Nonmonotonic response and light-cone freezing in fermionic systems under quantum quenches from gapless to gapped or partially gapped states

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The properties of prototypical examples of one-dimensional fermionic systems undergoing a sudden quantum quench from a gapless state to a (partially) gapped state are analyzed. By means of a generalized Gibbs ensemble analysis or by numerical solutions in the interacting cases, we observe an anomalous, nonmonotonic response of steady-state correlation functions as a function of the strength of the mechanism opening the gap. In order to interpret this result, we calculate the full dynamical evolution of these correlation functions, which shows a freezing of the propagation of the quench information (light cone) for large quenches. We argue that this freezing is responsible for the nonmonotonous behavior of observables. In continuum noninteracting models, this freezing can be traced back to a Klein-Gordon equation in the presence of a source term. We conclude by arguing in favor of the robustness of the phenomenon in the cases of nonsudden quenches and higher dimensionality.

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Nonequilibrium quantum physics is at the heart of most relevant applications of solid-state physics, such as transistors and lasers [1–3]. More fundamentally, one of the main difficulties in studying many-body nonequilibrium quantum physics is represented by the unavoidable interactions that any quantum system has with its surroundings. This coupling is difficult to control and causes an effectively nonunitary evolution, even on short time scales [4]. The recent advent of cold atom physics [5] has allowed not only the access of quantum systems characterized by weak coupling to the environment, but also the engineering of Hamiltonians which show nonergodic behavior [6,7], the so-called integrable systems [8]. Moreover, in the context of cold atom physics, it is possible to manipulate the parameters of the Hamiltonian in a time-dependent and controllable fashion [7,9–12]. The combination of these three ingredients gave rise to a renewed interest in the physics of quantum quenches [13–17], which led to the birth of a new thermodynamic ensemble, the generalized Gibbs ensemble (GGE) [18–23]. Quantum quenches have been studied in a wide range of systems with the property that a change in a parameter of the Hamiltonian deeply affects the physical properties of the system itself. Interaction quenches in Luttinger liquids [24–36] and magnetic field quenches in the one-dimensional (1D) Ising model [37–47] are prominent examples in this direction. Furthermore, at the level of free fermions, quantum quenches between gapped phases characterized by different Chern numbers have also been studied [48–51]. However, not much attention has been devoted to the study of quantum quenches between gapless and gapped states. A notable exception is represented by quantum quenches from a Luttinger liquid to a sine-Gordon model [52–61] and quantum time mirrors [62]. However, the characterization of the main features of gapless-to-gapped quantum quenches is still lacking.

In this manuscript we consider paradigmatic examples of gapless 1D systems which get partially or completely gapped by a change in the parameters of the Hamiltonian. Namely, a spin-orbit coupled (SOC) quantum wire in the presence of an applied magnetic field [63–66] and a chain of spinless 1D fermions. For the latter, the gapping quench mechanism is either induced by a staggered potential (SP) or by the sudden switch-on of fermion-fermion interactions [67].

When the quench does not involve interactions, we consider both the lattice models and their continuum counterparts, describing the low-energy sector of a wide class of 1D systems. In these cases, the Hamiltonian H can be written as $H(t) = \sum_k \Psi_k^\dagger[\mathcal{H}_k + \theta(t)\Delta\sigma^x]\Psi_k$, where $\mathcal{H}_k$ is a family of 2×2 matrices indexed by the (quasi-) momentum k, characterized by a gapless spectrum. Here, σ^x is the first Pauli matrix in the usual representation, Δ is the strength of the gap-opening mechanism, and $\theta(t)$ is the Heaviside function. Finally, $\Psi_k^\dagger = (d_{a,k}^\dagger, d_{b,k}^\dagger)$ is a two-component momentum-resolved Fermi spinor.

In the case of the SOC wire, the indexes a,b represent the spin projection along the quantization axis and Δ is proportional to the applied magnetic field. In the case of the SP model, the former labels left or right movers while Δ is proportional to the strength of the staggered potential. For the noninteracting cases we demonstrate that the quantity $M = \sum_k \langle \Psi_k^\dagger \sigma^x \Psi_k\rangle_{GGE}/n$, where $\langle\cdot\rangle_{GGE}$ denotes the average on the associated GGE and n is the total number of particles in the system, exhibits a maximum for a finite value of Δ and tends to the gapless value for $\Delta \to \infty$, meaning that the observable does not feel the quench for strong quenches. The same behavior also characterizes the scenario of gap opened by fermion-fermion interactions, both in the integrable and in the nonintegrable case. In the nonintegrable case, however, the results should be intended as valid in a long-lived prethermal state. In order to interpret the result, we study, in the continuous noninteracting models, the time dependence of the correlation function $\mathcal{G}(x,t) = \langle \Psi^\dagger(x,t)\sigma^x\Psi(0,t)\rangle$, where $\Psi^\dagger(x,t)$ is the Fermi spinor, the average is performed with respect to the prequench ground state, and $\lim_{t\to\infty}\mathcal{G}(0,t) = M$. For small quenches $\mathcal{G}(x,t)$ shows the propagation of a light cone conveying the information of the quench through the system, while for large quenches the light cone *freezes*. In fact, $\mathcal{G}(x,t)$ is governed by a Klein-Gordon (KG) equation with a mass term $\propto\Delta^2$ and a source term $\propto\Delta$: the source term is responsible for finite values of M, while the mass term acts as a stiffness which hinders the generation and propagation of the cone. The same behavior occurs also when interactions are quenched, providing strong evidence that the freezing of the light cone is responsible for the nonmonotonous behavior of observables. Finally, we conclude by analyzing the generality of the results.

We begin analyzing the four noninteracting models, indexed by $i = 1,..,4$. For the SOC quantum wire on the lattice we have $\mathcal{H}_k^{(1)} = 2[1-\cos(k)]\sigma^0 + \alpha\sin(k)\sigma^z$ (with $\sigma^0 = I_{2\times2}$), and the gap-opening time-dependent mechanism is given by the magnetic field $\Delta^{(1)} = B$. Here, the lattice constant has been set to 1 and α represents the spin-orbit coupling. The corresponding low-energy continuous theory is obtained by replacing $\mathcal{H}_k^{(1)}$ with $\mathcal{H}_k^{(2)} = k^2\sigma^0 + \alpha k\sigma^z$, with gap-opening parameter $\Delta^{(2)} = B$. We also discuss the SP model, with $\mathcal{H}_k^{(3)} = -2J\cos(k)\sigma^z$, and $\Delta^{(3)}$ the strength of the staggered potential. In this case, the sum is restricted to positive k only. To obtain a low-energy theory for the SP model we expand around $k = \pi/2$, obtaining a Dirac cone with velocity $2J$ with a gap-opening term $\Delta^{(4)} = \Delta^{(3)}$. The Hamiltonian density is $\mathcal{H}_k^{(4)} = -2Jk\sigma^z$.

We assume that, before the quench, the chemical potential is set to zero and the system is in its zero-temperature equilibrium ground state. We define $|\Phi_0^{(i)}(0)\rangle$ as the i-system ground state at $t = 0$ of the corresponding prequench Hamiltonian. We introduce the unitary transformation $U_{0,k}^{(i)}$ satisfying $U_{0,k}^{(i)}\mathcal{H}_k^{(i)}U_{0,k}^{(i)\dagger} = \mathrm{diag}\{\epsilon_{+,0,k}^{(i)},\epsilon_{-,0,k}^{(i)}\}$, with $\epsilon_{-,0,k}^{(i)} \leqslant \epsilon_{+,0,k}^{(i)}$ $\forall k$, to get

$$\big|\Phi_0^{(i)}(0)\big\rangle = \prod_{k_1^{(i)}}^{k_2^{(i)}} \big(U_{0,k}^{(i)\dagger}\Psi_k^{(i)\dagger}\big)_2|0^{(i)}\rangle. \tag{1}$$

Here, $|0^{(i)}\rangle$ is the vacuum of the ith Hamiltonian, $k_{1/2}^{(i)}$ are fixed by the condition that only states with negative and zero energy are occupied, and the subscript 2 means that the second component of the spinor has to be considered. Note that the choice of the occupation of the zero energy modes is of no importance for the following since all results will be evaluated in the thermodynamic limit. Although $k_{1/2}^{(i)}$ are computed exactly in the calculations, here we report only the approximated relations $k_1^{(1)} \simeq k_1^{(2)} = -\alpha$, $k_2^{(1)} \simeq k_2^{(2)} = \alpha$ and $k_1^{(3)} = k_1^{(4)} = 0$, $k_2^{(3)} = k_2^{(4)} = \pi$ [68].

In order to get the time evolution of the system for $t > 0$, we introduce a second unitary operator $U_{1,k}^{(i)}$ related to the postquench Hamiltonian by $U_{1,k}^{(i)}[\mathcal{H}_k^{(i)} + \Delta^{(i)}\sigma^x]U_{1,k}^{(i)\dagger} = \mathrm{diag}\{\epsilon_{+,1,k}^{(i)},\epsilon_{-,1,k}^{(i)}\}$, with $\epsilon_{-,1,k}^{(i)} \leqslant \epsilon_{+,1,k}^{(i)}$ $\forall k$. In the Heisenberg representation, the time evolution of the systems is thus encoded in the Fermi spinor,

$$\Psi_k^{(i)}(t) = U_{1,k}^{(i)\dagger}\mathrm{diag}\{e^{-i\epsilon_{+,1,k}^{(i)}t},e^{-i\epsilon_{-,1,k}^{(i)}t}\}U_{1,k}^{(i)}\Psi_k^{(i)}(0). \tag{2}$$

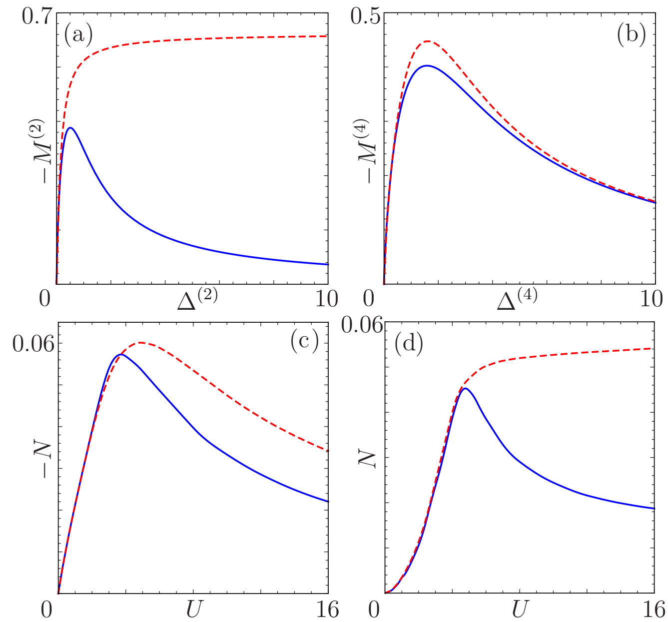

FIG. 1. (a) $M^{(2)}$ for the continuum SOC wire as a function of $\Delta^{(2)}$ with $\alpha = 1$; (b) $M^{(4)}$ for the continuum SP model as a function of $\Delta^{(4)}$ and $J = 1$; (c) N for an interaction-quenched fermion chain as a function of U with $V = 0$ and $J = 1$; (d) N for an interaction-quenched fermion chain as a function of U with $V = U$ and $J = 1$. In all panels, the solid lines represent the quenched long-time limit, the dashed lines show the results evaluated with an effective thermal model (see text).

Long after the quench, each of the four systems considered reaches a steady state which is locally described by a GGE [18]. The latter is constructed by considering as conserved quantities the occupation numbers $n_{k,j=1,2}^{(i)}$ of the energy levels of the corresponding postquench Hamiltonian,

$$n_{k,j=1,2}^{(i)} = \big(\Psi_k^{(i)\dagger}U_{1,k}^{(i)\dagger}\big)_j\big(U_{1,k}^{(i)}\Psi_k^{(i)}\big)_j. \tag{3}$$

Here, the subscript j on the right-hand side means that the jth component of the spinor must be considered. The GGE density matrices are hence given by

$$\rho^{(i)} = \frac{e^{-\sum_{k,j}\lambda_{k,j}n_{k,j}^{(i)}}}{\mathrm{Tr}\{e^{-\sum_{k,j}\lambda_{k,j}n_{k,j}^{(i)}}\}}. \tag{4}$$

The Lagrange multipliers $\lambda_{k,j}$ are fixed by the condition $\langle\Phi_0^{(i)}(0)|n_{k,j}^{(i)}|\Phi_0^{(i)}(0)\rangle = \mathrm{Tr}\{n_{k,j}^{(i)}\rho^{(i)}\}$.

We can now compute the observables of interest. We first focus on $M^{(i)} = \sum_k\langle\Psi_k^{(i)\dagger}\sigma^x\Psi_k^{(i)}\rangle_{GGE}/n^{(i)}$, with $n^{(i)}$ the total number of particles in the ith system. All quantities can be evaluated analytically, and the resulting expressions can be found in the Supplemental Material [69].

The results for the continuum SOC and SP models are shown by solid lines in Fig. 1, panels (a) and (b), as $\Delta^{(i)}$ is increased. In both cases, $M^{(i)}$ is nonmonotonous, increasing up to a maximum before dropping to the prequench value. The results for the two lattice models are qualitatively similar and are reported in the Supplemental Material [69]. A first interpretation of the phenomenon is the following: For infinitesimal $\Delta^{(i)}$ we do not expect any difference between a sudden quench and an adiabatic switching on of the gap-opening mechanism. Thus, the systems begins to magnetize. On the other hand, when $\Delta^{(i)}$ strongly exceeds the kinetic energy, $M^{(i)}$ is conserved and hence it remains at its prequench value. A maximum for finite $\Delta^{(i)}$ is thus expected.

We compare the GGE results with those obtained by an effective thermal ensemble at a given effective temperature $\beta_{\rm eff}^{-1}(\Delta^{(i)})$. The latter is obtained by solving

$$\left\langle \Phi_0^{(i)}(0) \middle| H_{pq}^{(i)} \middle| \Phi_0^{(i)}(0) \right\rangle = \frac{\mathrm{Tr}\left\{ e^{-\beta_{\rm eff}(\Delta^{(i)})(H_{pq}^{(i)} - \mu(\Delta^{(i)}) n^{(i)})} H_{pq}^{(i)} \right\}}{\mathrm{Tr}\{ e^{-\beta_{\rm eff}(\Delta^{(i)})(H_{pq}^{(i)} - \mu(\Delta^{(i)}) n^{(i)})} \}} \tag{5}$$

for the effective temperature. Here, $H_{pq}^{(i)} = H^{(i)}(t > 0)$ is the postquench Hamiltonian and $\mu(\Delta^{(i)})$ is the Lagrange multiplier ensuring particle number conservation. The effective-temperature magnetization is shown in Figs. 1(a) and 1(b) as a dashed line. While for the SP model there is a good qualitative agreement, for the SOC wire the disagreement is dramatic as the effective-temperature magnetization saturates to a nonzero value. The mechanism behind the nonmonotonous behavior of $M^{(i)}$ is hence not effective heating.

To get a deeper understanding, we now focus on the continuum models and introduce the Green's function

$$\mathcal{G}^{(i)}(x,t) = \left\langle \Phi_0^{(i)}(0) \middle| \Psi^{(i)\dagger}(x,t) \sigma^x \Psi^{(i)}(0,t) \middle| \Phi_0^{(i)}(0) \right\rangle. \tag{6}$$

Clearly, $\mathcal{G}^{(i)}(0,t) = M^{(i)}(t)$. $\mathcal{G}^{(i)}(x,t)$ satisfies, for $t > 0$, an inhomogeneous KG equation

$$\left(\partial_x^2 - \frac{1}{4u_i^2} \partial_t^2 \right) \mathcal{G}^{(i)}(x,t) = \lambda_i^2 \mathcal{G}^{(i)}(x,t) + \lambda_i \phi_i(x), \tag{7}$$

where $\lambda_i = \Delta^{(i)}/u_i$ (with $u_2 = \alpha, u_4 = J$) and the source term is $\phi_i(x) = i \partial_x \langle \Psi^{(i)\dagger}(x,0) \mathcal{M}^{(i)} \Psi^{(i)}(0,0) \rangle_0$, with $\mathcal{M}^{(2)} = \sigma^z$ and $\mathcal{M}^{(4)} = \sigma^y$. Equation (7) is solved with the prequench boundary-value condition $\mathcal{G}^{(i)}(x,0) = 0$. Equation (7) supports a steady-state solution for $t \to \infty$ and it can be checked (see Supplemental Material [69]) that $\lim_{t \to \infty} \mathcal{G}^{(i)}(0,t) = M^{(i)}$. Therefore, analyzing the time evolution of $\mathcal{G}^{(i)}(x,t)$ can shed light on the dynamics leading to the nonmonotonous magnetization. This is shown in Fig. 2 for the SOC wire model, in the case of (a) small quench $\Delta^{(2)} = 0.3$ and (b) large quench $\Delta^{(2)} = 2$. For a small quench, $\mathcal{G}^{(2)}(x,t)$ exhibits a typical light-cone behavior [17,24,25] and information of the quench is therefore able to propagate throughout the system. This leaves a finite "trail" in $x = 0$, which eventually results in a finite value of $M^{(2)}$ at large times. On the other hand, the response of $\mathcal{G}^{(2)}(x,t)$ to the "shock" induced by a large quench is dramatically different. Indeed, in this regime, $\mathcal{G}^{(2)}(x,t)$ is characterized by weakly damped and almost stationary oscillations both in space and in time, which strongly hinders the propagation of the information through the system and leads to both a slowdown and an overall suppression of the light cone. The Green's function $\mathcal{G}^{(2)}(x,t)$ oscillates around its prequench initial one and reaches in the long-time limit a value very close to the latter. This phenomenon can thus be interpreted as an effective freezing of the light cone. The same qualitative behavior is observed also for the continuum SP model, not shown here. We thus attribute the emergence of the nonmonotonous behavior of $M^{(i)}$ as a function of $\Delta^{(i)}$ to

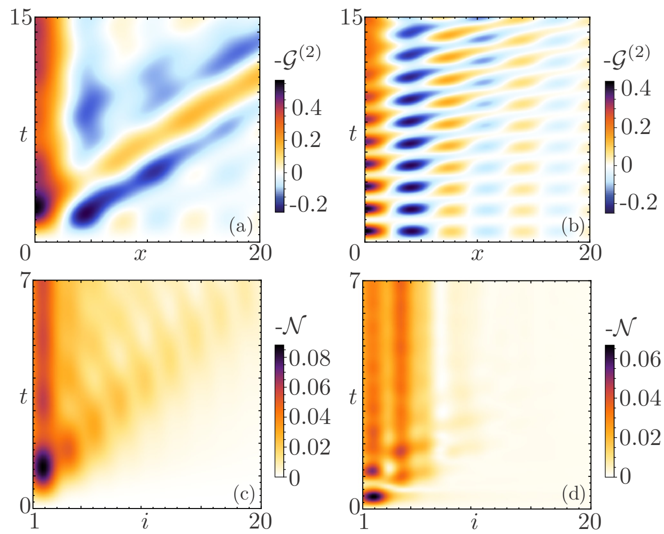

FIG. 2. (a) Density plot of $-\mathcal{G}^{(2)}(x,t)$ as a function of x and t for $\Delta^{(2)} = 0.3$ and $\alpha = 1$; (b) same as in (a) but for $\Delta^{(2)} = 2$ and $\alpha = 1$. (c) Density plot of $-\mathcal{N}(i,t)$ as a function of i and t for $U = 2$, $V = 0$, and $J = 1$; (d) same as in (c) but for $U = 16$, $V = 0$, and $J = 1$.

the competition between the propagating and freezing regimes. The identification of a freezing regime for large quenches constitutes a crucial result and, as we shall show in the last part of the paper, is a quite universal and robust feature of gap-opening sudden quench models. It represents a new concept in the physics of quantum quenches: Even though the gap is not able to dynamically introduce a length scale in the correlation functions, it dramatically influences the light-cone propagation.

In order to get a picture of the effect, it is worth noticing that, in a simple mechanical interpretation [73], the KG equation represents the transverse vibrations of a string driven by a force $\propto \lambda_i$, embedded into an elastic medium of elastic constant $\propto \lambda_i^2$. When the medium is slack, vibrations can propagate almost without disturbance, while in a stiff medium the wave propagation is strongly suppressed. The turning point turns out to be for $\lambda_i \approx 1$, which corresponds to the location of the maximum of $M^{(i)}$ shown in Figs. 1(a) and 1(b). Therefore, when the gap becomes comparable to the average kinetic energy scale, the freezing of the light cone begins to occur. Thanks to Wick's theorem, a similar behavior characterizes all higher-order correlators. This issue is relevant, since some of those correlators are either easier to numerically evaluate in the interacting systems we will analyze or are experimentally more accessible.

We now turn to the lattice model described by $\mathcal{H}_k^{(3)}$ where, instead of switching on a staggered potential, a sudden quench of the nearest-neighbor interaction U and/or of the next-to-nearest-neighbor interaction V is performed. For $U,V \gtrsim J$ interactions can open a gap in the spectrum. In addition, when $V > 0$ the model is nonintegrable. We turn to a numerical evaluation employing the density-matrix renormalization group (DMRG) technique [74–77].

Since the model is invariant under rotations in the spinor space, we analyze the long-time (stationary) limit N of the correlation function $\mathcal{N}(1,t)$, defined by $\mathcal{N}(i,t) = \langle (n_0(t) - 1/2)(n_i(t) - 1/2) \rangle_0 - \langle (n_0(0) - 1/2)(n_i(0) - 1/2) \rangle_0$, where

$n_i(t)$ is the (time-resolved) occupation number of the ith site and $\langle\cdot\rangle_0$ represents the average with respect to the prequench ground state. Results are shown in Fig. 1(c) for the integrable case $V = 0$ (solid line). N follows the same qualitative behavior of the magnetization in the noninteracting models, rising for small quenches up to a maximum value. As the gap size increases over the crossover point, N begins to decrease and tends (not shown) to the prequench value for very large U. Even when integrability is lost, as is the case of Fig. 1(d) ($V = U$, solid line), the qualitative picture remains, on the accessible time scales, the same. In both cases, a description in terms of an effective temperature (dashed lines) fails to reproduce the results.

Also in this model a competition between a propagation and a freezing regime for the light cone occurs. To show this fact, we consider the correlation function $\mathcal{N}(i,t)$, shown in Figs. 2(c) and 2(d). For small quenches with $U \leqslant 2J$ one clearly observes a propagation of the quench information spreading through the system. On the other hand, large quenches with $U > 2J$ display a sharp freezing of the light cone. Thus, the freezing of the light cone is a generic feature of systems subject to quenches opening large gaps in the spectrum.

To further support the idea that this mechanism is robust and represents a generic feature, we have checked that the results obtained here are valid even in higher dimensions. We have considered the paradigmatic case of a quench of a magnetic field applied to a Rashba-coupled two-dimensional electron gas [78]. Also in this case, the long-time magnetization shows a nonmonotonous behavior as a function of the magnetic field, increasing to a maximum before eventually turning to the prequench value for large quenches, see Supplemental Material [69]. The results are robust even with respect to the rapidity of the quench. We have studied the continuum SOC wire model when the magnetic field linearly ramps from 0 (see Supplemental Material [69] for details). For longer ramps, the asymptotic value of $M^{(2)}$ for large $\Delta^{(2)}$ increases, but the nonmonotonous behavior of the magnetization persists.

In conclusion, a nonmonotonic behavior of observables characterizes a wide range of gapless to gapped or partially gapped quantum quenches, both for sudden and nonsudden protocols, integrable and nonintegrable models, and not only in one spatial dimension. It is the hallmark of a peculiar phenomenon, namely, the freezing of the light cone, which conveys the quench information through the system. This freezing results in a state described by a GGE which differs from effective thermal states, in some cases even dramatically, thus providing an experimentally accessible way to test the GGE physics. In noninteracting models, the freezing of the light cone is captured by a KG equation, which provides an intuitive interpretation of the behavior of the system in terms of a simple continuum mechanical model. As a limit for the universality of the physics described, we point out that we do not expect to observe the effects when the gap is opened by merging of crossings, as relevant, for example, for Weyl semimetals [79], or for the models discussed in Refs. [80] and [81]. A static fermion-fermion interaction, which could be taken into account by means of bosonization [82–85] or DMRG [74,75], for instance, is expected to renormalize the gap to larger values [86], so we expect the phenomenon to persist with a shifted and renormalized maximum [87].

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