Hanle model of a spin-orbit coupled Bose-Einstein condensate of excitons in semiconductor quantum wells

S. V. Andreev[1,*] and A. V. Nalitov[1,2]
[1]*ITMO University, St. Petersburg 197101, Russia*
[2]*Science Institute, University of Iceland, Dunhagi-3, IS-107 Reykjavik, Iceland*

(Received 29 May 2017; revised manuscript received 27 March 2018; published 26 April 2018)

We present a theoretical model of a driven-dissipative spin-orbit coupled Bose-Einstein condensate of indirect excitons in semiconductor quantum wells (QW's). Our steady-state solution of the problem shares analogies with the Hanle effect in an optical orientation experiment. The role of the spin pump in our case is played by Bose-stimulated scattering into a linearly-polarized ground state and the depolarization occurs as a result of exchange interaction between electrons and holes. Our theory agrees with the recent experiment [A. A. High *et al.*, Phys. Rev. Lett. **110**, 246403 (2013)], where spontaneous emergence of spatial coherence and polarization textures have been observed. As a complementary test, we discuss a configuration where an external magnetic field is applied in the structure plane.

DOI: 10.1103/PhysRevB.97.165139

I. INTRODUCTION

Spin-orbit (SO) coupled Bose-Einstein condensation (BEC) has been under intense investigation in atomic gases during the past few years [1]. A key feature of this new type of BEC is the macroscopic occupation of single-particle states with nonzero momentum [2], with dramatic consequences for possible quantum phases and transitions between them. Among theoretically predicted are the stripe phase [3,4], the thermodynamically stable meron [5], and the many-body "cat" state [6]. Practical implementation of the latter could pave the way towards fault tolerant quantum computation at room temperature [7].

In solids the bosonic quasiparticles which display the SO interaction are excitons and polaritons—bound electron-hole pairs in a semiconductor media coupled to light [8]. Important distinction of these systems from their atomic counterpart is their driven-dissipative character. In most cases polaritonic condensates are strongly out-of-equilibrium, behaving essentially as lasers. In contrast, excitons can establish a kinetic equilibrium with respect to the interactions needed for formation of a Bose-Einstein condensate [9]. Promising candidates for creation and manipulation of interacting degenerate Bose gases in semiconductors are the so-called indirect excitons, formed of spatially separated electrons and holes [10].

Recently, spontaneous onset of extended spatial coherence and polarization textures have been observed in the photoluminescence (PL) of indirect excitons in coupled semiconductor quantum wells (QW's) [11]. The textures at different locations in the QW plane appear simultaneously at the same critical temperature and their orientation is pinned to the crystallographic axes. Whereas the former observation suggests a second-order phase transition [12], the latter fact allows one to unambiguously identify the SO interaction as playing a crucial role in these phenomena [11,13]. Until now, however, no link between formation of an exciton condensate in the momentum space and the spin texture has been established.

The present paper is aimed at filling this gap in the understanding of exciton BEC. We propose an ideal gas model of SO coupled exciton condensates accounting for the radiative decay and pumping from a reservoir. The emergent physics is reminiscent of the Hanle effect, which consists in suppression of the spin polarization induced by a continuous wave (cw) circularly polarized laser in a transverse magnetic field [14,15]. In our case, stimulated scattering of excitons into a linearly-polarized ground state (GS) is followed by coherent precession ("depolarization") of the condensate pseudospin in an effective magnetic field due to the electron-hole exchange interaction. The stationary configuration (pseudospin rotation angle) is determined by the balance between the radiative decay rate and the pumping from the reservoir. The model can be straightforwardly generalized to account for an external magnetic field. In the Faraday configuration (the magnetic field is applied perpendicularly to the structure plane) our theory reproduces the rotation of the linear polarization texture observed in the experiment [11]. As a complementary test of our model, we discuss the Voigt geometry (in-plane magnetic field).

II. GROUND STATE

Our starting point is the following single-particle SO-coupled Hamiltonian

$$\hat{H} = \begin{pmatrix} E_k & \beta_h k e^{-i\varphi} & \beta_e k e^{i\varphi} & 0 \\ \beta_h k e^{i\varphi} & E_k & 0 & \beta_e k e^{i\varphi} \\ \beta_e k e^{-i\varphi} & 0 & E_k & \beta_h k e^{-i\varphi} \\ 0 & \beta_e k e^{-i\varphi} & \beta_h k e^{i\varphi} & E_k \end{pmatrix}, \quad (1)$$

*serguey.andreev@gmail.com

which is obtained by the Kronecker summation of the corresponding Dresselhaus Hamiltonians for the constituent electrons and holes [16]. Here $E_k = \hbar^2k^2/2m$ is the exciton kinetic energy and φ is the angle between the exciton wave vector $\boldsymbol{k}$ and the [100] crystal axis (chosen as the x axis). The lowest energy eigenstates of the Hamiltonian (1) are the plane waves, characterized with the dispersion

$$E(k) = \frac{\hbar^2k^2}{2m} - k(\beta_e + \beta_h) \tag{2}$$

and the following spin structure

$$|\mathrm{GS}\rangle = \tfrac{1}{2}(1, \quad -e^{\mathrm{i}\varphi}, \quad -e^{-\mathrm{i}\varphi}, \quad 1)^{\mathrm{T}}. \tag{3}$$

The dispersion (2) has a characteristic minimum at $k_0 = m(\beta_e + \beta_h)/\hbar^2$. The φ-dependent part $\langle\varphi| = 1/2(-e^{-i\varphi}, -e^{i\varphi})$ in $|\mathrm{GS}\rangle$ corresponds to the bright exciton state and can be used to construct a pseudospin $\boldsymbol{S} = 1/2\,\langle\varphi|\,\boldsymbol{\sigma}\,|\varphi\rangle$ which defines the polarization of the light emitted by the exciton (here $\boldsymbol{\sigma}$ is the Pauli vector). Thus, the linear polarization degrees are given by $\rho_l \equiv (I_x - I_y)/(I_x + I_y) = 2S_x/\langle\varphi|\varphi\rangle$ and $\rho_{l'} \equiv (I_{x'} - I_{y'})(I_{x'} + I_{y'}) = 2S_y/\langle\varphi|\varphi\rangle$ with the frame (x',y') being rotated by $\pi/4$ with respect to (x,y). One can easily see that the light emitted by the state (3) is polarized along the wave vector $\boldsymbol{k}$.

In the work [11] a vortex of linear polarization and a four-leaf circular polarization texture have been observed around the localized bright spots (LBS's) in the exciton PL pattern. The bright spots are characterized by high intensities and partially suppressed coherence of the exciton PL. As has been recently pointed out by one of us [16], an LBS surrounded by a coherent spin-polarized halo represents an exciton cloud trapped in an external potential [17]. The potential can be assumed to be of a harmonic type

$$V_{\mathrm{ho}}(x,y) = \frac{m\left(\omega_x^2x^2 + \omega_y^2y^2\right)}{2}, \tag{4}$$

with $\omega_{x,y}$ being the harmonic oscillator frequencies. Below we present an ideal gas model of an exciton condensate in the 2D harmonic trap (4), where we let $\omega_x = \omega_y \equiv \omega$ for simplicity. The model reproduces the main features observed in the experiment.

The two-body interactions, while not being explicitly taken into account in our calculations, ensure the important relation

$$\tau_{\boldsymbol{k}} \ll \tau \tag{5}$$

between the characteristic momentum relaxation time $\tau_{\boldsymbol{k}}$ and the lifetime τ of excitons. In a classical exciton gas the elastic collisions are known to govern the relaxation of spin according to the Dyakonov-Perel scenario [18,19]. The collisions fluctuate the Dresselhaus effective fields, which results in loss of an artificially created spin polarization on the timescale $\tau_s \sim (\Omega_e^2\tau_{\boldsymbol{k}})^{-1}$, with $\hbar\Omega_e \sim \beta_e k_T$ and k_T being the thermal de Broglie wave vector of excitons.

In our model, instead of an external optical pumping we assume a cold reservoir of unpolarized electrons and holes, which bind into (initially) unpolarized excitons. In a bosonic ensemble of excitons being in dynamical equilibrium with the cold bath the condition (5) favors formation of a metastable Bose-Einstein condensed phase [1]. Spin polarization of the condensate builds up spontaneously due to the coupling of the exciton spin to the momentum, encoded in the Hamiltonian (1). The interactions stabilize the phase of the order parameter and freeze the Dresselhaus fields. As a result, the exciton spin relaxation becomes suppressed [20],

$$\tau_s \to \infty. \tag{6}$$

This picture differs dramatically from that assumed in the earlier interpretations [11,13]. The excitons forming coherent regions do not occupy the ground state of the SO-coupled Hamiltonian (1) in those models. The polarization of the exciton gas originates from a coherent superposition of the eigenstates of (1). Coherent dynamics of the spin is observed in space due to propagation of the excitons from a source. Thus, in Ref. [11] the authors assume a ring of a classical exciton gas being formed at the boundary of an LBS. The excitons propagate from the ring in the radial direction with the electron and hole spins precessing around the Dresselhaus fields. Symmetric distribution of the spins is obtained by imposing the same initial condition for the excitons leaving the ring and by fixing the magnitude and direction of their velocities (ballistic propagation). The latter condition is achieved by eliminating the collisions from consideration.

The symmetry of our problem results from an interplay between the symmetry of the ground state of the Hamiltonian (1) and the external confinement. The SO-coupled condensate has more complex structure than just the usual lowest-energy state of the harmonic oscillator, typical for the potential (4). In the limit of strong SO coupling $k_0l_{\mathrm{ho}} \gg 1$, where $l_{\mathrm{ho}} = \sqrt{\hbar/m\omega}$ is the harmonic oscillator length, the condensate wave function can be obtained as follows. In the momentum space representation the harmonic potential $V_{\mathrm{ho}}(x,y)$ can be regarded as a kinetic energy operator of a fictitious particle moving in the potential energy landscape (2). The corresponding Schrodinger equation reads

$$\left[-\frac{m\omega^2\nabla_{\boldsymbol{k}}^2}{2} + E(k)\right]\Psi(\boldsymbol{k}) = E\Psi(\boldsymbol{k}). \tag{7}$$

By using the substitution $\Psi_l(\boldsymbol{k}) = e^{il\varphi}f_l(k)\,|\mathrm{GS}\rangle\,/\sqrt{k}$ with $|\mathrm{GS}\rangle$ given by (3) and integrating out the angular degree of freedom, we obtain

$$\frac{m\omega^2}{2}\left[-\frac{\mathrm{d}^2}{\mathrm{d}k^2} + \frac{1}{k^2}\left(l^2 + \frac{1}{4}\right)\right]f_l = (E_l - E(k))f_l. \tag{8}$$

Developing the radial motion in equation (8) in the vicinity of k_0 we have for the eigenenergies:

$$E_{n,l} = E(k_0) + \frac{m\omega^2}{2k_0^2}\left(l^2 + \frac{1}{4}\right) + \hbar\omega\left(\frac{1}{2} + n\right), \tag{9}$$

where n and l are the radial and the orbital integer quantum numbers, respectively. The new ground state corresponds to $n = l = 0$ and is described by the following wave function

$$\Psi_0(\mathbf{k}) = \pi^{-1/4}\sqrt{\frac{l_{\mathrm{ho}}}{2\pi k}}\exp\left(-\frac{\hbar(k-k_0)^2}{2m\omega}\right)|\mathrm{GS}\rangle\,. \tag{10}$$

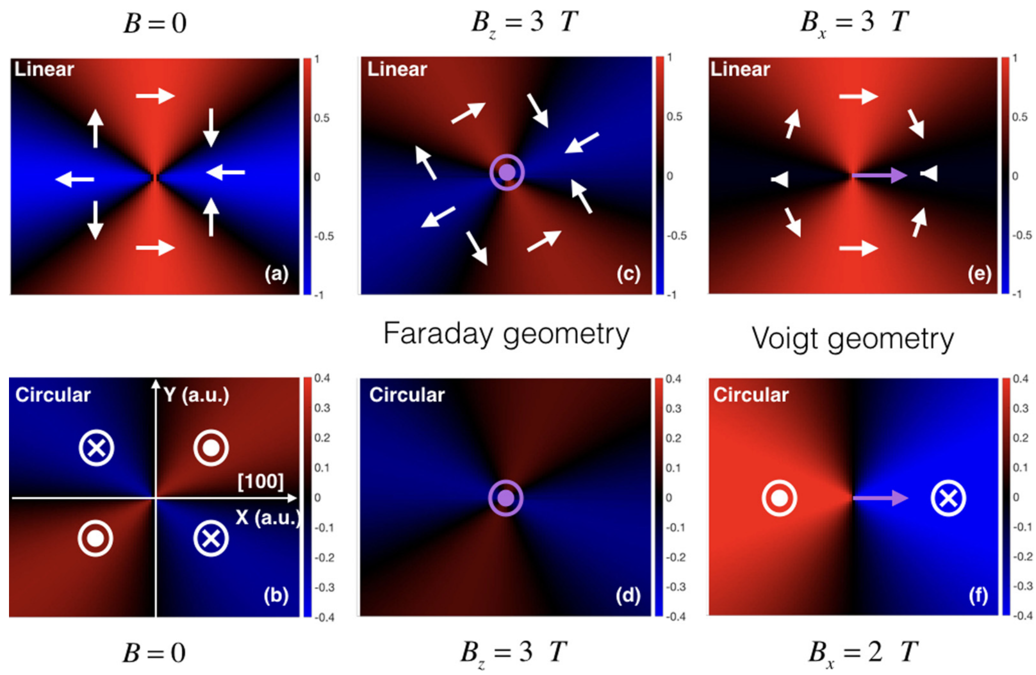

FIG. 1. Textures of the linear [ρ_l, shown in (a), (c), and (e)] and circular [ρ_c, shown in (b), (d), and (f)] polarization degrees of the light emitted by an exciton condensate in a trap. The coordinate axes [shown by white arrows in (b)] are chosen along the main crystallographic axes of the sample, consistently with the experiment [11]. For $B = 0$ the condensate pseudospin $\boldsymbol{S}$ [white arrows in (a)] rotates by 4π when going around the trap center. This corresponds to the 2π vortex of linear polarization observed in the experiment. The electron-hole exchange acts as an effective magnetic field acting on $\boldsymbol{S}$ along the x axis. The y component of $\boldsymbol{S}$ precesses around this field. In the steady state this results in the appearance of domains with positive and negative circular polarization [the four-leaf polarization texture in (b)], which corresponds to $S_z > 0$ (schematically depicted by $\odot$) and $S_z < 0$ (depicted by $\otimes$). The magnetic field applied along z (purple $\odot$, $B_z > 0$) rotates the linear polarization texture (c) and suppresses the circular polarization ρ_c (d). In contrast, an in-plane magnetic field applied along x (purple arrow) enhances $|\rho_c|$ [simultaneously transforming the four-leaf pattern into a two-leaf one, (f)] and destroys the negative part of ρ_l (e). We take $\tau = 0.1$ ns [21], $\beta_e = \beta_h = 0.5\ \mu\text{eV} \times \mu m$, $g_e = 0.1$, $g_h = 0.15$, $\delta = 1\ \mu$eV, and $S_0 = 1$. The values of the parameters $\beta_{e,h}$, $g_{e,h}$, and δ are of the same order of magnitude as those used in the previous models [11,13] and are typical for the structures under consideration.

In the coordinate space the GS wave function reads

$$\Psi_0(\mathbf{r}) = \frac{1}{\sqrt{2\pi k_0}} \begin{pmatrix} J_0(k_0 r) \\ J_1(k_0 r)e^{\mathrm{i}(\phi-\pi/2)} \\ J_1(k_0 r)e^{-\mathrm{i}(\phi-\pi/2)} \\ J_0(k_0 r) \end{pmatrix}, \tag{11}$$

with ϕ now being the polar angle of the radius-vector $\mathbf{r}$. The spin structure of the ground state (10) at the angle φ in the $\boldsymbol{k}$ space is reproduced in (11) at the angle $\phi = \varphi + \pi/2$. The pseudospin $\boldsymbol{S}$ constructed from the bright part of (11) lies in the QW plane and rotates by 4π when going around the LBS. In terms of the quantities ρ_l and $\rho_{l'}$ this corresponds to the 2π vortex of linear polarization observed in the experiment [Fig. 1(a)].

III. DYNAMICAL CORRECTION TO THE GROUND STATE

In order to obtain a finite circular polarization $\rho_c = 2S_z$ we need to go beyond the equilibrium model (1) and account for the driven-dissipative nature of the system. In fact, our assumption of the kinetic equilibrium (5) by no means implies a full thermodynamic equilibrium. The SO coupling results in the formation of a linearly-polarized condensate (10), but relaxation with respect to $\boldsymbol{k}$-independent spin interactions may take time longer than the exciton lifetime τ. Among those latter type of interactions are the electron-hole exchange and the Zeeman interaction with an external magnetic field. These interactions are thus expected to yield a dynamical correction

$$\hat{H}' = \begin{pmatrix} -\frac{1}{2}g_2\mu_B B_z & 0 & -\frac{1}{2}g_e\mu_B B_x & 0 \\ 0 & \frac{1}{2}g_1\mu_B B_z & -\delta & -\frac{1}{2}g_e\mu_B B_x \\ -\frac{1}{2}g_e\mu_B B_x & -\delta & -\frac{1}{2}g_1\mu_B B_z & 0 \\ 0 & -\frac{1}{2}g_e\mu_B B_x & 0 & \frac{1}{2}g_2\mu_B B_z \end{pmatrix} \tag{12}$$

to the Hamiltonian (1). Here $g_1 = g_h - g_e$ and $g_2 = g_e + g_h$ with $g_{e,h}$ being the electron (hole) g factors, μ_B is the Bohr magneton, and δ is the exchange constant [22]. The in-plane heavy-hole g factor is assumed to be equal to zero (we neglect possible random stresses typical for the samples of moderate quality [23]).

A proper description of the spin dynamics of a four-component system in the presence of decay and coherent pumping from a reservoir can be done by using the Lindblad equation for the spin density matrix

$$i\hbar\frac{d\hat{\rho}}{dt} = [\hat{H} + \hat{H}', \hat{\rho}] + \frac{i\hbar S_0}{\tau}\hat{\rho}_{\mathrm{GS}} - \frac{i\hbar}{\tau}\hat{\rho}, \tag{13}$$

where

$$\hat{\rho}_{\mathrm{GS}} = |\mathrm{GS}\rangle\langle\mathrm{GS}| \tag{14}$$

with $|\mathrm{GS}\rangle$ being defined by (3), and S_0/τ is the stimulated scattering rate being proportional to the number of particles in the condensate. In Eq. (13) we assume the conditions (5) and (6). In the steady-state configuration one has

$$\frac{d\hat{\rho}}{dt} = 0, \tag{15}$$

which yields a system of 16 coupled linear equations on the matrix elements of $\hat{\rho}$. The three components of the exciton pseudospin and the related linear and circular polarization degrees can then be obtained by using the formulas

$$\begin{aligned} 2S_x &= I\rho_l = \rho_{-1+1} + \rho_{+1-1}, \\ 2S_y &= I\rho_{l'} = \mathrm{i}(\rho_{-1+1} - \rho_{+1-1}), \\ 2S_z &= I\rho_c = \rho_{+1+1} - \rho_{-1-1}, \end{aligned} \tag{16}$$

where $I = \rho_{-1-1} + \rho_{+1+1}$.

The results of the solution of Eq. (15) are presented in Fig. 1. The observed phenomenology is reminiscent of the Hanle effect in an optical orientation experiment [15]. The role of the spin pump in our case is played by the boson stimulated scattering into the linearly-polarized GS [Fig. 1(a)]. The exchange interaction between the electrons and holes acts as an effective magnetic field of the magnitude 2δ oriented along the x axis in the structure plane. This field rotates the condensate pseudospin, so that the latter acquires a nonzero component S_z along the growth direction [Fig. 1(b)]. The rotation angle in the steady-state configuration (15) is determined by the balance between the radiative decay of the condensate and the pump from the reservoir. An external magnetic field applied along the z axis (Faraday geometry) rotates the pseudospin component lying in the structure plane [Fig. 1(c)], which yields the rotation of the linear polarization vortex observed in the experiment [11].

The effect of an in-plane magnetic field (Voigt geometry) is more subtle. Measurements of the exciton polarization in this configuration could provide a good test of our theory. As an example we show transformation of the linear [Fig. 1(e)] and circular [Fig. 1(f)] polarization textures by a magnetic field along the x axis with $B_x = 3$ T and $B_x = 2$ T, respectively. The field enhances the absolute degree of the circular polarization $|\rho_c|$ [simultaneously transforming the four-leaf spatial pattern into a two-leaf one] and destroys the negative part of ρ_l.

Let us now briefly discuss a possible effect of the two-body interactions on the properties of the ground state. Clearly, the rotational symmetry of the problem is preserved when the interactions are adiabatically switched on. A net repulsive interaction is expected to result in population of the states with larger angular momentum l and extrusion of the condensate to the peripheric region of the trap as to reduce the density in the center [24]. On the other hand, if one of the interaction channels admits a resonance, an increase of the chemical potential in the center may result in formation of a condensate of biexcitons, distinguished from the exciton condensate by a suppressed coherence of the emitted light [25]. In practice, both effects may be present simultaneously, complementing each other. An interplay between the SO interaction and resonant pairing of bosons is a challenging question which will be addressed in our future studies of many-body quantum phases of excitons.

IV. CONCLUSIONS

To conclude, we have shown that the experimentally observed polarization textures in the PL of indirect excitons can be interpreted in terms of the SO-coupled exciton Bose-Einstein condensation. In contrast to the previous considerations [11,13] based on the Dyakonov-Perel view of spins precessing around the Dresselhaus fields [18], in our model the spins align along the fields due to the quantum Bose statistics of excitons. The collisions between the excitons freeze the Dresselhaus fields and stabilize the symmetry of the order parameter.

In the absence of full thermodynamic equilibrium the dynamics of the condensate spin can be described by the Hanle-like equation on the spin density matrix (13). Stimulated scattering of the excitons into the linearly-polarized ground state is followed by "depolarization" of the condensate pseudospin by a static (effective) field not coupled to the exciton momentum. The stationary configuration (pseudospin rotation angle) is determined by the balance between the radiative decay and the stimulated scattering rate. The model can be adopted to the atomically thin layered materials [26], as well as to high-quality semiconductor microcavities [27].

ACKNOWLEDGMENTS

S.V.A. acknowledges M. Glazov and other participants of Low-Dimensional Seminar at Ioffe Institute for their remarks. I also thank D. Genkina, I. Spielman, and S. Stringari for their interest in the subject. The work has been supported by Russian Science Foundation (Grant No. 16-12-10503) and Russian Foundation for Basic Research (Grant No. 18-32-00434).

[1] L. P. Pitaevskii and S. Stringari, *Bose-Einstein Condensation and Superfluidity* (Oxford University Press, Oxford, 2016).
[2] Y.-J. Lin, K. Jiménez-Garcia, and I. B. Spielman, Nature (London) **471**, 83 (2011).
[3] Yun Li, Lev P. Pitaevskii, and Sandro Stringari, Phys. Rev. Lett. **108**, 225301 (2012).
[4] The stripe phase has recently been observed experimentally, see Jun-Ru Li, Jeongwon Lee, Wujie Huang, Sean Burchesky, Boris Shteynas, Furkan Cagi Top, Alan O. Jamison and Wolfgang Ketterle, Nature (London) **543**, 91 (2017).
[5] Ryan M. Wilson, Brandon M. Anderson, and Charles W. Clark, Phys. Rev. Lett. **111**, 185303 (2013).
[6] V. Galitski and I. B. Spielman, Nature (London) **494**, 49 (2013).
[7] T. D. Stanescu, B. Anderson, and V. Galitski, Phys. Rev. A **78**, 023616 (2008).
[8] E. L. Ivchenko, *Optical Spectroscopy of Semiconductor Nanostructures* (Alpha Science International, Harrow, U.K., 2005).
[9] In this paper we assume the excitonic gases formed of thermalized electrons and holes. In principle, the typical lifetime of an indirect exciton is sufficiently long for cooling of the

photoexcited excitons down to the temperature of the cold lattice. The cooling process occurs via emission of LA phonons, and the interactions redistribute the energy according to the equilibrium thermodynamics [L. V. Butov, A. L. Ivanov, A. Imamoglu, P. B. Littlewood, A. A. Shashkin, V. T. Dolgopolov, K. L. Campman, and A. C. Gossard, Phys. Rev. Lett. **86**, 5608 (2001); A. L. Ivanov, P. B. Littlewood, and H. Haug, Phys. Rev. B **59**, 5032 (1999)].

[10] L. V. Butov, J. Phys.: Condens. Matter **19**, 295202 (2007).

[11] A. A. High, A. T. Hammack, J. R. Leonard, Sen Yang, L. V. Butov, T. Ostatnicky, M. Vladimirova, A. V. Kavokin, T. C. H. Liew, K. L. Campman, and A. C. Gossard, Phys. Rev. Lett. **110**, 246403 (2013).

[12] S. V. Andreev, A. A. Varlamov, and A. V. Kavokin, Phys. Rev. Lett. **112**, 036401 (2014).

[13] A. V. Kavokin, M. Vladimirova, B. Jouault, T. C. H. Liew, J. R. Leonard, and L. V. Butov, Phys. Rev. B **88**, 195309 (2013); D. V. Vishnevsky, H. Flayac, A. V. Nalitov, D. D. Solnyshkov, N. A. Gippius, and G. Malpuech, Phys. Rev. Lett. **110**, 246404 (2013).

[14] W. T. Z. Hanle, Phys. **30**, 93 (1924).

[15] *Optical Orientation*, edited by F. Meyer and B. P. Zakharchenya (North-Holland, Amsterdam, 1984).

[16] S. V. Andreev, Phys. Rev. B **94**, 165308 (2016).

[17] Note that the localizing potential exists only for excitons but not for unbound electrons and holes.

[18] M. Dyakonov and V. Perel, Sov. Phys. Solid State **13**, 3023 (1972); M. Glazov and E. Ivchenko, J. Exp. Theor. Phys. **99**, 1279 (2004).

[19] J. R. Leonard, Y. Y. Kuznetsova, S. Yang, L. V. Butov, T. Ostatnicky, A. Kavokin, and A. C. Gossard, Nano Lett. **9**, 4204 (2009).

[20] One may expect the quantum many-body correlations in the condensate to counteract any other source of spin decoherence as well.

[21] The radiative lifetime of the condensate is expected to be shorter than the lifetime of a classical exciton gas, see B. Deveau, N. Roy, K. Satzke, B. Sermage, and D. S. Katzer, Phys. Rev. Lett. **67**, 2355 (1991).

[22] G. E. Pikus and F. G. Pikus, Solid State Commun. **89**, 319 (1994).

[23] S. V. Andreev, B. R. Namozov, A. V. Koudinov, Yu. G. Kusrayev, and J. K. Furdyna, Phys. Rev. B **80**, 113301 (2009).

[24] S. Sinha, R. Nath, and L. Santos, Phys. Rev. Lett. **107**, 270401 (2011).

[25] S. V. Andreev, Phys. Rev. B **92**, 041117(R) (2015); **94**, 140501(R) (2016).

[26] A. K. Geim and I. V. Grigorieva, Nature (London) **499**, 419 (2013).

[27] Y. Sun, P. Wen, Y. Yoon, G. Liu, M. Steger, L. N. Pfeiffer, K. West, D. W. Snoke, and K. A. Nelson, Phys. Rev. Lett. **118**, 016602 (2017).