

Rashba sandwiches with topological superconducting phases

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We introduce a versatile heterostructure harboring various topological superconducting phases characterized by the presence of helical, chiral, or unidirectional edge states. Changing parameters, such as an effective Zeeman field or chemical potential, one can tune between these three topological phases in the same setup. Our model relies only on conventional nontopological ingredients. The bilayer setup consists of an s -wave superconductor sandwiched between two two-dimensional electron gas layers with strong Rashba spin-orbit interaction. The interplay between two different pairing mechanisms, proximity induced direct and crossed Andreev superconducting pairings, gives rise to multiple topological phases. In particular, helical edge states occur if crossed Andreev superconducting pairing is dominant. In addition, an in-plane Zeeman field leads to a two-dimensional gapless topological phase with unidirectional edge states, which were previously predicted to exist only in noncentrosymmetric superconductors. If the Zeeman field is tilted out of the plane, the system is in a topological phase hosting chiral edge states.

DOI: [10.1103/PhysRevB.97.195421](https://doi.org/10.1103/PhysRevB.97.195421)**I. INTRODUCTION**

The discovery that certain properties of quantum states of matter can be captured in terms of topological invariants [1–3] immune to microscopic details of a system has triggered enormous interest in the exploration of topological phases of matter [4–7]. Currently, a great effort is put into the search for localized Majorana quasiparticles that are predicted to appear in one-dimensional topological superconductors (TSCs) [8–22]. Two-dimensional (2D) TSCs are the particle-hole symmetric analogs of the experimentally more extensively studied topological insulators [23–47]. One-dimensional (1D) TSCs have been the subject of intense experimental research [48–65], while 2D TSCs are not yet so well developed experimentally. However, various setups hosting chiral [66–74] and helical [75–84] superconducting edge states were proposed theoretically. In addition to gapped TSCs, there exist gapless TSCs, which are predicted to be realized in nodal superconductors with mixed singlet-triplet pairing and Rashba spin-orbit interaction (SOI) or in various heterostructures [85–95]. The majority of these proposals involve topological insulators and/or unconventional superconductors with p -, d -wave pairing symmetry.

In this work, we propose a bilayer heterostructure which can be brought into all the 2D topological phases mentioned above without the need of including topological materials and/or unconventional superconductors (SCs). The setup is composed of only conventional components: an s -wave superconductor sandwiched between two two-dimensional electron gas (2DEG) layers with strong Rashba SOI (see Fig. 1). The proximity to the superconductor induces superconducting correlations in the 2DEG layers with direct and crossed Andreev pairings [96–107]. In the former case, a Cooper pair tunnels into one layer, whereas, in the latter case, a Cooper pair splits and the electrons tunnel to opposite layers, respectively. When crossed Andreev pairing is dominant, the system is in a gapped phase with a Kramers pair of helical edge states, i.e., it is a helical TSC.

Interestingly, if an effective Zeeman field is introduced, e.g., due to externally applied magnetic fields or due to the ordering of magnetic impurities, the bilayer setup has the potential to realize either a chiral or a gapless TSC. If the Zeeman field lies in the plane, then, above a certain critical field strength, the system enters a gapless topological phase with unidirectional edge states. Unidirectional edge states, which are states that propagate in the same direction on opposite edges, appear on the edge orthogonal to the direction of the in-plane field [93]. If the Zeeman field points out of the plane with an angle larger than a model parameter dependent threshold, the system enters again a fully gapped phase with chiral edge states. In contrast to the unidirectional states, the chiral edge states propagate in opposite directions on opposite edges and appear on all boundaries of the system.

To the best of our knowledge, none of the theoretical proposals for realizing 2D TSCs by solely including conventional nontopological ingredients was reported to be able to obtain all of the topological phases mentioned above.

The paper is organized as follows. In Sec. II, we introduce the effective model describing our setup and discuss its bulk properties in the absence of a Zeeman field. In Sec. III, we show in which parameter regimes the system is a helical TSC, characterize the spin and charge properties of the helical edge states, and derive the effective low-energy Hamiltonian for the edge states. In Sec. IV, we study the possible topological phases in the presence of a Zeeman field. We show that the helical TSC, protected by time-reversal symmetry, gets immediately destroyed, and the topological phases with chiral and gapless edge states become accessible.

II. MODEL

We consider a bilayer setup consisting of two 2DEG layers with strong Rashba SOI coupled to an s -wave superconductor (see Fig. 1). Each layer is characterized by the SOI strength α_τ , where we label the upper (lower) layer by the index $\tau = 1$ ($\tau = \bar{1}$). In the following, we restrict the discussion to the case

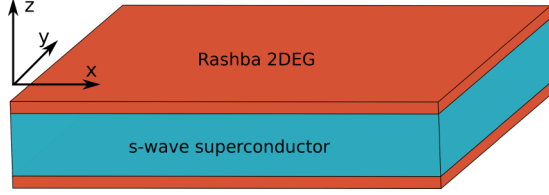


FIG. 1. Sketch of the bilayer setup consisting of an s -wave superconductor (blue) sandwiched between two two-dimensional electron gas layers (red) with strong Rashba spin-orbit interaction.

$\alpha_1 > \alpha_{\bar{1}} > 0$. The z axis is normal to the layers and $\mathbf{k} = (k_x, k_y)$ is the in-plane momentum. The Hamiltonian describing the two uncoupled layers reads in momentum space as

$$H_\tau = \sum_{\sigma, \sigma'} \int d^2\mathbf{k} \psi_{\tau\sigma, \mathbf{k}}^\dagger h_{\tau\sigma\sigma'}(\mathbf{k}) \psi_{\tau\sigma', \mathbf{k}}, \quad (1)$$

where $h_{\tau\sigma\sigma'}(\mathbf{k}) = [\epsilon_k - \mu_\tau + \alpha_\tau \mathbf{g} \cdot \boldsymbol{\sigma}]_{\sigma\sigma'}$ with $\epsilon_k = \hbar^2 |\mathbf{k}|^2 / 2m$ and $\mathbf{g} = (k_y, -k_x, 0)$. The field operator $\psi_{\tau\sigma, \mathbf{k}}$ annihilates an electron in the layer $\tau = \{1, \bar{1}\}$ with spin projection $\sigma = \{1, \bar{1}\}$ and momentum \mathbf{k} . For simplicity, in what follows, we assume that the chemical potentials μ_τ are tuned to the spin-orbit energy of the respective layer $E_{so, \tau} = m\alpha_\tau^2 / 2\hbar^2$. This assumption allows us to solve the problem analytically. A detuning of the chemical potential $\delta\mu_\tau$ (away from $E_{so, \tau}$) leads to a suppression of the crossed Andreev pairing. However, as we have checked numerically, all results remain valid as long as $\delta\mu_\tau$ is the smallest energy scale, $\delta\mu_\tau < \min(\Delta_D, \Delta_C, \Delta_Z)$, where Δ_Z is the Zeeman energy to be introduced later.

The proximity-induced superconductivity opens gaps in the spectrum of the bilayer system and is responsible for the topological phase. Generally, there are two types of superconducting terms: direct and crossed Andreev pairing terms of strength Δ_D and Δ_C , respectively. The direct (crossed Andreev) proximity-induced superconductivity induces coupling between two electrons from the same layer (from two different layers),

$$H_D = \frac{\Delta_D}{2} \sum_{\tau, \sigma, \sigma'} \int d^2\mathbf{k} (\psi_{\tau\sigma, \mathbf{k}}^\dagger [i\sigma_2]_{\sigma\sigma'} \psi_{\tau\sigma', -\mathbf{k}}^\dagger + \text{H.c.}), \quad (2)$$

$$H_C = \frac{\Delta_C}{2} \sum_{\tau, \sigma, \sigma'} \int d^2\mathbf{k} (\psi_{\tau\sigma, \mathbf{k}}^\dagger [i\sigma_2]_{\sigma\sigma'} \psi_{\bar{\tau}\sigma', -\mathbf{k}}^\dagger + \text{H.c.}). \quad (3)$$

Without loss of generality we assume $\Delta_C, \Delta_D > 0$ throughout this work.

The total Hamiltonian is given by $H = H_1 + H_{\bar{1}} + H_D + H_C$ and can be rewritten in terms of Pauli matrices $H = \frac{1}{2} \int d^2\mathbf{k} \Psi_{\mathbf{k}}^\dagger h(\mathbf{k}) \Psi_{\mathbf{k}}$ with

$$h(\mathbf{k}) = \epsilon_k \eta_3 + (\alpha_+ + \alpha_- \tau_3)(\sigma_1 k_y - \eta_3 \sigma_2 k_x) - \Delta_D \eta_2 \sigma_2 - \Delta_C \tau_1 \eta_2 \sigma_2, \quad (4)$$

where we have introduced $\alpha_\pm = (\alpha_1 \pm \alpha_{\bar{1}})/2$ and the Pauli matrices τ_i, η_i , and σ_i acting in layer, particle-hole, and spin space, respectively. One can check that H is time-reversal invariant with the time-reversal operator given by $\Theta = -i\sigma_2 \mathcal{K}$,

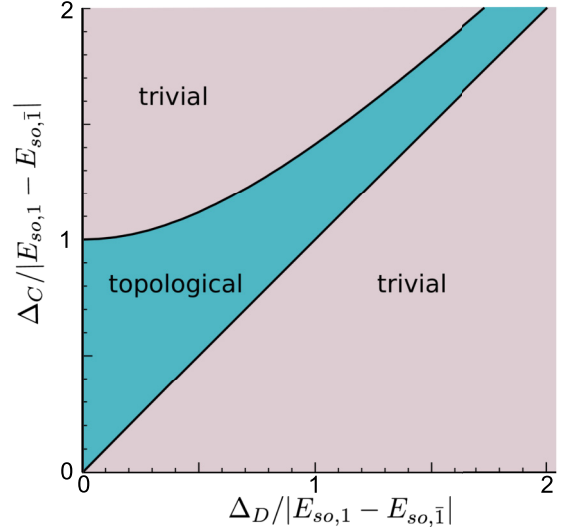


FIG. 2. Topological phase diagram as a function of direct (Δ_D) and crossed (Δ_C) Andreev superconducting pairing amplitude. Topological phase transitions occur for $\Delta_D = \Delta_C$ and $\Delta_C^2 = \Delta_D^2 + (E_{so,1} - E_{so,\bar{1}})^2$ (black lines). The topological phase (blue area) hosts a Kramers pair of edge states, whereas in the trivial phase there are no edge states. The larger the difference of the spin-orbit energies of the two layers, the larger is the topological region.

where \mathcal{K} is the complex conjugation operator. The particle-hole symmetry operator is given by $\mathcal{P} = \eta_1$. Therefore, H belongs to the DIII symmetry class which has a \mathbb{Z}_2 classification for 2D systems [108]. The bulk spectrum of the bilayer setup is given by

$$E_{\pm, \pm}^2(k) = \tilde{\epsilon}_{\pm, k}^2 + \alpha_\pm^2 k^2 + \Delta_D^2 + \Delta_C^2 \pm 2\sqrt{\alpha_\pm^2 k^2 (\tilde{\epsilon}_{\pm, k}^2 + \Delta_C^2) + \Delta_C^2 \Delta_D^2}, \quad (5)$$

where $\tilde{\epsilon}_{\pm, k} = \epsilon_k \pm \alpha_\pm k$. The bulk spectrum is gapped except in two special cases. First, if $\Delta_C = \Delta_D$, the bulk gap closes at $k = 0$. Second, if $\Delta_C = \tilde{\Delta}_C$, where $\tilde{\Delta}_C^2 = \Delta_D^2 + (E_{so,1} - E_{so,\bar{1}})^2$, the bulk gap closes at $k = 2m\alpha_+/\hbar^2$. Here the bulk gap is not closed at one point in momentum space, as in the first case, but along a circular nodal line. Both bulk gap closings correspond to a topological phase transition. The topological phase diagram is presented in Fig. 2, and contains one topological and two trivial regions. It is obvious to see that the system is in a trivial phase when $\Delta_C = 0$ and $\Delta_D > 0$. The system is therefore in the trivial phase for the parameter regime $\Delta_C < \Delta_D$. In the regime $\Delta_D < \Delta_C < \tilde{\Delta}_C$, the system is in the topological phase, where, as we will show below, a Kramers pair of edge states exists at each edge. For large Δ_C , i.e., $\Delta_C > \tilde{\Delta}_C$, the edge states disappear, which is again a trivial phase.

III. HELICAL TOPOLOGICAL SUPERCONDUCTOR

A. Helical edge states

In order to confirm the phase diagram obtained from the bulk spectrum in the previous section, we now investigate a finite-size system and focus on the properties of the edges. We first solve the problem numerically by implementing a

tight-binding model for the bilayer setup [109]. Without loss of generality, the layers are taken to be finite along the y direction, of length L (N_y lattice sites separated by lattice constant a),

$$\begin{aligned}
 H_\tau &= \frac{1}{2} \sum_{k_x, n} \left\{ \sum_{\sigma} (-t c_{k_x, \tau(n+1)\sigma}^\dagger c_{k_x, \tau n \sigma} + t c_{-k_x, \tau n \sigma} c_{-k_x, \tau(n+1)\sigma}^\dagger + [-t \cos(k_x a_x) + \mu_\tau/2] c_{k_x, \tau n \sigma}^\dagger c_{k_x, \tau n \sigma} \right. \\
 &\quad - [-t \cos(k_x a_x) + \mu_\tau/2] c_{-k_x, \tau n \sigma} c_{-k_x, \tau n \sigma}^\dagger + \text{H.c.}) + \tilde{\alpha}_\tau [i (c_{k_x, \tau(n+1)\uparrow}^\dagger c_{k_x, \tau n \downarrow} - c_{k_x, \tau(n-1)\uparrow}^\dagger c_{k_x, \tau n \downarrow}) + 2i \sin(k_x a_x) c_{k_x, \tau n \uparrow}^\dagger c_{k_x, \tau n \downarrow} \\
 &\quad \left. + i (c_{-k_x, \tau n \downarrow} c_{-k_x, \tau(n+1)\uparrow}^\dagger - c_{-k_x, \tau n \downarrow} c_{-k_x, \tau(n-1)\uparrow}^\dagger) - 2i \sin(k_x a_x) c_{-k_x, \tau n \downarrow} c_{-k_x, \tau n \uparrow}^\dagger + \text{H.c.} \right\}, \\
 H_C &= \frac{\Delta_C}{2} \sum_{k_x} \sum_{n, \tau} (c_{k_x, \tau n \sigma}^\dagger [i \sigma_2]_{\sigma \sigma'} c_{-k_x, \bar{\tau} n \sigma'}^\dagger + \text{H.c.}), \quad H_D = \frac{\Delta_D}{2} \sum_{k_x} \sum_{n, \tau} (c_{k_x, \tau n \sigma}^\dagger [i \sigma_2]_{\sigma \sigma'} c_{-k_x, \tau n \sigma'}^\dagger + \text{H.c.}). \quad (6)
 \end{aligned}$$

Here, t denotes the hopping amplitude. The operator $c_{k_x, \tau n \sigma}$ acts on an electron at position $y = na$ in the layer $\tau (= -\bar{\tau})$ with momentum k_x and spin projection σ . The spin-flip hopping amplitude is related to the SOI parameter by $\tilde{\alpha} = \alpha/2a_y$ [110]. Solving the Hamiltonian H , one finds that there are no edge states in the regimes $\Delta_C < \Delta_D$ and $\Delta_C > \tilde{\Delta}_C$ [see Fig. 3(b)], which confirms that these parameter regimes correspond to the trivial phases (see Fig. 2). In the regime $\Delta_D < \Delta_C < \tilde{\Delta}_C$ both edges host a Kramers pair of subgap states. These states are localized on the edges and have a linear dispersion around $k_x = 0$ [see Fig. 3(a)]. This is the hallmark of a helical TSC and confirms our expectation that this parameter regime corresponds to the topological phase.

Further, we investigate the spin and charge properties of the edge states. Since in our setup spin and charge are not conserved quantities, we calculate the expectation value of the spin (charge) operator \mathbf{S} (Q). In the following, $\langle O \rangle_\beta = \langle \Phi_\beta | O | \Phi_\beta \rangle$ denotes the expectation value for some operator O , where the ket $|\Phi_\beta\rangle$ describes the two edge states labeled by $\beta \in \{1, 2\}$ (for further details we refer to Appendix A). At $k_x = 0$, where the system can be mapped to the 1D analog of our setup [111, 112], the edge states have zero average spin and charge. This is consistent with previous works. For all other values of k_x the expectation values are generally nonzero and, as is expected, for fixed energy the two Kramers partners have the same charge. The average of the x component of

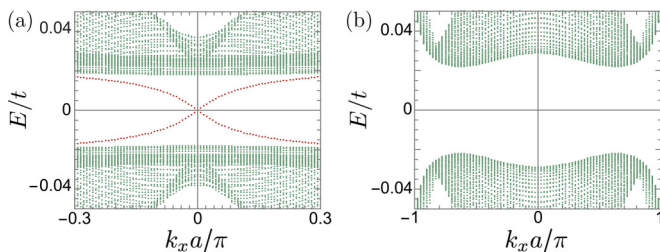


FIG. 3. Energy spectrum of the bilayer setup in (a) the topological phase ($\Delta_C/t = 0.11$) and (b) the trivial phase ($\Delta_C/t = 0.22$). In both cases the bulk states (green) have a spectral gap, while a Kramers pair of edge states (red) is only present in (a). The edge states are localized at both edges and have a linear dispersion around $k_x = 0$. The numerical parameters are chosen as $N_y = 300$, $\mu/t = -4$, $\alpha_1/t = 0.55$, $\alpha_1/t = 0.35$, and $\Delta_D/t = 0.06$.

and translationally invariant along the x direction, allowing us to use k_x as a good quantum number. The Hamiltonian for this setup is given by $H = H_1 + H_{\bar{1}} + H_D + H_C$ with

the spin vanishes, $\langle S_x \rangle = 0$, for all values of k_x , while the remaining components satisfy $\langle S_i(k_x) \rangle_1 = -\langle S_i(-k_x) \rangle_2$ (see Fig. 4). In this sense, the edge states are helical and protected from backscattering by time-reversal symmetry. Note that the edge states on the left and the right edge are connected by reflection symmetry, where the symmetry operator is given by $\mathcal{I} = \sigma_y$. Thus, the wave functions of the edge states on opposite edges and their properties are related by \mathcal{I} . Therefore, states on

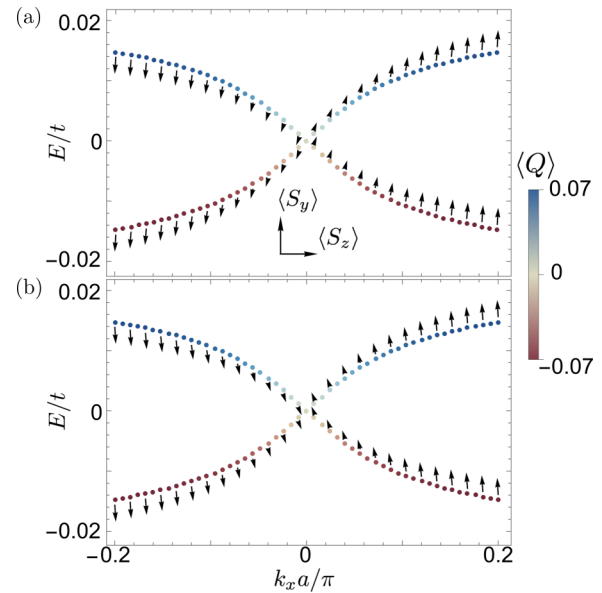


FIG. 4. Energy spectrum of helical edge states (a) on the left edge ($y = 0$) and (b) on the right edge ($y = L$). The average of the charge operator is encoded in the coloring of the data points and given in units of e . One can see that for a given energy $\langle Q(k_x) \rangle = \langle Q(-k_x) \rangle$ as is expected, since time reversal does not invert the sign of the charge. The average spin as a function of k_x is encoded in the black arrows. The average spin component along the x axis is zero for all momenta, while the other components are nonzero, and for the Kramers partner the relation $\langle S_i(k_x) \rangle_1 = -\langle S_i(-k_x) \rangle_2$ holds. The wave functions on opposite edges are related by the reflection symmetry operator $\mathcal{I} = \sigma_y$. Thus, states on opposite edges, which propagate in the same direction, have the same average spin along the y axis, whereas their average spins along the z axis are opposite. The numerical parameters are the same as in Fig. 3.

opposite edges that propagate in the same direction have the same average spin projection on the y axis. In contrast, their average spin projections on the z axis are opposite (differ by a minus sign) (see Fig. 4).

B. Effective low-energy Hamiltonian

As shown in the previous section, the topological phase hosts a Kramers pair of helical edge states with a linear dispersion around $k_x = 0$ on both edges. Next, we derive the effective low-energy Hamiltonian describing the properties of these states localized at the left ($y = 0$) edge. As in the tight-binding model, we assume that the system is translationally invariant along the x direction and we solve an effectively 1D Hamiltonian parametrized by k_x . For the moment, we assume that the right edge ($y = L$) is infinitely far away. This assumption allows us to treat the edge states as if the width of the sample was much larger than the localization length ξ of the edge states. Due to particle-hole symmetry, the edge states are at zero energy at $k_x = 0$ [see Fig. 3(a)], and we first determine the wave functions at this special point. In a next step, we treat the k_x terms perturbatively for $k_x \xi \ll 1$, and keep only terms linear in k_x . Using $h(0, k_y)$ [see Eq. (4)] we obtain two wave functions Φ_β ($\beta \in \{1, 2\}$) in the strong SOI regime for $\alpha_1 \gg \alpha_{\bar{1}}$ (see Appendix B). These two wave functions correspond to a Kramers pair of edge states and exist only if $\Delta_D < \Delta_C < \tilde{\Delta}_C$, which is consistent with our previous results. The wave functions have support in momentum space around $k_{F\tau} = 2k_{s\sigma\tau}$ and $k_{F,i} = 0$. The corresponding localization length ξ is determined by the bulk gaps at these Fermi points [113] and, thus, is given by the maximum of four length scales: $\xi_\tau = \hbar v_{F,\tau} / \Delta_D$ and

$$\xi_\pm = \frac{2\hbar v_{F,1} v_{F,\bar{1}}}{\sqrt{\Delta_D^2 (v_{F,1} - v_{F,\bar{1}})^2 + 4\Delta_C^2 v_{F,1} v_{F,\bar{1}} \pm \Delta_D (v_{F,1} + v_{F,\bar{1}})}}, \quad (7)$$

respectively, where we have introduced the Fermi velocities $v_{F\tau} = \alpha_\tau / \hbar$.

For $k_x \xi \ll 1$, the perturbation term linear in k_x is given by $h_{k_x} = (\alpha_+ + \alpha_- \tau_3) \eta_3 \sigma_2 k_x$ [see Eq. (4)], and the first-order correction to the energy can be found by calculating the matrix elements $\langle \Phi_\beta | h_{k_x} | \Phi_{\beta'} \rangle$. As a result the effective low-energy Hamiltonian is given by

$$h_{\text{eff}} = \hbar \tilde{v} \beta_3 k_x. \quad (8)$$

Here β_3 is the third Pauli matrix acting in the low-energy subspace spanned by Φ_β and \tilde{v} the effective Fermi velocity. The effective Hamiltonian h_{eff} has a form typical for helical TSCs. The helical edge states have a linear dispersion inside the bulk gap.

IV. EFFECT OF A ZEEMAN FIELD: 2D GAPLESS VS CHIRAL SUPERCONDUCTING TOPOLOGICAL PHASE

Having discussed the helical topological superconducting phase in the previous section, we now investigate the possible topological phases in the presence of an effective Zeeman field, which could arise due to the presence of polarized magnetic impurities in both layers, similarly, as was already

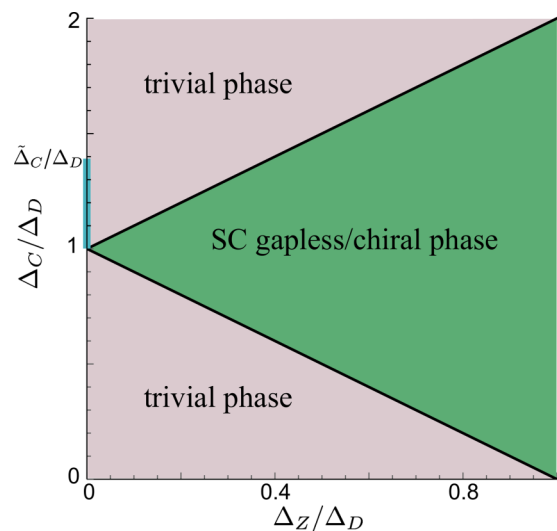


FIG. 5. Topological phase diagram as a function of the Zeeman energy (Δ_Z) and the crossed Andreev superconducting pairing amplitude (Δ_C). Topological phase transitions occur for $\Delta_C = |\Delta_D \pm \Delta_Z|$ (black lines) and for $\Delta_C = \tilde{\Delta}_C$ and $\Delta_Z = 0$. In the absence of a Zeeman field there exists a helical topological phase for $\Delta_D < \Delta_C < \tilde{\Delta}_C$, which is indicated by the blue line. We note that $\tilde{\Delta}_C$ depends on both Δ_D and the difference in SOI energies, and in this plot we assume $\Delta_D = |E_{s\sigma,1} - E_{s\sigma,\bar{1}}|$ such that $\tilde{\Delta}_C / \Delta_D = \sqrt{2}$. In the green region the system is in a 2D superconducting gapless (chiral) phase for an in-plane (out-of-plane) Zeeman field. For details on the transition from the gapless to the chiral phase as a function of the out-of-plane angle we refer to the main text.

discussed in the literature for magnetic islands on superconductors, Weyl semimetals, or quantum anomalous Hall effect [73,74,114–119]. We consider an effective Zeeman field in the direction determined by the unit vector $\mathbf{n} = (\cos \phi \cos \theta, \sin \phi \cos \theta, \sin \theta)^T$, then the Zeeman term reads

$$H_Z = \Delta_Z \sum_{\tau} \sum_{\sigma, \sigma'} \int d^2 \mathbf{k} \psi_{\tau\sigma, \mathbf{k}}^\dagger [\mathbf{n} \cdot \boldsymbol{\sigma}]_{\sigma\sigma'} \psi_{\tau\sigma', \mathbf{k}}, \quad (9)$$

where Δ_Z denotes the Zeeman energy.

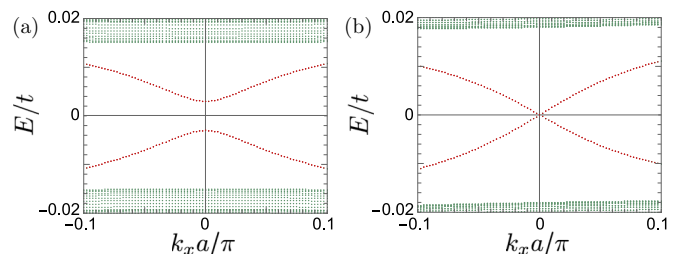


FIG. 6. Energy spectrum in the presence of an in-plane Zeeman field aligned (a) in the x direction ($\phi = 0$) and (b) in the y direction ($\phi = \pi/2$). Green (red) dots represent bulk (edge) states. The edge states become gapped once the Zeeman field has a component along the respective edge. The size of the gap opened in the edge state spectrum is given by the projection of the Zeeman field on the propagation direction, $\Delta E = \Delta_Z \cos \phi$. The numerical parameters are chosen as $N_x = 800$, $\alpha_1/t = 0.35$, $\alpha_{\bar{1}}/t = 0.15$, $\mu = -4t$, $\Delta_D/t = 0.06$, $\Delta_C/t = 0.12$, and $\Delta_Z/t = 0.006$.

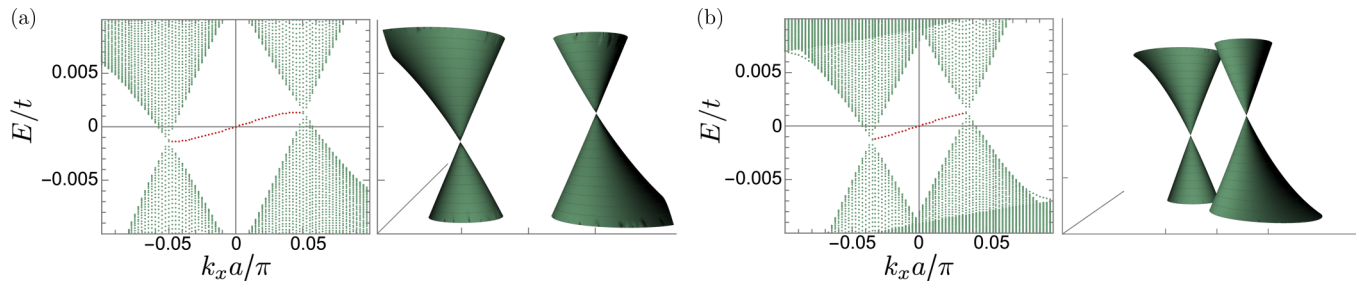


FIG. 7. Energy spectra in the presence of an in-plane Zeeman field (a) $\phi = \pi/2$ and (b) $\phi = \pi/4$. In the left subfigures the bulk (edge) states are represented by green (red) dots and obtained from the tight-binding model. In the right subfigure, we show the bulk spectrum of the translationally invariant system. The line connecting the centers of two Weyl cones is orthogonal to the direction of the Zeeman field. The degenerate edge states (one per edge) connect the two Weyl cones. The numerical parameters are chosen as $N_y = 2000$, $\alpha_1/t = 0.35$, $\alpha_l/t = 0.15$, $\mu = -4t$, $\Delta_D/t = 0.05$, $\Delta_C = \Delta_D$, and $\Delta_Z/t = 0.025$.

Again, we first focus on the phase diagram in the presence of an in-plane Zeeman field ($\theta = 0$), which breaks time-reversal symmetry (see Fig. 5). If $\Delta_Z > \Delta_D$, the superconductivity is strongly suppressed and the system is gapless. If $\Delta_C = |\Delta_Z \pm \Delta_D|$, the Zeeman term leads to a closing of the bulk gap at $\mathbf{k} = 0$ (black lines in Fig. 5). If $\Delta_C < \Delta_D$ and $\Delta_Z = 0$ the system is in a trivial phase as was shown above, and therefore, in the whole region $\Delta_C < |\Delta_Z - \Delta_D|$ of the phase diagram, the system is in the trivial phase. In the regime $\Delta_D < \Delta_C < \Delta_C$ and $\Delta_Z = 0$ the system is in the helical TSC phase with a Kramers pair of gapless edge states (blue line in Fig. 5). When $\Delta_Z > 0$, time-reversal symmetry is broken and, thus, the helical edge states are no longer protected against backscattering. If the Zeeman field has a component along a given edge, it leads to a coupling between the Kramers partners and thereby gaps out the edge states. The size of the gap is given by the projection of the Zeeman field on the given edge, which for our tight-binding model translates into $\Delta E = \Delta_Z \cos \phi$ (see Fig. 6). In the special case of $\phi = \pi/2$, the pair of edge states propagating along the x axis stay gapless. However, if the system was finite in both x and y directions, one can make use of the spatial symmetries of the setup to conclude that the spectrum of the edge states propagating along the y direction is gapped. Thus, we conclude that a weak in-plane Zeeman field leads to a gap in the spectrum of the helical edge states at

least along one of the edges, so the region $\Delta_C > |\Delta_Z + \Delta_D|$ corresponds to a trivial phase.

Next, we analyze the parameter region of the phase diagram that is bounded by the two gap closing lines $\Delta_C = |\Delta_Z \pm \Delta_D|$ (green area in Fig. 5). In this regime, the bulk spectrum contains two Weyl cones. Their position in the Brillouin zone is determined by the polar angle ϕ of the Zeeman field: the line connecting the two Weyl cones is orthogonal to the direction of the effective Zeeman field (see insets in Fig. 7). Localized edge states connect the two Weyl nodes in momentum space, where they coexist with bulk modes. Since the nodes are not at zero energy the edge states have a finite group velocity (see Fig. 7). There is one state per edge and they have the same direction of propagation on opposite edges, i.e., the system hosts unidirectional edge states [90,93]. As is the case for Weyl semimetals, in order to determine whether edge states appear on a given edge, one has to project the Weyl nodes onto the edge direction; edge states appear only on edges where the Weyl nodes are not projected onto the same point. For our tight-binding model, this means that the unidirectional edge states do not appear on the edge along the x direction if $\phi = 0$. A gap can be opened in the 2D Weyl spectrum if a perturbation proportional to σ_z is added, i.e., an effective Zeeman field has an out-of-plane component along the z direction. Thus, it is interesting to investigate the evolution of the edge states as

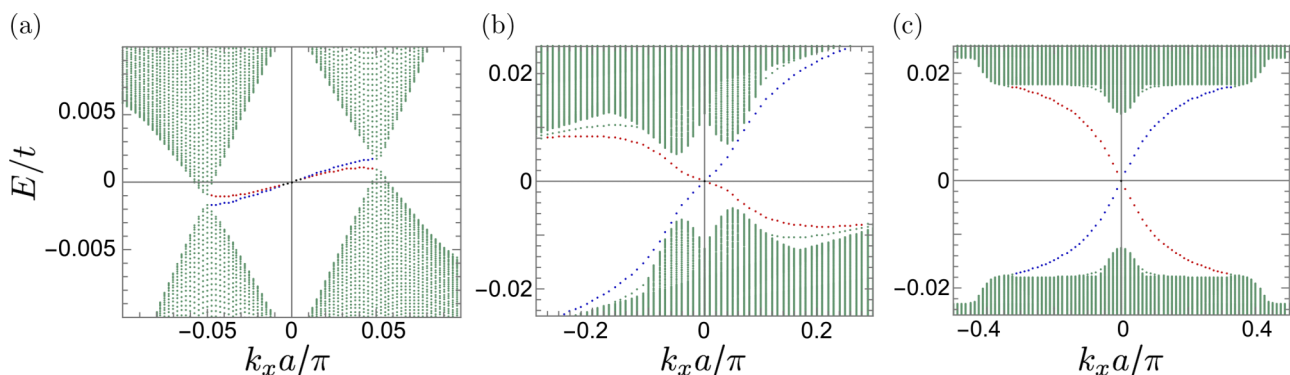


FIG. 8. Energy spectra in the presence of an out-of-plane Zeeman field for $\phi = \pi/2$ and (a) $\theta = \pi/90$, (b) $\theta = \pi/6$, and (c) $\theta = \pi/2$. The Zeeman field component along the z direction leads to an opening of a gap in the bulk spectrum (green dots) at the position of the nodes. The gap increases with the out-of-plane angle θ . At the same time the degeneracy of the edge states (see Fig. 7) gets lifted as the out-of-plane angle increases, and the edge states become chiral, i.e., counterpropagating on opposite edges (red and blue dots) for $\theta > \pi/15$. The other numerical parameters are the same as in Fig. 7.

the out-of-plane angle θ of the Zeeman field is increased. The larger the angle the larger is the gap, which is opened in the spectrum of Weyl cones. For small θ , the bulk spectrum stays gapless and the edge states remain unidirectional although their twofold degeneracy gets lifted (see Fig. 8). For the parameter settings used in our numerics this holds for $\theta \leq \pi/15$. If θ is larger than this threshold, the bulk spectrum is fully gapped and the edge states evolve from unidirectional to chiral edge states (see Fig. 8). We conclude that a 2D gapless topological phase with unidirectional edge states is only achieved if the out-of-plane angle of the Zeeman field is “small.” otherwise the system is a fully gapped chiral topological superconductor.

V. CONCLUSIONS

We have shown how three distinct topological superconducting phases can be engineered in a heterostructure composed of two 2DEG layers with strong Rashba SOI and an s -wave superconductor sandwiched between these two layers. Such a setup could be realized in semiconductor superlattices [120–122] or using (quasi-) 2D materials with strong SOI [123–127].

In the time-reversal symmetric case, a pair of helical edge states exists when crossed Andreev superconducting pairing is dominant. When time-reversal symmetry is broken by a Zeeman field, the system can potentially be in two distinct topological phases. If the field is in plane or if the out-of-plane component is small enough, a gapless topological superconducting phase with unidirectional edge states can be realized. If the out-of-plane angle is large enough, the system can be tuned into a gapped topological superconducting phase with chiral edge states. It is well known that the TSCs with a bulk gap are the most stable against disorder. The helical TSC is, due to Kramers theorem, stable against nonmagnetic disorder, while a chiral TSC is stable against both nonmagnetic and magnetic disorder. However, in both cases, this only holds as long as the disorder strength is not comparable to the size of the superconducting gap and Weyl cones are well separated, which is the case for long-range disorder [128].

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APPENDIX A: NUMERICAL EVALUATION OF CHARGE AND SPIN EXPECTATION VALUES

In the main text we discussed the expectation values of the spin (charge) operator for the helical edge states. In this appendix we complete the discussion by providing the detailed expressions. Solving numerically the tight-binding model defined by Eq. (6), we obtain $8N_y$ energy states for each fixed momentum k_x , i.e., the energy states can be parametrized as $E_n(k_x)$ with the corresponding eigenfunc-

tions $\Phi_n(j, k_x)$, where $n \in \{1, \dots, 8N_y\}$ is the band index and $j \in \{1, \dots, N_y\}$ denotes the lattice site. Here, Φ_n is an eight-component spinor in Nambu space written in the basis $(c_{k_x 1 \uparrow n}, c_{k_x 1 \downarrow n}, c_{-k_x 1 \uparrow n}^\dagger, c_{-k_x 1 \downarrow n}^\dagger, c_{k_x \bar{1} \uparrow n}, c_{k_x \bar{1} \downarrow n}, c_{-k_x \bar{1} \uparrow n}^\dagger, c_{-k_x \bar{1} \downarrow n}^\dagger)$.

The average spin and charge of the n th energy eigenstate with k_x fixed is computed as [129]

$$\begin{aligned} \langle \mathbf{S}(k_x) \rangle_n &= \sum_j \Phi_n^\dagger(j, k_x) \mathbf{S} \Phi_n(j, k_x), \\ \langle Q(k_x) \rangle_n &= \sum_j \Phi_n^\dagger(j, k_x) Q \Phi_n(j, k_x), \end{aligned} \quad (\text{A1})$$

where \mathbf{S} (Q) is the spin (charge) operator measured in units of $\hbar/2$ (e) and represented in Nambu space $S_i = \text{diag}(\sigma_i, -\sigma_i^T, \sigma_i, -\sigma_i^T)$ [$Q = \text{diag}(\sigma_0, -\sigma_0, \sigma_0, -\sigma_0)$], where σ_i are the Pauli matrices and σ_0 is the 2×2 identity matrix.

APPENDIX B: DERIVATION OF THE EFFECTIVE LOW-ENERGY HAMILTONIAN

In this appendix we present the detailed calculation of the wave functions at $k_x = 0$ and derive the effective low-energy Hamiltonian describing helical edge states [see Eq. (8) in Sec. III].

The wave functions at $k_x = 0$ should be related by time-reversal symmetry Θ and also can be chosen to respect the particle-hole symmetry [111]. As a result, they can be represented in the basis $(\psi_{1\uparrow}, \psi_{1\downarrow}, \psi_{1\uparrow}^\dagger, \psi_{1\downarrow}^\dagger, \psi_{\bar{1}\uparrow}, \psi_{\bar{1}\downarrow}, \psi_{\bar{1}\uparrow}^\dagger, \psi_{\bar{1}\downarrow}^\dagger)$ in the following form [113]:

$$\Phi_1(y) = \begin{pmatrix} f_1(y) \\ g_1(y) \\ f_1^*(y) \\ g_1^*(y) \\ f_{\bar{1}}(y) \\ g_{\bar{1}}(y) \\ f_{\bar{1}}^*(y) \\ g_{\bar{1}}^*(y) \end{pmatrix}, \quad \Phi_2(y) = \begin{pmatrix} g_1^*(y) \\ -f_1^*(y) \\ g_1(y) \\ -f_1(y) \\ g_{\bar{1}}^*(y) \\ -f_{\bar{1}}^*(y) \\ g_{\bar{1}}(y) \\ -f_{\bar{1}}(y) \end{pmatrix}. \quad (\text{B1})$$

We start with $h(0, k_y)$ [see Eq. (4)], which after the unitary transformation $U = e^{-(i\pi/4)\sigma_2}$ reads

$$\begin{aligned} h(0, k_y) &= \frac{\hbar^2 k_y^2}{2m} \eta_3 + (\alpha_+ + \alpha_- \tau_3) \sigma_3 k_y - \Delta_D \eta_2 \sigma_2 \\ &\quad - \Delta_C \tau_1 \eta_2 \sigma_2. \end{aligned} \quad (\text{B2})$$

In the next step, we linearize [130] the spectrum around the Fermi points $k_i = 0$, $k_{F\tau} = \pm 2k_{s\sigma, \tau}$ such that the field operators $\psi_{\tau\sigma}$ are approximated as

$$\begin{aligned} \psi_{\tau\uparrow} &= L_{\tau\uparrow}(y) + R_{\tau\uparrow}(y) e^{ik_{F\tau} y}, \\ \psi_{\tau\downarrow} &= L_{\tau\downarrow}(y) e^{-ik_{F\tau} y} + R_{\tau\downarrow}(y), \end{aligned} \quad (\text{B3})$$

where $R_{\tau\sigma}(y)$ [$L_{\tau\sigma}(y)$] are slowly varying right (left) moving fields with spin projection σ along the x axis (due to the unitary transformation U above) in the τ layer. In the limit of strong SOI energy and $\alpha_1 \gg \alpha_{\bar{1}}$, the crossed Andreev pairing term only couples fields at $k = 0$ [111]. As a result, the linearized

Hamiltonian reads

$$\begin{aligned} \bar{h} = & v_{F1} \hat{k} \frac{1 + \tau_3}{2} \rho_3 + v_{F\bar{1}} \hat{k} \frac{1 - \tau_3}{2} \rho_3 - \Delta_D \eta_2 \sigma_2 \rho_1 \\ & - \Delta_C \tau_1 \eta_2 (\sigma_2 \rho_1 - \sigma_1 \rho_2) / 2, \end{aligned} \quad (\text{B4})$$

with $\hat{k} = -i\hbar\partial_y$ the momentum operator around the Fermi points and ρ_i acting in left/right mover space. We make the ansatz $\phi_\xi(y) = \phi_\xi e^{-y/\xi}$ with ϕ_ξ being a 16-component vector and search for decaying zero-energy solutions satisfying $\bar{h}\phi_\xi = 0$. We find eight decaying eigenmodes with localization lengths given by

$$\begin{aligned} \xi_1 &= \frac{\hbar v_{F,1}}{\Delta_D}, \quad \xi_2 = \frac{\hbar v_{F,\bar{1}}}{\Delta_D}, \\ \xi_3 &= \frac{2\hbar v_{F,1} v_{F,\bar{1}}}{-\Delta_D(v_{F,1} + v_{F,\bar{1}}) + \sqrt{\Delta_D^2(v_{F,1} - v_{F,\bar{1}})^2 + 4\Delta_C^2 v_{F,1} v_{F,\bar{1}}}}, \\ \xi_4 &= \frac{2\hbar v_{F,1} v_{F,\bar{1}}}{\Delta_D(v_{F,1} + v_{F,\bar{1}}) + \sqrt{\Delta_D^2(v_{F,1} - v_{F,\bar{1}})^2 + 4\Delta_C^2 v_{F,1} v_{F,\bar{1}}}}. \end{aligned} \quad (\text{B5})$$

By imposing vanishing boundary condition $\Phi_\beta(0) \stackrel{!}{=} 0$ on a linear combination of these eigenmodes, we find that the edge state wave functions are determined by

$$\begin{aligned} f_1(y) &= i g_1^*(y) = -i(e^{-y/\xi_4} - e^{-ik_{F1}y} e^{-y/\xi_1}), \\ f_{\bar{1}}(y) &= i g_{\bar{1}}^*(y) = -\frac{i}{g_-} (e^{-ik_{F\bar{1}}y} e^{-y/\xi_2} - e^{-y/\xi_4}), \end{aligned} \quad (\text{B6})$$

with

$$g_- = \frac{\Delta_D(v_{F1} - v_{F\bar{1}}) + \sqrt{\Delta_D^2(v_{F1} - v_{F\bar{1}})^2 + 4\Delta_C^2 v_{F1} v_{F\bar{1}}}}{2\Delta_C v_{F1}}. \quad (\text{B7})$$

In the limit $v_{F1} = v_{F\bar{1}}$ this solution reproduces the wave functions of the Kramers pair of Majorana fermions found in Ref. [111]. We note that if $\Delta_D > \Delta_C$, eight eigenmodes are linearly independent and their linear combination cannot satisfy vanishing boundary condition. Therefore, edge states do not exist in this regime.

For $k_x \xi \ll 1$, the perturbation linear in k_x reads $h_{k_x} = (\alpha_+ + \alpha_- \tau_3) \eta_3 \sigma_2 k_x / 2$. The first-order correction to the energy is calculated by evaluating the matrix elements $\langle \Phi_n | h_{k_x} | \Phi_{n'} \rangle$. Using Eqs. (B1) and (B6), we arrive at

$$\begin{aligned} \langle \Phi_1 | h_{k_x} | \Phi_1 \rangle &= -2k_x \sum_{\tau=\{1,\bar{1}\}} \alpha_\tau \int_0^\infty dy ([f_\tau^*]^2 + [f_\tau]^2) \\ &= -\langle \Phi_2 | h_{k_x} | \Phi_2 \rangle, \\ \langle \Phi_1 | h_{k_x} | \Phi_2 \rangle &= -2ik_x \sum_{\tau=\{1,\bar{1}\}} \alpha_\tau \int_0^\infty dy ([f_\tau^*]^2 - [f_\tau]^2) \\ &= \langle \Phi_2 | h_{k_x} | \Phi_1 \rangle, \end{aligned} \quad (\text{B8})$$

where for compactness we express the matrix elements in terms of f_1 ($f_{\bar{1}}$) only. Decomposing the functions $f_\tau(y) = \text{Re}[f_\tau(y)] + i\text{Im}[f_\tau(y)]$ into real and imaginary parts we rewrite the matrix elements above as

$$\begin{aligned} \langle \Phi_1 | h_{k_x} | \Phi_1 \rangle &= -4k_x \sum_{\tau=\{1,\bar{1}\}} \alpha_\tau \int_0^\infty dy ([\text{Re}(f_\tau)]^2 - [\text{Im}(f_\tau)]^2), \\ \langle \Phi_1 | h_{k_x} | \Phi_2 \rangle &= -8k_x \sum_{\tau=\{1,\bar{1}\}} \alpha_\tau \int_0^\infty dy \text{Re}(f_\tau) \text{Im}(f_\tau), \end{aligned} \quad (\text{B9})$$

which now can be seen to be purely real. For completeness we give the expressions for the decomposition of f_τ into real and imaginary parts

$$\begin{aligned} \text{Re}(f_1) &= \sin(k_{F1}y) e^{-y/\xi_1}, \quad \text{Im}(f_1) = \cos(k_{F1}y) e^{-y/\xi_1} - e^{-y/\xi_4}, \\ \text{Re}(f_{\bar{1}}) &= -\frac{\sin(k_{F\bar{1}}y) e^{-y/\xi_2}}{g_-}, \quad \text{Re}(f_{\bar{1}}) = \frac{1}{g_-} [e^{-y/\xi_4} - \cos(k_{F\bar{1}}y) e^{-y/\xi_2}]. \end{aligned} \quad (\text{B10})$$

Plugging these equations into (B9), we find the following effective Hamiltonian:

$$h_{\text{eff}} = \begin{pmatrix} \langle \Phi_1 | h_{k_x} | \Phi_1 \rangle & \langle \Phi_1 | h_{k_x} | \Phi_2 \rangle \\ \langle \Phi_2 | h_{k_x} | \Phi_1 \rangle & \langle \Phi_2 | h_{k_x} | \Phi_2 \rangle \end{pmatrix} = \begin{pmatrix} A & B \\ B & -A \end{pmatrix} k_x, \quad (\text{B11})$$

where the real constants A and B are found to be given by

$$\begin{aligned} A &= -4 \left[\frac{2\alpha_1 \xi_1}{1 + k_{F1}^2 \xi_1^2} + 2\alpha_1 \xi_4 - \frac{8\alpha_1 \xi_1 \xi_4 (\xi_1 + \xi_4)}{(\xi_1 + \xi_4)^2 + k_{F1}^2 \xi_1^2 \xi_4^2} + \frac{2\alpha_{\bar{1}} \xi_2}{g_-^2 (1 + k_{F\bar{1}}^2 \xi_2^2)} + \frac{2\alpha_{\bar{1}} \xi_4}{g_-^2} - \frac{8\alpha_{\bar{1}} \xi_2 \xi_4 (\xi_2 + \xi_4)}{g_-^2 [(\xi_2 + \xi_4)^2 + k_{F\bar{1}}^2 \xi_2^2 \xi_4^2]} \right], \\ B &= 2 \left[\alpha_1 k_{F1} \left(\frac{4\xi_1^2 \xi_4^2}{(\xi_1 + \xi_4)^2 + k_{F1}^2 \xi_1^2 \xi_4^2} - \frac{\xi_1^2}{1 + k_{F1}^2 \xi_1^2} \right) + \frac{\alpha_{\bar{1}} k_{F\bar{1}}}{g_-^2} \left(\frac{4\xi_2^2 \xi_4^2}{(\xi_2 + \xi_4)^2 + k_{F\bar{1}}^2 \xi_2^2 \xi_4^2} - \frac{\xi_2^2}{1 + k_{F\bar{1}}^2 \xi_2^2} \right) \right]. \end{aligned} \quad (\text{B12})$$

We find that this matrix is diagonal in the basis Φ^\pm with eigenvalues $\pm \hbar \tilde{v} k_x$, where $\tilde{v} = \sqrt{A^2 + B^2} / \hbar$ and

$$\Phi^\pm = \frac{B \Phi_1 \pm (\sqrt{A^2 + B^2} \mp A) \Phi_2}{\sqrt{B^2 + (\sqrt{A^2 + B^2} \mp A)^2}}. \quad (\text{B13})$$

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