Fluctuations of tunneling currents in photonic and polaritonic systems

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Here we develop the nonequilibrium Green's function formalism to analyze the fluctuation spectra of the boson tunneling currents. The approach allows us to calculate the noise spectra in both equilibrium and nonequilibrium conditions. The proposed general formalism is applied to several important realizations of boson transport, including the tunneling transport between two reservoirs and the case where the boson current flows through the intermediate region between the reservoirs. Developed theory can be applied for the analysis of the current noise in waveguides, coupled optical resonators, quantum microcavities, etc., where the tunneling of photons, exciton-polaritons, or excitons can be realized.

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I. INTRODUCTION

Studies of fluctuations in condensed-matter physics are attracting increasing interest nowadays [1–4]. The intensity and spectral behavior of noise provide important information about the energy spectrum fine structure, kinetics of charge carriers, and response to external fields [5–11]. For example, electric current fluctuations in equilibrium systems are controlled by electron conductivity, the key parameter in electronic transport, while spin noise is related to the magnetic susceptibility of the system, in accordance with the fluctuation-dissipation theorem [12]. External perturbations caused, e.g., by static and alternating electric and magnetic fields or optical excitation bring the system out of thermodynamical equilibrium and, generally, break the relation between the noise spectra and any linear-response function [13–17]. It makes noise spectroscopy a complementary and highly sensitive tool to study transport and spin effects in nonequilibrium systems.

The role of fluctuations increases with a decrease in the system size and dimensionality, making current and spin noise in nanosystems very prominent. This is because the size reduction dictates, as a rule, a decrease in the number of involved particles N, thus enhancing the relative magnitude of noise $\sim \frac{1}{N^{1/2}}$. That is why fluctuations play an especially important role in the tunneling phenomena [3,18–22].

While the tunneling current noise for electrons, i.e., fermions, has already been studied in detail (see Refs. [3,4,23–27] for a review), the role of fluctuations in the tunneling of bosons has received much less attention [28–32]. However, progress in the fabrication of photonic crystals, waveguides, and microcavities, achievements in experimental and theoretical studies of light propagation and light-matter coupling in such systems as well as the prospects of applications of semiconductor photonic structures in optical communications and information processing, opens up the prospects of studies of fluctuation phenomena in bosonic systems. Recently, bosonic analogs of electronic tunneling junctions based, e.g., on pillar microcavities with or without embedded quantum dots, diodes [33,34], transistors [35–37], and interferometers [38] based on plasmons and exciton-polaritons have been demonstrated theoretically and experimentally. These advancements attracted a lot of attention to transport and tunneling of bosons. It makes the problem of current noise in the tunneling of bosons, namely, photons, plasmon-polaritons, excitons, and exciton-polaritons, highly important. For device applications experimental and theoretical studies of the so-called photon blockade effect in light transport are highly important [39–41].

Physically, the key difference between bosons and fermions is related to their statistics: while fermions cannot occupy a single quantum state by more than one particle, the occupation number of bosons in a given state is not limited. Therefore, the particle number fluctuations of bosons and fermions within a given quantum state are already very different, let alone the noise of their fluxes.

This work aims at the development of the theory of noise in the process of boson tunneling. We use the nonequilibrium Green's function technique and study fluctuations of boson fluxes for tunneling between the reservoirs. The method can be applied for both close-to-equilibrium and strongly nonequilibrium conditions. We study several experimentally relevant situations where (i) the reservoirs are connected through a tunneling link, (ii) there is an intermediate empty cavity between the reservoirs, and (iii) a two-level system is placed in the intermediate cavity. This paper is organized as follows. Section II presents the general formalism and addresses the simplest possible case of tunneling between the two reservoirs (leads). The case of a cavity between the reservoirs is studied in Sec. III, and the situation where the cavity contains a two-level

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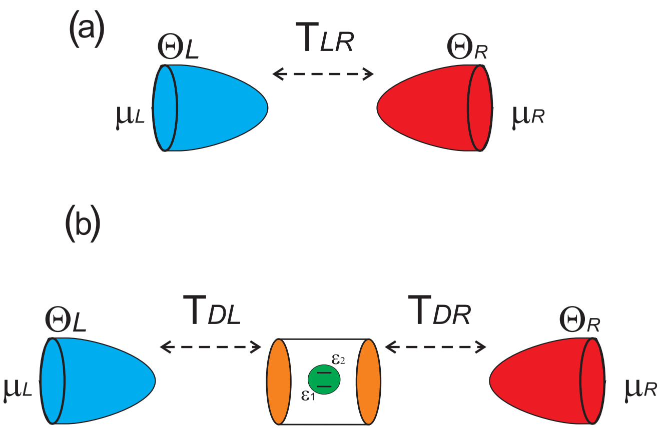

FIG. 1. Schemes of the boson transport models. (a) Simplest scheme of the tunneling between two semi-infinite reservoirs. (b) Scheme of the transport through the intermediate region containing a two-level system situated between two semi-infinite reservoirs. Here $\mu_{L,R}$ are the chemical potentials in the leads, $\Theta_{L,R}$ are the temperatures in the leads, and T_{LR}, T_{DL}, and T_{LD} denote the coupling constants; see text for details.

system (e.g., a quantum dot) is addressed in Sec. IV. The main results are summarized in Sec. V.

II. BOSON TRANSPORT BETWEEN TWO LEADS

The minimal model of the bosonic transport is the system formed by two semi-infinite reservoirs (leads) coupled via a tunneling junction as illustrated in Fig. 1(a). Such a structure can be realized for photons in two coupled cavities or in a quantum microcavity with a metallic gate on top which provides a given landscape of exciton-polaritons potential energy. The Hamiltonian describing the system under study [32]

$$\hat{H} = \hat{H}_L + \hat{H}_R + \hat{H}_{\rm tun} \tag{1}$$

consists of $\hat{H}_L$ and $\hat{H}_R$ parts describing the noninteracting states in the left, L, and right, R, reservoirs,

$$\hat{H}_L = \sum_\alpha \varepsilon_\alpha c_\alpha^\dagger c_\alpha, \quad \hat{H}_R = \sum_\beta \varepsilon_\beta c_\beta^\dagger c_\beta, \tag{2}$$

and the tunneling part

$$\hat{H}_{\rm tun} = \sum_{\alpha,\beta} T_{LR}(c_\alpha^\dagger c_\beta + c_\beta^\dagger c_\alpha) \tag{3}$$

responsible for the coupling of the leads. Here the subscript α (β) corresponds to the states in the left (right) lead with energy $\varepsilon_{\alpha,\beta} = \hbar\omega_{\alpha,\beta}$, and operators $c_{\alpha,\beta}^\dagger$ ($c_{\alpha,\beta}$) describe processes of the creation (annihilation) of the bosons in the leads. In Eq. (3) T_{LR} is the real tunnel coupling constant between the reservoirs assumed to be independent of the boson states α, β. Note that, in contrast to the bosonic transport analysis performed in [32], we will not introduce u-u-type coupling between the leads. This means that anomalous terms in the tunneling Hamiltonian like $c_\alpha^\dagger c_\beta^\dagger$ will be omitted and processes of simultaneous creation and annihilation of two bosons in different leads will not be considered.

Our goal is to calculate the particle current (flux) between the leads and, ultimately, the fluctuations of the current. By analogy with electronic systems it is natural to define the boson current as the change in the total number of bosons in the reservoir per unit of time. Taking the right reservoir for definiteness, we have for the variation of the total particle number there $N_R = \sum_\beta c_\beta^\dagger c_\beta$:

$$I_R(t) = \langle \dot{N}_R(t)\rangle = \left\langle \frac{d}{dt}\sum_\beta c_\beta^\dagger(t) c_\beta(t)\right\rangle. \tag{4}$$

Here the overdot denotes the time derivative, and angular brackets denote averaging over the reservoir states.

It is convenient to use, following Ref. [32], the Heisenberg equation of motion for the N_R operator in the form

$$\dot{N}_R(t) = -\frac{i}{\hbar} T_{LR} \sum_{\alpha\beta} [c_\beta^\dagger c_\alpha - c_\alpha^\dagger c_\beta]. \tag{5}$$

We introduce the nonequilibrium lesser Keldysh Green's function [42,43] as a correlator of boson creation and annihilation operators:

$$G_{LR}^<(t,t') = -\frac{i}{\hbar}\langle c_\alpha(t) c_\beta^\dagger(t')\rangle. \tag{6}$$

This function is a matrix, with the subscripts L and R running, respectively, through states α and β of the bosons in the left and right reservoirs. Making use of Eq. (6), an expression for the photon current between two semi-infinite leads in the frequency representation can be obtained by means of the nonequilibrium diagram technique formalism:

$$I_R(\omega) = 2\,\mathrm{Re}\sum_{R=\beta} T_{LR}\int \frac{d\omega}{2\pi} G_{LR}^<(\omega). \tag{7}$$

Following the standard nonequilibrium Keldysh Green's functions formalism, one can derive the system of Dyson equations, which determine the lesser Green's function $G_{LR}^<(\omega)$:

$$G_{LR}^<(\omega) = \sum_{L=\beta}\left[G_{LL}^{0r}(\omega) T_{LR} G_{RR}^<(\omega) + G_{LL}^{0<}(\omega) T_{LR} G_{RR}^a(\omega)\right], \tag{8a}$$

$$G_{RR}^<(\omega) = G_{RR}^{0<}(\omega) + G_{RR}^{0r}(\omega) T_{LR}\sum_{L=\alpha} G_{LR}^<(\omega) + G_{RR}^{0<}(\omega) T_{LR}\sum_{L=\alpha} G_{LR}^a(\omega), \tag{8b}$$

$$G_{LR}^a(\omega) = G_{LL}^{0a}(\omega) T_{LR}\sum_{R=\beta} G_{RR}^a(\omega), \tag{8c}$$

$$G_{RR}^a(\omega) = G_{RR}^{0a}(\omega) + G_{RR}^{0a}(\omega) T_{LR}\sum_{L=\alpha} G_{LR}^a(\omega). \tag{8d}$$

Superscripts r and a correspond to retarded and advanced Green's functions. The equilibrium Green's functions in the system of equations (8) are given by Fourier transformation of functions, namely,

$$G^{0r}_{LL}(t,t') = -\frac{i}{2\omega_\alpha}\theta(t-t')[e^{-i\omega_\alpha(t-t')} - e^{i\omega_\alpha(t-t')}], \quad (9a)$$

$$G^{0<}_{RR}(t,t') = -\frac{i}{2\omega_\beta}\{n_R(\varepsilon_\beta)e^{-i\omega_\beta(t-t')} + [1+n_R(\varepsilon_\beta)]e^{i\omega_\beta(t-t')}\}, \quad (9b)$$

$$G^{0<}_{LL}(t,t') = -\frac{i}{2\omega_\alpha}\{n_L(\varepsilon_\alpha)e^{-i\omega_\alpha(t-t')} + [1+n_L(\varepsilon_\alpha)]e^{i\omega_\alpha(t-t')}\}, \quad (9c)$$

$$G^{0a}_{RR}(t,t') = -\frac{i}{2\omega_\beta}\,\theta(t-t')[e^{-i\omega_\beta(t-t')} - e^{i\omega_\beta(t-t')}]. \quad (9d)$$

Here

$$n_{L(R)}(\varepsilon_{\alpha(\beta)}) = \frac{1}{e^{(\varepsilon_{\alpha(\beta)}-\mu_{L(R)})/k\Theta_{L(R)}} - 1} \quad (10)$$

are the bosonic occupation numbers in the left (L) and right (R) leads, which are assumed to be in equilibrium hereafter; μ_L and μ_R are chemical potentials; k is the Boltzmann constant; and $\Theta_{L(R)}$ is the temperature of each lead.

Introducing the densities of states $\nu_{L(R)}(\omega)$ in the left (right) leads and taking into account that $\sum_{L(R)=\alpha(\beta)} G^{0r}_{LL(RR)}(\omega) = -i\pi\nu_{L(R)}(\omega)$ and $\sum_{L(R)=\alpha(\beta)} G^{0<}_{LL(RR)}(\omega) = 2i\pi\nu_{L(R)}(\omega)n_{L(R)}(\omega)$, one can obtain from Eqs. (8) the following relations:

$$\sum_{\alpha,\beta} G^{<}_{LR}(\omega) = \frac{2T^2_{LR}\nu_L(\omega)\nu_R(\omega)}{[1+T^2_{LR}\nu_L(\omega)\nu_R(\omega)]^2}[n_L(\omega)-n_R(\omega)], \quad (11)$$

$$G^{<}_{LL}(\omega) = \frac{2i\nu_L(\omega)}{[1+T^2_{LR}\nu_L(\omega)\nu_R(\omega)]^2} \times [n_L(\omega) + T^2_{LR}\nu_L(\omega)\nu_R(\omega)n_R(\omega)]. \quad (12)$$

An expression for the Green's function $G^{<}_{RR}(\omega)$ can be obtained from Eq. (12) by interchanging the subscripts L and R.

As a result, the current between the leads can be expressed in the form

$$I_R = \int_0^\infty \mathcal{T}(\omega)[n_L(\omega)-n_R(\omega)]d\omega, \quad (13)$$

where

$$\mathcal{T}(\omega) = \frac{2T^2_{LR}\nu_L(\omega)\nu_R(\omega)}{[1+T^2_{LR}\nu_L(\omega)\nu_R(\omega)]^2} = \frac{2\zeta(\omega)}{[1+\zeta(\omega)]^2} \quad (14)$$

describes the transition probability for a particle with energy ω from the left to the right reservoir per unit of time and $\zeta(\omega) = T^2_{LR}\nu_L(\omega)\nu_R(\omega)$. For a weak tunneling link between the reservoirs where $\zeta(\omega) \ll 1$, Eq. (13) immediately follows from an evaluation of the difference in the fluxes of particles moving from the left lead to the right lead and leaving the right lead and moving into the left one. The local density of states in the reservoirs is modified by the flowing of photon current, which is why the denominator appears in the general expression (14) for the transition probability. Moreover, Eq. (12) demonstrates that states in one of the reservoirs contribute to the local nonequilibrium boson distribution in another reservoir. Naturally, Eq. (13) is antisymmetric with the replacement of $L \leftrightarrow R$; this is because in the steady state the number of particles leaving from the left reservoir and going to the right one equals exactly the number of particles arriving in the right reservoir from the left one. Note that in the thermal equilibrium between the reservoirs, $n_R(\omega) = n_L(\omega)$, and the current vanishes.

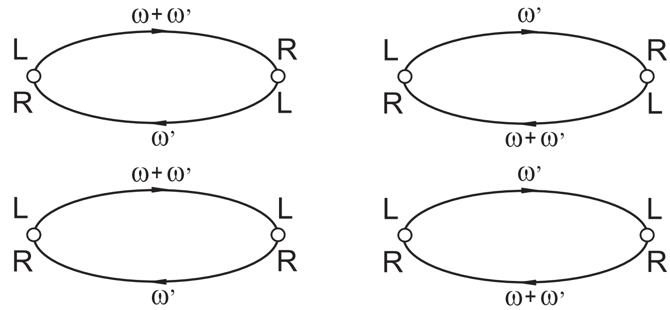

FIG. 2. Diagrams contributing to the boson current noise spectra in the case of the transport between two semi-infinite reservoirs. Open circles denote the tunnel coupling constant T_{LR}.

Now let us turn to the evaluation of the boson current fluctuations. The current noise is characterized by the set of correlation functions

$$S_{ij}(t,t') = \langle I_i(t)I_j(t')\rangle - \langle I_i(t)\rangle\langle I_j(t')\rangle, \quad (15)$$

where $i,j = L,R$ and the current operators are defined, by analogy with Eq. (4), as $I_i = \dot{N}_i(t)$ through the derivative of the particle number in the reservoir. It follows from the general principles of statistical physics that the correlation functions (15) depend in the steady state only on the difference of times $\tau = t' - t$ and, moreover, $S_{ij}(t,t') = S_{ji}(t,t')$ due to time-reversal symmetry [12,44]. It is instructive to introduce the Fourier transform of the correlation functions in the form

$$\tilde{S}_{ij}(\omega) = \int_{-\infty}^{\infty} S_{ij}(t,t+\tau)e^{i\omega\tau}d\tau. \quad (16)$$

Although in the steady state the current through the structure is the same, the current fluctuations are not homogeneous in space in general. Hence, generally, $\tilde{S}_{LL}(\omega) \neq \tilde{S}_{RR}(\omega) \neq \tilde{S}_{LR}(\omega) = \tilde{S}_{RL}(\omega)$, although at $\omega = 0$ all components of $\tilde{S}_{ij}(0)$ are the same, just like in electronic structures [26,43]. In what follows we focus on $\tilde{S}_{LR}(\omega) = \tilde{S}_{RL}(\omega)$.

The correlation function in the frequency domain can be obtained from the sum of diagrams shown in Fig. 2. By means of the Keldysh diagram technique one can get the following expression for the photon current noise spectra:

$$\tilde{S}_{LR}(\omega) = T^2_{LR}\int d\omega'[G^{<}_{LL}(\omega+\omega')G^{>}_{RR}(\omega') + G^{<}_{LR}(\omega+\omega')G^{>}_{LR}(\omega') + (L \leftrightarrow R)]. \quad (17)$$

Consequently, substituting Eqs. (11) and (12) into Eq. (17), we arrive at

$$\begin{aligned}\tilde{S}_{LR} = &\int d\omega'\mathcal{T}(\omega+\omega')\mathcal{T}(\omega')\\ &\times\{[n_L(\omega+\omega')+\zeta(\omega+\omega')n_R(\omega+\omega')][1+n_R(\omega')\\ &+\zeta(\omega')(n_L(\omega')+1)]+(L\to R)\}\\ &+2\int d\omega'\mathcal{T}(\omega+\omega')\mathcal{T}(\omega')[n_L(\omega')-n_R(\omega')]\\ &\times[n_L(\omega+\omega')-n_R(\omega+\omega')].\end{aligned}\tag{18}$$

Equation (18) is the central result of this section. It allows us to calculate the fluctuations of the current of bosons. The first two lines of Eq. (18) are nonzero in the thermal equilibrium conditions and account for the fluctuations of the current which arise from a momentary imbalance of the occupancies of states in the left and right leads which is present even in thermal equilibrium [12]. While the current is zero on average since bosons tunnel back and forth, resulting in the contributions to the current of opposite signs, the mean square of the current is nonzero. The last line of Eq. (18) describes the additional contribution to current noise in the presence of current flow. For $\zeta \ll 1$ Eq. (18) reduces to the results of previous works [28,29]. Particularly, for $n_L(\omega)=n_R(\omega)\equiv n(\omega)$, where

$$n(\omega)=\frac{1}{\exp\left(\frac{\omega-\mu}{k_B\Theta}\right)-1}$$

is the Bose-Einstein distribution function, the equilibrium contribution to the current noise reads

$$\tilde{S}_{LR}^{(\text{eq})}=2\int d\omega'\mathcal{T}(\omega+\omega')\mathcal{T}(\omega')n(\omega+\omega')[1+n(\omega')],\tag{19}$$

which is proportional to the mean-square fluctuation of the boson number in the state $\propto n(1+n)$. Note that for fermions, in agreement with Refs. [28,43], the mean-square fluctuation of the particle number is $\propto n(1-n)$.

It is instructive to analyze two contributions to the boson current noise independently. In order to derive simplified analytical expressions we assume a two-dimensional density of states in the reservoirs $\nu_L(\omega)=\nu_R(\omega)\equiv\nu_0$ that is independent of the energy and assume tunneling to be weak enough, $\zeta\ll 1$. In this case, $\mathcal{T}(\omega+\omega')\mathcal{T}(\omega')$ reduces to a frequency-independent factor $4T_{LR}^2\nu^2$. Let us start from the equilibrium contribution to the current noise. The integral in Eq. (19) can be reduced to, with $x=\omega/k_B\Theta$, $x'=\omega'/k_B\Theta$,

$$\tilde{S}_{LR}^{(\text{eq})}=8T_{LR}^2\nu^2k_B\Theta\times\int_0^\infty\frac{\exp\left(x'-\frac{\mu}{k_B\Theta}\right)dx'}{\left[\exp\left(x'-\frac{\mu}{k_B\Theta}\right)-1\right]\left[\exp\left(x+x'-\frac{\mu}{k_B\Theta}\right)-1\right]}.$$

Making a change in the variable $\exp(x')=y$, it can be found analytically with the result

$$\tilde{S}_{LR}^{(\text{eq})}=8T_{LR}^2\nu_0^2\frac{\omega-k_B\Theta\ln\left(\frac{n(0)}{n(\omega)}\right)}{1-\exp(\omega/k_B\Theta)}.\tag{20}$$

Particularly, at $\omega\to 0$ the noise intensity is given by $\tilde{S}_{LT}^{(\text{eq})}=4T_{LR}^2\nu_0^2k_B\Theta n(0)$, and it monotonously decreases with an increase in the frequency. At high frequencies the current noise has the asymptotic form $\tilde{S}_{LR}^{(\text{eq})}=4T_{LR}^2\nu_0^2[k_B\Theta\ln n(0)-\mu]\exp(-\omega/k_B\Theta)$. Here $\Theta\equiv\Theta_L=\Theta_R$ and $\mu\equiv\mu_L=\mu_R$. This analysis demonstrates that the characteristic frequencies in the current noise in equilibrium conditions are given by the temperature of the reservoirs.

The nonequilibrium fluctuations of the current can be most conveniently analyzed in the limiting case where one of the reservoirs is empty, i.e., $n_R(\omega)=0$, $n_L(\omega)\equiv n(\omega)$. Again, for the constant density of states in the reservoirs and weak tunneling we obtain

$$\begin{aligned}\tilde{S}_{LR}^{(\text{neq})}&=2\int d\omega'\mathcal{T}(\omega+\omega')\mathcal{T}(\omega')n(\omega+\omega')n(\omega'),\\ &8T_{LR}^2\nu^2k_B\Theta\\ &\times\int_0^\infty\frac{dx'}{\left[\exp\left(x'-\frac{\mu}{k_B\Theta}\right)-1\right]\left[\exp\left(x+x'-\frac{\mu}{k_B\Theta}\right)-1\right]}.\end{aligned}\tag{21}$$

Again, after standard transformations we obtain

$$\tilde{S}_{LR}^{(\text{neq})}=8T_{LR}^2\nu_0^2\times\left[k_B\Theta\ln\left(1-e^{(\mu-\omega)/k_B\Theta}\right)+\frac{\omega-k_B\Theta\ln\left(\frac{n(0)}{n(\omega)}\right)}{1-\exp(\omega/k_B\Theta)}\right].\tag{22}$$

This current-induced noise contribution has a frequency dependence similar to that of the equilibrium noise, decreasing monotonously $\propto\exp(\omega/k_B\Theta)$ with an increase in the frequency. The magnitude of this contribution increases with an increase in the current in the system.

In order to illustrate general expression (18) we plotted in Fig. 3 numerically calculated current noise spectra for the same temperature of reservoirs and different chemical potentials. The numerical results after Eq. (18) shown by the solid curves agree well with analytical expressions (20) and (22) (see dotted lines). The inset illustrates the results for the "three-dimensional" density of states $\nu(\omega)\propto\omega^2$ (dashed line) and, for comparison, the constant density of states (solid line). Note that, qualitatively, all dependencies are quite similar. The characteristic frequency where the noise drops is given by the typical temperature of the reservoir.

III. BOSON TRANSPORT THROUGH THE NONINTERACTING CENTRAL REGION

Let us now consider the situation when an intermediate system (central region) is placed between the leads, as illustrated in Fig. 1(b). In this section we consider the minimal model of the "noninteracting" central region; that is, the intermediate system acts as a cavity for the bosons. The Hamiltonian of the system then has the following form:

$$\hat{H}=\hat{H}_L+\hat{H}_R+\hat{H}_D+\hat{H}_{\text{tun}}.\tag{23}$$

Here the Hamiltonians $\hat{H}_L$ and $\hat{H}_R$ describe the states in the reservoirs and are given by Eq. (2); the central-region part $\hat{H}_D$ describes the states in the intermediate system. For simplicity we assume that there is just one energy level in the intermediate system with energy ε_D and present the Hamiltonian in the form

$$\hat{H}_D = \varepsilon_D c_D^\dagger c_D \tag{24}$$

and the tunneling part

$$\hat{H}_{\text{tun}} = \sum_\alpha T_{DL}(c_\alpha^\dagger c_D + c_D^\dagger c_\alpha) + \sum_\beta T_{DR}(c_\beta^\dagger c_D + c_D^\dagger c_\beta). \tag{25}$$

The boson creation and annihilation operators in the central (intermediate) reservoir are denoted $c_D^\dagger$ and c_D, respectively. The first and second sums in Eq. (25) correspond to the particle transitions from the left lead to the central region and from the central region to the right lead, respectively, and T_{DL} and T_{DR} are the corresponding tunnel coupling constants.

Following the approach developed in Sec. II and making use of the Heisenberg equation of motion

$$\dot{N}_R(t) = -\frac{i}{\hbar} T_{LR} \sum_\alpha [c_\beta^\dagger c_D - c_D^\dagger c_\beta], \tag{26}$$

we express the boson current in the form

$$I_R(\omega) = 2\,\mathrm{Re} \sum_{R=\beta} T_{DR} \int \frac{d\omega}{2\pi} G_{DR}^<(\omega). \tag{27}$$

Similarly, the current noise spectrum in the presence of the noninteracting central region is described by the diagrams

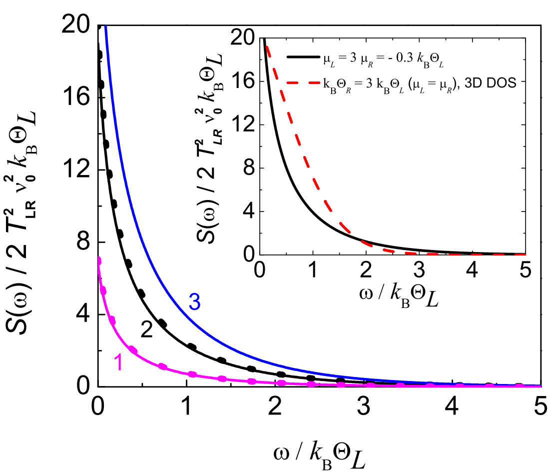

FIG. 3. Boson current noise spectra for tunneling between two reservoirs. For curves 1, the equilibrium situation is considered, where $n_R(\omega) = n_L(\omega) \equiv n(\omega)$. Here $\mu_L = \mu_R = -0.1k_B\Theta_L$; the solid curve is plotted after Eq. (18), and the dotted curve is plotted after Eq. (22). For curves 2, the nonequilibrium situation is considered, where one of the reservoirs is empty, $n_R(\omega) = 0$, $n_L(\omega) = n(\omega)$. Here $\mu_L = \mu_R = -0.1k_B\Theta_L$; the solid curve is plotted after Eq. (18), and the dotted curve is plotted after Eq. (20). Curve 3 shows the result for $\mu_L = 3\mu_R = -0.3k_B\Theta_L$ plotted after Eq. (18). The inset shows the result plotted after Eq. (18): the solid curve corresponds to $\mu_L = 3\mu_R = -0.3k_B\Theta_L$; the dashed curve demonstrates the results obtained for the three-dimensional density of states in the reservoirs and $k_B\Theta_R = 3k_B\Theta_L$ and $\mu_L = \mu_R$.

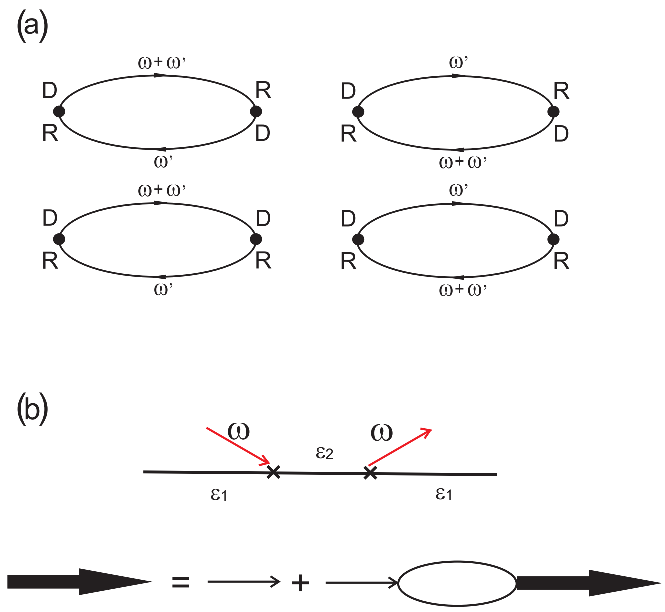

FIG. 4. (a) Diagrams showing the boson current noise spectra in the case of the transport through the noninteracting intermediate region situated between two semi-infinite reservoirs; (b) lowest-order diagrams corresponding to the boson Green's function corrections caused by the presence of interaction with the two-level system in the intermediate region [Fig. 1(c)]. Solid circles denote the coupling constants T_{DL} and T_{DR}; crosses denote the interaction constant with the two-level system g.

depicted in Fig. 4(a) and reads

$$\tilde{S}_{LR}(\omega) = T_{DR}^2 \int d\omega' [G_{RR}^<(\omega+\omega')G_{DD}^>(\omega') + G_{DR}^<(\omega+\omega')G_{DR}^>(\omega') + (D \leftrightarrow R)]. \tag{28}$$

Correspondingly, the lesser Green's function $G_{DR}^<(\omega)$ satisfies the Dyson equation:

$$G_{DR}^<(\omega) = G_{DD}^r(\omega) T_{DR} G_R^{0<}(\omega) + G_{DD}^<(\omega) T_{DR} G_R^{0a}(\omega), \tag{29}$$

where the Green's functions of the intermediate region can be expressed as

$$G_{DD}^r(\omega) = \frac{1}{\omega - \varepsilon_D + i(\gamma_1 + \gamma_2)}, \tag{30a}$$

$$G_{DD}^<(\omega) = 2i n_D(\omega)\,\mathrm{Im}\, G_{DD}^r(\omega), \tag{30b}$$

with

$$\gamma_{1(2)} = -i\pi \sum_{R(L)} T_{DR(L)}^2 G_{R(L)}^{0r}(\omega), \tag{31}$$

$$n_D(\omega) = \frac{n_R(\omega)\gamma_1 + n_L(\omega)\gamma_2}{\gamma_1 + \gamma_2}. \tag{32}$$

It follows from Eqs. (30) that the Green's function of the intermediate region has a standard form, with γ_1 and γ_2 defined in Eq. (31) being the partial contributions to the damping of the intermediate states due to the bosons tunneling into the reservoirs. Here we disregard the tunneling-induced energy renormalization, assuming that it is already accounted for in

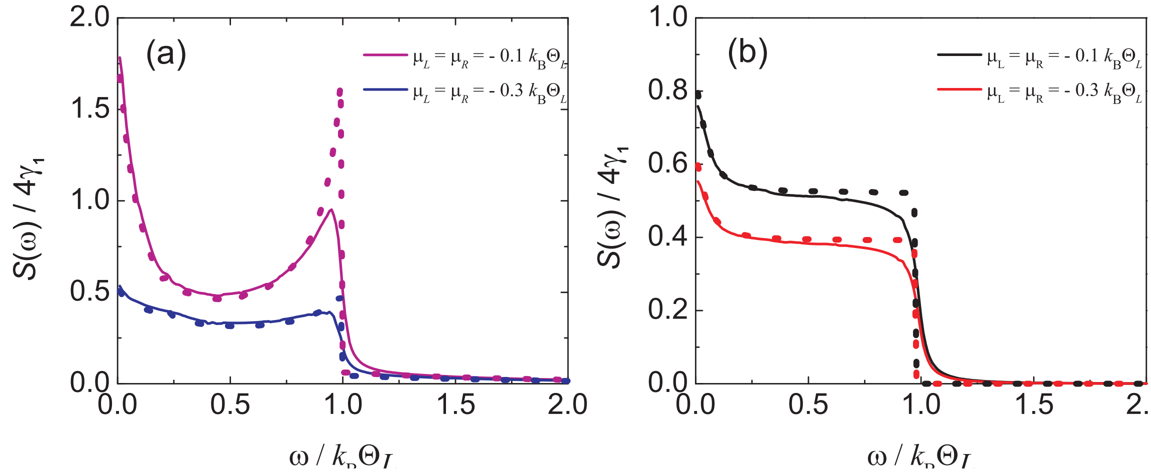

FIG. 5. Boson current noise spectra for tunneling through the noninteracting central region. (a) The equilibrium situation is considered, where $n_R(\omega) = n_L(\omega) = n_D(\omega) \equiv n(\omega)$. Solid curves are plotted after Eq. (33), and dotted curves are plotted after Eqs. (34) and (35). (b) The nonequilibrium situation is considered, where one of the reservoirs is empty, $n_R(\omega) = 0$, $n_L(\omega) = n(\omega)$. Solid curves are plotted after Eq. (33), and dotted curves are plotted after Eqs. (37)–(40). For all the curves parameters $k_B\Theta_L = 1$, $\varepsilon_D = 1$, $\gamma_1 = 0.02$, and $\gamma_2 = 0.01$ are the same.

the value of ε_D; thus, $\gamma_{1,2}$ are real. Accordingly, the occupation of the intermediate region is given by the ratio of the incoming and outgoing particle rates. Equations (30), (31), and (32) are derived in the regime of sequential tunneling where the particle leaves, e.g., the left lead, enters the intermediate system, and then goes to the right lead before the next particle arrives into the intermediate part of the system. Otherwise, the accumulation of the particles in the intermediate state should be taken into account. Finally, substituting expressions for Green's functions $G^r_{DD}(\omega)$ and $G^<_{DD}(\omega)$ into Eq. (29), one can directly obtain the boson current noise spectrum in the case when the intermediate region is present:

$$\begin{aligned}\mathcal{S}(\omega) = &-4\gamma_1 \int d\omega' \{\mathrm{Im}\, G^r_{DD}(\omega') n_R(\omega+\omega')[1+n_D(\omega')] + \mathrm{Im}\, G^r_{DD}(\omega'+\omega) n_D(\omega+\omega')[1+n_R(\omega')]\} \\ &+4\gamma_1^2 \int d\omega'\, \mathrm{Im}\, G^r_{DD}(\omega')\, \mathrm{Im}\, G^r_{DD}(\omega'+\omega)\{n_R(\omega+\omega')[1+n_D(\omega')] + n_D(\omega+\omega')[1+n_R(\omega')]\} + \frac{4\gamma_1^2\gamma_2}{(\gamma_1+\gamma_2)} \\ &\times \int d\omega'\, \mathrm{Im}\, G^r_{DD}(\omega')\, \mathrm{Im}\, G^r_{DD}(\omega'+\omega)\{[n_R(\omega+\omega') - n_L(\omega+\omega')][1+n_D(\omega')] + n_D(\omega+\omega')[n_R(\omega') - n_L(\omega')]\} \\ &+8\gamma_1^2 \int d\omega'\, \mathrm{Im}\, G^r_{DD}(\omega')\, \mathrm{Im}\, G^r_{DD}(\omega'+\omega)[n_D(\omega') - n_R(\omega')][n_D(\omega'+\omega) - n_R(\omega'+\omega)]. \end{aligned} \tag{33}$$

Here, as in Sec. II, two contributions to the current noise are clearly seen: the first two lines of Eq. (33) do not vanish in the equilibrium conditions, while the two last lines of Eq. (33) are proportional to the current flowing through the system.

In order to illustrate the results it is instructive to consider the equilibrium system where $n_R(\omega) = n_L(\omega) = n_D(\omega) \equiv n(\omega)$. Taking into account that for sufficiently weak tunneling $\gamma_{1,2} \ll \varepsilon_D$ the imaginary part of the intermediate-system Green's function, Eq. (30a), can be replaced by the Dirac δ function as

$$\mathrm{Im}\, G^r_{DD}(\omega') = -\pi\delta(\omega - \varepsilon_D),$$

which describes resonant transmission through an intermediate region. Thus, for $\gamma_{1,2} \ll \Theta$ the first line of Eq. (33) assumes the form

$$\begin{aligned} S_1^{(\mathrm{eq})}(\omega) = 4\pi\gamma_1\{&n(\omega+\varepsilon_D)[1+n(\varepsilon_D)] \\ &+\theta(\varepsilon_D - \omega)n(\varepsilon_D)[1+n(\varepsilon_D-\omega)]\}, \end{aligned} \tag{34}$$

where $\theta(x)$ is the Heaviside step function. The contribution of the second line in Eq. (30a) can also be evaluated analytically in the limit of weak damping $\gamma_1,\gamma_2 \ll \varepsilon_D,\Theta$ making use of the following relation:

$$\int d\omega'\, \mathrm{Im}\, G^r_{DD}(\omega')\, \mathrm{Im}\, G^r_{DD}(\omega'+\omega) = \frac{2\pi(\gamma_1+\gamma_2)}{\omega^2 + 4(\gamma_1+\gamma_2)^2}.$$

As a result, for the second line of Eq. (33) we obtain

$$\begin{aligned} S_2^{(\mathrm{eq})}(\omega) = \frac{8\pi\gamma_1^2(\gamma_1+\gamma_2)}{\omega^2+4(\gamma_1+\gamma_2)^2}\{&n(\omega+\varepsilon_D)[1+n(\varepsilon_D)] \\ &\times\theta(\varepsilon_D-\omega)n(\varepsilon_D)[1+n(\varepsilon_D-\omega)]\}. \end{aligned} \tag{35}$$

To illustrate the nonequilibrium case we, as in Sec. II, consider the situation where one of the reservoirs is empty, $n_R(\omega) = 0$, $n_L(\omega) = n(\omega)$. It follows from Eq. (32) that

$$n_D(\omega) = n(\omega)\frac{\gamma_2}{\gamma_1+\gamma_2} \tag{36}$$

and the equilibrium contribution is given by the first two lines in Eq. (33) and can be written as

$$S_1^{(\mathrm{eq})}(\omega) = \frac{4\pi\gamma_1\gamma_2}{\gamma_1+\gamma_2} n(\varepsilon_D)\theta(\varepsilon_D-\omega), \tag{37}$$

$$S_2^{(\mathrm{eq})}(\omega) = \frac{8\pi\gamma_1^2\gamma_2}{(\gamma_1+\gamma_2)[\omega^2+4(\gamma_1+\gamma_2)^2]} \times [n(\omega+\varepsilon_D)+n(\varepsilon_D)\theta(\varepsilon_D-\omega)]. \tag{38}$$

The last two contributions in Eq. (33) relevant for the nonequilibrium situation can be calculated in the same way:

$$S_1^{(\mathrm{neq})}(\omega) = -\frac{4\pi\gamma_1^2\gamma_2}{\omega^2+4(\gamma_1+\gamma_2)^2} \times \left\{ n(\omega+\varepsilon_D)\left[1+\frac{2\gamma_2}{\gamma_1+\gamma_2}n(\varepsilon_D)\right] + \theta(\varepsilon_D-\omega)n(\varepsilon_D)\left[1+\frac{2\gamma_2}{\gamma_1+\gamma_2}n(\varepsilon_D-\omega)\right]\right\}, \tag{39}$$

$$S_2^{(\mathrm{neq})}(\omega) = \frac{8\pi\gamma_1^2\gamma_2^2}{(\gamma_1+\gamma_2)[\omega^2+4(\gamma_1+\gamma_2)^2]} \times [n(\omega+\varepsilon_D)n(\varepsilon_D)+\theta(\varepsilon_D-\omega)n(\varepsilon_D)n(\varepsilon_D-\omega)]. \tag{40}$$

The above expressions demonstrate that the noise of the boson flux basically has two features: one is at $\omega = 0$, and the other one is at $\omega = \varepsilon_D$. This is the result of the resonant tunneling of bosons through the intermediate region: The presence of the "resonant" state in the cavity naturally gives rise to the fluctuations at the frequency corresponding to this state. The calculation results are shown in Fig. 5. As we are interested only in the boson transport properties, we consider the situation when the population in the right lead exceeds the population in the left lead (due to the imbalance of chemical potentials or temperatures in the leads) and analyze the current noise spectra in the one-by-one boson tunneling regime. Moreover, to prevent the pair creation and annihilation processes in the intermediate region we focus on the case when the following ratio between system parameters is present: $T_{DL} \geqslant T_{DR}$ and, consequently, $\gamma_2 \geqslant \gamma_1$. The numerical results are in good agreement with the analytical calculations.

IV. BOSON TRANSPORT THROUGH THE INTERACTING CENTRAL REGION

Let us now briefly discuss the results of our approach in the presence of interaction in the central region. We consider a situation where a two-level system, such as a quantum dot, is placed in the intermediate region. Accounting for the bosons' interaction with the two-level electronic system, the Hamiltonian $\hat{H}_D$ can be recast in the following form:

$$\hat{H}_D = \varepsilon_D c_D^\dagger c_D + g(a_1^\dagger a_2 c_D + a_2^\dagger a_1 c_D^\dagger). \tag{41}$$

It describes photon absorption and emission processes due to the presence of electron-photon interaction. Here the operators $a_{1,2}^\dagger$ ($a_{1,2}$) are the electron creation (annihilation) in the energy level $\varepsilon_{1,2}$ of the two-level system, and g is the electron-photon coupling constant.

In order to calculate photon current noise spectra one can apply the general expression (33) derived in Sec. III considering the presence of interaction in the central part. This means that the Green's function $G^r_{DD}(\omega)$ should be modified due to the presence of electron-photon interaction processes. Diagrams illustrating the interaction processes and photon Green's function renormalization are shown in Fig. 4(b). Now the Green's function of the central region can be written as

$$G^r_{DD} = \frac{1}{\omega - \varepsilon_D - \frac{g^2}{\omega+\varepsilon_1-\varepsilon_2+i\Gamma} + i(\gamma_1+\gamma_2)}. \tag{42}$$

Here Γ is the damping of the electronic transition between states 2 and 1 unrelated to the interaction with the bosons. Such an approach is valid provided that the mean number of bosons in the intermediate region is substantially smaller than 1.

Here two important limits can be identified [45]. In the first one, which is known as a *weak-coupling regime*, $g \ll \gamma_1 + \gamma_2, \Gamma$, the presence of a two-level system does not qualitatively affect the energy spectrum, resulting in a slight shift and renormalization of the damping of the boson mode in the intermediate state. In the weak-coupling regime the boson current noise spectrum is very similar to that obtained in Sec. III (see Fig. 5). By contrast, in the *strong-coupling regime*, where $g \gg \gamma_1 + \gamma_2, \Gamma$, new eigenstates like exciton-polaritons in microcavities are formed, and the Green's function (42) has two distinct poles at $\omega = \Omega_\pm$, which are the roots of the quadratic equation

$$(\Omega_\pm - \varepsilon_D)(\Omega_\pm - \varepsilon_2 + \varepsilon_1) = g^2.$$

For example, if $\varepsilon_2 - \varepsilon_1 = \varepsilon_D$, the pole energies are $\varepsilon_D \pm g$. In this case two features in the current noise spectra, similar to those shown in Fig. 4 at $\omega = \varepsilon_D$, are expected now at $\omega = \Omega_\pm$.

V. CONCLUSION

To conclude, we have developed the Green's function theory to calculate the fluctuation spectra of the boson tunneling currents. This formalism generalizes, to a certain extent, the results obtained previously for fermionic [3,24] and bosonic [28,29] systems on transport fluctuations in the strongly nonequilibrium case. The theory allows us to calculate the noise spectra in both equilibrium and nonequilibrium conditions. The general formalism is applied to several important situations, including the tunneling transport between two reservoirs and the case where the bosons pass through the intermediate region between the reservoirs. Such systems can be experimentally realized, e.g., in waveguide structures or microcavity systems operating in the strong-coupling regime.

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