Rapid Communications

Tunneling probe of fluctuating superconductivity in disordered thin films

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Disordered thin films close to the superconductor-insulator phase transition (SIT) hold the key to understanding quantum phase transition in strongly correlated materials. The SIT is governed by superconducting quantum fluctuations, which can be revealed, for example, by tunneling measurements. These experiments detect a spectral gap, accompanied by suppressed coherence peaks, on both sides of the transition. Here we describe the insulating side in terms of a fluctuating superconducting field with finite-range correlations. We perform a controlled diagrammatic resummation and derive analytic expressions for the tunneling differential conductance. We find that short-range superconducting fluctuations suppress the coherence peaks even in the presence of long-range correlations. Our approach offers a quantitative description of existing measurements on disordered thin films and accounts for tunneling spectra with suppressed coherence peaks.

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Introduction. Superconducting thin films have attracted recently a lot of attention due to the possibility of observing a direct superconductor-to-insulator transition (SIT) [1–3]. The SIT is considered as an excellent example of a quantum phase transition [4]: It occurs at temperature $T = 0$ and is driven by a nonthermal tuning parameter g. Experimentally, the SIT can be driven by a wide variety of g's, such as thickness [5–16], magnetic [11,12,17–26] or electric fields [27], chemical composition [28,29], and disorder [25,30]. Near the quantum critical point $g = g_c$, the system is governed by quantum fluctuations [31–35] and cannot be described in terms of classical Ginzburg-Landau theories [36–39].

In the vicinity of the SIT, experiments show an intriguing behavior of the superconducting energy gap Δ. Traditionally, Δ is determined by fitting the tunneling conductivity to a phenomenological extension of the BCS theory, which takes into account an effective energy broadening Γ and is known as the Dynes formula [40],

$$\frac{dI}{dV}(V) = \mathrm{Re}\left[\frac{V - i\Gamma}{\sqrt{(V - i\Gamma)^2 - \Delta^2}}\right]. \tag{1}$$

This procedure is very useful in extracting values of Δ as a function of temperature for a relatively clean superconductor. In these materials, Δ shrinks to zero as T approaches the critical temperature T_c, in agreement with the predictions of the BCS theory [41]. In contrast, in disordered thin films, tunneling experiments have revealed that Δ smoothly evolves across the transition [42,43] and through T_c [44].

In addition to this nonconventional behavior of Δ, superconducting thin films show a significant deviation of the experimentally measured density of states (DOS) from Eq. (1) due to a considerable suppression of the coherence peaks at the gap edges [43,44]. A similar disagreement was observed in high-temperature superconductors [45,46]. This discrepancy can be accounted for by assuming the two Γ's that appear in the numerator and in the denominator of Eq. (1) are different [47–49], but this approach lacks physical insight (see the Supplemental Material [50]).

In this Rapid Communication we suggest an alternative approach which relates the experimental findings to a well-defined theoretical model. Instead of considering an effective energy broadening, we include superconducting fluctuations by postulating a bosonic field $\Delta(\mathbf{r},t)$ with a finite correlation length. By summing the contributions of short-range (SR) and long-range (LR) fluctuations, we obtain an excellent agreement with experimental curves on the insulating side of the SIT. Our results demonstrate that there are two important length scales: One is the effective size of a superconducting island ξ_{sc}, and the other is the typical size of quantum fluctuations ξ_{fluc}, which diverges at the SIT.

Superconducting fluctuations. We begin our analysis by introducing the Hamiltonian $H = H_0 + H_\Delta$, where $H_0 = \sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k}} c^\dagger_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma}$ describes free electrons (quasiparticles) with a Fermi surface at $\varepsilon_{\mathbf{k}} = 0$. Superconducting fluctuations are represented by a randomly fluctuating bosonic field $\Delta(\mathbf{r},t)$ coupled to the fermions by

$$H_\Delta = \Delta(\mathbf{r},t) c^\dagger_\uparrow(\mathbf{r},t) c^\dagger_\downarrow(\mathbf{r},t) + \mathrm{H.c.} \tag{2}$$

The effects of small fluctuations of Δ on the superconducting side of the transition were analyzed self-consistently in Refs. [51–53]. In this Rapid Communication, we instead focus on the insulating side of the transition where the superconducting order parameter averages to zero $\langle \Delta(\mathbf{r},t) \rangle = 0$. We model finite-range superconducting fluctuations by a free field with two-point correlations,

$$C(\mathbf{r} - \mathbf{r}', t - t') = \langle \Delta(\mathbf{r},t) \Delta^*(\mathbf{r}',t') \rangle. \tag{3}$$

The function C describes the decay of the superconducting correlations and tends to zero at long distances and long times. Our phenomenological model can be justified by the numerical solution of the attractive Hubbard model with on-site disorder [32,54,55]. These earlier studies showed that, on the insulating side of the transition, the interplay between disorder

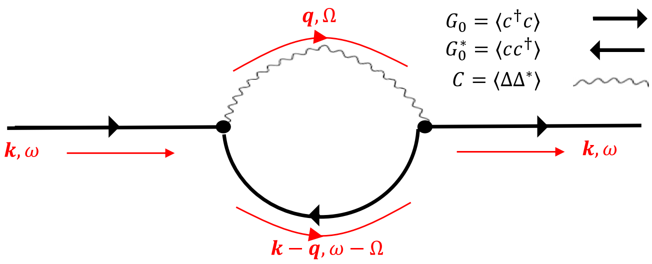

FIG. 1. One-loop diagram—second-order perturbation in $\Delta(\mathbf{r},t)$. The black thick arrows represent charge (i.e., the right arrow for a particle and the left arrow for a hole), whereas the thin red ones represent momentum and energy.

and interactions gives rise to a superconducting gap with short-range correlations.

We now derive the relation between $C(\mathbf{r}-\mathbf{r}',t-t')$ and tunneling measurements. The tunneling differential conductivity is proportional to [56]

$$\frac{dI}{dV}(V) \propto \int_{-\infty}^{\infty} d\omega\, \rho(V+\omega) f'(\omega). \tag{4}$$

Here V is the voltage bias, $f'(\omega) = df/d\omega$ is the derivative of the Fermi-Dirac distribution function, and $\rho(\omega)$ is the DOS of the sample. Equation (4) assumes a constant DOS of the tip and at $T=0$ simply reduces to $dI/dV \propto \rho(V)$. Within the Green's function formalism in Nambu space [57], $\rho(\omega) = -(1/\pi)\langle \mathrm{Im}\{\mathrm{Tr}[\sum_{\mathbf{k}} G^{\mathrm{ret}}(\mathbf{k},\omega)]\}\rangle$, where $G^{\mathrm{ret}}(\mathbf{k},\omega)$ is the retarded Green's function, Tr is the trace in Nambu space, Im is the imaginary part, and $\langle\cdots\rangle$ implies an average over the fluctuations of the superconducting field $\Delta(\mathbf{r},t)$.

Our first step involves a Dyson resummation of the one-loop contributions shown in Fig. 1, whose two vertices represent the coupling term (2). By performing a trace over particle and hole contributions, we find (see the Supplemental Material [58])

$$\mathrm{Tr}[G^{\mathrm{ret}}(\mathbf{k},\omega)] = \frac{\omega + i0^+}{(\omega+i0^+)^2 - \varepsilon_{\mathbf{k}}^2 - \mathcal{D}^2(\mathbf{k},\omega)}. \tag{5}$$

Here we defined the pairing-fluctuations' function $\mathcal{D}$ as

$$\mathcal{D}^2(\mathbf{k},\omega) = \int d^2\mathbf{q} \int d\Omega \frac{\omega + i0^+ + \varepsilon_{\mathbf{k}}}{i\Omega + \omega + i0^+ + \varepsilon_{\mathbf{k}-\mathbf{q}}} C(\mathbf{q},\Omega), \tag{6}$$

with $C(\mathbf{q},\Omega) = \int d^2\mathbf{r}\, dt\, C(\mathbf{r},t) e^{i\mathbf{q}\cdot\mathbf{r} - i\Omega t}$.

Because the Green's function in Eq. (5) is strongly peaked at $\mathbf{k} = \mathbf{k}_F$, we can approximate the density of states as

$$\rho(\omega) = \mathrm{Re}\left[\frac{\omega}{\sqrt{\omega^2 - \mathcal{D}^2(\omega)}}\right], \tag{7}$$

where we defined $\mathcal{D}(\omega) \equiv \mathcal{D}(\mathbf{k}_F,\omega)$.

Equation (7) is analogous to (1), but involves the frequency-dependent $\mathcal{D}(\omega)$ instead of the quasiparticles' lifetime Γ. Note that, if the correlation function $C(\mathbf{r},t)$ does not decay in space and time (i.e., the BCS limit), its Fourier transform is $C(\mathbf{q},\Omega) = \Delta_0^2 \delta(\mathbf{q})\delta(\Omega)$. In this case, Eq. (6) yields a frequency-independent $\mathcal{D}^2(\omega) = \Delta_0^2$, and one recovers the well-known result.

Our approach has some similarities to Refs. [36–38] where superconducting fluctuations with a finite lifetime were considered as well. These authors were interested in the thermal regime $T > T_c$ where superconducting fluctuations are weak and lead to small deviations of the density of states. As a consequence, their approximation scheme does not recover the diverging density of states predicted by BCS. The Dyson resummation employed in the present Rapid Communication allows us to consider strong superconducting fluctuations and get closer to the SIT. See also Ref. [59] for a microscopic model describing the effect of superconducting fluctuations close to the SIT and their effects on the DOS.

In what follows, for simplicity, we will generically assume that the correlations are time independent, i.e.,

$$C(\mathbf{q},\Omega) = C(\mathbf{q})\delta(\Omega). \tag{8}$$

This assumption is justified if the collective-mode velocity v is much smaller than the Fermi velocity v_F (see the Supplemental Material [60]). Assuming a quadratic dispersion relation $\varepsilon_{\mathbf{k}} = (k^2 - k_F^2)v_F/2k_F$ and assuming that $C(\mathbf{q})$ decays to zero at $q \sim k_F$, we obtain

$$\mathcal{D}^2(\omega) = \int d^2\mathbf{q} \frac{\omega}{\omega - v_F q_x} C(\mathbf{q}). \tag{9}$$

Equations (7) and (9) are the key theoretical results of our analysis, and we will now use them to model the density of states of disordered superconductors under various assumptions in the form of $C(\mathbf{q})$. In the following, the films are assumed to be thin enough such that both $\mathbf{k}$ and $\mathbf{q}$ can be treated as two dimensional.

Short range vs long range. In order to understand the effects of superconducting fluctuations on the density of states, we consider correlation functions decaying over a typical inverse length scale q_0. This quantity can be associated with the average size of superconducting islands in the granular materials and with an emergent electronic granularity of amorphous materials [31–35]. In the specific case of $C(\mathbf{q}) = \frac{\Delta_0^2}{\pi^{3/2} v_F q_0^2} \exp(-q^2/q_0^2)$, we can analytically solve the integral in Eq. (9) to find

$$\mathcal{D}^2(\omega) = \Delta_0^2 \frac{1}{v_F q_0} \exp\left(-\frac{\omega^2}{v_F^2 q_0^2}\right)\omega\left[\mathrm{erfi}\left(\frac{\omega}{v_F q_0}\right) - i\right]. \tag{10}$$

Here erfi is the imaginary error function, which is a real function. The real and imaginary parts of Eq. (10) are shown in the upper panels of Figs. 2(a)–2(c) and the corresponding DOS in the lower panels. Note that the real part of $\mathcal{D}^2(\omega)$ closely resembles the local density of states $\rho(\omega)$, but these two quantities have a different physical meaning: The former actually needs to be substituted in Eq. (7) to deliver the latter. We observe that both $\mathrm{Re}[\mathcal{D}^2]$ and $\mathrm{Im}[\mathcal{D}^2]$ are peaked at a typical energy scale $v_F q_0$. The effect of superconducting fluctuations on the DOS changes dramatically, depending on the ratio between $v_F q_0$ and Δ_0.

Let us consider two extreme cases, which we denote by long range (LR) and short range (SR), respectively. The former occurs for $v_F q_0 \sim v_F \xi_{\mathrm{fluc}}^{-1} \ll \Delta_0$. In this case, $\mathcal{D}^2(\omega)$ is approximately constant, and we recover the BCS limit [see Fig. 2(a)]. On the other hand, when $v_F q_0 \sim v_F \xi_{\mathrm{sc}}^{-1} \gg \Delta_0$, the

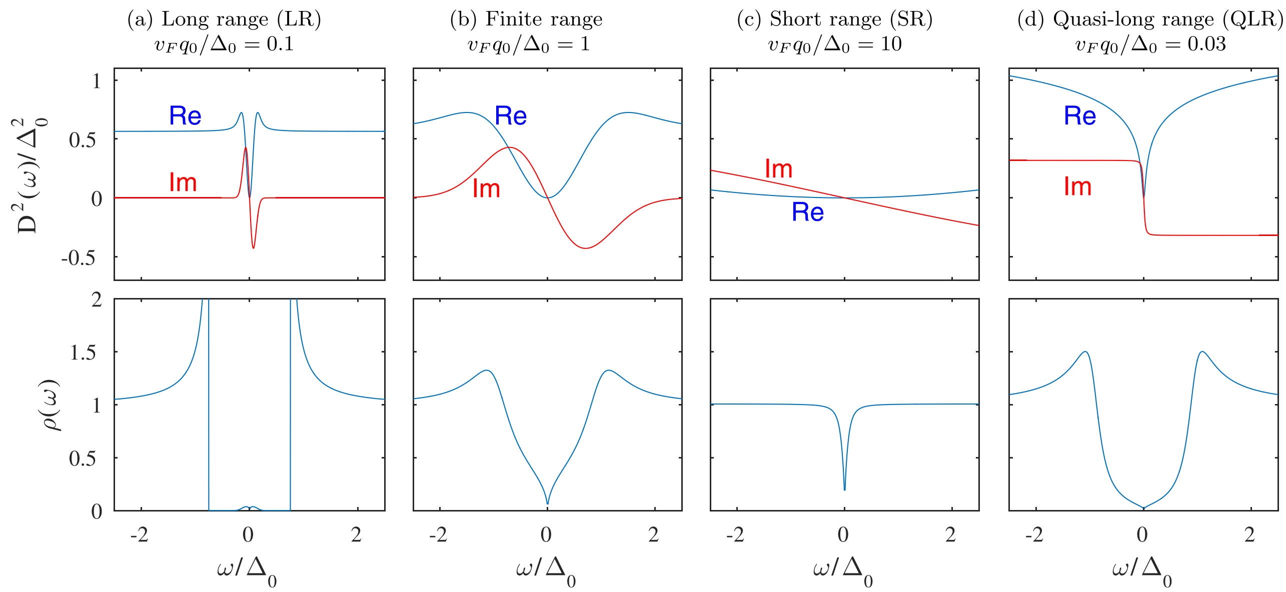

FIG. 2. Upper panel: (a)–(c) Real and imaginary parts of Eq. (10) for different values of $v_F q_0/\Delta_0$. (d) Real and imaginary parts of Eq. (12). Lower panel: the corresponding density of states, Eq. (7).

fluctuations are short ranged. In this regime,

$$\mathcal{D}^2(\omega) \approx -i\omega\gamma \tag{11}$$

is purely imaginary, and the DOS shows a deep without coherence peaks [see Fig. 2(c)]. The distinction between LR and SR fluctuations does not depend on the specific choice of $C(\mathbf{q})$ and can be related to the Anderson limit of superconductivity [55,61]. The crossover between these two regimes occurs when Δ_0 is on the order of the typical energy level spacing of a superconducting island of size $1/q_0$ (the superconducting correlation length), i.e., $\Delta_0 \sim v_F q_0$. As q_0 increases, the coherence peaks become less pronounced, and their positions move to higher energies [62,63] (see the Supplemental Material [64]).

In the vicinity of the SIT, superconducting fluctuations are described by a universal critical theory [65], which in its simplest form is given by $C(\mathbf{q}) = \frac{\Delta_0^2}{\pi^3}\frac{1}{q^2+q_0^2}$, where $q_0 = 1/\xi_{\rm fluc}$ tends to zero at the transition. In this case, which we denote as QLR, we can again solve analytically Eq. (9) to obtain

$$\mathcal{D}^2(\omega) = \frac{\Delta_0^2}{\pi^2}\frac{\omega}{\sqrt{v_F^2q_0^2+\omega^2}}\left[\ln\left(\frac{\sqrt{v_F^2q_0^2+\omega^2}+\omega}{\sqrt{v_F^2q_0^2+\omega^2}-\omega}\right) - i\pi\right]. \tag{12}$$

As shown in Fig. 2(d), for $q_0 \to 0$, the real part of Eq. (12) diverges logarithmically, whereas the imaginary part is proportional to $\mathrm{sgn}(\omega)$. The resulting DOS resembles the LR situation and is very weakly dependent on the infrared cutoff q_0. As we will see below, QLR superconducting fluctuations give a better description of the experiment than true LR correlations.

In actual materials, one generically expects to find a combination of superconducting fluctuations with long-range and short-range correlations. The former are universal and determine the emergent properties of the material, whereas the latter depend on the microscopical details and are often neglected. In contrast to this common practice, we find that short-range correlations strongly affect the density of states (see Fig. 3): Although the correlation functions denoted by LR and SR + LR have the same asymptotic behavior [subplot (a)], the corresponding DOS are very different [subplot (b)].

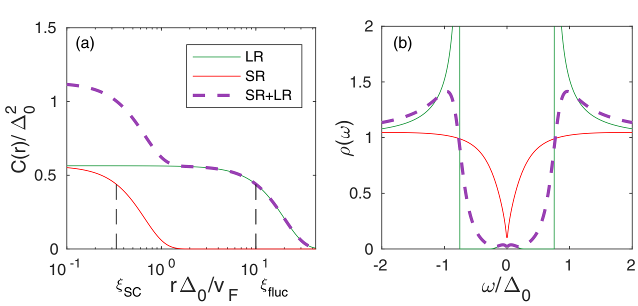

FIG. 3. (a) Normalized spatial correlations of the superconducting fluctuations $C(r)$. (b) Density of states $\rho(\omega)$ for different types of superconducting correlations. Note that adding SR fluctuations ($v_F q_0/\Delta_0 = 3$) suppresses the peaks even in the presence of LR superconducting correlations ($v_F q_0/\Delta_0 = 0.1$).

TABLE I. Comparison between different correlation functions, i.e., Dynes, SR + LR, and SR + QLR.

	DOS	Best fit (meV)	χ^2
Dynes	Equation (1)	$\Delta_0 = 0.73$ $\Gamma = 0.16$	0.046
SR + LR	Equation (7) with $\mathcal{D}^2(\omega) =$ Eqs. (11) + (10)	$\Delta_0 = 0.67$ $\gamma = 0.22$ $v_F q_0 = 0.22$	0.022
SR + QLR	Equation (7) with $\mathcal{D}^2(\omega) =$ Eqs. (11) + (12)	$\Delta_0 = 0.69$ $\gamma = 0.11$ $v_F q_0 = 0.022$	0.011

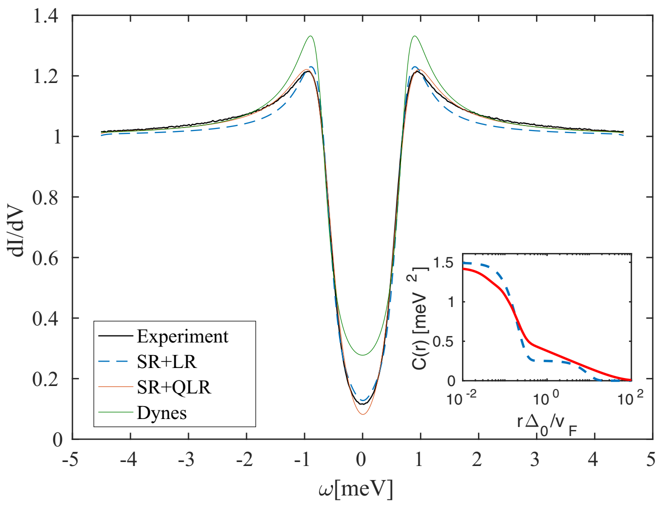

FIG. 4. Comparison between theoretical predictions and actual measurements performed on an insulating thin film close to the SIT. The best theoretical fits for each curve are presented in Table I. The best fit to the experiment is given by SR + QLR. The inset shows the corresponding superconducting correlation functions ($\xi_{SC} = \frac{1}{3}q_0$).

Comparison with experiments. We now compare our theoretical calculations with the tunneling measurement of Ref. [43], performed on an InO film at $T = 1$ K on the insulating side of the SIT. Note that, in order to isolate the superconducting contribution to the DOS, the experimental raw data were normalized by the tunneling spectra at a high magnetic field. The results of our analysis are summarized in Table I and Fig. 4 where we show the best-fitting parameters and the minimal normalized χ^2 distribution between theory and experiment. We find that the sum of short-range and long-range superconducting fluctuations is required to obtain a good fit. Furthermore, a detailed analysis reveals that the long-range part is best described by Eq. (12) ($\chi^2 = 0.011$) rather than Eq. (10) ($\chi^2 = 0.022$), in agreement with the expected critical behavior of the SIT [65].

Discussion. In this Rapid Communication we studied the effects of superconducting fluctuations on the tunneling conductivity of disordered thin films, focusing on the insulating side of the SIT. The common approach, known as the Dynes formula (1), relies on a phenomenological parameter Γ that describes the inverse lifetime of the quasiparticles. In this Rapid Communication, we showed that the experiments are better fit by a theory of free electrons, coupled to superconducting fluctuations with finite-range correlations. By using a controlled diagrammatic approach, we derived a simple expression that connects the correlations of the superconducting fluctuations to the tunneling spectra Eqs. (6) and (7). This result has potential applications that go beyond the present Rapid Communication, including quantum as well as classical superconducting phase transitions. Our analytical results show that, generically, short-range fluctuations lead to tunneling spectra with reduced or absent coherence peaks even in the presence of long-range superconducting correlations.

By comparing our analytic expressions to experimental measurements, we find that, in disordered thin films, the superconducting fluctuations are given by the sum of two components. The long-range component is associated with universal fluctuations close to the SIT quantum critical point, characterized by a diverging length scale ξ_{fluc}. Accordingly, the experimental data are best fit by a critical theory with $q_0 \sim 1/\xi_{\text{fluc}} \ll k_F$ (see the last row of Table I, taking into account that $v_F k_F \sim 1$ eV). In contrast, the short-range component is determined by the microscopic details of the material. Specifically, short-range correlations are expected to play a predominant role in amorphous materials where Cooper pairs are localized by disorder. In granular materials, on the other hand, the superconducting correlations decay over a much longer range, set by the typical scale of the grains. This distinction can explain why the Dynes formula fits well experiments on granular Pb films [9] but does not fit amorphous InO films [43]. The distinction between short-range and long-range fluctuations can bridge the long-standing controversy between the fermionic and the bosonic approach to the SIT [55]. On a broader prospective, our approach contributes to the understanding of puzzling spectrometric experiments in unconventional superconductors. Specifically, we find that, although SR fluctuations contribute to the local superconducting gap, they generically lead to a tunneling spectra with suppressed coherence peaks in analogy to the experimental observations in the pseudogap regime of underdoped cuprates (see, for example, Refs. [47,48,66,67]).

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