Rapid Communications

Analytic solution of the domain-wall nonequilibrium stationary state

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We consider the out-of-equilibrium dynamics generated by joining two domains with arbitrary opposite magnetizations. We study the stationary state which emerges by the unitary evolution via the spin-1/2 XXZ Hamiltonian, in the gapless regime, where the system develops a stationary spin current. Using the generalized hydrodynamic approach, we present a simple formula for the space-time profile of the spin current and the magnetization exact in the limit of long times. As a remarkable effect, we show that the stationary state has a strongly discontinuous dependence on the strength of interaction as confirmed by the exact analytic expression of the Drude weight that we compute. These features allow us to give a qualitative estimation for the transient behavior of the current which is in good agreement with numerical simulations. Moreover, we analyze the behavior around the edge of the magnetization profile, and we argue that, unlike the XX free-fermionic point, interactions always prevent the emergence of a Tracy-Widom scaling.

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Introduction. Recent experimental developments with cold atoms [1] have given a new perspective to the study of nonequilibrium transport under coherent evolution. As an example, the measurement of conductances well beyond the regime of linear response has provided clear examples of the thermoelectric effect [2,3]. The simplest protocol to induce an out-of-equilibrium behavior is the one of the quantum quenches in which the system is prepared in an equilibrium state of the initial Hamiltonian $\boldsymbol{H}_0$, which is suddenly switched to $\boldsymbol{H}$, thus inducing a nontrivial time evolution [4–7]. Then, in describing the long-time dynamics, a fundamental role is played by the conserved quantities of $\boldsymbol{H}$, i.e., the set of local (or quasilocal [8]) operators $\{\boldsymbol{Q}_k\}$ satisfying $[\boldsymbol{Q}_k,\boldsymbol{H}]=0$. As the system is isolated, the expectation value of these conserved quantities remains constant during the evolution. For homogeneous systems, these conditions are sufficient to predict the exact behavior of any local observable at long times: This is based on assuming equilibration to the *generalized Gibbs ensemble* (GGE), which results from the maximization of entropy given the constraints imposed by conserved quantities [9]. This principle suggests a dichotomy between generic models for which a finite number of conserved quantities exist (i.e., the Hamiltonian and a few others) and integrable ones, which instead present an infinite number of them [10]. Nowadays, the GGE scheme has been validated by several experiments [11–18] and theoretical works, employing free theories [19–21], integrability [22–27] and numerical methods [9,28–30].

However, the study of transport requires considering more generic situations where, for instance, an initial spatial inhomogeneity is used to induce particle or energy flow. The simplest examples are *local quenches* where $\boldsymbol{H}_0$ and $\boldsymbol{H}$ differ only in a finite region of space due, for instance, to the presence of a localized defect [31–37]. In particular, in the *partitioned protocol*, the initial density matrix is factorized into two halves, i.e., $\rho_0=\rho_L\otimes\rho_R$, which are suddenly connected, inducing an out-of-equilibrium dynamics around the junction [38,39]. This problem was well understood in noninteracting theories [40–51] (with even mathematically rigorous treatments [52,53]) and field theories [38,39,54–63] even in higher dimensions [64–66]. However, for interacting models, only numerical approaches [67–75] and approximate results were available [76–80]. Whereas at extremely long-times $v_0t\gg L$ (with L as the system length and v_0 as the maximal velocity [81–83]), one expects the system to become homogeneous, the most interesting regime is the one where $a\ll v_0t\ll L$ (with a as the typical microscopic length). In this regime, conserved quantities are dynamically exchanged between different portions of space, and therefore the simple knowledge of their initial values is not enough to characterize the local behavior of the steady state. Nevertheless, conserved quantities must still satisfy a continuity equation $\partial_t\boldsymbol{q}_k(x,t)+\partial_t\boldsymbol{j}_k(x,t)=0$ with $\boldsymbol{q}_k(x,t)$ as the local density of $\boldsymbol{Q}_k$ and $\boldsymbol{j}_k(x,t)$ as the corresponding current. This condition was recently employed [84,85] to derive a *generalized hydrodynamic description* (GHD) applicable to a large class of one-dimensional integrable models [86–96]. For the partitioned protocol, this description becomes exact, and at long times, a *local quasistationary state* (LQSS) emerges in which local observables only depend on the scaling variable $\zeta=x/t$.

In this Rapid Communication, we consider the XXZ spin 1/2 in the gapless regime, prepared in the partitioned initial state composed by two domains of arbitrary opposite magnetizations. We solve the hydrodynamic equations, obtaining simple analytic expressions for the magnetization and spin current profiles. This represents an example of an out-of-equilibrium steady state of an interacting quantum system, admitting an explicit exact solution. Remarkably, the stationary spin current exhibits a strongly discontinuous behavior as a function of the anisotropy as a result of the peculiar structure of bound states in the model. In Ref. [42], the same setting has been considered for the XX model [10]: The density profile around

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the fastest particles shows a remarkable correspondence with the distribution of the largest eigenvalue of a random Gaussian matrix, the Tracy-Widom distribution [97]. It was an open question whether this behavior was universal and could survive in the presence of interactions. Here, we provide a definitive and negative answer to this question. Note that, even though the numerical solution of the GHD equation for this setting was already possible [85], our conclusion is fully grounded on the analytic expressions that we derive.

The model. We consider the XXZ Hamiltonian,

$$\boldsymbol{H} = \sum_{i=-(L/2)}^{(L/2)-1} \left[\boldsymbol{s}_i^x \boldsymbol{s}_{i+1}^x + \boldsymbol{s}_i^y \boldsymbol{s}_{i+1}^y + \Delta\left(\boldsymbol{s}_i^z \boldsymbol{s}_{i+1}^z - \frac{1}{4}\right)\right], \quad (1)$$

where L is the length of the chain and $\boldsymbol{s}_i^\alpha$'s are spin-1/2 operators each acting on the local Hilbert space at site i. We focus on the gapless phase, thus specializing $\Delta = \cos(\gamma)$ with $\gamma = \pi\, Q/P$, where Q and P are two coprime integers with $1 \leqslant Q < P$. The ratio Q/P admits a finite continued fraction representation $Q/P = [0; \nu_1, \nu_2, \dots, \nu_\delta]$ with length δ. For any finite L such a model is exactly solvable via the Bethe-ansatz method [98–100]. In the thermodynamic limit (i.e., when $L \to \infty$ with fixed particle density), a generic thermodynamic state can be fully characterized by a set of functions $\{\rho_j(\lambda), \rho_j^h(\lambda)\}$ with $j \in \{1, \dots, \ell\}$, $\ell = \sum_{j=1}^{\delta} \nu_j$ and $\lambda \in [-\infty, \infty]$. These functions, also known as "root densities," describe different species of quasiparticles (different "strings") and are solutions of a system of coupled nonlinear integral equations [98–100]. We can associate with each string j a given *parity* $\upsilon_j \in \{-1,1\}$, *length* $n_j \in \{1, \dots, P-1\}$, and *sign* $\sigma_j \in \{-1,1\}$. The *filling factors* are introduced as the ratios $\vartheta_j(\lambda) \equiv \rho_j(\lambda)/[\rho_j(\lambda) + \rho_j^h(\lambda)]$; we refer the reader to the Supplemental Material [101] for further details.

The local quasistationary state redux. Starting from a partitioned initial-state $\varrho_0 = \varrho_L \otimes \varrho_R$ and unitarily evolving this state under the Hamiltonian (1), the general formal solution of the LQSS reads [85]

$$\vartheta_{j,\zeta}(\lambda) = \vartheta_j^L(\lambda)\Theta[v_{j,\zeta}(\lambda) - \zeta] + \vartheta_j^R(\lambda)\Theta[\zeta - v_{j,\zeta}(\lambda)], \quad (2)$$

in terms of the scaling variable $\zeta = x/t$ with $\Theta(z)$ being the Heaviside step function.

The functions $\vartheta_j^{L/R}(\lambda)$ are the filling factors which describe the homogeneous stationary state emerging on the very far left/right part of the system. Equation (2) formally identifies, for each type of quasiparticle the related stationary distribution function. This solution admits a very simple geometrical interpretation: for any value of j and λ, starting from $\zeta = -\infty$, the left bulk stationary description extends up to $\zeta_j^*(\lambda)$ such that $v_{j,\zeta^*}(\lambda) = \zeta^*$, thereafter, $\vartheta_{j,\zeta}(\lambda)$ suddenly jumps to the right bulk stationary description. In practice this formal solution explicitly depends on the dressed velocity $v_{j,\zeta}(\lambda) = e'_{j,\zeta}(\lambda)/p'_{j,\zeta}(\lambda)$, where $e'_{j,\zeta}(\lambda)$ and $p'_{j,\zeta}(\lambda)$, respectively, are the dressed energy and momentum derivative. For a generic thermodynamic state described by a set of filling factors $\{\vartheta_j(\lambda)\}$ and a generic conserved charge $\boldsymbol{Q}$ with single-particle eigenvalues $\mathfrak{q}_j(\lambda)$, the dressing is obtained solving

$$q'_j(\lambda) = \mathfrak{q}'_j(\lambda) - \sum_k \int d\mu\, T_{j,k}(\lambda - \mu)\sigma_k \vartheta_k(\mu) q'_k(\mu), \quad (3)$$

where we chose the convention to use calligraphic notation for bare quantities $\mathfrak{q}_j(\lambda)$. Introducing the function,

$$a_n^{(\upsilon)}(\lambda) = \frac{\upsilon}{\pi} \frac{\sin(\gamma n)}{\cosh(2\lambda) - \upsilon\, \cos(\gamma n)}, \quad (4)$$

the kernel $T_{j,k}(\lambda)$ assumes the form

$$T_{j,k}(\lambda) = \left(1 - \delta_{n_j,n_k}\right) a_{|n_j - n_k|}^{(\upsilon_j \upsilon_k)}(\lambda) + 2a_{|n_j - n_k|+2}^{(\upsilon_j \upsilon_k)}(\lambda) + \dots + 2a_{n_j + n_k - 2}^{(\upsilon_j \upsilon_k)}(\lambda) + a_{n_j+n_k}^{(\upsilon_j \upsilon_k)}(\lambda), \quad (5)$$

whereas the bare eigenvalues for the energy and the momentum derivative are

$$\mathfrak{e}_j(\lambda) = -\pi\, \sin(\gamma) a_j(\lambda), \quad \mathfrak{p}'_j(\lambda) = 2\pi a_j(\lambda), \quad (6)$$

where we defined $a_j(\lambda) \equiv a_{n_j}^{(\upsilon_j)}(\lambda)$. Note that, as the dressing operation (3) is performed over the state $\vartheta_j(\lambda) = \vartheta_{j,\zeta}(\lambda)$, the solution for the LQSS has to be found self-consistently in such a way that it keeps the form in Eq. (2) with its own dressed velocity $v_{j,\zeta}(\lambda)$. Therefore, in general, the dressed velocity will depend on the scaling variable ζ via the state $\vartheta_{j,\zeta}(\lambda)$.

From the thermodynamic Bethe ansatz (TBA) description of the LQSS, we can easily evaluate the expectation value of a generic charge-density $\boldsymbol{q} = \boldsymbol{Q}/L$ and its current,

$$\langle \boldsymbol{q} \rangle_\zeta = \sum_k \int \frac{d\lambda}{2\pi} \mathfrak{q}_k(\lambda) \sigma_k p'_{k,\zeta}(\lambda) \vartheta_{k,\zeta}(\lambda), \quad (7)$$

$$\langle \boldsymbol{j}_{\boldsymbol{q}} \rangle_\zeta = \sum_k \int \frac{d\lambda}{2\pi} \mathfrak{q}_k(\lambda) v_{k,\zeta}(\lambda) \sigma_k p'_{k,\zeta}(\lambda) \vartheta_{k,\zeta}(\lambda). \quad (8)$$

Opposite magnetization domains. The system is initially prepared into two halves with infinite temperatures and opposite values of magnetic-field h in the $\hat{z}$ direction, namely,

$$\varrho_0 \equiv \varrho_L(h) \otimes \varrho_R(-h) = \frac{e^{2h S_L^z}}{Z_L} \otimes \frac{e^{-2h S_R^z}}{Z_R}, \quad (9)$$

where $\boldsymbol{S}_{L/R}^z = \sum_{i \in L/R} \boldsymbol{s}_i^z$ is the $\hat{z}$ component of the total spin in the left/right part of the system.

A generic thermodynamic state $\varrho_{L/R}(h)$ is stationary under the unitary evolution induced by its own XXZ Hamiltonian. It admits a TBA description in terms of constant filling factors $\vartheta_j^{(h)}$ (i.e., independent of the rapidity λ), which satisfy the major properties (see Ref. [101] for the complete definition of $\vartheta_j^{(h)}$, $\forall\, j \in \{1, \dots, \ell\}$),

$$\vartheta_j^{(h)} = \vartheta_j^{(-h)} \quad j < \ell - 1, \quad \vartheta_{\ell-1}^{(h)} = 1 - \vartheta_\ell^{(-h)}. \quad (10)$$

In the limit $h \to \infty$ the state ϱ_0 reduces to the domain-wall (DW) $|\Uparrow\rangle \otimes |\Downarrow\rangle$ product state with $\vartheta_j^{|\Uparrow\rangle} = 0$ and $\vartheta_j^{|\Downarrow\rangle} = \delta_{j,\ell} + \delta_{j,\ell-1}$ for $j = 1, \dots, \ell$.

The full analytic solution. Now if we consider the protocol generated attaching two states with h (left) and $-h$ (right), the $\zeta \to \pm\infty$ boundary conditions in Eq. (2) read $\vartheta_j^L(\lambda) = \vartheta_j^{(h)}$, $\vartheta_j^R(\lambda) = \vartheta_j^{(-h)}$. Thanks to the symmetries (10) of the boundary filling factors, when constructing the LQSS, only the filling factors $\vartheta_{j,\zeta}(\lambda)$ corresponding to the last two strings $j = \ell - 1$ and $j = \ell$ may depend on ζ. In order to fix them, we need to determine the dressed velocities. Using that $T_{\ell,k}(\lambda) = -T_{\ell-1,k}(\lambda)$ and $a_\ell(\lambda) = -a_{\ell-1}(\lambda)$, we have

$p'_{\ell,\zeta}(\lambda) = -p'_{\ell-1,\zeta}(\lambda)$ implying $v_{\ell,\zeta}(\lambda) = v_{\ell-1,\zeta}(\lambda)$. As the last two strings always have opposite *signs*, i.e., $\sigma_{\ell-1} = -\sigma_\ell$, we can reduce Eq. (3) for the dressed momentum derivative to

$$p'_{j,\zeta}(\lambda) = \mathfrak{p}'_j(\lambda) - \sum_{k\leqslant \ell-2} \sigma_k \vartheta_k^{(h)} \int d\mu\, T_{j,k}(\lambda-\mu) p'_{k,\zeta}(\mu)$$
$$- \sigma_\ell \int d\mu [\vartheta_{\ell,\zeta}(\mu) - \vartheta_{\ell-1,\zeta}(\mu)] T_{j,\ell}(\lambda-\mu) p'_{\ell,\zeta}(\mu),$$

which does not depend on the space-time scaling variable ζ since $\vartheta_{\ell,\zeta}(\mu) - \vartheta_{\ell-1,\zeta}(\mu) = \vartheta_\ell^{(h)} - \vartheta_{\ell-1}^{(h)}$. From now on we discard the subscript ζ whenever it will be superfluous. As a consequence of the last result, we can calculate the dressed momentum derivative solving

$$p'_j(\lambda) = \mathfrak{p}'_j(\lambda) - \sum_k \sigma_k \vartheta_k^{(h)} \int d\mu\, T_{j,k}(\lambda-\mu) p'_k(\mu), \quad (11)$$

which correspond to evaluate the dressing on the left thermodynamic state $\varrho_L(h)$. Note that the dressing can be equivalently evaluated in the right part of the system as it is even in sign of the magnetic field. Equation (11) can be solved in a Fourier transform, reducing to an algebraic system of linear equations. For the last two strings the dressing operation reduces to a simple rescaling of the bare quantities, i.e.,

$$p'_\ell(\lambda) = \mathcal{R}(h)\mathfrak{p}'_\ell(\lambda), \quad p'_{\ell-1}(\lambda) = \mathcal{R}(h)\mathfrak{p}'_{\ell-1}(\lambda), \quad (12)$$

with the following rescaling factor:

$$\mathcal{R}(h) \equiv \frac{\tanh(h)}{2} \frac{\sinh[(n_\ell + n_{\ell-1})h]}{\sinh(n_\ell h)\sinh(n_{\ell-1}h)}, \quad (13)$$

where as expected $\mathcal{R}(-h) = \mathcal{R}(h)$. As a consequence, the quasiparticle velocity of the last two strings is not changed by the dressing operation. It can therefore be expressed in terms of the undressed momentum as follows:

$$\mathfrak{v}_\ell = \frac{v_\ell \sin(\gamma)}{\sin(n_\ell \gamma)} \sin(\mathfrak{p}_\ell) = \zeta_0 \sin(\sigma_\ell \mathfrak{p}_\ell), \quad (14)$$

with $\zeta_0 \equiv \sin(\gamma)/\sin(\pi/P)$ and $\sigma_\ell \mathfrak{p}_\ell(\lambda)$ as a strictly increasing function in $[-\pi/P,\pi/P]$. Therefore, the velocity $\mathfrak{v}_\ell(\lambda) \in [-\sin(\gamma), \sin(\gamma)]$. The explicit form of the LQSS for the last two strings thus reads (for $j \in \{\ell-1,\ell\}$)

$$\vartheta_{j,\zeta}(\lambda) = \vartheta_j^{(h)} \Theta(\sigma_j \mathfrak{p}_j - \mathfrak{p}^*_\zeta) + \vartheta_j^{(-h)} \Theta(\mathfrak{p}^*_\zeta - \sigma_j \mathfrak{p}_j), \quad (15)$$

where $\mathfrak{p}^*_\zeta \equiv \arcsin[\zeta/\zeta_0]$. From this, using $\mathrm{Tr}[s^z \varrho_L(h)] = \tanh(h)/2$ and $\mathcal{R}(h) = \tanh(h)/(1 - \vartheta_\ell^{(h)} - \vartheta_{\ell-1}^{(h)})$, we can easily evaluate the magnetization and spin current profile inside the light-cone $\zeta \in [-\sin(\gamma), \sin(\gamma)]$,

$$\langle s^z \rangle_\zeta = -\frac{\tanh(h)}{2\pi/P} \arcsin\left(\frac{\zeta}{\zeta_0}\right), \quad (16a)$$

$$\langle \boldsymbol{j}_{s^z} \rangle_\zeta = \frac{\tanh(h)}{2\pi/P} \zeta_0 \left[\sqrt{1 - \frac{\zeta^2}{\zeta_0^2}} - \cos\left(\frac{\pi}{P}\right)\right], \quad (16b)$$

which are simply related to one another via the continuity equation $\zeta\, \partial_\zeta \langle s^z \rangle_\zeta = \partial_\zeta \langle \boldsymbol{j}_{s^z} \rangle_\zeta$. Interestingly, the way in which the magnetic-field h enters in the stationary solutions is almost trivial: Indeed, Eqs. (16) coincide with the DW solutions ($h \to \infty$) simply rescaled by the factor of $\tanh(h)$. Moreover

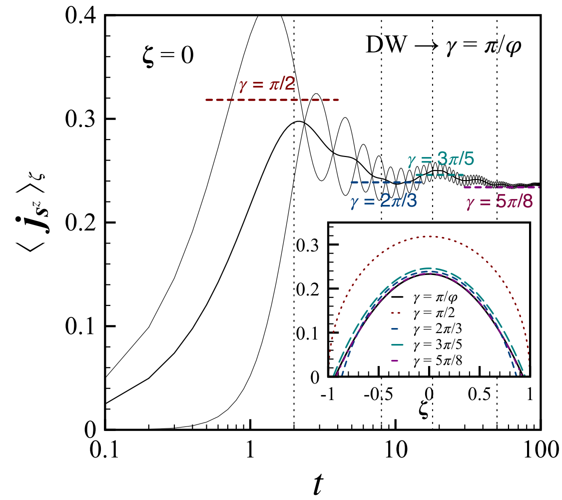

FIG. 1. (Main) Spin current at the junction, i.e., $\langle \boldsymbol{j}_{s^z} \rangle_{\zeta=0}$ for a quench from the DW with $\gamma = \pi/\varphi$. The time-dependent density-matrix renormalization-group (DMRG) data for the spin current between lattice sites (0,1) and (1,2) (the thin black lines) and their average (the thick black line) are compared with the stationary values associated with the rational approximation of the golden ratio (the horizontal dashed lines). The dotted vertical lines represent the typical time scale at which the current passes from one rational approximation to the next one. (The inset) The analytic stationary profile for the golden ratio is compared with different rational approximations.

in this limit $\mathcal{R}(h) \to 1$, showing that for the DW initial state, no dressing occurs.

The anisotropy dependence. It is interesting to investigate how the interaction strength Δ affects the stationary state. Both current and magnetization profiles have an explicit dependence on the denominator P of γ/π: As one can pick two arbitrarily close values of $\gamma = \pi Q/P$ and $\tilde{\gamma} = \pi \tilde{Q}/\tilde{P}$ with very different values of P and $\tilde{P}$, the magnetization and current profiles exhibit jumps in correspondence of any rational γ/π, corresponding to a dense subset of $\Delta \in [-1,1]$. Nevertheless, the continuation to irrational values is well defined taking $P \to \infty$ with γ finite. In such a limit, the current profile reduces to (for $\gamma/\pi \in \mathbb{R}/\mathbb{Q}$)

$$\langle \boldsymbol{j}_{s^z} \rangle_\zeta^{(\mathbb{R}/\mathbb{Q})} = \frac{\tanh(h)}{4}\left[\sin(\gamma) - \frac{\zeta^2}{\sin(\gamma)}\right], \quad (17)$$

and the magnetization behaves linearly in ζ. For any irrational number γ/π, although the large time limit will be characterized by the stationary values in (17), we expect the relaxation dynamics to spend long times on the rational approximations of such an irrational value, i.e., the truncated continued fractions $[0; \nu_1, \ldots, \nu_n]$. The ideal case to verify this hypothesis corresponds to all $\nu_k = 1$'s, i.e., $\gamma = \pi/\varphi$ with $\varphi \equiv (1+\sqrt{5})/2$, the *golden ratio*. Its nth order rational approximation is given by $\varphi = \lim_{n\to\infty} \varphi_n$ with $\varphi_n \equiv F_{n+1}/F_n$ and F_n as the Fibonacci sequence. In Fig. 1, the numerical data for the spin current exhibit plateaus corresponding to subsequent approximations of φ. The time δt spent in each plateau can be estimated by a simple dimensional argument, i.e., $\delta t \simeq (\Delta_{n+1} - \Delta_n)^{-1} \simeq \varphi^{2n}$, with $\Delta_n = \cos \pi/\varphi_n$.

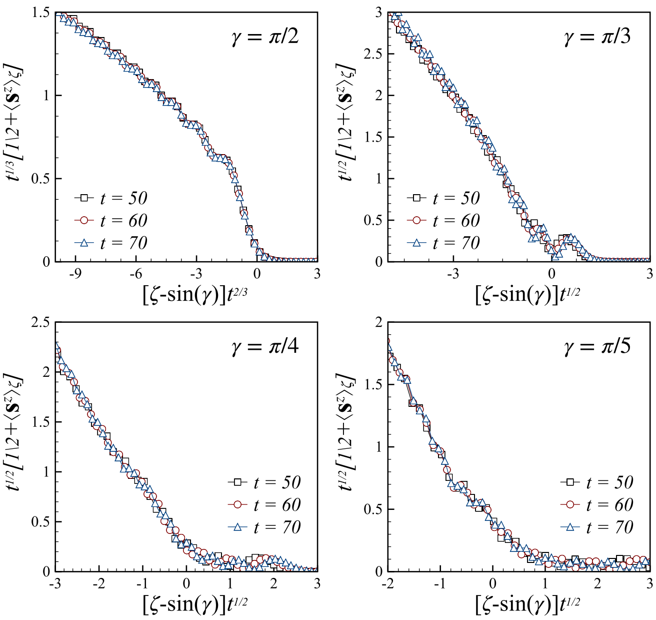

FIG. 2. Scaling of the particle density profile at the edge of the light cone for different values of the interactions and DW initial condition. As expected, in the free-fermion case the scaling of the data is governed by the Airy kernel [97]. However, when interactions are turned on, the behavior becomes purely diffusive preventing a Tracy-Widom-like scaling.

Remarkably, our exact result definitively gives analytical confirmation to the tightness of the bound in Ref. [102] for the spin Drude weight $\mathcal{D}_{s^z}$, numerically corroborated in Refs. [92,93]. In the linear-response regime, indeed, the spin Drude weight gives the magnitude of the singular part of the spin conductivity, therefore signaling ballistic transport [103–108]. Integrating the current (16b) over ζ, for $\beta \to 0$ we obtain [92,93]

$$(16/\beta)\mathcal{D}_{s^z} = \zeta_0^2\left[1 - \frac{\sin(2\pi/P)}{2\pi/P}\right]. \tag{18}$$

This result exactly coincides with the lower bound obtained in Ref. [102], proving that it is in fact saturated.

Absence of a Tracy-Widom distribution. The profiles in (16) exhibit a smooth dependence on the scaling variable ζ apart from the edges of the light cone, i.e., $\zeta = \pm\sin(\gamma)$ where the derivatives are nonanalytic. In particular, one has

$$\frac{\partial_\zeta \langle \boldsymbol{j}_{s^z}\rangle_{\zeta=\sin(\gamma)}}{\tanh(h)} = -\frac{\tan(\pi/P)}{2\pi/P}, \tag{19}$$

which remains finite for any value of γ but $\pi/2$, i.e., the free-fermion point, where it diverges indicating a square-root singularity. The absence of such a singularity for $|\zeta| = \sin(\gamma)$ is a proof that the edges of the front cannot be described by a Tracy-Widom scaling [42,97] as soon as $\Delta \neq 0$ and the model is interacting. Indeed, if the magnetization near the edge followed from an Airy process [109], then it necessarily would have a square-root singularity for $|\zeta| = \sin(\gamma)$. We investigated numerically the scaling of the magnetization profile at the light-cone boundaries, analyzing $\langle \boldsymbol{\rho}\rangle_\zeta \equiv 1/2 + \langle \boldsymbol{s}^z\rangle_\zeta$. Figure 2 indicates that $t^\alpha \langle \boldsymbol{\rho}\rangle_\zeta$ converges to a nontrivial scaling function of the variable $X \equiv \frac{x \pm t\,\sin(\gamma)}{t^\alpha}$ with $\alpha = 1/3$ only at $\Delta = 0$ and $\alpha = 1/2$ otherwise in the whole gapless regime (see Ref. [101] for additional checks).

Beyond generalized hydrodynamics and diffusion. An exact analysis of the edge scaling goes beyond the limits of the GHD [110]. However, due to the absence of dressing for the DW initial conditions, we can construct a free-fermion model that reproduces Eqs. (16), see Ref. [101]. We conjecture that this free-fermion model is also able to capture the edge behavior. For simplicity, we take $Q = 1$, yielding $0 < \gamma \leqslant \pi/2$; then the magnetization profile (16a) is obtained from the fermion density of a model where particles have the dispersion relation $\varepsilon(p) = -\cos(p)$. The initial state of the fermions is factorized in real space. At $t = 0$, on the semi-infinite line $x < 0$ all the single-particle energy levels with momenta $|p| \leqslant \pi$ but $|p| \notin [\gamma, \pi - \gamma]$ are occupied, there are instead no particles on the right half of the chain. The fermion density obtained from such an initial state in the limit $x \to \infty$ at fixed $\zeta = x/t$ coincides with $1/2 + \langle s^z\rangle_\zeta$ and the magnetization profile as in (16a).

The peculiarity of the initial state is crucial because, if $\gamma \neq \pi/2$, no particles travel at the maximal velocity. In this case, an asymptotic analysis of the fermionic model confirms that $\alpha = 1/2$ is the correct scaling exponent and gives a prediction for the scaling function at the edge [101,111]. Finally, we observe that, within this picture, in the isotropic limit $\gamma \to 0$, i.e., $\Delta \to 1^-$, the magnetization profile is expected to be a scaling function of the ratio $\frac{x}{\sqrt{t}}$ for all values of h thus signaling a diffusive behavior [67,112–115]. A similar diffusive scaling has been obtained in Ref. [116] with slow corrections which might justify the anomalous scaling in Ref. [114].

Conclusions. We considered the emblematic nonequilibrium protocol generated by joining two domains with opposite magnetization. We were able to find a full analytic solution for the LQSS, obtaining a closed expression for both the magnetization and the spin current stationary profiles. Interestingly, our analytic results show a nowhere continuous behavior as a function of the interaction Δ. Moreover, our exact solution provides an example of a physical system where interactions prevent the emergence of a Tracy-Widom scaling. Such a conclusion has been supported by numerical DMRG simulations, that indicate a diffusive edge scaling throughout the critical regime, as long as $\Delta \neq 0$.

The simple stationary state, presented here, is a promising starting point to derive a continuous field-theory description of the LQSS, thus extending the results of Refs. [47,62,63] in the presence of interactions. For instance, an interesting outcome would be the study of the entanglement dynamics [117].

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