Periodic array of quantum rings strongly coupled to circularly polarized light as a topological insulator

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We demonstrate theoretically that a strong high-frequency circularly polarized electromagnetic field can turn a two-dimensional periodic array of interconnected quantum rings into a topological insulator. The elaborated approach is applicable to calculate and analyze the electron energy spectrum of the array, the energy spectrum of the edge states, and the corresponding electronic densities. As a result, the present theory paves the way to optical control of the topological phases in ring-based mesoscopic structures.

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I. INTRODUCTION

Symmetries play a crucial role in modern science, since they determine physical properties of various systems. Particularly, the translational and inversion symmetries together with the time-reversal symmetry define the electronic structure of solids. If one of the symmetries is broken, the electron energy spectrum complicates and the electron system can reach topologically nontrivial phases [1]. So, the breaking of inversion symmetry in semiconductor structures can lead to the transition from the normal semiconducting state to the topological insulator—the matter that behaves as an insulator in its interior but whose edges contain conducting electronic states [2–4]. The breaking of time-reversal symmetry—for instance, by application of a magnetic field—radically changes the electron energy spectrum as well, resulting in the discrete set of Landau levels within the bulk and the chiral edge states at the boundaries. In turn, the Coulomb interaction can further complicate the physical picture leading to the incompressible Laughlin states and other exotic phases of matter [5].

It has been recently shown that topologically nontrivial phases may arise not only in the condensed-matter electronic systems but also in photonic structures [6,7] or in the strongly coupled light-matter systems based on cavity polaritons [8–11]. Moreover, in ringlike nanostructures (particularly, in mesoscopic ballistic rings) the coherent coupling of electrons to a circularly polarized electromagnetic field leads to the appearance of an effective $U(1)$ gauge field, which breaks the time-reversal symmetry. Particularly, this field results in the physical nonequivalence of clockwise and counterclockwise electron rotations in an irradiated quantum ring (QR) similarly to a stationary magnetic field [12–14]. The field-controlled interference of the electron waves corresponding to these rotations opens a way to an optical tuning of QR arrays [15]. Currently, QRs are actively studied both experimentally and theoretically as a basis for various nanoelectronic applications [16]. Developing this scientific trend in the present paper, we demonstrate theoretically that a circularly polarized field can turn a two-dimensional (2D) array of interconnected QRs into a topological insulator. It should be noted that topological properties of electrons strongly coupled to light are under consideration currently for various condensed-matter systems, including graphene, semiconductor structures, etc. (see, e.g., Refs. [17–21]). Therefore, the subject of the present research fits well the growing tendencies in condensed-matter physics.

The paper is organized as follows. In Sec. II, we elaborate the theory describing the stationary electronic properties of an irradiated array of QRs. In Sec. III, we derive the electron energy spectrum of the array, calculate the spectrum of electronic edge states and the corresponding electronic densities, and analyze the found edge states within the formalism based on the Chern numbers. The last section contains the Acknowledgments.

II. MODEL

Let us consider the 2D periodic array of interconnected QRs, which are irradiated by a circularly polarized electromagnetic wave with the electric field amplitude, $\widetilde{E}_0$, and the frequency, ω (see Fig. 1). In what follows, the field frequency, ω, is assumed to satisfy the two conditions. First, the field frequency should be far from interband resonant frequencies of electrons in the periodic array. Under this off-resonant condition, one can neglect the interband absorption of the field. Second, the field frequency should satisfy the condition $\omega\tau \gg 1$, where τ is the electron life time restricted by intraband scattering processes. Under this high-frequency condition, the intraband (Drude) absorption of the field by electrons can also be neglected. Since the wave is both off-resonant and high-frequency ("dressing field" within the conventional terminology of quantum optics), the absorption of the field does not erode the effects under consideration (for more details, see also the discussion in Refs. [22,23]). Correspondingly, the effects caused by the dressing field substantially differ from the effects induced

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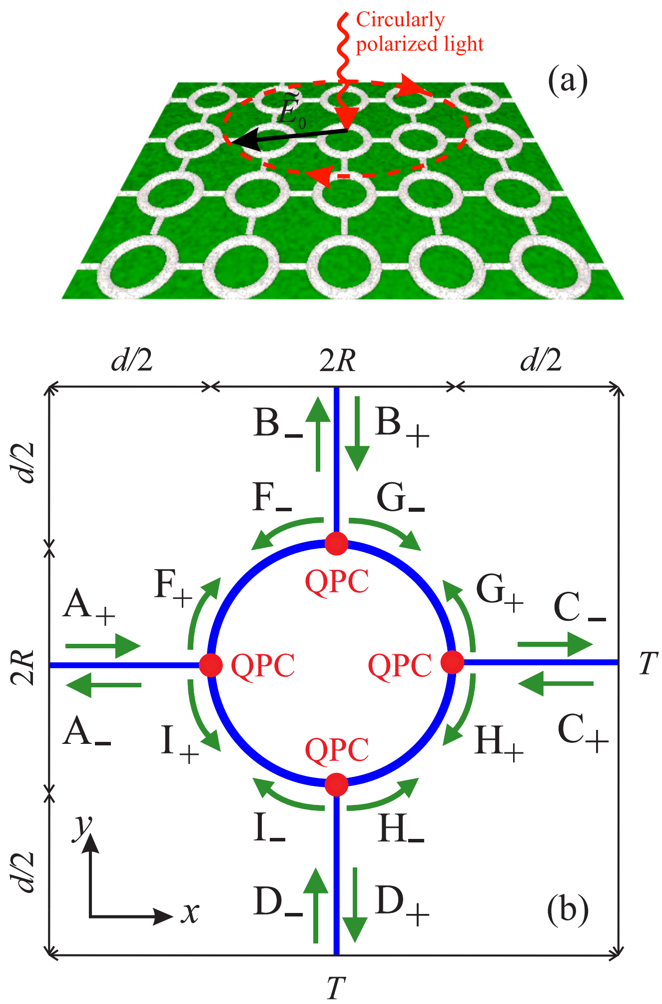

FIG. 1. Sketch of the system under consideration: (a) The 2D periodic array of quantum rings (QRs) irradiated by a circularly polarized electromagnetic wave with the electric field amplitude $\widetilde{\mathbf{E}}_0$. (b) The elementary cell of the 2D array with the period T, which consists of a QR with the radius R, four leads with the length $d/2$ and four quantum point contacts (QPC). The green arrows correspond to electron waves traveling in different ways with the amplitudes $A_\pm, B_\pm, C_\pm, D_\pm, F_\pm, G_\pm, H_\pm, I_\pm$.

in QRs by absorption of light (see, e.g., Refs. [24–27]). Physically, the circularly polarized dressing field breaks the time-reversal symmetry and, therefore, effects on the electron system in the QR similarly to a stationary magnetic field [12]. In particular, electronic properties of a dressed QR can be described by the effective stationary Hamiltonian,

$$H_{\text{eff}} = \frac{(\hat{p}_\phi - eA_{\text{eff}})^2}{2m_e}, \tag{1}$$

where $\hat{p}_\phi$ is the operator of electron momentum in the QR, and

$$A_{\text{eff}} = \frac{eE_0^2}{2Rm_e\omega^3} \tag{2}$$

is the artificial $U(1)$ gauge potential produced by the interaction between electrons in the QR and circularly polarized photons of the dressing field [13]. It follows from Eqs. (1)–(2) that the effective vector potential, A_{eff}, is responsible for the optically induced Aharonov-Bohm effect [13,14] and results in the phase shift of an electron wave traveling between nearest quantum point contacts (QPCs),

$$\phi_0 = \frac{e\pi RA_{\text{eff}}}{2\hbar} = \frac{\pi e^2\widetilde{E}_0^2}{4m_e\hbar\omega^3}. \tag{3}$$

Within the conventional scattering matrix approach [28,29], the amplitudes of electronic waves propagating in the array, $A_\pm, B_\pm, C_\pm, D_\pm, F_\pm, G_\pm, H_\pm, I_\pm$, satisfy the following set of equations:

$$\begin{pmatrix} A_-e^{-iqd/2} \\ F_+ \\ I_+ \end{pmatrix} = S\begin{pmatrix} A_+e^{iqd/2} \\ F_-e^{i(\pi qR/2-\phi_0)} \\ I_-e^{i(\pi qR/2+\phi_0)} \end{pmatrix}, \tag{4}$$

$$\begin{pmatrix} B_+e^{-iqd/2} \\ F_- \\ G_- \end{pmatrix} = S\begin{pmatrix} B_-e^{iqd/2} \\ F_+e^{i(\pi qR/2+\phi_0)} \\ G_+e^{i(\pi qR/2-\phi_0)} \end{pmatrix}, \tag{5}$$

$$\begin{pmatrix} C_-e^{-iqd/2} \\ G_+ \\ H_+ \end{pmatrix} = S\begin{pmatrix} C_+e^{iqd/2} \\ G_-e^{i(\pi qR/2+\phi_0)} \\ H_-e^{i(\pi qR/2-\phi_0)} \end{pmatrix}, \tag{6}$$

$$\begin{pmatrix} D_+e^{-iqd/2} \\ H_- \\ I_- \end{pmatrix} = S\begin{pmatrix} D_-e^{iqd/2} \\ H_+e^{i(\pi qR/2+\phi_0)} \\ I_+e^{i(\pi qR/2-\phi_0)} \end{pmatrix}, \tag{7}$$

where the scattering matrix is

$$S = \begin{pmatrix} \sqrt{1-2\varepsilon^2} & \varepsilon & \varepsilon \\ \varepsilon & \frac{-(1+\sqrt{1-2\varepsilon^2})}{2} & \frac{(1-\sqrt{1-2\varepsilon^2})}{2} \\ \varepsilon & \frac{(1-\sqrt{1-2\varepsilon^2})}{2} & \frac{-(1+\sqrt{1-2\varepsilon^2})}{2} \end{pmatrix}, \tag{8}$$

where ε is the electron transmission amplitude through QPCs $(0 \leqslant \varepsilon \leqslant 1/\sqrt{2})$, $\hbar q = \sqrt{2m_eE}$ is the electron momentum, m_e is the electron effective mass, E is the electron energy, and ϕ_0 is the field-induced phase shift Eq. (3).

Applying the Bloch theorem to the considered periodic array of QRs, we arrive at the equations

$$\begin{pmatrix} C_- \\ C_+ \end{pmatrix} = e^{ik_xT}\begin{pmatrix} A_+ \\ A_- \end{pmatrix}, \tag{9}$$

$$\begin{pmatrix} B_+ \\ B_- \end{pmatrix} = e^{ik_yT}\begin{pmatrix} D_- \\ D_+ \end{pmatrix}, \tag{10}$$

where $\mathbf{k} = (k_x,k_y)$ is the electron wave vector originated from the periodicity of the array and $T = d + 2R$ is the period of the array. Mathematically, Eqs. (4)–(7) and Eqs. (9) and (10) form the homogeneous system of 16 linear algebraic equations for the 16 amplitudes $A_\pm, B_\pm, C_\pm, D_\pm, F_\pm, G_\pm, H_\pm, I_\pm$. Solving the secular equation of this algebraic system numerically, we arrive at the sought electron energy spectrum of the irradiated array, $E(\mathbf{k})$, which is discussed below.

III. RESULTS AND DISCUSSION

The first six energy bands, $E(\mathbf{k})$, of the 2D infinite periodic array of QRs with transparent QPCs ($\varepsilon = 1/\sqrt{2}$) are plotted in Figs. 2(a)–2(c) for various directions Γ-X-M-Γ in the Brillouin zone of the array [see Fig. 2(d)] and various phase incursions, ϕ_0. In the absence of the dressing field ($\phi_0 = 0$), the electron energy spectrum plotted in Fig. 2(a) has no gaps in the density of electronic states and consists of two flat bands (1 and 6),

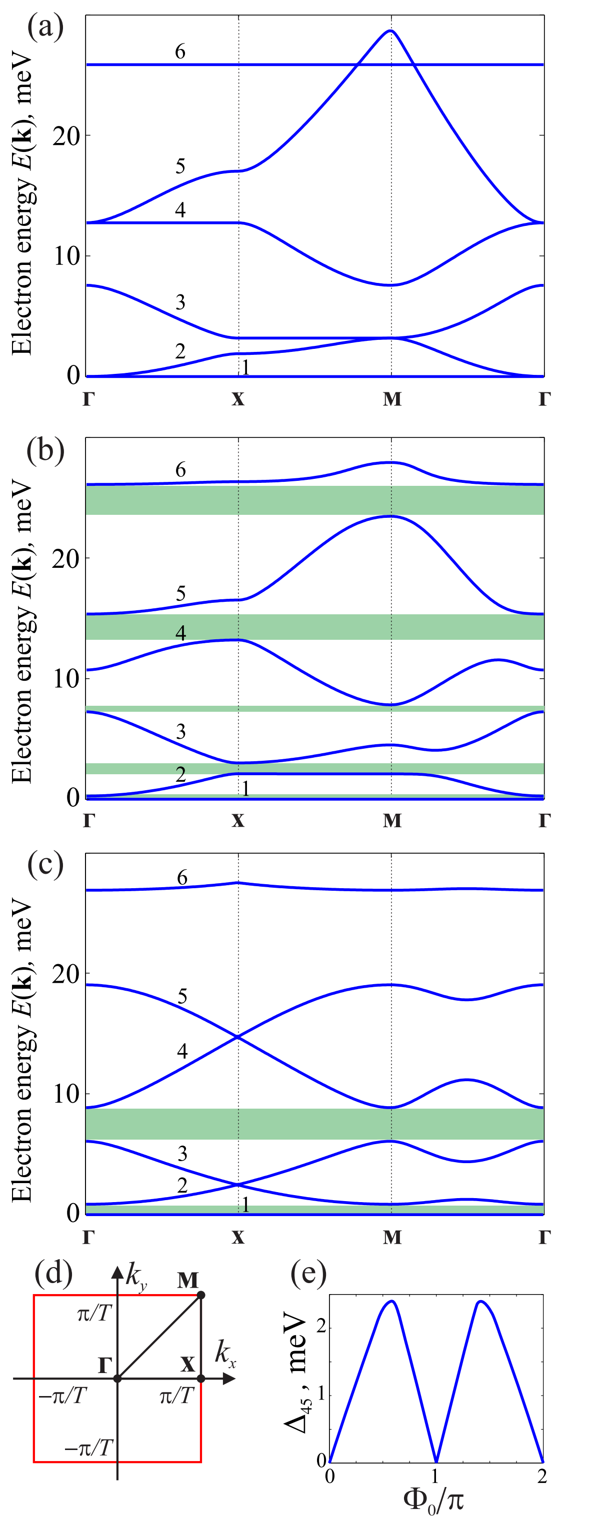

FIG. 2. Electron energy spectrum, $E(\mathbf{k})$, of the first six bands (1–6) in the 2D infinite periodic array of GaAs-based QRs with the radius $R = 30$ nm, the period $T = 100$ nm, and the electron transmission amplitude, $\varepsilon = 1/\sqrt{2}$, in the presence of a circularly polarized electromagnetic field with the different phase incursions, ϕ_0: (a) $\phi_0 = 0$, (b) $\phi_0 = \pi/9$, (c) $\phi_0 = \pi/4$. The green strips depict the field-induced band gaps, the plot (d) shows the first Brillouin zone of the 2D periodic array of QRs and the plot (e) presents the dependence of the band gap, Δ_{45}, on the field-induced phase incursion along a whole QR, $\Phi_0 = 4\phi_0$.

which correspond to electron states localized within QRs, and four bands (2–5) corresponding to delocalized electron states propagating along the array. The dressing field ($\phi_0 \neq 0$), first, results in coupling between localized and delocalized electron states [see the dispersions of the sixth band in Figs. 2(a) and 2(b)] and, second, induces the energy gaps between different bands [see the green strips in Figs. 2(b)–2(c)]. In what follows, the gaps between the bands with the numbers i and j are denoted as Δ_{ij}. It should be noted that the values of the field-induced gaps depend periodically on the dressing field as it is shown in Fig. 2(e). Let us demonstrate that edge states—both topologically protected and unprotected—appear within these field-induced band gaps.

Following the conventional theory of topological insulators [4], we have to fold the Brillouin zone pictured in Fig. 2(d) as a torus T^2. Then the Chern number corresponding to the nth band of the considered periodic array is defined as

$$C_n = \frac{1}{2\pi i}\int_{T^2} d^2k F_{xy}(\mathbf{k}), \tag{11}$$

where $F_{xy}(\mathbf{k}) = \partial A_y/\partial k_x - \partial A_x/\partial k_y$ is the field strength associated with the Berry connection, $A_j(\mathbf{k}) = \langle n(\mathbf{k})|\frac{\partial}{\partial k_j}|n(\mathbf{k})\rangle$ is the vector potential of the field, and $|n(\mathbf{k})\rangle$ is the normalized Bloch wave function of the nth band [4]. Applying Eq. (11) to the Bloch functions found from Eqs. (4)–(10), we arrive at the Chern numbers of the considered bands: $C_1 = 0$, $C_2 = -1$, $C_3 = 1$, $C_4 = 1$, and $C_5 = -1$. According to the bulk-boundary correspondence [4], the sum of the Chern numbers of the bands below a certain gap is equal to the number of topologically protected edge modes in the gap (per each boundary):

$$N = \left|\sum_i C_i\right|. \tag{12}$$

It follows from Eq. (12) that two branches of the topologically protected edge states must exist within each of the two nonzero band gaps, Δ_{23} and Δ_{45}, if the array of QRs is of finite size along one dimension. To find the energy spectrum of the branches, let us restrict the consideration by the array, which is infinite along the x axis and includes only five QRs along the y axis [see Fig. 3(a)]. The energy spectrum of electrons in this array can be easily calculated within the approach developed in Sec. II with the only difference: The elementary cell of the finite array pictured in Fig. 3(a) consists of five QRs, where the edge rings (with the numbers 1 and 5) have three QPCs unlike the others. The energy spectrum of lowest bands of delocalized electrons in this array, $E(\mathbf{k})$, is plotted in Fig. 3(b). As a first consequence of finite size of the array along the y axis, each delocalized band of the 2D infinite array [see the branches 2–5 pictured in Figs. 2(a)–2(c)] is split into five subbands in Fig. 3(b). As a second consequence, the branches of edge states within the band gaps appear [see the red, blue, and green curves in Fig. 3(b)]. The edge character of these branches follows clearly from the calculated electron density, $\mathcal{R}$, which has its maximum at edge QRs with the numbers 1 and 5 [see Figs. 3(c)–3(e)]. It follows from the aforesaid that the branches of edge states plotted in red and blue—which connects two adjacent bands—are topologically protected since their numbers satisfy the condition Eq. (12). On the

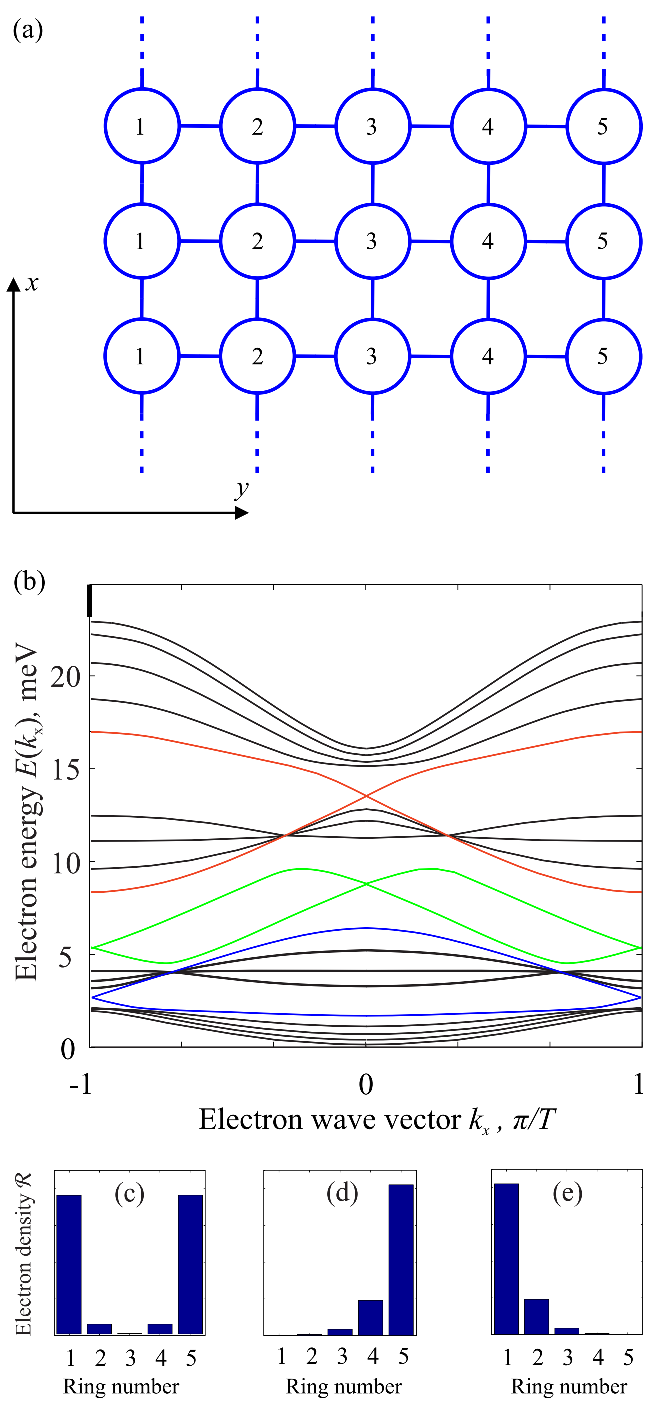

FIG. 3. Electronic properties of the finite array of QRs. (a) Sketch of the finite array of QRs, which is infinite along the x axis and consists of five QRs along the y axis. (b) The energy bands, $E(\mathbf{k})$, of the finite array consisting of GaAs-based QRs with the radius $R = 30$ nm, the period $T = 100$ nm, and the electron transmission amplitude, $\varepsilon = 1/\sqrt{2}$, in the presence of a circularly polarized electromagnetic field with the phase incursions, $\phi_0 = \pi/9$. The red and blue lines correspond to the topologically protected edge states, whereas the green lines correspond to unprotected ones. (c)–(e) Distribution of the electron density, $\mathcal{R}$, in QRs with the different numbers for the state corresponding to the intersection of the red branches at $k = 0$ in the plot (c), the state corresponding to the upper red branch at $k_x T/\pi = -0.007$ in the plot (d), and the state corresponding to the upper red branch at $k_x T/\pi = 0.007$ in the plot (d).

contrary, the edge states plotted in green are not topologically protected since they correspond to the zero number Eq. (12).

It should be noted that the band gap, Δ_{34}, can also be opened by varying the electron transmission amplitude through QPCs, ε. Namely, $\Delta_{34} \neq 0$, if the transmission amplitude is $\varepsilon < 1/\sqrt{2}$. However, this gap opening does not result in topological edge states since the corresponding number Eq. (12) is zero. As a consequence, only field-induced band gaps can turn the periodic array of QRs into a topological insulator and, therefore, the strong electron coupling to a circularly polarized dressing field is crucial for the effect under consideration. Physically, this follows from the fact that the circularly polarized dressing field effects an electron system in a QR similarly to a stationary magnetic field (see Eqs. (1)–(3) and the detailed discussion in Refs. [12–14]). As a consequence, formation of the edge electron states similar to those appearing in the quantum Hall effect takes place in the considered periodic array of irradiated QRs.

Finalizing the discussion, we have to formulate the conditions of observability of the predicted effects. From this viewpoint, there are the two fundamental restrictions: (i) the mean free path of electrons for inelastic scattering processes should be much greater than the array period T and (ii) the time of an electron traveling through a ring should be much more than the field period, $2\pi/\omega$. In the considered case of GaAs-based QRs with the Fermi energy of meV scale and the array period $T \sim 10^{-5}$ cm, the both conditions can be satisfied for a dressing field near the THz frequency range. Since the calculated field-induced band gaps are of meV scale for such a dressing field with the intensity $I \sim$ kW/cm^2, the discussed topological edge states can be detected in state-of-the-art experiments. It should be noted also that the present theory can be applied to describe the same array in the presence of a stationary uniform magnetic field directed perpendicularly to the array: One need only to replace the phase shift Eq. (3) with the shift arisen from the magnetic field.

In conclusion, we developed the theory of electronic properties of the 2D periodic array of interconnected ballistic QRs interacting with an off-resonant circularly polarized high-frequency electromagnetic wave (dressing field). It was demonstrated that the Aharonov-Bohm effect induced by the dressing field substantially modifies the electron energy spectrum of the array, opening band gaps and producing edge states—both topologically protected and unprotected—within the field-induced gaps. As a result, the light-induced topological insulator appears. The present theory paves the way to optical control of the electronic properties of QR-based mesoscopic structures.

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