Vanishing Hall conductance in the phase-glass Bose metal at zero temperature

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Motivated in part by numerical simulations [H. G. Katzgraber and A. P. Young, Phys. Rev. B **66**, 224507 (2002); J. M. Kosterlitz and N. Akino, Phys. Rev. Lett. **81**, 4672 (1998); **81**, 4672 (1998)] that reveal that the energy to create a defect in a gauge or phase glass scales as L^θ with $\theta < 0$ for two dimensions, thereby implying a vanishing stiffness, we reexamine the relevance of these kinds of models to the Bose metal in light of the new experiments [N. P. Breznay and Kapitulnik (unpublished); Y. Wang, I. Tamir, D. Shahar, and N. P. Armitage, arXiv:1708.01908 [cond-mat.supr-con]], which reveal that the Hall conductance is zero in the metallic state that disrupts the transition from the superconductor to the insulator in two-dimensional (2D) samples. Because of the particle-hole symmetry in the phase-glass model, we find that bosonic excitations in a phase-glass background generate no Hall conductance at the Gaussian level. Furthermore, this result persists to any order in perturbation theory in the interactions. We show that when particle-hole symmetry is broken, the Hall conductance turns on with the same power law as does the longitudinal conductance. This prediction can be verified experimentally by applying a ground plane to the 2D samples.

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Because of the canonical relationship between phase and particle number, bosons are traditionally thought to either condense in an eigenstate of phase (superconducting) or insulate as dictated by particle number eigenstates. Indeed, the initial experiments [1–3] seemed to conform to the predictions [4] of the phase-only XY model that only at the critical point do bosons exhibit the quantum of resistance of $h/4e^2$. However, subsequent experiments [5–8] indicated that there is nothing special about the value of the resistance at the critical point, thereby calling into question the relevance or accuracy of the prediction of the phase-only model that only bosons on the brink of localization conduct with the quantum of resistance for charge $2e$ carriers. More importantly, since 1989 [8–21], a state with apparent finite $T \to 0$ resistivity appeared immediately upon the destruction of superconductivity. Although questions of thermometry were raised regarding the initial [5] observation, the leveling of the resistance persisted in the magnetic-field tuned transition in MoGe [9,13,18], Ta [10,22], InO_x [19,23], and $NbSe_2$ [16,20]. The key contribution of the magnetic-field tuned data was to clarify that the intervening state occurred well below H_{c2}. Consequently, if these observations constitute a true metallic state at $T = 0$, the charge carriers must be $2e$ bosons that lack phase coherence. As a result, the insulator above H_{c2} is mediated by the breaking of the Cooper pairs.

The newer observations of the Bose metal in cleaner samples with either gate [12] or magnetic-field tuning [20] tell us three things. First, in the field-effect transistors [12] composed of ion-gated ZrNCl crystals, the superconducting state that obtains for gate voltages exceeding 4 V is destroyed [12] for perpendicular magnetic fields as low as $0.05T$. The authors [12] attribute this behavior to weak pinning of vortices, and hence they reach the conclusion that throughout most of the vortex state, be it a liquid or a glass, a metallic state obtains. Second, in the $NbSe_2$ samples, essentially crystalline materials, the resistance turns on [20] continuously as $\rho \approx (g - g_c)^\alpha$, where g_c is the critical value of the tuning parameter for the onset of the metallic state. Similar results have also been observed in MoGe [14]. Third, in InO_x and TaN_x, the Hall conductance is observed [23] to vanish throughout the Bose metallic state, thereby indicating that particle-hole symmetry is an intrinsic feature of this state. In strong support of this last claim are the recent experiments demonstrating that the cyclotron resonance vanishes in the Bose metallic state [24].

While there have been numerous proposals for a Bose metal [25–30], a state with a finite resistance at $T = 0$, the new experiments greatly constrain possible theoretical descriptions. In light of the new experimental findings, we reexamine the phase-glass model we proposed several years ago [25,31,32], which we demonstrated, using the Kubo formula in the collision-dominated (or hydrodynamic) regime, to have a finite $T \to 0$ resistivity that turns on as $\rho \approx (g - g_c)^\alpha$, as highlighted in the experiments on $NbSe_2$ [20]. While questions [33] regarding the phase stiffness of the phase glass have been raised, numerical simulations all indicate [34–36] that the energy to create a defect in a two-dimensional (2D) phase or gauge glass scales as L^θ, where $\theta = -0.39$. Hence, the stiffness is nonexistent. In three dimensions [34–36], $\theta > 0$ and a stiffness obtains. Consequently, such glass states are candidates to explain the vortex glass [37,38]. In addition, $\theta < 0$ [34–36] in 2D is consistent with the experimental finding [12] in ion-gated ZrNCl, an extreme 2D system, and that the resultant vortex state is indeed metallic and not a true superconductor.

In this paper, we show that the Hall conductance in the phase-glass model vanishes as observed experimentally as a result of an inherent particle-hole symmetry. As shown previously [29], any amount of dirt in a 2D superconductor induces $\pm J$ disorder, where J is the Josephson coupling. Consequently, a disordered superconductor is closer to a disordered XY model rather than a dirty superfluid. Justifiably, the starting point for analyzing the experiments is the disordered XY model. Since we wish to calculate the Hall conductance from the Kubo formula, we consider

$$H = -E_C \sum_i \left(\frac{\partial}{\partial \theta_i}\right)^2 - \sum_{\langle i,j\rangle} J_{ij}\cos(\theta_i - \theta_j - A_{ij}), \quad (1)$$

the phase glass in a perpendicular magnetic field, where $A_{ij} = e^*/\hbar \int_i^j \mathbf{A} d\mathbf{l}$ ($e^* = 2e$), E_C is the constant on site, and J_{ij} is the strength of the Josephson couplings, which are randomly distributed according to $P(J_{ij}) = 1/\sqrt{2\pi J^2}\exp[(J_{ij} - J_0)^2/2J^2]$, with nonzero mean J_0. In terms of the phase on each island, we introduce the vector $S_i = (\cos(\theta_i), \sin(\theta_i))$. This will allow us to to recast the interaction term in the random Josephson Hamiltonian as a spin problem with random magnetic interactions, $\sum_{\langle i,j\rangle} J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j$. Let $\langle\cdots\rangle$ and $[\cdots]$ represent averages over the thermal degrees of freedom and over the disorder, respectively. In the superconductor, not only $\langle S_{i\nu}\rangle$ but also $[\langle S_{i\nu}\rangle]$ acquire a nonzero value. In the phase (or spin) glass, however, $\langle S_{i\nu}\rangle \neq 0$ but $[\langle S_{i\nu}\rangle] = 0$. As we have shown previously [39,40], the Landau theory for this problem is obtained by using replicas to average over the disorder and the identity $\ln[Z] = \lim_{n\to 0}([Z^n] - 1)/n$ to obtain the zero replica limit. The quartic and quadratic spin-spin interaction terms that arise from the disorder average can be decoupled by introducing the auxiliary real fields,

$$Q^{ab}_{\mu\nu}(\vec{k},\vec{k}',\tau,\tau') = \langle S^a_\mu(\vec{k},\tau)S^b_\nu(\vec{k}',\tau')\rangle \quad (2)$$

and $\Psi^a_\mu(\vec{k},\tau) = \langle S^a_\mu(\vec{k},\tau)\rangle$, respectively. Here the superscripts indicate the replica indices, and the subscripts indicate the components of the spin. To simplify our notation, we will introduce the one-component complex field $\psi^a = (\Psi^a_1,\Psi^a_2)$.

Taking into account the effects of the magnetic field $\mathbf{B} = B\hat{z}$, we will use the Landau gauge $\mathbf{A} = (0,Bx,0)$ and rewrite ψ as a sum over different Landau levels,

$$\psi^a(l,x,y,\omega,p_y) = C^a_{l,p_y}(\omega)\phi_l\left(x - \frac{\hbar p_y}{e^* B}\right)e^{ip_y y}, \quad (3)$$

where ϕ_l is the normalized eigenstate of the harmonic oscillator. The relevant part of the free energy consists of the purely bosonic degrees of freedom and their coupling to the phase-glass sector, which is controlled by the Edwards-Anderson order parameter. This free energy has been derived previously [25,39]. To tailor the expressions to a calculation of the Hall conductance, we expand the ψ degrees of freedom in terms of the Landau levels. The resulting free energy per replica is then

$$\begin{aligned}\mathcal{F}_\psi[C,Q] = &\sum_{a,l,p_y,\omega_n}\left(m_H^2\left(l+\frac{1}{2}\right)+\omega_n^2+m^2\right)\left|C^a_{l,p_y}(\omega_n)\right|^2 \\ &-\frac{1}{\kappa t}\sum_{\substack{a,b,l\\ p_y,\omega_n,\omega_n'}} C^a_{l,p_y}(\omega_n)C^{b*}_{l,p_y}(\omega_n')Q^{ab}(l,p_y,\omega_n,\omega_n') \\ &+\frac{U}{2}\sum_{a,l_i,\omega_{ni},p_{yi}}|\psi^a(l_i,x,y,\omega_{ni},p_{yi})|^4, \end{aligned} \quad (4)$$

where $C^a_{l,m}$ describes bosonic excitations with charge $2e$; κ, t, and U are standard Landau theory parameters [25,40]; m^2 is an inverse correlation length; ω_n are the Matsubara frequencies; and $m_H^2 = \frac{e^*}{c\hbar}B$. We have left the interaction in terms of ψ for simplicity, with $i = (1,2,3,4)$. There is also a contribution to the free energy from terms only proportional to Q. In our analysis, we will only be treating the ψ field dynamically, and so these terms can be ignored. As shown previously [31,40], the spin-glass order parameter is of the form

$$\begin{aligned}Q^{ab}(l,p_y,\omega_1,\omega_2) = &\beta(2\pi)^2\delta_{l,0}\delta_{p_y,0}\big[-\eta|\omega_1|\delta_{\omega_1+\omega_2,0}\delta^{ab} \\ &+\beta\delta_{\omega_1,0}\delta_{\omega_2,0}q^{ab}\big],\end{aligned} \quad (5)$$

where $\eta = 1/\kappa^2\tau$ and q^{ab} is a symmetric ($q^{ab} = q$ for all a,b) ultrametric matrix. Due to the factor of $|\omega_n|$, the dynamic critical exponent of this system is $z = 2$, and as a result particle-hole symmetry is a natural consequence.

Substituting Eq. (5) into the free energy, we obtain

$$\begin{aligned}\mathcal{F}_\psi[C] = &\sum_{a,l,m,\omega_n}\left[m_H^2\left(l+\frac{1}{2}\right)+\omega_n^2+\eta|\omega_n|+m^2\right]\left|C^a_{l,p_y}(\omega_n)\right|^2 \\ &-\beta q^{ab}\sum_{a,b,l,p_y,\omega_n} C^a_{l,p_y}(\omega_n)C^{b*}_{l,p_y}(\omega_n) \\ &+\frac{U}{2}\sum_{a,l,p_y}|\psi^a(l,x,y,p_y)|^4,\end{aligned} \quad (6)$$

where we have shifted $q \to q\kappa t$. The propagator for the Gaussian part of the theory is given by

$$\begin{aligned}G^{ab}(l,p_y,\omega_n) &= G_0(l,\omega_n)\delta^{ab} + \beta G_0^2(l,\omega_n)q^{ab}, \\ G_0(l,\omega_n) &= \left[m_H^2\left(l+\tfrac{1}{2}\right)+\omega_n^2+\eta|\omega_n|+m^2\right]^{-1}.\end{aligned} \quad (7)$$

As is well known, G_0, the propagator in the presence of Ohmic dissipation [41,42], is insufficient to describe the metallic state. Such physics originates from the characteristic double-trace deformation the spin-glass term induces in the full Gaussian propagator, G^{ab}. Note that $G(l,\omega_n)$ is symmetric under $\omega_n \to -\omega_n$. This will be referred to as particle-hole symmetry from here on.

To find the Hall conductance for this system, we will use the Kubo formula

$$\begin{aligned}\sigma_H(i\omega_\nu) = \sigma_{xy}(i\omega_\nu) = &\frac{\hbar}{\omega_\nu}\int d^2(x-x')\int d(\tau-\tau') \\ &\times\frac{\partial^2[Z^n]}{\partial A_x(x,\tau)\partial A_y(x',\tau')}e^{i\omega_\nu(\tau-\tau')}.\end{aligned} \quad (8)$$

For our system, this simplifies to

$$\begin{aligned}\sigma_H(i\omega_\nu) = &\frac{i(e^*m_H)^2}{2\omega_\nu\hbar\beta}\sum_{\substack{a,b,l,l',p_y,p_y',\\ p_y'',\omega_n,\omega_n',\omega_n''}}\int d\tau\, e^{i\omega_\nu\tau}\sqrt{(l+1)(l'+1)} \\ &\times\big\langle\big[C^a_{l,p_y}(\omega_n)C^{a*}_{l+1,p_y}(\omega_n)+C^a_{l+1,p_y}(\omega_n)C^{a*}_{l,p_y}(\omega_n)\big] \\ &\times\big[C^b_{l',p_y'}(\omega_n')C^{b*}_{l'+1,p_y''}(\omega_n'') \\ &-C^b_{l'+1,p_y'}(\omega_n')C^{b*}_{l',p_y''}(\omega_n'')\big]\big\rangle.\end{aligned} \quad (9)$$

At the Gaussian level, p_y has no effect, and so for the following calculations we will suppress it. Using Eq. (7), we then have

$$\sigma_H(i\omega_\nu) = \frac{i(e^* m_H)^2}{2\omega_\nu \hbar \beta} \sum_{a,b,l\omega_n} (l+1) \times [G^{ab}(l,\omega_n)G^{ab}(l+1,\omega_n+\omega_\nu) - G^{ab}(l+1,\omega_n)G^{ab}(l,\omega_n+\omega_\nu)]. \quad (10)$$

It has already been shown that in an array of Josephson junctions without random couplings, the Hall conductance vanishes as $T \to 0$ [43]. As such, we will only be considering terms in Eq. (10) that are proportional q. Since $q^{ab}q^{ab}$ vanish in the $n \to 0$ limit, we will only consider terms involving $q^{ab}\delta^{ab} = q$. After using the δ functions to sum over ω_n, we then have for the Hall conductance

$$\sigma_H = \frac{i(e^* m_H)^2}{2\omega_\nu \hbar} \sum_{a,b,l} (l+1) q [G_0(l,-\omega_\nu)G_0^2(l+1,0) + G_0(l+1,\omega_\nu)G_0^2(l,0) - G_0(l+1,-\omega_\nu)G_0^2(l+1,0) - G_0(l,\omega_\nu)G_0^2(l+1,0)]. \quad (11)$$

Due to the particle-hole symmetry of the propagator, the Hall conductance then vanishes independent of ω_ν and $\lim_{\omega_\nu \to 0} \sigma_H(i\omega_\nu) = 0$. After taking the $T \to 0$ limit, we conclude that at the Gaussian level, the Hall conductance at $T = 0$ vanishes.

We now look at the role of interactions. To do this, we will consider the exact propagator, the exact four-point vertex, and show that even with interactions, the Hall conductance still vanishes as $T \to 0$. In doing this, we are assuming that all effects can be resummed into a new propagator and a new vertex. Taking into account the quartic interaction term in the free energy, we rewrite the exact propagator in the $n \to 0$ limit in the form

$$\mathcal{G}^{ab}(l,\omega_n,p_y) = \widetilde{G}(l,\omega_n,p_y)\delta^{ab} + \beta q^{ab} g(l,p_y)\delta_{\omega_n,0}, \quad (12)$$

where we have split the propagator into a diagonal component involving $\widetilde{G}$ and an off-diagonal component involving g [31]. Since all diagrams have particle-hole symmetry, we can conclude that $\widetilde{G}$ must also have particle-hole symmetry. Since β only couples to the diagonal component in the original free energy, we conclude that the diagonal components of Eq. (12) must be independent of β.

The exact four-point propagator is given by

$$\Gamma(l_i,\omega_j,p_{yk}) = \frac{U}{\beta} \sum_{l_1,\omega_j,p_{yk}} \delta_{\Sigma\omega_i,0}\delta_{\Sigma p_{yk},0} f(l_i,\omega_j,p_k) \times C_{l_1,p_{y1}}(\omega_1)C^*_{l_2,p_{y2}}(\omega_2)C_{l_3,p_{y3}}(\omega_3)C^*_{l_4,p_{y4}}(\omega_4), \quad (13)$$

where the exact form of $f(n_l,\omega_j,p_{yk})$ is unknown, but since any diagram can be rotated or switched, we expect f to be independent of the order of its parameters [31]. Due to the particle-hole symmetry of all diagrams, we can also conclude that $f(n_i,\omega_j) = f(n_i,-\omega_j)$. If we use the exact propagator and the exact vertex, there are two diagrams that contribute to $\sigma_H(i\omega_\nu)$; see Fig. 1.

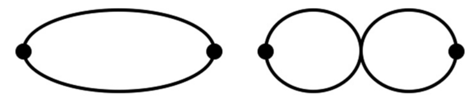

FIG. 1. (a) The nonvertex and (b) vertex diagrams that contribute to the Hall conductance. Here the propagators are the exact propagator, and the vertex is the exact vertex.

The diagram in Fig. 1(a) has a contribution of

$$\sigma_{H1} = \frac{i(e^* m_H)^2}{2\omega_\nu \hbar \beta} \sum_{a,b,l\omega_n,p_y} [\mathcal{G}^{ab} A_1(l)(l,\omega_n,p_y) \times \mathcal{G}^{ab}(l+1,\omega_n+\omega_\nu,p_y)] - [\mathcal{G}^{ab}(l+1,\omega_n,p_y) \times \mathcal{G}^{ab}(l,\omega_n+\omega_\nu,p_y)], \quad (14)$$

where $A_1(l)$ is a dimensionless function of l. Using Eq. (12), we can expand Eq. (14) into terms proportional to $\delta^{ab}\delta^{ab}$, terms proportional to $\beta q^{ab}\delta^{ab}$, and terms proportional to $\beta^2 q^{ab}q^{ab}$. The terms proportional to $\delta^{ab}\delta^{ab}$ vanish in the $T \to 0$ limit, and the $\beta^2 q^{ab}q^{ab}$ terms vanish in the $n \to 0$ limit (which is taken before the $T \to 0$ limit). So the only terms remaining are proportional to $\beta q^{ab}\delta^{ab} = \beta q$. Evaluating these terms, we find that, due to the particle-hole symmetry of $\widetilde{G}$, this diagram does not contribute to the Hall conductance.

Similarly, the diagram in Fig. 1(b) yields

$$\sigma_{H2} = \frac{U(e^* m_H)^2}{2\omega_\nu \hbar \beta^2} \sum_{\substack{a,b,l,l' \\ \omega_n,\omega_n,p_y,p'_y}} A_2(l,l') f(l,l',\omega_n,\omega'_n,p_y,p'_y) \times \mathcal{G}^{ab}(l,\omega_n,p_y)\mathcal{G}^{ab}(l+1,\omega_n+\omega_\nu,p_y) \times \mathcal{G}^{bc}(l'+1,\omega'_n,p'_y)\mathcal{G}^{bc}(l',\omega'_n+\omega_\nu,p'_y) + \mathcal{G}^{ab}(l+1,\omega_n,p_y)\mathcal{G}^{ab}(l,\omega_n+\omega_\nu,p_y) \times \mathcal{G}^{bc}(l'+1,\omega'_n,p'_y)\mathcal{G}^{bc}(l',\omega'_n+\omega_\nu,p'_y), - (l' \leftrightarrow l'+1), \quad (15)$$

where $A_2(l,l')$ is dimensionless and is symmetric in l and l'. For this diagram, let us first fix the values of l and l'. If we then expand the propagators as we did with the propagators in the first diagram and invoke particle-hole symmetry of f and $\mathcal{G}$, we find that the contribution from each l and l' vanishes independently of ω_ν. Thereby, the second diagram also does not contribute to the Hall conductance. From this, we can conclude that the Hall conductance will remain 0 at all levels in perturbation theory. Consequently, the phase-glass model of the Bose metal is consistent with the vanishing of the Hall conductance even in the presence of interactions. In the Appendix, we explicitly carry out the calculations to linear order in U and show that these contributions vanish.

We now consider the effects of breaking the particle-hole symmetry of this system. This can be done by including a term $i\lambda\psi^*\partial_\tau\psi$ in the free energy. This changes the propagator to

$$G_0(l,\omega_n,p_y) = \left[m_H^2\left(l+\tfrac{1}{2}\right)+\omega_n^2+\eta|\omega_n|+i\lambda\omega_n+m^2\right]^{-1}. \quad (16)$$

This term breaks particle-hole symmetry. Without the effects of dissipation ($\eta = 0$), the number of particles at finite temperature is given by $N(\omega^{\pm}) = [\exp(\beta\omega^{\pm}) - 1]^{-1}$, where $\omega^{\pm} = \mp\lambda + \sqrt{\lambda^2 + m^2}$ [43]. Particle-hole symmetry is thereby restored at $\lambda = 0$.

We will first look at the Hall conductance of this system at the Gaussian level (again suppressing p_y). Using the Kubo formula, Eq. (10), we find that

$$\sigma_H(i\omega_\nu) = \frac{\lambda(e^* m_H^2)^2}{\hbar} \sum_{a,b,l} q(l+1)\big[G_0(l,-\omega_\nu)G_0(l,\omega_\nu) \times G_0^2(l+1,0) + G_0(l+1,-\omega_\nu) \times G_0(l+1,\omega_\nu)G_0^2(l,0)\big]. \tag{17}$$

Taking the limits $\omega_\nu \to 0$ and $T \to 0$, and then evaluating the sum, we find

$$\sigma_H(i\omega_\nu) = \frac{\lambda q\left(e^* m_H^2\right)^2}{\hbar m^4}\left(\frac{2}{x} - \frac{\Psi\left(1,\frac{x+2}{2x}\right)}{x^3}\right), \tag{18}$$

where $x = \frac{m_H^2}{m^2}$, and $\Psi(1,x)$ is the first digamma function. In the low-magnetic-field regime ($x \ll 1$), the Hall conductance is approximately

$$\sigma_H(i\omega_\nu) = \frac{\lambda q 4 e^{2*}}{3\hbar m^4}\left(1 + \frac{m_H^2}{m^2}\right). \tag{19}$$

In the high-magnetic-field regime ($x \gg 1$), the Hall conductance is

$$\sigma_H(i\omega_\nu) = \frac{\lambda q e^{*2}}{\hbar m_H^4}\left(2 + \frac{\pi^2 m^2}{m_H^2}\right). \tag{20}$$

So in the case of broken particle-hole symmetry, there is a nonvanishing Hall conductance for all ranges of the magnetic field. The Hall conductance also scales algebraically throughout this range. This is contrary to the results from a nonrandom array of Josephson junctions, where it was shown that the Hall conductance vanishes when $T \to 0$ even in the case of a broken particle-hole [43] term.

In the presence of a broken particle-hole symmetry, the longitudinal conductance of this system is given by

$$\sigma_{xx}(i\omega_\nu) = \frac{\eta q(e^* m_H)^2}{\hbar m^4}\left(\frac{2}{x} - \frac{\Psi\left(1,\frac{x+2}{2x}\right)}{x^3}\right), \tag{21}$$

which is unchanged from the particle-hole symmetric case [31]. Thus if both $\lambda \neq 0$ and $\eta \neq 0$, we see that the longitudinal and Hall conductances have the same algebraic scaling, a falsifiable prediction of this theory.

We will now look at corrections to the Hall conductance arising from quartic interactions at linear order in U. These contributions come from the following diagrams in Fig. 2. The effects from Fig. 2(a) can be expressed as a redefinition of the m term given in the $\omega_\nu \to 0$ and $T \to 0$ limit [32],

$$\widetilde{m}^2 = m^2 + \frac{U q m_H^2}{4\pi} \sum_l \left(G_{l,0}^{(0)}\right)^2 = \frac{U q}{4\pi m_H^2}\Psi(1, x + 1/2). \tag{22}$$

For the high-magnetic-field regime, the correction to the mass in Eq. (22) is approximately $\frac{\pi U q}{8 m_H^2}(1 + \frac{\Psi(2,1/2)m^2}{2\pi m_H^2})$, and in the low-magnetic-field regime it is $\frac{Uq}{4\pi m^2}$.

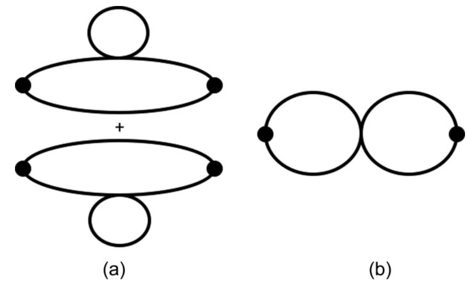

FIG. 2. The two diagrams that contribute to the Hall conductance in the presence of a broken particle-hole symmetry: (a) the loop correction that will be expressed as a rescaling of the mass, and (b) the vertex correction. The propagator shown here is at the Gaussian level, and the vertex is that which appears in the free energy.

To evaluate Fig. 2(b), we will use Eq. (3) to write the interaction term as

$$\Gamma = \frac{U}{2} \sum_{l_i,\omega_j,p_{yk}} C^a_{l_1,p_{y1}}(\omega_1) C^{*a}_{l_2,p_{y2}}(\omega_2) C^a_{l_3,p_{y3}}(\omega_3) C^{*a}_{l_4,p_{y4}}(\omega_4) \times \phi_{l_1}\left(x - \frac{\hbar p_{y1}}{e^* B}\right)\phi_{l_2}\left(x - \frac{\hbar p_{y2}}{e^* B}\right)\phi_{l_3}\left(x - \frac{\hbar p_{y3}}{e^* B}\right) \times \phi_{l_4}\left(x - \frac{\hbar p_{y4}}{e^* B}\right)\delta_{\Sigma\omega_j,0}\delta_{\Sigma p_{yk},0}. \tag{23}$$

Inserting Eq. (23) into Eq. (9) and evaluating the sums and integrating, we find that the contribution is zero due to the orthogonality of ψ_l and ψ_{l+1}. This calculation will be explicitly done in the Appendix. As a result, to linear order in U the only correction to the Hall conductance comes from the rescaling of the mass term.

In conclusion, we have shown that the vanishing of the Hall conductance found in experiments is consistent with the phase-glass model. Even if interactions are considered, the Hall conductance remains zero. This is a consequence of the fact that the system obeys a particle-hole symmetry. However, if the particle-hole symmetry is broken explicitly, we see that there is a nonvanishing Hall conductance that persists even at zero temperature. This finite Hall conductance at zero temperature is a result of both breaking the particle-hole symmetry and the glassy nature of the system. Furthermore, the Hall conductance scales in the same way as the longitudinal conductance of this system and the phase glass with particle-hole symmetry. This falsifiable prediction can be confirmed by ground-plane experiments and should offer a new window into the true nature of the ground state of the Bose metal.

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APPENDIX

Here we will briefly show that the contribution to the Hall conductance is 0 at linear order in U if there is particle-hole symmetry. Furthermore, we will also explicitly show that the vertex diagram, Fig. 2(b), does not contribute independently from the actual form of the Gaussian propagator. The linear order corrections to the Hall conductance are shown in Fig. 2. It has been shown that this loop correction can be expressed as a correction to the mass term of the free energy [32],

$$\delta m^2 = \frac{UTm_H^2}{4\imath} \sum_{\omega_n,l} \left[G_0(l,\omega_n) + \beta\delta_{\omega_n,0} q G_0^2(l,\omega_n)\right]. \tag{A1}$$

Since this affects the particle-hole symmetry of the propagator, the result from the Gaussian level shows that this contribution is zero. We now will explicitly calculate the contribution from the diagram in Fig. 2(b). In full, the contribution to the Hall conductance is

$$\begin{aligned}\sigma_H(i\omega_\nu) = {}& \frac{i(e^* m_H)^2}{2\omega_\nu \hbar\beta} \sum_{\substack{a,b,c,l,l',p_y,p_y',\\ p_y'',\omega_n,\omega_n',\omega_n''}} \int dz \int d\tau e^{i\omega_\nu\tau} \sqrt{(l+1)(l'+1)} \Big\langle \left[C^a_{l,p_y}(\omega_n) C^{a*}_{l+1,p_y}(\omega_n) + C^a_{l+1,p_y}(\omega_n) C^{a*}_{l,p_y}(\omega_n)\right] \\ & \times \left[C^b_{l',p_y'}(\omega_n') C^{b*}_{l'+1,p_y''}(\omega_n'') - C^b_{l'+1,p_y'}(\omega_n') C^{b*}_{l',p_y''}(\omega_n'')\right] \frac{U}{2} \sum_{l_i,\omega_j,p_{yk}} C^c_{l_1,p_{y1}}(\omega_1) C^{*c}_{l_2,p_{y2}}(\omega_2) C^c_{l_3,p_{y3}}(\omega_3) C^{*c}_{l_4,p_{y4}}(\omega_4) \Big\rangle \\ & \times \phi_{l_1}\left(z - \frac{\hbar p_{y1}}{e^* B}\right) \phi_{l_2}\left(z - \frac{\hbar p_{y2}}{e^* B}\right) \phi_{l_3}\left(z - \frac{\hbar p_{y3}}{e^* B}\right) \phi_{l_4}\left(z - \frac{\hbar p_{y4}}{e^* B}\right) \delta_{\Sigma\omega_j,0} \delta_{\Sigma p_{yk},0}.\end{aligned} \tag{A2}$$

We will now fix l and l' and focus on the contribution of

$$\begin{aligned}B(l_i) = {}& \sum_{\substack{a,b,p_y,p_y',\\ p_y'',\omega_n,\omega_n',\omega_n''}} \int dz \Big\langle C^a_{l_1,p_y}(\omega_n) C^{a*}_{l_2,p_y}(\omega_n) C^b_{l_3,p_y'}(\omega_n') C^{b*}_{l_4,p_y''}(\omega_n'') \frac{U}{2} \sum_{l_i',\omega_j,p_{yk}} C^a_{l_1',p_{y1}}(\omega_1) C^{*a}_{l_2',p_{y2}}(\omega_2) C^a_{l_3',p_{y3}}(\omega_3) C^{*a}_{l_4',p_{y4}}(\omega_4) \Big\rangle \\ & \times \phi_{l_1'}\left(z - \frac{\hbar p_{y1}}{e^* B}\right) \phi_{l_2'}\left(z - \frac{\hbar p_{y2}}{e^* B}\right) \phi_{l_3'}\left(z - \frac{\hbar p_{y3}}{e^* B}\right) \phi_{l_4'}\left(z - \frac{\hbar p_{y4}}{e^* B}\right) \delta_{\Sigma\omega_j,0} \delta_{\Sigma p_{yk},0}.\end{aligned} \tag{A3}$$

Equation (A2) is then simply

$$\begin{aligned}\sigma_H(i\omega_\nu) = {}& \frac{i(e^* m_H)^2}{2\omega_\nu \hbar\beta} \sum_{l,l'} \int dz \int d\tau e^{i\omega_\nu\tau} \sqrt{(l+1)(l'+1)} [B(l,l+1,l',l'+1) + B(l+1,l,l',l'+1) \\ & - B(l,l+1,l'+1,l') - B(l+1,l,l'+1,l')].\end{aligned} \tag{A4}$$

Returning to Eq. (A3) and using Wick's theorem and the fact that $\langle C^a_{l,p_y}(\omega) C^{*b}_{l',p_y'}(\omega')\rangle = G^{ab}(l,p_y,\omega)\delta_{l,l'}\delta_{p_y,p_y'}\delta_{\omega,\omega'}$, we then have

$$\begin{aligned}B(l_i) = {}& 2U \sum_{\substack{a,b,p_y,p_y',\\ p_y'',\omega_n,\omega_n',\omega_n''}} \int dz G^{ac}(l_1,p_y\omega_n) G^{ac}(l_2,p_y,\omega_n) G^{bc}(l_3,p_y',\omega_n') G^{bc}(l_4,p_y''\omega_n'') \\ & \times \phi_{l_1}\left(z - \frac{\hbar p_y}{e^* B}\right) \phi_{l_2}\left(z - \frac{\hbar p_y}{e^* B}\right) \phi_{l_3}\left(z - \frac{\hbar p_{y'}}{e^* B}\right) \phi_{l_4}\left(z - \frac{\hbar p_{y''}}{e^* B}\right) \delta_{\omega_n'-\omega_n'',0} \delta_{p_y'-p_y'',0}.\end{aligned} \tag{A5}$$

Summing over p_y, we find that there is a term in the propagator that is independent of p_y (see the expression defining the full propagator in the text). So the only p_y dependence is from the ϕ functions. Therefore,

$$B(l_i) \propto \sum_{p_y} \phi_{l_1}\left(z - \frac{\hbar p_y}{e^* B}\right) \phi_{l_2}\left(z - \frac{\hbar p_y}{e^* B}\right) = \delta_{l_1,l_2}, \tag{A6}$$

where we have used the orthonormality of the eigenfunctions of the harmonic oscillator. Plugging Eq. (A6) into Eq. (A4) we see that $\lim_{\omega_\nu \to 0} \sigma_H(\omega_\nu) = 0$. This is true for any propagator, provided that it is independent of p_y. Since this is true of the phase glass, both with and without particle-hole symmetry, we conclude that Fig. 2(b) does not contribute in either case.

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