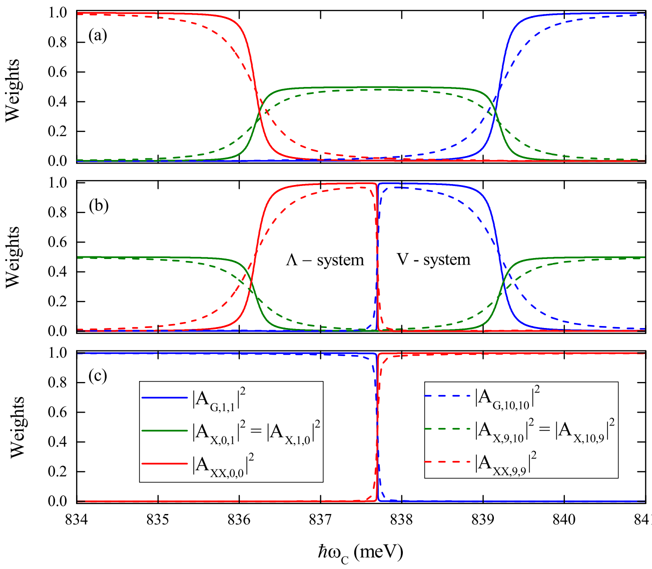

FIG. 1. The weights of the ground state and exciton and biexciton states associated to three dressed states of the QD-cavity system as function of ω_c. Each figure (a)–(c) corresponds to one dressed state. The mixing of these states also depends on the photon numbers n_+,n_- except for the resonant cases $\omega_c = \omega_X = 839.2$ meV and $\omega_c = \omega_{XX} = 836.15$ meV where $|A_{\nu,n_+,n_-}|^2 = |A_{\nu,n'_+,n'_-}|^2$. Other parameters: $\hbar g_c = 0.05$ meV.

numbers $n_\pm$ are presented in Fig. 1 as functions of ω_c. The weights of the fourth state are omitted because they do not depend on frequency and one has $|A_{X_\uparrow,n_+,n_--1}|^2 = |A_{X_\downarrow,n_+-1,n_-}|^2 = 1/2$ and $|A_{G,n_+,n_-}|^2 = |A_{XX,n_+-1,n_--1}|^2 = 0$. It is not difficult to observe that away from $\omega_c = 837.75$ meV the three dressed states describe either a three-level Λ system (if $\omega_c < 837.5$ meV) or a V system (if $\omega_c > 838$ meV). In the case of the Λ system, the mixing of the ground state $|G,n_1,n_2\rangle$ is negligible while for the V system the biexciton state is decoupled from the other states. Note that the weights in Fig. 1 also depend on the photon numbers except for the resonance points. The dressed states and the associated energies of the Λ system at the biexciton resonance ω_{XX} are explicitly given in the Appendix. In particular if $\omega_c = \omega_{XX}$ one recovers in Figs. 1(a) and 1(b) the analytical results $|A_{X_\uparrow,n_+,n_--1}|^2 = |A_{X_\downarrow,n_+-1,n_-}|^2 = 1/4$ and $|A_{XX,n_+-1,n_--1}|^2 = 1/2$.

We solve the master equation numerically in the free states $\{|\nu,n_+,n_-\rangle\}$ by suitably truncating the photon number $N = n_+ + n_-$ to a maximum value N_0, such that the N_0-photon population $P_{N_0}(t)$ is so small that it can be safely neglected for all values of the cavity losses $\hbar\kappa > 5$ μeV. The corresponding numerical results are stable for a photon cutoff $N_0 = 20$. The steady-state values of various quantities (e.g., the current, average photon number) are obtained numerically as the long-time limit of the statistical averages with respect to the reduced density matrix $\rho(t)$.

As a first characterization of the transport through the QD-cavity system, we present in Fig. 2(a) the steady-state current J_S as function of the frequency ω_c. The chemical potentials of the reservoirs were chosen such that $\mu_c > W_{\nu\nu'}$ and $\mu_v < W_{\nu\nu'}$ for all many-body configurations $\{\nu,\nu'\}$. In this regime, the electrons enter the QD only from the left contact and tunnel

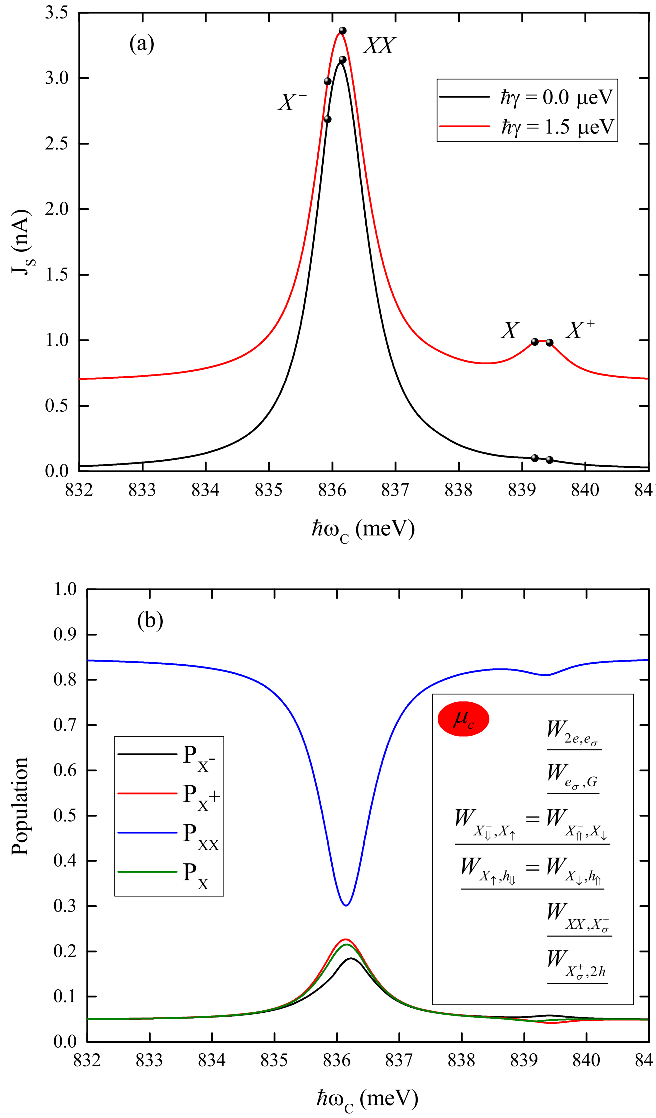

FIG. 2. (a) The steady-state current J_S as a function of the cavity frequency. The bullets mark the resonance frequencies. (b) The steady-state populations of relevant MB configurations as function of ω_c at $\hbar\gamma = 1.5$ μeV. Right inset: the relevant charging energies $W_{\nu\nu'}$ are located below the chemical potential μ_c (see also the discussions in the main text). σ denotes the electronic spin of the s-shell single-particle state. Other parameters: $\hbar\kappa = 0.5$ meV, $\Gamma = 13$ μeV, $\hbar g_c = 0.05$ meV, $\mu_c = 625$ meV, $\mu_v = -275$ meV.

out only to the right contact. In Fig. 2(b) we schematically show the charging energies $W_{\nu\nu'}$ describing transitions to the s-shell states in the conduction band. All charging energies associated to tunneling into the p-shell states are higher than μ_c and were not included in the calculation.

Away from resonance frequencies, a background current around 0.65 nA that passes through the system is mainly due to the nonradiative losses into the leaky modes; it disappears if $\gamma = 0$. The clear peak around $\omega_c = 836$ meV is due to the transitions $XX \to X_\sigma$ and $X^-_{\Uparrow(\Downarrow)} \to e_{\uparrow(\downarrow)}$; note that their corresponding frequencies ω_{XX} and ω^-_X are separated by 0.25 meV. A second much smaller peak appears around $\omega_c = 839$ meV and is associated to the exciton ground state and positive

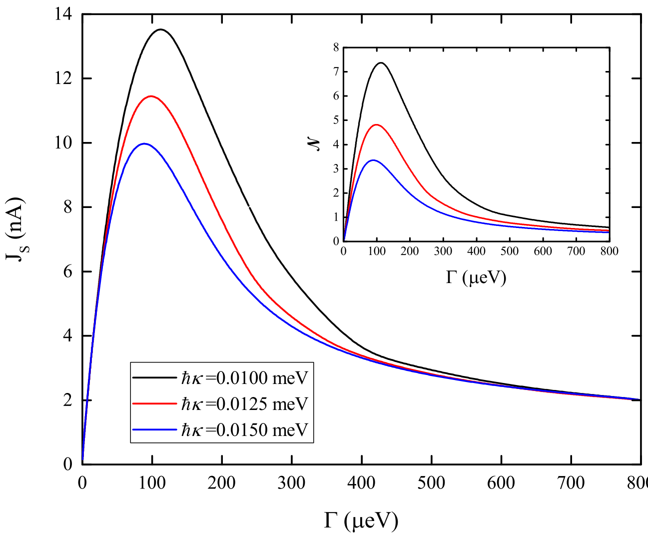

FIG. 3. The steady-state current as a function of the tunnel coupling Γ. Inset: the steady-state mean number of photons $\mathcal{N}$ as a function of the tunneling parameter Γ. Other parameters: $\mu_c = 625$ meV, $\mu_v = -275$ meV, $\hbar g_c = 0.05$ meV, $\hbar\gamma = 0.0015$ meV.

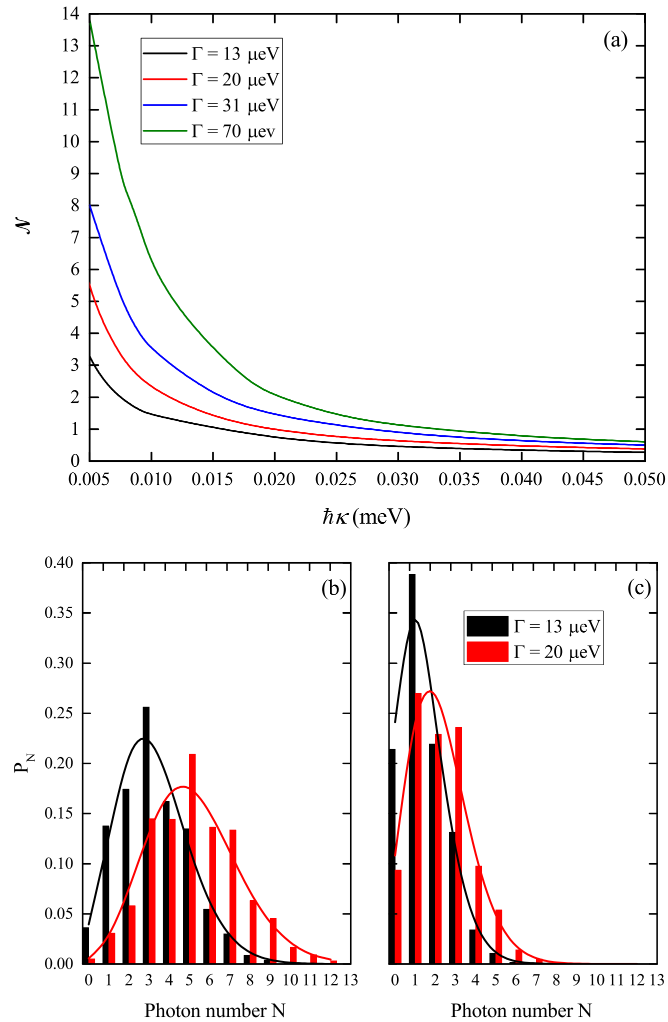

FIG. 4. (a) The steady-state average number of photons $\mathcal{N}$ as a function of the cavity losses. The coupling to reservoirs Γ is fixed. The sudden drop of $\mathcal{N}$ covers a very narrow range of κ. The steady-state occupation of the N-photon states at low cavity losses (b) $\hbar\kappa = 0.005$ meV and (c) $\hbar\kappa = 0.01$ meV for two values of the coupling to reservoirs Γ. The solid curves are fit to Poisson distribution. Other parameters: $\mu_c = 625$ meV, $\mu_v = -275$ meV, $\hbar\gamma = 0.0015$ meV, $\hbar g_c = 0.05$ meV.

trion-one-hole recombination. The separation between the two peaks is roughly given by the biexciton binding energy E_b.

The rather poor transport response of the system at $\omega_c = \omega_X$ is caused by the rapid filling of the biexciton states $|XX,n_+,n_-\rangle$ on the one hand and by their weak recombination on the other hand (recall that $\omega_{XX} = \omega_X - E_b$). To confirm this scenario, we looked at the relevant steady-state populations [see Fig. 2(b)]. One notices a high steady-state population of the biexciton state ($P_{XX} \sim 0.85$) at $\omega_c = \omega_X$; this also corresponds to a negligible average photon number $\mathcal{N} \sim 0.01$ (not shown). In contrast, as ω_c sweeps the biexciton resonance, P_{XX} drops while the populations of the bright exciton $P_X = P_{X_\uparrow} + P_{X_\downarrow}$ and trions $P_{X^\pm}$ increase. The mechanism behind this behavior when $\omega_c \sim \omega_{XX}$ is easy to grasp. On the one hand, the strong biexciton recombination feeds the population of the bright excitons. On the other hand, the positive/negative trions are generated from any exciton state (i.e., bright or dark) by injecting one more electron/hole. The sequence of charging energies in Fig. 2(b) suggests that the filling of the negative trions can be reduced if one tunes $\mu_c < W_{X^-_\Downarrow X_\uparrow} = W_{X^-_\Uparrow X_\downarrow}$. This also inhibits the biexciton population via the channel $X^- \to XX$. Similarly, eliminating the other biexcitonic channel $X^+_\sigma \to XX$ requires $\mu_c < W_{XX,X^+_\sigma}$ but also reduces the recombination processes as the bright excitons are no longer filled ($\mu_c < W_{X_\uparrow h_\Downarrow}$). Therefore, the interplay between the intraband and interband Coulomb interactions does not allow us to disregard the biexciton state in transport calculations.

The same features are observed for other values of the tunneling strength Γ. In the following, we shall focus on the resonant regime $\omega_c = \omega_{XX}$. Next, we investigated the effect of the tunneling strength Γ on the steady-state current. The closed formula of $J_c^{\rm in}$ [see Eq. (22)] naively suggests a linear dependence on the tunneling rate $\Gamma/\hbar$. When looking at Fig. 3 one observes a more complex behavior.

At small or moderate coupling, the steady-state current indeed increases linearly as function of Γ. By further increasing Γ the current reaches a maximum and then decreases. The inset in Fig. 3 establishes the clear correspondence between the stationary current and the mean photon number, as $\mathcal{N}(\Gamma)$ shows the same nonmonotonous dependence on the tunneling strength. By increasing the coupling Γ more photons are generated and stored within the cavity. However, beyond a critical value $\mathcal{N}(\Gamma)$ decreases monotonously since the tunneling processes eventually trigger decoherence between the lasing levels [19]. This self-quenching clearly induces the transport features shown in Fig. 3(a). Note that the location of the maxima associated to different values of κ do not coincide; they are shifted to lower couplings Γ as the cavity losses increase. This happens because both Γ and κ contribute to dephasing.

Figure 4(a) presents the steady-state mean photon number as function of the cavity losses parameter κ for several values of Γ. As long as Γ is very small, the recombination processes

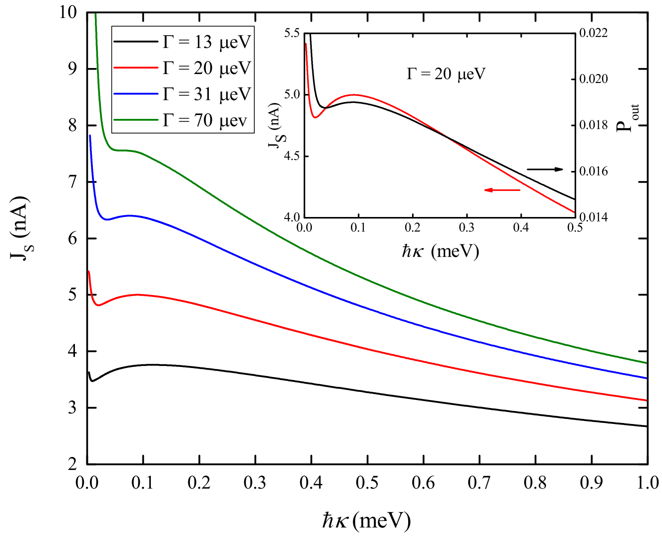

FIG. 5. The steady-state current J_S as function of cavity losses κ at several values of the tunneling strength Γ. As Γ decreases, J_S displays a nonmonotonous behavior. The frequency of the cavity mode matches ω_{XX}. The inset shows the same behavior of J_S and the output power (see the discussion in the text). Other parameters: $\hbar g_c = 0.05$ meV, $\mu_c = 625$ meV, $\mu_v = -275$ meV, $\hbar\gamma = 0.0015$ meV.

are less effective because of very slow tunneling processes and $\mathcal{N}(\kappa)$ does not exceed 3 even for high-quality cavities. One observes that in the strong optical coupling regime $g_c > \kappa$ the steady-state average $\mathcal{N}(\kappa)$ drops very fast especially for $\Gamma > 30\ \mu$eV. In particular, the cavity accommodates up to 13.5 photons for $\Gamma = 70\ \mu$eV and $\hbar\kappa = 0.005$ meV. At $\hbar\kappa = 0.02$ meV, most of the photons are lost through leaky modes such that $\mathcal{N} \sim 3$. The decrease of $\mathcal{N}$ slows down as $\hbar\kappa > 0.02$ meV. As expected, in the weak coupling regime $g_c < \kappa$ the cavity does not accumulate photons anymore and we find that $\mathcal{N} < 0.1$ for $\hbar\kappa > 0.2$ meV.

Since the average number $\mathcal{N}$ does not provide detailed information on the photon statistics, it is useful to analyze the occupation of the N-photon states in the steady-state regime. We observe in Fig. 4(b) that for $\Gamma = 13\ \mu$eV, the states with more than five photons are poorly populated, while for $\Gamma = 20\ \mu$eV the one- and two-photon states are nearly depleted although they substantially contribute to the transient regime (not shown). The system is mostly described by states with a number of photons close to the average number $\mathcal{N}$; more precisely, at $\Gamma = 20\ \mu$ eV the N-photon states whose population is larger that 0.1 correspond to $3 \leqslant N \leqslant 7$. For $\hbar\kappa = 0.01$ meV, the states with only few photons are more likely to be populated. Indeed, at $\Gamma = 13\ \mu$eV one observes in Fig. 4(c) that the steady-state is mostly described by one-, two-, and three-photon configurations. The comparison with the Poisson distribution [see Figs. 4(b) and 4(c)] shows that the system is driven close to coherent light.

We finally investigate the dependence of the current J_S as a function of the cavity losses κ. Figure 5 shows $J_S(\kappa)$ for several values of the tunneling strength Γ. At fixed coupling to the reservoirs, the steady-state curves of $J_S(\kappa)$ clearly resemble van der Waals–type "isotherms."

If $\Gamma > 60\ \mu$eV, the current decreases almost uniformly as κ increases. Rather surprisingly, as the coupling to the contacts decreases below $\Gamma = 60\ \mu$eV, a nonmonotonous behavior emerges. Indeed, the inset in Fig. 5 allows us to distinguish three transport regimes at $\Gamma = 20\ \mu$eV. Although J_S still drops rapidly for $\hbar\kappa \in [0.005, 0.025]$ meV (first regime), then it increases until $\hbar\kappa \sim 0.1$ meV (second regime) meV, and eventually uniformly decreases (third regime). Also, the local minimum of $J_S(\kappa)$ shifts to lower κ as Γ decreases. At first glance, this complex behavior of the steady-state current is not easy to predict, especially in view of the fact that $\mathcal{N}$ is a monotonously decreasing function of κ [see Fig. 4(a)].

A quite similar complex dependence is found for the output power $P_{\rm out} = \kappa\mathcal{N}$ which follows closely the shape of the J_S (see inset in Fig. 5). This is not surprising since the power supplied to the system by the current injection is essentially recovered in the field coming out from the cavity. Small differences arise from unimportant power losses by emission into nonlasing modes (γ is small), and from nonresonant optical processes, which have a low efficiency. On the other hand, in the steady state $P_{\rm out}$ is equal to the net photon generation in the cavity and, therefore, the explanation of its behavior should rely on the interplay between photon emission and absorption processes. Initially, the strong drop in the photon number with increasing κ leads to a decrease of $P_{\rm out}$. Later, with $\mathcal{N}$ leveling off to small values [see Fig. 4(a)], the photon generation decrease continues, due to the strong dephasing associated with large cavity losses.

The surprising effect is the intermediate interval of increasing behavior. It is clear from $P_{\rm out}$ being the product of the increasing κ and the decreasing $\mathcal{N}$ that the whole picture is the result of competing tendencies. In the case of this middle interval, the careful examination of different contributions shows that photon emission (stimulated and spontaneous) and absorption are to a great extent compensating each other, so that the net result depends on finer contributions to their leading terms. Such corrections contain carrier-photon correlation effects, which turn out to play an important role. For instance, the term describing stimulated emission from the biexciton state $\mathrm{Tr}\{\rho \sum_{n_+,n_-} |XX,N\rangle\langle XX,N|\}$ differs significantly from the factorized form $P_{XX}\mathcal{N}$. As a consequence, the increasing behavior remains beyond simple, intuitive explanations in terms of carrier and photon populations.

IV. CONCLUSIONS

We presented an open system approach to the photon-assisted transport in self-assembled quantum dots embedded in a single-mode cavity. It is argued that the Lindblad terms responsible for the system-lead tunneling cannot be evaluated in terms of the noninteracting states, as it is often found in the literature. We specify the conditions under which the renormalization of the tunneling energies due to matter-photon coupling can be neglected, but we find that the Coulomb intradot interaction is essential. As a result, the transport Lindblad terms are expressed in terms of jump operators associated to pairs of tunnel-coupled many-body states.

Our numerical results are obtained by taking into account all many-body configurations associated to the lowest-energy s-shell single-particle states. We find that the largest steady-state current passing through the system is achieved when the frequency of the cavity mode matches the biexciton resonant frequency. In contrast, around the exciton resonance the current does not differ much from the off-resonant background value because the biexciton state is continuously fed by the source reservoir and its decay into bright excitons is an off-resonant process. Under electrical injection trions and dark excitons are easily populated and contribute to the current, hence, one cannot disregard them as it is safely done in the optically pumped systems.

Unexpected features of the steady-state current as a function of the cavity losses are explained by the delicate interplay of the emission and absorption processes in the presence of carrier-photon correlations.

ACKNOWLEDGMENT

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APPENDIX: THE Λ SYSTEM

Here, we derive analytical expressions for the dressed states and energies of the Λ system introduced in Sec. III. We also discuss in more detail how to achieve condition (A8) in order rewrite the master equation in the free-states basis.

We assume for simplicity that frequency of the cavity is set to the biexciton resonance, that is $\omega_c = \omega_{XX}$; the effect of a detuning could be equally included and would only complicate the notation. As stated in Sec. III, the biexciton binding energy inhibits the mixing of the ground state in a dressed state. In this case, the optically coupled bright excitons and the biexciton can be seen as an effective three-level Λ system. The associated dressed states can be obtained by diagonalizing the matrix:

$$\hat{H}_\Lambda = \begin{pmatrix} \mathcal{E}^0_{X_\downarrow,n_+-1,n_-} & 0 & -i\hbar g_c\sqrt{n_-} \\ 0 & \mathcal{E}^0_{X_\uparrow,n_+,n_--1} & i\hbar g_c\sqrt{n_+} \\ i\hbar g_c\sqrt{n_-} & -i\hbar g_c\sqrt{n_+} & \mathcal{E}^0_{XX,n_+-1,n_--1} \end{pmatrix}. \quad \text{(A1)}$$

The three dressed states of the Λ system and the associated energies can be calculated analytically (see, e.g., [38]). For further use, we give explicit expressions for the energies:

$$\mathcal{E}_{0,n_+,n_-} = \mathcal{E}^0_{X_\downarrow,n_+-1,n_-} = \mathcal{E}^0_{X_\uparrow,n_+,n_--1}, \quad \text{(A2)}$$

$$\mathcal{E}_{\pm,n_+,n_-} = E_{X_\downarrow} + \hbar\omega(n_+ + n_- - 1) \pm \Delta, \quad \text{(A3)}$$

where we introduced the Rabi splitting $\Delta = \hbar g_c\sqrt{n_+ + n_-}$. The dressed states within the neutral subspace (i.e., $q = 0$) are as follows:

$$|\varphi_{G,n_+,n_-}\rangle = |G,n_+,n_-\rangle,$$
$$|\varphi_{0,n_+,n_-}\rangle = \alpha|X_\downarrow,n_+-1,n_-\rangle + \beta|X_\uparrow,n_+,n_--1\rangle,$$
$$|\varphi_{+,n_+,n_-}\rangle = \frac{1}{\sqrt{2}}\{|XX,n_+-1,n_--1\rangle - i\beta|X_\downarrow,n_+-1,n_-\rangle + i\alpha|X_\uparrow,n_+,n_--1\rangle\},$$
$$|\varphi_{-,n_+,n_-}\rangle = \frac{1}{\sqrt{2}}\{|XX,n_+-1,n_--1\rangle + i\beta|X_\downarrow,n_+-1,n_-\rangle - i\alpha|X_\uparrow,n_+,n_--1\rangle\},$$

where we introduced the coefficients $\alpha = \sqrt{n_-/(n_+ + n_-)}$ and $\beta = \sqrt{n_+/(n_+ + n_-)}$. For the simplicity of writing we tacitly dropped out the dependence of these parameters on the photon numbers n_+ and n_-.

On the other hand, the subspace with $q = -1$ contains linear combinations of negative trions $X^-_\Uparrow, X^-_\Downarrow$ and one-electron states $e_\uparrow, e_\downarrow$. Two of these dressed states are

$$|\varphi^\uparrow_{+,n_+,n_-}\rangle = \cos\theta|e_\uparrow,n_+,n_-\rangle + i\sin\theta|X^-_\Uparrow,n_+-1,n_-\rangle,$$
$$|\varphi^\uparrow_{-,n_+,n_-}\rangle = -\sin\theta|e_\uparrow,n_+,n_-\rangle + i\cos\theta|X^-_\Uparrow,n_+-1,n_-\rangle, \quad \text{(A4)}$$

where $\theta = \arctan\sqrt{(\Delta'-\delta')/(\Delta'+\delta')}$ and the detuning $\delta' = \mathcal{E}^0_{X^-_\Uparrow,n_+-1,n_-} - \mathcal{E}^0_{e_\uparrow,n_+,n_-}$. Also, $\Delta' = \sqrt{4\hbar^2 g_c^2 n_+ + (\delta')^2}$. For further use, let us also write the energy of these dressed states:

$$\mathcal{E}^\uparrow_{\pm,n_+,n_-} = E_{X^-_\Uparrow} + \hbar\omega_{XX}(n_+ + n_- - 1) \pm \frac{\Delta'}{2} - \frac{\delta'}{2}. \quad \text{(A5)}$$

By noticing that $E_{X^-_\Uparrow} - E_{e_\uparrow} = \hbar\omega_{XX} + E_b + V_{ee} - V_{eh}$ one finds that the detuning with respect to the biexciton resonance $\delta' = V_{eh} - V_{hh}$ depends on the Coulomb interaction.

To see how condition (13) explicitly works, let us select from Eq. (11) the term proportional to $f_c(\mathcal{E}_p - \mathcal{E}_{p'})T^{c,\uparrow}_{pp'}$. If p' spans the neutral subspace $q(p') = 0$, it follows from the definition of $T^{c,\uparrow}_{pp'}$ that the tunneling-in processes contributing to the master equation are formally given by $c^\dagger_\uparrow|X_\downarrow\rangle = |X^-_\Uparrow\rangle$ and $c^\dagger_\uparrow|G\rangle = |e_\uparrow\rangle$. Moreover, φ_p runs over the states $|\varphi^\uparrow_{\pm,n_+,n_-}\rangle$ [see Eq. (A4)] which belong to the subspace $q = -1$. When writing the dressed states in terms of bare states, the contributions of the transition $X_\downarrow \to X^-_\Uparrow$ to Eq. (11) can be written as follows:

$$M_{X^-_\Uparrow X_\downarrow} := \sum_{\nu',\lambda'}\sum_{\{\mathrm{ph}\}\#} |\nu',n'_+,n'_-\rangle\langle\lambda',m'_+,m'_-| \sum_{p,p'} f_c(\mathcal{E}_p - \mathcal{E}_{p'}) A^{(p)}_{\nu',n'_+,n'_-}\overline{A}^{(p)}_{X^-_\Uparrow,n_+,n_-} A^{(p')}_{X_\downarrow,n_+,n_-}\overline{A}^{(p')}_{\lambda',m'_+,m'_-},$$

where {ph #} is a shorthand notation for the various photon numbers and p and p' run over the subspaces mentioned above. Now, we notice that from (A2) and (A5) one obtains

$$\mathcal{E}^\uparrow_{\pm,n_+,n_-} - \mathcal{E}_{0,n_+,n_-} = W_{X^-_\Uparrow X_\downarrow} \pm \frac{\Delta'}{2} - \frac{\delta'}{2}, \quad \text{(A6)}$$

$$\mathcal{E}^\uparrow_{\pm,n_+,n_-} - \mathcal{E}_{\pm,n_+,n_-} = W_{X^-_\Uparrow X_\downarrow} \mp \Delta \pm \frac{\Delta'}{2} - \frac{\delta'}{2}. \quad \text{(A7)}$$

We notice that the interaction with the cavity mode induces dressed-states-dependent shifts of the transition energy $W_{X_\Uparrow^- X_\downarrow}$ between the exciton and trion states. In particular, Δ and Δ' contain the Rabi splittings associated to different photon numbers. These shifts are much smaller than $W_{X_\Uparrow^- X_\downarrow}$ which roughly equals the energy of the single-particle state involved in the tunneling processes. By using the realistic parameters introduced in Sec. III (i.e., the Coulomb parameters, the many-body energies and the coupling strength g_c) we have checked that it suffices to set the chemical potential μ_c just few meV away from $W_{X_\Uparrow^- X_\downarrow}$ such that

$$\begin{aligned} f_c(\mathcal{E}^{\uparrow}_{\pm,n_+,n_-} - \mathcal{E}_{0,n_+,n_-}) &= f_c(\mathcal{E}^{\uparrow}_{\pm,n_+,n_-} - \mathcal{E}_{\pm,n_+,n_-}) \\ &= f_c(W_{X_\Uparrow^- X_\downarrow}), \end{aligned} \tag{A8}$$

which is nothing but a particular case of the condition given in Eq. (13). Then, $f_c(W_{X_\Uparrow^- X_\downarrow})$ can be factored out, and by using the unitarity of the transformations $\nu \leftrightarrow \varphi_p$ on the subspaces $\{\varphi^{\uparrow}_{\pm,n_+,n_-}\}$ and $\{\varphi_{\pm,n_+,n_-},\varphi_{0,n_+,n_-}\}$ we find that $M_{X_\Uparrow^- X_\downarrow}$ simplifies to

$$M_{X_\Uparrow^- X_\downarrow} = f_c(W_{X_\Uparrow^- X_\downarrow})|X_\Uparrow^-\rangle\langle X_\downarrow|. \tag{A9}$$

In the same way one can show that the terms arising from the pair $\{|e_\uparrow\rangle,|G\rangle\}$ reduce to $M_{e_\uparrow G} = f_c(W_{e_\uparrow G})|e_\uparrow\rangle\langle G|$. By collecting similar conditions for all pairs of subspaces coupled by the tunneling Hamiltonian, one recovers Eq. (14).

The validity of the off-resonant master equation does not depend on the exactly solvable Λ system we used here, as it is clear that there always exists a unitary transformation switching between the dressed and free states. If analytical expressions of the dressed-states energies are not available, the conditions required for the off-resonant master equation can only be checked numerically after calculating $\mathcal{E}_p$.

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