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Correlational latent heat by nonlocal quantum kinetic theory

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A kinetic equation of nonlocal and noninstantaneous character unifies the achievements of transport in dense quantum gases with the Landau theory of quasiclassical transport in Fermi systems. Large cancellations in the off-shell motion appear, which are usually hidden in non-Markovian behaviors. The remaining corrections are expressed in terms of shifts in space and time that characterize the nonlocality of the scattering process. In this way, it is possible to recast quantum transport into a quasiclassical picture. In addition to the quasiparticle, the balance equations for density, momentum, energy, and entropy also include correlated two-particle contributions beyond the Landau theory. The medium effects on binary collisions are shown to mediate the latent heat, i.e., an energy conversion between correlation and thermal energy. For Maxwellian particles with time-dependent s-wave scattering, the correlated parts of the observables are calculated and a sign change of the latent heat is reported at a universal ratio of scattering length to the thermal de Broglie wavelength. This is interpreted as a change from correlational heating to cooling.

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I. OVERVIEW ABOUT NONLOCAL KINETIC THEORY

A. Extended quasiparticle picture

To extend the validity of the Boltzmann equation to moderately dense gases, Clausius and Boltzmann included the space nonlocality of binary collisions [1]. After one century, virial corrections won new interest as they can be incorporated into Monte Carlo simulation methods [2]. The microscopic theory of nonlocal corrections to the collision integral has been pioneered within the theory of gases by many authors [3–19].

In this paper, a correlational latent heat in strongly coupled nonequilibrium systems is reported based on the development of a nonlocal extension of the Boltzmann equation. The latter one is reviewed below in order to make the reader acquainted with the consistent extensions of the quantum Boltzmann equation and nonequilibrium thermodynamics beyond the Landau theory. The result is then reported in Sec. II as an actual analytical calculation of all corrections to the quantum Boltzmann equation for a model of time-dependent s-wave scattering. This highlights the nonlocal corrections derived earlier and might be of importance to nonequilibrium quantum systems in cold atoms, plasma physics, and nuclear physics.

In the limit of small scattering rates, the transport equation for the Green's function is converted into a kinetic equation of Boltzmann type by the extended quasiparticle approximation derived for small scattering rates [20–22]. One introduces an effective quasiparticle distribution f from which the Wigner distribution ρ can be constructed,

$$\rho[f]=f+\int\frac{d\omega}{2\pi}\frac{\wp}{\omega-\varepsilon}\frac{\partial}{\partial\omega}[(1-f)\sigma^<_\omega-f\sigma^>_\omega]. \tag{1}$$

Here, $\sigma^>$ and $\sigma^<$ denote the self-energies describing all correlations and ε is the quasiparticle energy. The limit of small scattering rates was first introduced by Craig [23] and an inverse relation $f[\rho]$ was constructed [24]. For equilibrium nonideal plasmas this approximation has been employed by Refs. [25,26] and has been used under the name of the generalized Beth-Uhlenbeck approach by Ref. [27] in nuclear matter for studies of the correlated density. The name "extended quasiparticle approximation" was finally used for the study of the mean removal energy and high-momenta tails of Wigner's distribution [28]. The nonequilibrium form has been derived as the modified Kadanoff and Baym ansatz [29].

B. Nonlocal kinetic equation

The resulting quantum kinetic theory unifies the achievements of transport in dense gases with the quantum transport of dense Fermi systems [20–22,30]. The quasiparticle drift of Landau's equation,

$$\frac{\partial f_1}{\partial t}+\frac{\partial\varepsilon_1}{\partial\vec k}\frac{\partial f_1}{\partial\vec r}-\frac{\partial\varepsilon_1}{\partial\vec r}\frac{\partial f_1}{\partial\vec k}=I_1^{\rm in}-I_1^{\rm out}, \tag{2}$$

is connected with a dissipation governed by a nonlocal and noninstant scattering integral in the spirit of Enskog corrections. For the scattering out (lost term), it reads

$$\begin{aligned}I_1^{\rm out}=&\sum_b\int\frac{d^3p}{(2\pi\hbar)^3}\frac{d^3q}{(2\pi\hbar)^3}\frac{2\pi}{\hbar}\delta(\varepsilon_1+\varepsilon_2-\varepsilon_3-\varepsilon_4+2\Delta_E)\\&\times\left(1+\frac{1}{2}\frac{\partial\vec\Delta_2}{\partial\vec r}+\frac{\partial\varepsilon_2}{\partial r}\frac{\partial\vec\Delta_2}{\partial\omega}\right)_{\omega=\varepsilon_1+\varepsilon_2}f_1f_2(1-f_3-f_4)\\&\times\left|\mathcal{T}\left(\varepsilon_1+\varepsilon_2+\Delta_E,\vec k+\frac{\vec\Delta_K}{2},\vec p+\frac{\vec\Delta_K}{2},\vec q,\vec r+\vec\Delta_r,t+\frac{\Delta_t}{2}\right)\right|^2,\end{aligned} \tag{3}$$

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and the scattering in (gain term) by replacing $f \leftrightarrow 1-f$ and changing the signs of the shifts. The distribution functions and observables have the arguments

$$\begin{aligned}
\varepsilon_1 &\equiv \varepsilon_a(\vec k,\vec r,t),\\
\varepsilon_2 &\equiv \varepsilon_b(\vec p,\vec r+\vec\Delta_2,t),\\
\varepsilon_3 &\equiv \varepsilon_a(\vec k-\vec q+\vec\Delta_K,\vec r+\vec\Delta_3,t+\Delta_t),\\
\varepsilon_4 &\equiv \varepsilon_b(\vec p+\vec q+\vec\Delta_K,\vec r+\vec\Delta_4,t+\Delta_t),
\end{aligned}\tag{4}$$

with the transferred momentum $\vec q$.

In the scattering out (scattering in is analogous) one can see the distributions of quasiparticles f_1f_2 describing the probability of a given initial state characterized by momenta k and p for the binary collision partners. The hole distributions $(1-f)$ are the probability that the requested final states are empty. Both combine in the final-state occupation factors $(1-f_3)(1-f_4)+f_3f_4=1-f_3-f_4$. The scattering rate covers the energy-conserving δ function, and the differential cross section is given by the modulus of the T matrix $|\mathcal{T}|$ reduced by the wave-function renormalizations [31], which are higher order in small scattering rates.

The corrections to the quantum Boltzmann equation are expressed in terms of shifts in space and time that characterize the nonlocality of the scattering process [32]. These Δ's are derivatives of the scattering phase shift ϕ of the T matrix $\mathcal{T}_R=|\mathcal{T}|e^{i\phi}$, according to the following list,

$$\begin{aligned}
&\vec\Delta_K=\frac{1}{2}\frac{\partial\phi}{\partial\vec r},\quad \Delta_E=-\frac{1}{2}\frac{\partial\phi}{\partial t},\quad \Delta_t=\frac{\partial\phi}{\partial\omega},\\
&\vec\Delta_2=\frac{\partial\phi}{\partial\vec p}-\frac{\partial\phi}{\partial\vec q}-\frac{\partial\phi}{\partial\vec k},\quad \vec\Delta_3=-\frac{\partial\phi}{\partial\vec k},\\
&\vec\Delta_4=-\frac{\partial\phi}{\partial\vec q}-\frac{\partial\phi}{\partial\vec k},\quad \vec\Delta_r=\frac{1}{4}(\vec\Delta_2+\vec\Delta_3+\vec\Delta_4).
\end{aligned}\tag{5}$$

As special limits, this kinetic theory includes the Landau theory as well as the Beth-Uhlenbeck equation of state [33,34] which means correlated pairs. The medium effects on binary collisions are shown to mediate the latent heat, which is the energy conversion between correlation and thermal energy [22,35]. In this respect, the seeming contradiction between particle-hole symmetry and time-reversal symmetry in the collision integral was solved [36]. Compared to the Boltzmann equation, the presented form of virial corrections only slightly increases the numerical demands in implementations [37–39] since large cancellations in the off-shell motion, which are usually hidden in non-Markovian behaviors, appear. Details on how to implement the nonlocal kinetic equation into existing Boltzmann codes can be found in Ref. [39]. In this way, it is possible to recast quantum transport into a quasiclassical picture suited for simulations.

C. Balance equations

The balance equations for density, momentum, energy, and entropy include quasiparticle contributions and the correlated two-particle contributions beyond the Landau theory. A number of attempts have been made to modify the Boltzmann equation so that its equilibrium limit would cover at least the second virial coefficient [5,40,41]. Corrections to the Boltzmann equation have the form of gradients or nonlocal contributions to the scattering integral. The nature of two-particle correlations induces gradients and therefore nonlocal kinetic and exchange energies [42,43].

We multiply the kinetic equation (2) with a variable $\xi_1=1,\vec k,\varepsilon_1,-k_{\rm B}\ln[f_1/(1-f_1)]$ and integrate over momentum. It results in the equation of continuity, the Navier-Stokes equation, the energy balance, and the evolution of the entropy, respectively. All these conservation laws or balance equations for the mean thermodynamic observables have the form [44,45]

$$\frac{\partial\langle\xi^{\rm qp}+\xi^{\rm mol}\rangle}{\partial t}+\frac{\partial\big(\vec j_\xi^{\rm qp}+\vec j_\xi^{\rm mol}\big)}{\partial\vec r}=I^{\rm gain},\tag{6}$$

consisting of a quasiparticle part,

$$\xi^{\rm qp}=\int\frac{d^3k}{(2\pi\hbar)^3}\xi_1f_1,\tag{7}$$

and the correlated or molecular contribution,

$$\xi^{\rm mol}=\int\frac{d^3kd^3pd^3q}{(2\pi\hbar)^9}D\Delta_t\frac{\xi_1+\xi_2}{2}.\tag{8}$$

The latter one leads to the statistical interpretation as if two particles form a molecule. The rate of binary processes $D=|\mathcal{T}|^2 2\pi\delta(\varepsilon_1+\varepsilon_2-\varepsilon_3-\varepsilon_4+2\Delta_E)(1-f_3-f_4)f_1f_2/\hbar$ is weighed with the lifetime of the molecule Δ_t, respectively. We suppress the obvious sum over sort indices for the sake of legibility.

The usual quasiparticle currents of the observables read

$$\vec j_\xi^{\rm qp}=\int\frac{d^3k}{(2\pi\hbar)^3}\xi_1\frac{\partial\varepsilon_1}{\partial\vec k}f_1,\tag{9}$$

and the molecular currents we have obtained as [22,44,45]

$$\vec j_\xi^{\rm mol}=\frac{1}{2}\int\frac{d^3kd^3pd^3q}{(2\pi\hbar)^9}D(\xi_2\vec\Delta_2-\xi_3\vec\Delta_3-\xi_4\vec\Delta_4).\tag{10}$$

It is the balance of observables carried by the different spatial offsets.

To present explicit formulas, the total quasiparticle stress tensor formed by the quasiparticles reads

$$\Pi_{ij}^{\rm qp}=\int\frac{d^3k}{(2\pi\hbar)^3}\left(k_j\frac{\partial\varepsilon}{\partial k_i}+\delta_{ij}\varepsilon\right)f-\delta_{ij}E^{\rm qp},\tag{11}$$

with the quasiparticle energy functional [22],

$$\begin{aligned}
E^{\rm qp}=&\int\frac{d^3k}{(2\pi\hbar)^3}f_1(k)\frac{k^2}{2m}\\
&+\frac{1}{2}\int\frac{d^3kd^3p}{(2\pi\hbar)^6}f_1(k)f_2(p){\rm Re}\,\mathcal{T}(\varepsilon_1+\varepsilon_2,k,p,0),
\end{aligned}\tag{12}$$

instead of the Landau functional which is valid only in a local approximation.

The molecular contributions (10) to the stress tensor reads

$$\begin{aligned}
\Pi_{ij}^{\rm mol}=&\frac{1}{2}\int\frac{d^3kd^3pd^3q}{(2\pi\hbar)^9}D\\
&\times[(k_j-q_j)\Delta_{3i}+(p_j+q_j)\Delta_{4i}-p_j\Delta_{2i}].
\end{aligned}\tag{13}$$

This momentum tensor is the balance of the momenta carried by the corresponding spatial offsets weighted with the rate to form a molecule D.

For the density $\xi = 1$ we do not have a gain $I^{\rm gain}$. The momentum gain $\xi = k_j$ reads

$$I^{\rm gain}_{Kj} = \int \frac{d^3k d^3p d^3q}{(2\pi\hbar)^9} D\Delta_{Kj}. \tag{14}$$

Dividing and multiplying by Δ_t under the integral, we see that the momentum gain is the probability $D\Delta_t$ to form a molecule multiplied with the force $\vec{\Delta}_K/\Delta_t$ exercised during the delay time Δ_t from the environment by all other particles. This momentum gain (14) can be exactly recast together with the term of the drift into a spatial derivative [22,45],

$$\int \frac{d^3k}{(2\pi\hbar)^3} \varepsilon \frac{\partial f}{\partial \vec{r}_j} + I^{\rm gain}_{Kj} = \frac{\partial E^{\rm qp}}{\partial \vec{r}_j}, \tag{15}$$

of the quasiparticle energy functional (12). Analogously, the energy gain,

$$I^{\rm gain}_{E} = \int \frac{d^3k d^3p d^3q}{(2\pi\hbar)^9} D\Delta_{E}, \tag{16}$$

combines with the drift into the total time derivative of the quasiparticle energy functional (12),

$$\int \frac{d^3k}{(2\pi\hbar)^3} \varepsilon \frac{\partial f}{\partial t} - I^{\rm gain}_{E} = \frac{\partial E^{\rm qp}}{\partial t}. \tag{17}$$

The only remaining explicit gain is the entropy gain,

$$I^{\rm gain}_{S} = -\frac{k_B}{2} \int \frac{d^3k d^3p d^3q}{(2\pi\hbar)^9} D \ln \frac{f_3 f_4 (1-f_1)(1-f_2)}{(1-f_3)(1-f_4) f_1 f_2}, \tag{18}$$

while the momentum gain and energy gain are transferring kinetic into correlation parts and do not appear explicitly. In Ref. [44] it is proved that this entropy gain is always positive, establishing the Boltzmann H-theorem including single-particle and two-particle quantum correlations. In other words, the additional gain on the right-hand side of the balance equations (6) might be due to an energy or force feed from the outside or the entropy production by collisions.

II. MODEL WITH TIME-DEPENDENT INTERACTION

So far we have summarized the nonlocal extensions of the quantum Boltzmann equation. Now, as an exploratory example and result, we want to consider a pointlike interaction where the T matrix is dominated by the s-wave channel with a single time-dependent scattering length $a_{\rm sc}(t)$. One might think of cold atoms where the time-dependent magnetic field $\vec{B}(\vec{r},t)$ near the Feshbach resonance determines this scattering length [46],

$$a_{\rm sc}(\vec{r},t) \approx a\left(1 - \frac{\Gamma}{|\vec{B}(\vec{r},t)| - B}\right). \tag{19}$$

In such a way the binary T matrix becomes an externally controlled function of time and space. We consider two atoms with their center-of-the-mass momentum $K = k + p$ and their difference momentum $\kappa = \frac{m_b}{M}k - \frac{m_a}{M}p$, where $M = m_a + m_b$ and $\mu^{-1} = m_a^{-1} + m_b^{-1}$. The sum of energies before (k,p) and after the collision $(k-q, p+q)$ reads

$$\begin{aligned} \epsilon_1 + \epsilon_2 &= \frac{K^2}{2M} + \frac{\kappa^2}{2\mu}, \\ \epsilon_3 + \epsilon_4 &= \frac{K^2}{2M} + \frac{\kappa^2 + q^2}{2\mu} - \frac{\vec{q}\cdot\vec{\kappa}}{\mu}. \end{aligned} \tag{20}$$

In the dilute gas the medium effect on the binary interaction caused by the Pauli blocking is negligible so that we can use the free-space T matrix,

$$\mathcal{T}_R(t) = \frac{2\pi\hbar^2 a_{\rm sc}(t)}{\mu} \frac{1}{1 + i\frac{a_{\rm sc}(t)}{\hbar}\sqrt{2\mu(\Omega - K^2/2M)}}, \tag{21}$$

which one obtains by solving the T-matrix equation with a running coupling constant [47]. Due to the pointlike interaction, the corresponding set of Δ's given by Eqs. (5) becomes

$$\begin{aligned} \Delta_t(t) &= -\frac{a_{\rm sc}\mu}{\kappa\left(1 + a_{\rm sc}^2\kappa^2/\hbar^2\right)}, \\ \vec{\Delta}_K(t) &= \frac{1}{a_{\rm sc}}\frac{\partial a_{\rm sc}}{\partial \vec{r}}\frac{\kappa^2}{2\mu}\Delta_t, \quad \Delta_E(t) = -\frac{1}{a_{\rm sc}}\frac{\partial a_{\rm sc}}{\partial t}\frac{\kappa^2}{2\mu}\Delta_t, \\ \vec{\Delta}_3(t) &= \vec{\Delta}_4 = \frac{\vec{K}}{M}\Delta_t, \quad \vec{\Delta}_2 = 0. \end{aligned} \tag{22}$$

The value of $\vec{\Delta}_{3,4}$ is a free flight of the interacting pair during Δ_t.

Now we calculate all the thermodynamic quantities. Since the T matrix and all shifts are independent of the transferred momentum q, one can easily integrate

$$\int \frac{d^3q}{(2\pi\hbar)^3} \delta(\epsilon_1 + \epsilon_2 - \epsilon_3 - \epsilon_4) = \frac{\mu\kappa}{2\pi^2\hbar^3}. \tag{23}$$

For Maxwellian distributions of equal temperature T and densities of the two species n_a and n_b,

$$f_1 f_2 = n_a n_b \frac{(2\pi\hbar^2)^3}{T^3(\mu M)^{3/2}} e^{-\frac{K^2}{2MT} - \frac{\kappa^2}{2\mu T}}, \tag{24}$$

one can trivially perform the K integration since all shifts and the T matrix are only dependent on κ. Then, the correlated density, energy (8), and momentum tensor (13) become

$$\begin{aligned} n^{\rm mol}(t) &= \frac{1}{2}\int \frac{d\kappa}{(2\pi\hbar)^3}\tilde{D}, \quad \Pi^{\rm mol}_{ij}(t) = \delta_{ij} T n^{\rm mol}, \\ E^{\rm mol} &= \frac{1}{2}\int \frac{d\kappa}{(2\pi\hbar)^3}\tilde{D}\left(\frac{3}{2}T + \frac{\kappa^2}{2\mu}\right), \end{aligned} \tag{25}$$

and the energy gain (16) and momentum gain (14) are

$$\left.\begin{matrix} I^{\rm gain}_E(t) \\ \vec{I}^{\rm gain}_K(t) \end{matrix}\right\} = \int \frac{d\kappa}{(2\pi\hbar)^3}\tilde{D}\frac{\kappa^2}{2\mu}\left\{\begin{matrix} -\frac{\partial a_{\rm sc}}{\partial t}, \\ \frac{\partial a_{\rm sc}}{\partial \vec{r}}, \end{matrix}\right. \tag{26}$$

where we used

$$\tilde{D} = -\frac{4\pi^{5/2}\hbar^6}{\mu^3 T^3} n_a n_b x^3 \frac{e^{-\frac{\kappa^2}{2\mu T}}}{(1 + {a_{\rm sc}}^2\kappa^2/\hbar^2)^2}, \tag{27}$$

and introduce with the de Broglie wavelength $\lambda^2 = 2\pi\hbar^2/2\mu T$ the time-dependent variable

$$x^2(t) = 2\pi\frac{a_{\rm sc}^2(t)}{\lambda^2}. \tag{28}$$

All correlated currents are zero in this example.

The various κ integrations are straightforward with the help of the error function,

$$x\xi(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty dt \frac{e^{-\frac{t^2}{x^2}}}{1+t^2} = \sqrt{\frac{\pi}{2}} e^{x^{-2}} \operatorname{erfc}(x^{-1}). \tag{29}$$

The final results then read

$$\begin{aligned}
n^{\mathrm{mol}}(x) &= n_a n_b \frac{\pi\hbar^3}{(\mu T)^{3/2}} x^2 \xi'(x),\\
\Pi_{ij}^{\mathrm{mol}}(x) &= \delta_{ij} T n^{\mathrm{mol}}(x),\\
E^{\mathrm{mol}}(x) &= \frac{3}{2} n^{\mathrm{mol}}(x) T + T x^2 \left(\frac{n^{\mathrm{mol}}(x)}{x}\right)',\\
\left.\begin{matrix} I_E^{\mathrm{gain}}(x) \\ \vec{I}_K^{\mathrm{gain}}(x) \end{matrix}\right\} &= 2 T x^2 \left(\frac{n^{\mathrm{mol}}(x)}{x}\right)' \left\{\begin{matrix} -\frac{\partial a_{\mathrm{sc}}}{\partial t}, \\ \frac{\partial a_{\mathrm{sc}}}{\partial \vec{r}}, \end{matrix}\right.
\end{aligned} \tag{30}$$

where we denote the prime as a derivative with respect to x. The quasiparticle energy (12) becomes

$$E^{\mathrm{qp}}(x) = \tfrac{3}{2} T (n_a + n_b) + 4 T n^{\mathrm{mol}}(x), \tag{31}$$

which shows the expected three translational degrees of freedom for the free particles and eight degrees of freedom for the correlated molecules. The latter can be understood as twice the three translational degrees of freedom of the two colliding particles and two additional rotational degrees of freedom if the two particles form a bound state that is seen as a classical dumbbell.

The molecular energy behaves as

$$E^{\mathrm{mol}} = n^{\mathrm{mol}} T \left\{\begin{matrix} \frac{1}{2} + \frac{4}{\sqrt{\pi} x} + o(x^{-2}), \\ \frac{7}{2} - 6x^2 + o(x^3), \end{matrix}\right. \tag{32}$$

in the high- and low-temperature limit, respectively [Eq. (28)]. Compared with the quasiparticle part (31), the high-temperature limit (32) brings an additional degree of freedom by the correlational contribution. It can be seen as an additional internal degree of freedom such as the two-particle dumbbell state gains an additional third fictitious particle by correlations.

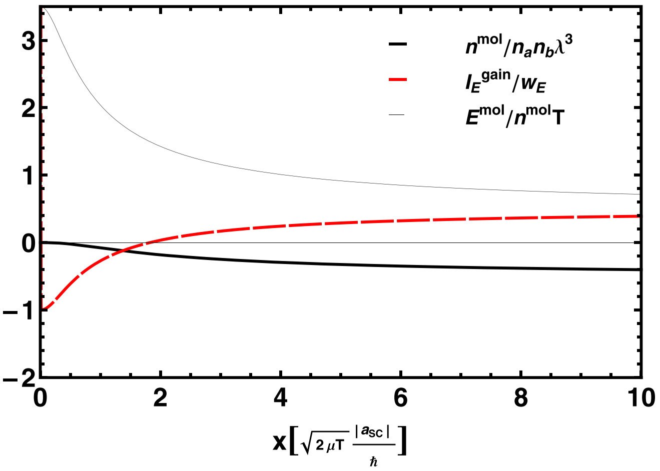

FIG. 1. The molecular density (8) (black thick line), energy gain (16) per external power (33) which is the latent heat (red dashed line), and correlation energy (8) (thin line) vs the dimensionless scattering length (28) for Maxwellian particles.

III. EFFECT OF EXTERNAL POWER FEED

In order to check the energy conservation (17) we consider the effect of the external power feed due to the time-dependent potential $V(t) = 2\pi\hbar^2 a_{\mathrm{sc}}(t)/\mu$. From the retarded and advanced T matrices in operator notation, $\mathcal{T}_{R/A}^{-1} = V(t)^{-1} - G_{R/A}$, it follows that one can write the real part of the T matrix, $\mathrm{Re}\,\mathcal{T} = (\mathcal{T}_R + \mathcal{T}_A)/2 = \mathcal{T}_R (V^{-1} - \mathrm{Re}\, G)\mathcal{T}_A$. From (12) we see therefore that the time-dependent potential leads to an extra feed,

$$w_E = -\frac{1}{2} \int \frac{d^3k d^3p}{(2\pi\hbar)^3} \frac{\partial V^{-1}}{\partial t} |\mathcal{T}|^2 f_1 f_2, \tag{33}$$

to the energy balance (16),

$$\frac{\partial E^{\mathrm{qp}}}{\partial t} = -I_E^{\mathrm{gain}} + w_E, \tag{34}$$

since $\epsilon = k^2/2m_a$ and f are independent of time here. For the considered point-interaction model we get

$$w_E = 4 T n^{\mathrm{mol}} \frac{\partial \ln a_{\mathrm{sc}}}{\partial t}. \tag{35}$$

On the other hand, the derivative of (31) can be calculated explicitly,

$$\frac{\partial E^{\mathrm{qp}}}{\partial t} = 4 T x (n^{\mathrm{mol}})' \frac{1}{a_{\mathrm{sc}}} \frac{\partial a_{\mathrm{sc}}}{\partial t} = -I_E^{\mathrm{gain}} + 4 T n^{\mathrm{mol}} \frac{\partial \ln a_{\mathrm{sc}}}{\partial t}, \tag{36}$$

and one sees how the extra feed (33) appears such that indeed (34) holds. This illustrates the proof of energy conservation (17).

The correlated density in (30) appears as the Beth-Uhlenbeck equation of state [33,48]. The correlated density is negative according to the fact that we can only describe attractive interactions with contact potentials [47]. Then, the two correlated particles form possible bound states. The density of correlated particles or molecules possesses the low-temperature limit,

$$n^{\mathrm{mol}} = -n_a n_b \lambda^3 \frac{\sqrt{\pi}}{4} x^3 + o(x^4) = -2\pi^2 n_a n_b a_{\mathrm{sc}}^3 + o(T^2), \tag{37}$$

which is just the expression of the second virial coefficient for hard spheres to the pressure. The high-temperature limit, $n^{\mathrm{mol}}(x) = -n_a n_b \lambda^3 \frac{\pi}{4} + o(x^{-1})$, vanishes with $T^{-3/2}$.

In Fig. 1 we plot the ratio of the energy gain (16) to the external power (33) with the analytical results (30). This has the merit to be a dimensionless quantity where the time derivative $\partial_t \ln a_{\rm sc}(t)$ drops out. It is now easy to see that the energy gain (16) and entropy gain (18) are linked for equilibrium distributions as

$$T I_S^{\rm gain} = I_E^{\rm gain}. \tag{38}$$

Since the latent heat is the temperature times the entropy difference occurring during a phase transition, we can consider the formation of short-living molecules here analogously. Therefore, the energy gain is the rate of latent heat. Dividing by the external power, we obtain the ratio of the latent heat to the interaction strength due to correlations.

It is remarkable that the energy gain relative to the external pumping changes sign at $x_0 \approx 1.8184$, which means $a_{\rm sc}/\lambda \approx 0.7254$. This value is independent of interaction and in this sense universal for such short-range interactions. For products of scattering length and temperatures smaller than this value the correlations lead to an opposite behavior, as expected from the feed, $I_E^{\rm gain}/w_E = -1 + 3x^2 + o(x)$, and we have correlational cooling. For high temperatures the gain approaches half of the external power $I_E^{\rm gain}/w_E = 1/2 - 2/\sqrt{\pi}x + o(x^{-2})$ and we see correlational heating.

IV. SUMMARY

To summarize, when strong correlations are formed there is a cancellation of off-shell processes in the kinetic equation, resulting in a proper extended quasiparticle picture. The remaining modifications of the quantum Boltzmann equation consist of the nonlocal collision scenario where the offsets are uniquely determined by the phase shift of the T matrix and the quasiparticle energies modifying the drift. Besides the quasiparticle parts of the Landau theory, the resulting balance equations also show explicit two-particle contributions of short-living molecules. The energy and momentum conservation is ensured due to an internal transfer of energy and momentum analogously to a latent heat. An explicit gain remains only for the entropy, which can be proved to be positive [44], ensuring Boltzmann's H-theorem. The single-particle entropy can decrease on the cost of the correlated part of entropy describing the two particles in a molecular state.

For an exploratory example of Maxwellian particles we find a sign change of the energy gain compared with the external power feed independent of the interaction. The interpretation as the rate of latent heat due to correlations is suggested, leading to correlational heating and cooling.

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