Rapid Communications

Compatible orders and fermion-induced emergent symmetry in Dirac systems

Lukas Janssen,[1] Igor F. Herbut,[2] and Michael M. Scherer[3]
[1]*Institut für Theoretische Physik, Technische Universität Dresden, 01062 Dresden, Germany*
[2]*Department of Physics, Simon Fraser University, Burnaby, British Columbia, Canada V5A 1S6*
[3]*Institut für Theoretische Physik, Universität Köln, 50937 Köln, Germany*

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We study the quantum multicritical point in a (2+1)-dimensional Dirac system between the semimetallic phase and two ordered phases that are characterized by anticommuting mass terms with $O(N_1)$ and $O(N_2)$ symmetries, respectively. Using ϵ expansion around the upper critical space-time dimension of four, we demonstrate the existence of a stable renormalization-group fixed point, enabling a direct and continuous transition between the two ordered phases directly at the multicritical point. This point is found to be characterized by an emergent $O(N_1 + N_2)$ symmetry for arbitrary values of N_1 and N_2 and fermion flavor numbers N_f as long as the corresponding representation of the Clifford algebra exists. Small $O(N)$-breaking perturbations near the chiral $O(N)$ fixed point are therefore irrelevant. This result can be traced back to the presence of gapless Dirac degrees of freedom at criticality, and it is in clear contrast to the purely bosonic $O(N)$ fixed point, which is stable only when $N < 3$. As a by-product, we obtain predictions for the critical behavior of the chiral $O(N)$ universality classes for arbitrary N and fermion flavor number N_f. Implications for critical Weyl and Dirac systems in 3+1 dimensions are also briefly discussed.

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Introduction. The interplay and competition of different ordering tendencies in many-body systems are the source of various exciting phenomena, including unconventional superconductivity, the nature of quantum spin liquids, and the physics of deconfined criticality. These notoriously challenging problems sometimes become theoretically accessible when an emergent higher symmetry can be found. The complex phase diagram of the high-T_c superconductors, for instance, has been argued to be deducible from an emergent symmetry in which the O(3) Néel and U(1) superconducting order parameters are combined into a five-tuplet which turns out to be a vector under O(5) [1]. Numerical simulations of the deconfined critical point between Néel and valence-bond-solid orders on the square lattice also find evidence for an emergent O(5) symmetry [2]. The emergence of this symmetry can be made natural by postulating duality relations between the bosonic gauge theory describing the deconfined critical point and certain fermionic theories [3,4]. Similarly, recent quantum Monte Carlo simulations of Dirac fermions in 2+1 dimensions find a direct and continuous transition between O(3) and $\mathbb{Z}_2$ ordered phases with an emergent O(4) symmetry at criticality [5].

Despite the general interest, however, a simple model in which emergent $O(N)$ symmetry with $N \geqslant 4$ can be explicitly shown appears to be still lacking. This is certainly true within the standard Landau-Ginzburg-Wilson approach in which a continuous quantum phase transition is assumed to be described by bosonic order-parameter fluctuations alone [6–8]: The purely bosonic $O(N)$ fixed point in $2 + 1$ dimensions is unstable under small perturbations that break the $O(N)$ symmetry for all $N > 3$, and presumably even the Heisenberg fixed point is unstable under a cubic anisotropy [9,10]. In this Rapid Communication, we demonstrate that the stability under symmetry-breaking perturbations significantly changes in the presence of gapless fermionic degrees of freedom. In particular, we demonstrate that the *chiral* $O(N)$ fixed point in which the bosonic order parameter is coupled to N_f flavors of massless Dirac fermions in $2 + 1$ dimensions is stable under perturbations that break the $O(N)$ symmetry. This adds a prime example to the general observation of fermion-induced symmetry enhancement in quantum critical Dirac systems [11–15]. Our results can be immediately applied to the triple point between the semimetallic and the O(3) Néel and U(1) Kekulé valence-bond-solid phases on the honeycomb lattice. The crucial ingredient here is the anticommuting nature of the corresponding Dirac mass terms, enabling us to combine them into a single order parameter that becomes a vector under O(5). The triple point is characterized by emergent O(5) symmetry and features a continuous and direct transition between the ordered phases as long as the system is tuned directly through the multicritical point. The corresponding universal exponents define the chiral O(5) universality class for which we provide estimates. A similar reasoning applies to the multicritical point between the Dirac semimetal and the O(3) and $\mathbb{Z}_2$ ordered phases [5].

Anticommuting Dirac masses. Consider the gapless Dirac Hamiltonian in $D = d + 1$ space-time dimensions,

$$\mathcal{H}_0(\vec{p}) = \alpha_i p_i, \quad i = 1, \ldots, d. \tag{1}$$

We assume the summation convention over repeated indices. The matrices α_i fulfill the Clifford algebra $\{\alpha_i, \alpha_j\} = 2\delta_{ij}\mathbb{1}_{d_\gamma}$ with d_γ being the dimension of the representation. The Hamiltonian $\mathcal{H}_0$ can be gapped out by adding (explicit or dynamical) mass terms,

$$\mathcal{H}_m = m_a \beta_a^\phi + m_b \beta_b^\chi, \quad a = 1, \ldots, N_1, \quad b = 1, \ldots, N_2. \tag{2}$$

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The mass operators β_a^ϕ and β_b^χ anticommute with $\mathcal{H}_0$ and among themselves. Their commutators $M_{aa'}^\phi = \frac{i}{2}[\beta_a^\phi,\beta_{a'}^\phi]$ and $M_{bb'}^\chi = \frac{i}{2}[\beta_b^\chi,\beta_{b'}^\chi]$ commute with $\mathcal{H}_0$ and generate an $\mathrm{O}(N_1)\oplus\mathrm{O}(N_2)$ symmetry under which the mass operators β_a^ϕ and β_b^χ transform as vectors. In this Rapid Communication, we assume the masses m_a and m_b to be *compatible*, i.e., their mass operators β_a^ϕ and β_b^χ also to mutually anticommute,

$$\{\beta_a^\phi,\beta_b^\chi\} = 0, \quad \text{for all } a,b. \tag{3}$$

The massive Hamiltonian $\mathcal{H}_0+\mathcal{H}_m$ has the spectrum $\varepsilon_{\vec{p}} = \pm\sqrt{p^2+m_a^2+m_b^2}$, and m_a and m_b do not compete [16].

The Dirac system with $\mathrm{O}(N_1)\oplus\mathrm{O}(N_2)$ anticommuting mass terms in d spatial dimensions requires N_1+N_2+d anticommuting $d_\gamma\times d_\gamma$ matrices. The number of Dirac fermion components d_γ is therefore [17],

$$d_\gamma \geqslant 2^{\lfloor (N_1+N_2+d)/2\rfloor}, \tag{4}$$

where $\lfloor\cdot\rfloor$ denotes the floor function. In the situation relevant for graphene we have $d=2$ and $d_\gamma=8$, and the maximal number of anticommuting mass terms is thus $N_1+N_2\leqslant 5$, which is consistent with the known classification of the 36 mass terms of spin-1/2 fermions on the honeycomb lattice [16]. We note that, in general, smaller (real) representations are possible if one employs a Nambu particle-hole construction, and the right-hand side of Eq. (4) is then to be replaced by $n/2$, where n is the dimension of the irreducible real representation of the Clifford algebra $C(d,N_1+N_2)$ [18].

Model. We study the system of gapless Dirac fermions Ψ and $\Psi^\dagger$ coupled to compatible order parameters $\phi\equiv\sum_a^{N_1}\phi_a\beta_a^\phi$ and $\chi\equiv\sum_b^{N_2}\chi_b\beta_b^\chi$ with anticommuting mass operators β_a^ϕ and β_b^χ. This is described by the Lagrangian

$$\mathcal{L}_\mathrm{F} = \Psi^\dagger[\partial_\tau+\mathcal{H}_0(-i\vec{\nabla})+g_1\phi+g_2\chi]\Psi, \tag{5}$$

with the Yukawa-type couplings g_1 and g_2 parametrizing the coupling to the fluctuating boson fields ϕ and χ. When radiative corrections are taken into account, the latter receive their own dynamics as well as bosonic self-interactions. We therefore include already from the outset all symmetry-allowed terms that may become generated by the fluctuations, up to fourth order in the fields,

$$\begin{aligned}\mathcal{L}_\mathrm{B} = {}& \tfrac{1}{2}\phi_a\big(-\partial_\mu^2+r_1\big)\phi_a+\tfrac{1}{2}\chi_b\big(-\partial_\mu^2+r_2\big)\chi_b\\ &+\lambda_1\big(\phi_a^2\big)^2+\lambda_2\big(\chi_b^2\big)^2+2\lambda_3\phi_a^2\chi_b^2,\end{aligned} \tag{6}$$

with $(\partial_\mu)\equiv(\partial_\tau,\vec{\nabla})$ and $\mu=0,1,\ldots,d$.

The action of the full system then is given by $S=\int d\tau\, d^d\vec{x}(\mathcal{L}_F+\mathcal{L}_B)$. This action describes a theory space with an explicit $\mathrm{O}(N_1)\oplus\mathrm{O}(N_2)$ symmetry generated by the $[N_1(N_1-1)+N_2(N_2-1)]/2$ commutators $M_{aa'}^\phi$ and $M_{bb'}^\chi$, where $a<a'=1,\ldots,N_1$ and $b<b'=1,\ldots,N_2$, respectively. It includes an $\mathrm{O}(N_1+N_2)$-invariant subspace, which is achieved by choosing $g_1=g_2$, $\lambda_1=\lambda_2=\lambda_3$, $r_1=r_2$. The additional generators promoting $\mathrm{O}(N_1)\oplus\mathrm{O}(N_2)$ to $\mathrm{O}(N_1+N_2)$ are the N_1N_2 operators $M_{ab}^{\phi\chi}=\frac{i}{2}[\beta_a^\phi,\beta_b^\chi]$. They rotate ϕ and χ into each other, allowing to construct a (N_1+N_2)-tuplet (ϕ_a,χ_b), which transforms as a vector under $\mathrm{O}(N_1+N_2)$.

The Yukawa-type couplings and the bosonic self-interactions have the scaling dimension $[g_1^2]=[g_2^2]=[\lambda_1]=[\lambda_2]=[\lambda_3]=3-d$ and therefore become simultaneously marginal in three spatial dimensions. In the following, we will mainly consider $1<d<3$ with a focus on $d=2$. In this case, all couplings become relevant in the renormalization-group (RG) sense. However, the ratios g_1^2/r_1 and g_2^2/r_2, describing the effective strengths of the interaction [19], are irrelevant for large enough r_1 and r_2, respectively. In this limit, the massive boson fields can be integrated out, leaving behind the stable noninteracting semimetal phase. The latter is trivially within the $\mathrm{O}(N_1+N_2)$-invariant subspace. Upon lowering r_1 (r_2) towards zero, on the other hand, the $\mathrm{O}(N_1)$ [$\mathrm{O}(N_2)$] part of the full symmetry group becomes spontaneously broken to a residual $O(N_1-1)$ [$O(N_2-1)$] symmetry, characterized by a finite vacuum expectation value of $\langle\phi_a\rangle\neq 0$ [$\langle\chi_b\rangle\neq 0$] for some a (b). Due to the Yukawa couplings g_1 and g_2, the condensation of the bosonic fields simultaneously opens a mass gap of the Dirac fermions. r_1 and r_2 are the tuning parameters for the corresponding quantum phase transitions. We are interested in the behavior of the fermionic *multicritical* point [20–23] in which both r_1 and r_2 are tuned to their critical values.

An important example in $d=2$, to which the above model can be immediately applied, is the physics of interacting spin-1/2 fermions on the honeycomb lattice. One of the 56 distinct five-tuplets of pairwise anticommuting mass operators consists of, for instance, the three components of the Néel order parameter and the two components of the Kekulé valence-bond-solid order parameter [16]. In the notation of Ref. [24], these masses read

$$\big(\beta_a^\phi\big)_{a=1-3}=\vec{\sigma}\otimes\gamma_0,\quad \big(\beta_b^\chi\big)_{b=1,2}=\mathbb{1}_2\otimes(i\gamma_0\gamma_3,i\gamma_0\gamma_5).$$

In the lattice model of Ref. [5], the O(2) symmetry in the Kekulé sector is explicitly broken down to $\mathbb{Z}_2\simeq\mathrm{O}(1)$ [25]. Note that the chiral $\mathrm{O}(N)$ fixed point in the presence of a small $\mathrm{O}(N)$-breaking perturbation can be understood as a multicritical point, and our model therefore applies also to this problem.

Flow equations. The existence of a unique upper critical dimension allows a controlled expansion in powers of $\epsilon=3-d$. Integrating over the momentum shell from Λ/b to Λ, we obtain the flow equations to the leading order,

$$\dot{g}_1^2=(\epsilon-\eta_1-2\eta_\Psi)g_1^2+2(N_1-2)g_1^4+2N_2g_1^2g_2^2, \tag{7}$$

$$\dot{g}_2^2=(\epsilon-\eta_2-2\eta_\Psi)g_2^2+2(N_2-2)g_2^4+2N_1g_1^2g_2^2, \tag{8}$$

where $\dot{g}_i^2\equiv\frac{dg_i^2}{d\ln b}$, $i=1,2$, and accordingly,

$$\dot{\lambda}_1=(\epsilon-2\eta_1)\lambda_1-4(N_1+8)\lambda_1^2-4N_2\lambda_3^2+N_\mathrm{f}g_1^4, \tag{9}$$

$$\dot{\lambda}_2=(\epsilon-2\eta_2)\lambda_2-4(N_2+8)\lambda_2^2-4N_1\lambda_3^2+N_\mathrm{f}g_2^4, \tag{10}$$

$$\begin{aligned}\dot{\lambda}_3={}&(\epsilon-\eta_1-\eta_2)\lambda_3-16\lambda_3^2-4(N_1+2)\lambda_1\lambda_3\\&-4(N_2+2)\lambda_2\lambda_3+N_\mathrm{f}g_1^2g_2^2,\end{aligned} \tag{11}$$

with fermion anomalous dimension $\eta_\Psi=(N_1g_1^2+N_2g_2^2)/2$ and order-parameter anomalous dimensions $\eta_i=2N_\mathrm{f}g_i^2$. In order to arrive at the above equations, we have rescaled the couplings as $g_i^2\Lambda^{\eta_i+2\eta_\Psi-\epsilon}/(8\pi^2)\mapsto g_i^2$ and $\lambda_i\Lambda^{2\eta_i-\epsilon}/(8\pi^2)\mapsto\lambda_i$ with $\eta_3=(\eta_1+\eta_2)/2$ and abbreviated

$N_\mathrm{f} := d_\gamma/4$. We have tuned both ϕ and χ to criticality by setting $r_1 = r_2 = 0$. This system of flow equations simplifies to various known results within respective limits: For $g_1, g_2 \to 0$, we recover the flow equations of the purely bosonic system with $\mathrm{O}(N_1) \oplus \mathrm{O}(N_2)$ symmetry [6,7,10]. For $N_1 = 1$ and $N_2 = 2$, Eqs. (7)–(10) also agree with the stability analysis of the $\mathbb{Z}_2 \times \mathrm{O}(2)$ Gross-Neveu-Yukawa theory [20,26]. Finally, in the isotropic limit $g_1 = g_2$ and $\lambda_1 = \lambda_2 = \lambda_3$, the flow equations reduce to the known equations for the chiral Ising, chiral XY, and chiral Heisenberg universality classes for $N_1 + N_2 = 1$, 2, and 3, respectively [24,27,28].

Stability of the isotropic fixed point. As the bosonic self-interactions do not feed back into the Yukawa-coupling sector, the latter can be solved independently. In addition to the noninteracting Gaussian fixed point at $g_1^2 = g_2^2 = 0$, the system of two coupled quadratic equations for g_1^2 and g_2^2 allows three interacting fixed points. Due to the homogeneity of the equations, two of them must be located on the g_1 and g_2 axes. These two *decoupled* fixed points describe the chiral $\mathrm{O}(N_1)$ and $\mathrm{O}(N_2)$ universality classes, respectively. The coupling to a second critical scalar field, however, is a relevant perturbation, and the decoupled fixed points are therefore unstable when both ϕ and χ are tuned to criticality. The topology of the flow then requires the third interacting fixed point to be stable. For symmetry reasons, it must be located on the bisectrix $g_1^2 = g_2^2$ and thus describes the chiral $\mathrm{O}(N_1 + N_2)$ universality class. This is the *isotropic* fixed point. It is generally expected that the chiral $\mathrm{O}(N)$ universality classes exist for all N and N_f compatible with Eq. (4) and the isotropic fixed point should therefore be located within the real coupling space $g_1^2 = g_2^2 > 0$. Small symmetry-breaking perturbations are always irrelevant, and $\mathrm{O}(N_1 + N_2)$ symmetry becomes emergent when both ϕ and χ are tuned to criticality, at least within the Yukawa sector. This general expectation is corroborated by the explicit evaluation of the one-loop flow: The isotropic fixed point is located at

$$g_1^{*2} = g_2^{*2} = \frac{\epsilon}{2N_\mathrm{f} + 4 - N} + O(\epsilon^2), \tag{12}$$

and it is characterized by the stability exponents, which determine the flow near the fixed point,

$$(\theta_1, \theta_2) = \left(-1, -\frac{2N_\mathrm{f} + 4}{2N_\mathrm{f} + 4 - N}\right)\epsilon + O(\epsilon^2), \tag{13}$$

with $N := N_1 + N_2$. Here, θ_1 corresponds to the flow within the $\mathrm{O}(N_1 + N_2)$ invariant subspace, whereas θ_2 corresponds to perturbations out of this subspace. For all N_1, N_2, and N_f compatible with Eq. (4), we have $2N_\mathrm{f} + 4 - N > 0$, and thus the isotropic fixed point is always real and stable within the Yukawa sector. We have explicitly checked that this remains true when the Nambu particle-hole construction is employed [18]. The flow diagram in the g_1-g_2 sector is depicted for the example of the chiral $\mathrm{O}(3) \oplus \mathrm{O}(2)$ theory with emergent O(5) in Fig. 1. Using Eq. (12), we find the corresponding bosonic couplings at the isotropic fixed point as $\lambda_1^* = \lambda_2^* = \lambda_3^* = \lambda^*$ with

$$\lambda^* = \frac{f_N(N_\mathrm{f}) + 4 - N - 2N_\mathrm{f}}{8(N+8)(2N_\mathrm{f} + 4 - N)}\epsilon + O(\epsilon^2), \tag{14}$$

where $f_N(N_\mathrm{f}) = 2N_\mathrm{f}\sqrt{1 + \frac{5N+28}{N_\mathrm{f}} + (\frac{N-4}{2N_\mathrm{f}})^2} > 2N_\mathrm{f} + N$. Consequently, the effective potential is real and bounded

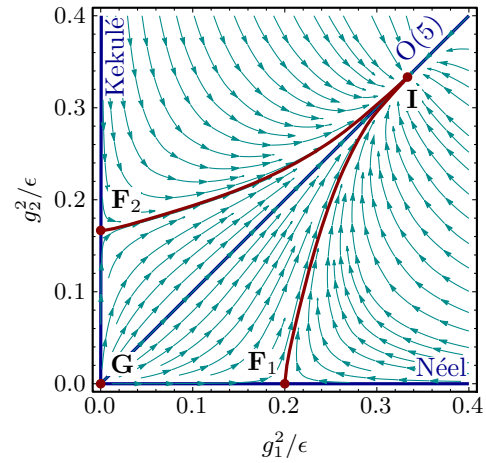

FIG. 1. RG flow diagram in the g_1^2-g_2^2 sector near the multicritical point between the O(3) Néel antiferromagnet and the O(2) Kekulé order using $d_\gamma = 4N_\mathrm{f} = 8$. The decoupled fixed points F_1 and F_2 describe the individual chiral Heisenberg and XY universality classes. The unique stable fixed point is the isotropic fixed point I, characterized by an emergent O(5) symmetry.

from below. The remaining stability exponents $(\theta_3, \theta_4, \theta_5)$, corresponding to the flow in the bosonic sector in the presence of g_1^* and g_2^*, are smaller than θ_1 and θ_2 for all N_1, N_2, and N_f compatible with Eq. (4), see Ref. [29]. The isotropic fixed point is therefore fully stable within the entire coupling space when both r_1 and r_2 are tuned to criticality. Consequently, the multicritical point between the Dirac semimetal and the gapped phases with $\mathrm{O}(N_1)$ and $\mathrm{O}(N_2)$ order parameters is characterized by an emergent $\mathrm{O}(N_1 + N_2)$ symmetry. The scaling behavior near the multicritical point is described by the chiral $\mathrm{O}(N)$ universality class with $N = N_1 + N_2$. The leading corrections to scaling correspond to a flow direction within the higher-symmetric subspace. The corresponding exponent is $\omega_1 = -\theta_1$. Small symmetry-breaking perturbations are strongly irrelevant and contribute to the corrections to scaling only at subleading order with a comparatively large exponent $\omega_2 = -\theta_2$. At the isotropic fixed point, we find the anomalous dimensions

$$(\eta_\phi, \eta_\Psi) = \frac{1}{2N_\mathrm{f} + 4 - N}\left(2N_\mathrm{f}, \frac{N}{2}\right)\epsilon + O(\epsilon^2), \tag{15}$$

with $\eta_\phi = \eta_1 = \eta_2$. The correlation-length exponent ν is obtained from the flow of the tuning parameters r_1 and r_2 at the isotropic fixed point and reads

$$\nu = \frac{1}{2} + \frac{2N_\mathrm{f}(N+14) + (N+2)[f_N(N_\mathrm{f}) + 4 - N]}{8(N+8)(2N_\mathrm{f} + 4 - N)}\epsilon + O(\epsilon^2). \tag{16}$$

Equations (15) and (16) generalize previous results for the chiral Ising, XY, and Heisenberg universality classes [24,28] to the chiral $\mathrm{O}(N)$ universality classes with arbitrary $N \in \mathbb{N}$.

Critical exponents from functional RG. Improved estimates for the critical exponents can be obtained from the nonperturbative functional RG (FRG) method. In particular, this is a suitable approach for calculations directly in 2+1 dimensions and has been shown to compare well to other

TABLE I. Chiral O(N) universality classes in 2+1 dimensions from functional RG and ϵ expansion for $d_\gamma = 4N_\mathrm{f} = 8$: Correlation-length exponent ν, anomalous dimensions η_ϕ and η_Ψ, and leading corrections-to-scaling exponent ω_1. For the chiral Ising, XY, and Heisenberg universality classes, various further estimates are known, see Ref. [28].

$N_\mathrm{f} = 2$		ν	η_ϕ	η_Ψ	ω_1
Chiral Ising	FRG	1.018	0.760	0.032	0.872
	ϵ^1	31/42	4/7	1/14	1
Chiral XY	FRG	1.160	0.875	0.062	0.878
	ϵ^1	4/5	2/3	1/6	1
Chiral Heisenberg	FRG	1.296	1.015	0.084	0.924
	ϵ^1	97/110	4/5	3/10	1
Chiral O(4)	FRG	1.364	1.159	0.091	1.017
	ϵ^1	1	1	1/2	1
Chiral O(5)	FRG	1.356	1.285	0.089	1.132
	ϵ^1	31/26	4/3	5/6	1

methods in the context of Gross-Neveu-type universality classes [23,30–40]. The central FRG equation is formulated in terms of the effective average action Γ_k, which interpolates between the microscopic action S at the UV cutoff $k = \Lambda$ and the full quantum effective action Γ in the infrared $k \to 0$ [41,42]. For an approximate solution, we expand Γ_k in powers of derivatives and truncate beyond the leading order,

$$\Gamma_k = \int_{\tau,\vec{x}} \left[Z_\Psi \bar{\Psi} \partial\!\!\!/ \Psi + g \bar{\Psi} \phi \Psi - \frac{Z_\phi}{2} \phi_a \partial_\mu^2 \phi_a + U(\rho) \right].$$

This constitutes the so-called local potential approximation. Here, $\partial\!\!\!/ \equiv \gamma^\mu \partial_\mu$ is the standard Dirac operator, and $\bar{\Psi} \equiv \Psi^\dagger \gamma^0$ denotes the Dirac conjugate. ϕ_a, $a = 1, \ldots, N$ are the components of an O(N) symmetric order-parameter field $\phi \equiv \sum_a^N \phi_a \beta_a^\phi$. It couples to the fermions via the scale-dependent Yukawa-type coupling g. The scale-dependent effective potential $U(\rho)$ is a functional of all symmetry-allowed boson self-interactions and as such depends only on the field-invariant $\rho = \frac{1}{2}\phi_a \phi_a$. Finally, we have introduced the wave-function renormalizations Z_Ψ and Z_ϕ, which also carry a scale dependence and are related to the anomalous dimensions via $\eta_\Phi = -(\partial_t Z_\Phi)/Z_\Phi$ with $\Phi \in \{\Psi, \phi\}$. Explicit expressions of the FRG flow equations for the dimensionless versions of the scale-dependent quantities $U(\rho)$ and g as well as for η_Ψ and η_ϕ are given in Ref. [29]. In practice, we expand $U(\rho)$ in a finite power series in ρ around the origin up to order ρ^6. This yields a closed set of algebraic fixed-point equations, which we solve numerically. For all N and compatible N_f tested, we find a unique stable fixed point, characterizing the corresponding chiral O(N) universality class. For the example of $N_\mathrm{f} = 2$, relevant to spin-1/2 fermions on the honeycomb lattice, our estimates for the universal exponents are given in Table I. The estimates from FRG and the leading-order ϵ expansion are in reasonable agreement for ν, η_ϕ, and ω_1. Significant differences appear in the fermion anomalous dimension η_Ψ.

Implications for 3+1 dimensions. In the limit of $\epsilon \to 0$, which applies to (3+1)-dimensional Weyl and Dirac systems, both the Yukawa couplings and the bosonic self-interactions become marginally irrelevant. They flow to zero, however, with fixed ratios $g_2/g_1 \to 1$ and $\lambda_{2,3}/\lambda_1 \to 1$, indicating emergent O($N$) symmetry also in this case.

Conclusions. We have demonstrated that the multicritical Dirac systems with compatible O(N_1) and O(N_2) order parameters are generically characterized by emergent O($N_1 + N_2$) symmetry. Within the first-order ϵ expansion, this result holds for all N_1 and N_2 and fermion flavor numbers N_f as long as the corresponding representation of the Clifford algebra exists. Put differently, the chiral O(N) universality classes are stable under any small perturbation that breaks O(N) symmetry. This conclusion is in surprising contrast to the purely bosonic O(N) universality classes in which symmetry-breaking perturbations destabilize the isotropic O(N) fixed point when $N \geqslant 3$ [6–10]. There, however, it is well known that the first-order ϵ expansion, when extrapolated to $\epsilon = 1$, significantly overestimates the stability of the isotropic fixed point. In the present system, by contrast, higher-loop corrections become suppressed for a large number of Dirac fermions N_f. Consequently, we expect our conclusion of emergent O(N) symmetry for all $N > 1$ to be true also in $d = 2$ as long as N_f is large enough. Within the first-order ϵ expansion, symmetry-breaking perturbations are strongly irrelevant also for small N_f, and one is therefore tempted to argue that higher-order corrections will ultimately not overturn the leading-order result also in this case. This conjecture deserves further investigation. The chiral O(N) universality classes are therefore in principle accessible in lattice Dirac systems without an explicit O(N) symmetry by tuning two parameters through a suitable multicritical point. The critical behavior is characterized by universal exponents for which we have given estimates from ϵ expansion and functional RG. Our result partly explains the recently observed emergent O(4) symmetry in simulations of a Dirac system with anticommuting mass terms [5]. There, however, evidence for emergent O(4) is found also significantly away from the multicritical point. This represents an interesting problem on its own.

Note added. After this Rapid Communication was submitted, a related preprint appeared on the arXiv [43] also demonstrating emergent O(N) symmetry in multicritical Dirac systems.

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