## Understanding photon sideband statistics and correlation for determining phonon coherence

Ding Ding,<sup>1,2</sup> Xiaobo Yin,<sup>1</sup> and Baowen Li<sup>1,\*</sup>

<sup>1</sup>Department of Mechanical Engineering, University of Colorado Boulder, Boulder, Colorado 80309, USA <sup>2</sup>Singapore Institute of Manufacturing Technology, 2 Fusionopolis Way, Singapore 138634

(Received 27 March 2017; published 18 January 2018)

Generating and detecting coherent high-frequency heat-carrying phonons have been topics of great interest in recent years. Although there have been successful attempts in generating and observing coherent phonons, rigorous techniques to characterize and detect phonon coherence in a crystalline material have been lagging compared to what has been achieved for photons. One main challenge is a lack of detailed understanding of how detection signals for phonons can be related to coherence. The quantum theory of photoelectric detection has greatly advanced the ability to characterize photon coherence in the past century, and a similar theory for phonon detection is necessary. Here, we reexamine the optical sideband fluorescence technique that has been used to detect high-frequency phonons in materials with optically active defects. We propose a quantum theory of phonon detection using the sideband technique and found that there are distinct differences in sideband counting statistics between thermal and coherent phonons. We further propose a second-order correlation function unique to sideband signals that allows for a rigorous distinction between thermal and coherent phonons. Our theory is relevant to a correlation measurement with nontrivial response functions at the quantum level and can potentially bridge the gap of experimentally determining phonon coherence to be on par with that of photons.

DOI: 10.1103/PhysRevB.97.020302

Phonons are packets of vibrational energy that share many similarities with their bosonic cousins: photons. Advances in nanofabrication have enabled many parallels between the development of photon and phonon control, such as photonic [1] versus phonoic crystals [2], optical [3] versus acoustic metamaterials [4], etc. Active manipulation of electromagnetic waves through light-matter interaction have led to the creation of nanoscale optical emitters [5] and gates [6] and similar progress has been made in controlling phonons using their interaction with matter especially in the realms of optomechanics [7] and phononic devices [8,9].

Just like photons, phonons span a vast frequency range, and, although techniques to control and sense lower-frequency coherent phonons have been well developed [10–19], heatcarrying coherent terahertz (THz) acoustic phonons have been harder to measure directly due to the small wavelength and numerous scattering mechanisms at these small wavelengths [20]. Recently, surface deflection techniques with ultrafast optics have been used to generate and detect phonons close to THz frequencies in materials [21–24]. Nanoscale material structures have also exhibited phonon coherence through their thermal conductivity measurements and predictions [25–31].

In the past, THz crystal phonons have been generated and detected in low-temperature experiments with defect doped crystals [32–34] or with absorption and emission sideband detection [33–38]. Sideband detection is attractive compared to both thermal conductivity measurements and optical deflection techniques due to its ability to directly access atomic length scales where THz phonon wavelengths reside. Furthermore,

sideband signals are universal in almost all materials with optically active defects (such as  $\text{Sm}^{2+}$  doped potassium bromide and chloride [34,36], magnesium fluoride [35], diamond [37], etc.). Last but not least, the energy of the phonons detected can be resolved precisely using an optical spectrometer, allowing for a precise and yet broadband phonon detection.

Sideband spectroscopy has been used widely as a means to detect nonequilibrium phonon population [37], phonon propagation [37], phonon transmission through interfaces [36], phonon band structures [35,39], etc. Extensive theoretical studies in the 1950s have allowed rigorous understanding of sideband line shapes and have been utilized to complement neutron-scattering phonon band-structure measurements [40,41]. Optical coherence has been well characterized using photon counting statistics [42] and the Hanbury-Brown-Twist (HBT) experiment [43] thanks to the rigorous development of the quantum theory of photoelectric detection [44]. However, in this Rapid Communication, we offer a quantum theory describing the sideband phonon detection response that will enable its application in measuring phonon coherence.

In this Rapid Communication, we use the basis of quantum theory of photodetection to rigorously characterize the phonon coherence using sideband detection. We conclude that sideband detection does not give a linear response but a nontrivial function of the phonon number. Using this function, we determine that sideband photon counting statistics distinctly defer between a coherent phonon and a thermal phonon distribution. Then, we propose a sideband HBT interferometer [43] for phonons and define a unique second-order correlation function to detect phonon coherence. It is important to emphasize that a HBT experiment can only be defined rigorously with a quantum theory of the specific detector response. Moreover, our approach can be generalized to other nontrivial detection

2469-9950/2018/97(2)/020302(6)

<sup>\*</sup>baowen.li@colorado.edu



FIG. 1. (a) Schematic of the sideband detection scheme. An incident optical beam gets absorbed and interacts with the local phonon population. The measured spectral properties of the transmitted photons with a grating spectrometer give information of the local phonon population. (b) Energy-level diagram of a defect electron with vibrational transitions for the ground state and excited state. Optical absorption couples the ground to the excited state where the zero-phonon line (ZPL) couples the same vibrational levels ( $\alpha = \beta$ ) whereas any other case  $\alpha \neq \beta$  implies that phonons are absorbed or emitted. The *S* parameter determines the average phonon energy absorbed. (c)  $\langle F(\alpha) \rangle$  versus the *S* parameter and average phonon number  $\langle \alpha \rangle$  for a thermal-state ensemble. (d)  $\langle F(\alpha) \rangle$  versus the *S* parameter and average phonon number  $\langle \alpha \rangle$  for a coherent-state ensemble. Both (c) and (d) have similar behaviors at larger average phonon numbers, but there are noticeable differences between them at lower *S* values.

responses [45] for rigorous quantum treatment of coherence measurements.

Figure 1(a) shows the schematic of the sideband detection setup consisting of a crystal with defects. An incident optical beam excites the electrons in the defects, which absorbs some of the photons. The transmitted light gets collected into a grating spectrometer which diffracts the collected signal by a frequency for detection. Figure 1(b) shows the energy-level diagram for an optically active defect [46]. The ground-state defect electron resides in the crystal field as a harmonic potential consisting of different vibrational levels  $\alpha$ , where  $\alpha$  denotes the phonon number. The excited state is assumed to have the same vibrational frequency and therefore the same harmonic curvature. Optical excitation coupling states of the same vibrational energy  $\alpha = \beta$  between the ground and the excited states is known as the ZPL [47]. The excited-state minimum in general is different from that of the ground state and thus sidebands appear about the ZPL. The additional energy between the ground-state minimum and the excited state assuming a Franck-Condon (FC) optical transition is determined by the Huang-Rhys parameter or the S parameter. Physically, the S parameter equals the average value of phonons absorbed or emitted per optical transition and is a measure of the interaction strength between the defect electron and the lattice phonons. Values of the *S* parameter can vary by from 0.01 to 40 depending on the material of interest [46], and we will consider a wide range of values for the *S* parameter. Relevant information on sideband theory from Refs. [46–48] is summarized in Sec. I of the Supplemental Material [49]. Here, we proceed with an important result derived from Eqs. (S6)–(S9) of the sideband theory which describes the overlap strength between the ground vibrational state  $|\alpha\rangle$  and the excited-state  $|\beta\rangle$  as

$$|\Theta_{\alpha\beta}|^{2} = \exp(-S_{i})\frac{\alpha!}{\beta!}S_{i}^{\beta-\alpha} \left|L_{\alpha}^{\beta-\alpha}(S_{i})\right|^{2}, \qquad (1)$$

where  $S_i$  is the *S* parameter for the *i*th normal mode and the localized excitation and  $L_n^m(z)$ 's are Laguerre polynomials [47]. Equation (1) is used to explain the shapes of absorption or emission spectra in sideband detection experiments [35].

At low temperatures, almost all absorption processes will be Stokes, i.e., the absorbed photon has a higher energy than the ZPL [38]. However, phonons also do not equilibrate so easily at low temperatures due to a lack of temperature-dependent scattering processes. Thus, any nonequilibrium phonons generated can persist for propagation distances up to millimeters, allowing the presence of anti-Stokes signals to be detected [37]. Nevertheless, no rigorous coherence characterizations have been carried out so far except from broadening inferences [37]. First, we recap the quantum theory of photoelectric detection in Sec. II of the Supplemental Material [49] and highlight an important result for the probability of one-photon photodetection at time  $t_0 + \Delta t$  which is  $P(t_0 + \Delta t) \sim \eta \langle \hat{a}_l^{\dagger} \hat{a}_l \rangle \Delta t$ where  $\langle \rangle$  means ensemble average,  $\hat{a}_l, \hat{a}_l^{\dagger}$  are photon annihilation and creation operators such that  $\langle \hat{a}_l^{\dagger} \hat{a}_l \rangle = I$ ,  $\Delta t$  is the measurement duration, and  $\eta$  is the overall experimental detection efficiency taking into account of electron wavefunction overlap among initial and final states, collection area, etc. [44]. This equation tells us that the probability of detecting a photon is directly proportional to its intensity.

We propose a quantum theory for sideband detection of phonons. Physically, two processes happen in sideband detection compared to a single process of electron-photon interaction. As shown in Fig. 1(b), there exists a electronphonon on the ground-state defect electron resulting in a harmonic potential of vibrational states with phonon number  $\alpha$  [46,50]. Then, the ground-state electron gets a FC excitation by an electromagnetic wave to the excited state [46,50]. In this Rapid Communication, we will focus on the one-phonon anti-Stokes absorption sideband which has been used for phonon detection [36]. The one-phonon ( $\beta = \alpha - 1$ ) overlap strength can be obtained from Eq. (1) and is defined as  $F(\alpha) =$  $\sum_{\beta} |\langle \beta | \hat{b}_j | \alpha \rangle|^2 = \exp(-S_i) \alpha |L_{\alpha}^{-1}(S_i)|^2 S_i^{-1}$  here, where  $\alpha$  is the ground vibrational state energy level in Fig. 1(b). *F* will be proportional to the sideband intensity of the one-phonon sideband signal. Figures 1(c) and 1(d) show the ensemble-averaged  $\langle F(\alpha) \rangle$  as a function of the average phonon number and S parameter for a coherent and thermal distribution. Although it is clear that  $\langle F(\alpha) \rangle$  is not a linear function of phonon population due to the oscillatory behavior of the Laguerre polynomials in Eq. (1)  $\langle F(\alpha) \rangle$ , we will show that the slight differences between Figs. 1(c) and 1(d) for the average signal can lead to observable differences in the detected sideband signal.

Phonon fields can be expressed in a quantized form, such as the photon fields [46,51] which describes electron-phonon coupling (see Eq. (S23) of the Supplemental Material [49]). We formulate the interaction Hamiltonian for the crystal phonon to the defect electron's phonon interaction as

$$H_I = \hat{d}\xi_l \hat{a}_l \sum_j \Lambda_j \hat{B}_j(t), \qquad (2)$$

where  $\hat{d}$  is the dipole operator,  $\hat{a}_l = \hat{a}_l e^{-i\omega_l t_1} + \hat{a}_l^{\dagger} e^{i\omega_l t_1}$  such that  $\hat{a}$  is the photon annihilation operators for photon absorption with frequency  $\omega_l$ ,  $\hat{B}_j = \{\hat{b}_j \exp[i(k_j r_0 - \omega_j t)] + \hat{b}_j^{\dagger} \exp[-i(k_j r_0 - \omega_k t)]\}$  for the defect phonon operators of the *j*th modes [35], and  $\xi_l$  and  $\Lambda_j$  are the products for the proportionality constant of the photon and phonon fields (defined in Eqs. (S16) and (S23) of the Supplemental Material [49]), respectively. This derivation is one dimensional but can easily be generalized to three dimensions. Also, we do not consider the polarization dependence of the lattice modes or the photon fields in this derivation for simplicity of illustrating the concept.

We can now relate our result to Eq. (1) for the absorption spectrum by using the anti-Stokes one-phonon overlap  $F(\alpha)$ .

After some algebra (Eqs. (S24) and (S35) of the Supplemental Material [49]), our sideband detection probability in Eq. (S31) is simplified further to

$$P(t,\Delta t) = \eta F(\alpha) \Delta t \,\delta[\omega_l - (\omega_b - \omega_a) + \omega_j], \qquad (3)$$

where  $\eta$  is once again the overall detection efficiency defined in Eq. (S36) of the Supplemental Material [49].

Now, let us consider the case of phonon sideband detection where we are not directly measuring the phonon intensity  $\alpha$  (Eqs. (S20)–(S22) of the Supplemental Material [49]) but a function of the sideband photon count  $F(\alpha)$  described by Eq. (3). Then, sideband count probability p for the sideband signal intensity can be formulated as Eq. (S37). In the long-time limit,  $\langle f \rangle \approx \eta \langle F \rangle T$  such that

$$p(f,t,T) \approx \frac{1}{f!} (\eta \langle F \rangle T)^f \exp(-\eta \langle F \rangle T), \qquad (4)$$

just like the *n* photon probability except that it is not a direct function of the phonon number  $\alpha$ . In the short-time limit,  $\langle f \rangle_{\alpha} \approx \eta F(t)T$  but since  $P(\alpha)$  is a discrete distribution of the phonon ensemble, we sum over all states  $\alpha$  to obtain

$$p(f,t,T) = \sum_{\alpha} \frac{1}{f!} [\eta F(\alpha)T]^{f} \exp[-\eta F(\alpha)T]P(\alpha)$$
$$= \left\langle \frac{1}{f!} (\eta FT)^{f} \exp(-\eta FT) \right\rangle_{\alpha}.$$
(5)

The difference between the sideband detection probability and the photon detection probability in Eqs. (S20)–(S22) of the Supplemental Material [49] is in the measured variable, but they share the same physical principle where short-time limit fluctuations arise due to the ensemble average of the whole detection probability rather than just the measured quantity. Here, we note the similar feature where a coherent state will lead to the same result in the short- and long-time limits [Eqs. (5) and (4)] but not for the thermal state.

Figure 2 shows the probability distribution p(f) for different values of the sideband count f with different time-scales  $\eta T$  under a coherent or thermal ensemble with an average phonon number of  $\langle \alpha \rangle = 10$ . Here we arbitrarily choose the value of  $\eta T$  to account for the short-time ( $\eta T = 0.1$ ) to the long-time limit ( $\eta T = 10$ ). Under these cases, we obtain two limits for the coherent ensemble and compare the thermal ensemble for different values of the S parameter. First, we notice that the S parameter strongly affects the sensitivity of the difference between a coherent and a thermal state. A small S parameter will lead to a virtually undetectable difference between the two. Physically, a small S parameter means less average phonons absorbed or emitted and thus less sensitivity. Second, higher values of S do not imply a greater difference in signal between coherent and thermal states. This is caused by the oscillatory nature of the Laguerre polynomials in Eq. (1). Third, coherent and thermal states show different probabilities in both the short- and the long-time limit unlike the case for *n*-photon probability. This is because the function  $\langle F(\alpha) \rangle$ will have different average values in Eq. (4) [evident from Figs. 1(c) and 1(d)] between a thermal and a coherent state for the same average phonon number even in the long-time limit. It is important to see that the probability of sideband detection for



FIG. 2. (a)–(c) Plots of Eqs. (5) and (4) labeled as short (blue) and long (red), respectively. The coherent-state ensemble is labeled with a cross. The probability of sideband detection for coherent phonons is orders of magnitude lower than thermal phonons just at a few sideband counts. The short- and long-time limits do not show distinct variation in probability except for the intermediate value of the *S* parameter. The average phonon number used here is  $\langle \alpha \rangle = 10$ .

coherent phonons is orders of magnitude lower than thermal phonons at just a few sideband counts. Photon probability measurements have been used to characterize coherent and nonclassical states [42], and we can likewise carry out similar measurements for sideband phonon detection.

A more well-known technique to characterize optical coherence is using a HBT experiment [43,44], and it is conceivable that a similar setup in Fig. 3(a) for sideband phonon detection is possible. Here, an optical excitation generates a phonon beam which becomes incident on a smooth oblique interface and gets partially scattered and transmitted. Two optical beams incident on different locations of the crystal then are used to create two sideband signals which are correlated with each other. Before examining this proposal further, let us first recap how the photon intensity correlation function looks

$$g^{(2)}(t,t+\tau) = \frac{\langle \hat{a}_l^{\dagger}(t)\hat{a}_l^{\dagger}(t+\tau)\hat{a}_l(t+\tau)\hat{a}_l(t)\rangle}{\langle \hat{a}_l^{\dagger}(t)\hat{a}_l(t)\rangle^2}.$$
 (6)



FIG. 3. (a) Schematic of the proposed measurement of the second-order phonon correlation using sideband detection. An optical excitation generates a phonon beam which gets split by a clean interface into two paths. Detection on each path with the sideband technique allows us to measure the second-order correlation defined in Eq. (7). (b)–(d) Second-order sideband correlation for different values of the *S* parameter and average phonon number  $\langle \alpha \rangle$  at  $\tau = 0$ . The red line indicates the coherent ensemble, and the blue line indicates the thermal ensemble. Note that the thermal state is always above the coherent state for all values of average phonon number  $\langle \alpha \rangle$  and *S* parameters.

The numerator in the intensity correlation in Eq. (6) is ordered with  $\hat{a}_l^{\dagger}$  before  $\hat{a}_l$  and time ordered such that a state  $\langle \alpha | \hat{a}_l^{\dagger} \hat{a}_l^{\dagger} \rightarrow \langle \alpha - 2 |$  (the same for  $\hat{a}_l \hat{a}_l | \alpha \rangle$ ), which implies two consecutive absorption processes by two photodetectors from the same state. The denominator just implies independent absorption events. For our case, we are measuring  $F(\alpha)$ , which is a function of the phonon number. This function projects the phonon number onto a function and prevents us from defining our second-order correlation in the same manner as in Eq. (6). Nevertheless, we can define a second-order correlation that describes the same physical process of correlated absorption events versus independent events. In this case, we have

$$g^{(2)}(t,t+\tau) = \frac{\sum_{\alpha} P(\alpha) \sum_{\beta,\beta'} \langle \alpha | \hat{b}_{j}^{\dagger}(t) | \beta \rangle \langle \beta | \hat{b}_{j}(t) | \alpha \rangle \langle \alpha - 1 | \hat{b}_{j}(t+\tau) | \beta' \rangle \langle \beta' | \hat{b}_{j}^{\dagger}(t+\tau) | \alpha - 1 \rangle}{\langle F[\alpha(t)] \rangle_{\alpha}^{2}}$$
$$= \frac{\langle F[\alpha(t)] F(\alpha[t+\tau) - 1] \rangle_{\alpha}}{\langle F[\alpha(t)] \rangle_{\alpha}^{2}}$$
(7)

Figure 3 plots the second-order correlation for the sideband detection defined in Eq. (7) at  $\tau = 0$  for different values of the *S* parameters. Note that we have the thermal ensemble being above the coherent ensemble for all values of average phonon intensity  $\langle \alpha \rangle$  for different values of the *S* parameter. This is in agreement with the behavior of the photon intensity correlation at time  $\tau = 0$  in Eq. (6). However, the photon intensity or other parameters unlike our sideband correlation here, which can vary by orders of magnitude for different *S* parameters and average phonon numbers.

Our Rapid Communication differs from those in optomechanics and nonlinear coherent phonon control [52]. Optomechanics typically requires coupling a specific mechanical mode that can be controlled coherently. It is remarkable that quantum coherence of phonons has been predicted [53-55] and observed [7] in this field. Here, we are proposing a detection scheme for intrinsic phonon modes in materials which are usually not coherent. Also, we only restrict our discussion to coherent and thermal states, although it is possible to consider other quantum states, such as Fock states and squeezed states [53–55]. For the field of nonlinear coherent phonon generation, an optical field directly couples to optical phonons [52] or zone-center acoustic phonons [56], and as a result of the phase matching, always results in coherent phonons being observed. Our Rapid Communication actually detects high-frequency acoustic phonons which are not capable of direct coupling to light through phase matching. Recent work of sharing some similarity to ours include phonon-mediated gate operations using defects in nitrogen-vacancy centers [57] and characterizing phonon coherence in thermal transport using correlation functions [27]. It is thus evident that characterizing high-frequency coherent acoustic phonons in materials using quantum-mechanical descriptions are only starting to be explored.

The low detection probability in Fig. 2 makes detecting beyond a sideband counts challenging especially in the shorttime limit. For instance, sideband detection of phonons in diamond [37] yields a detection probability of  $\sim 10^{-19}$ . However, detecting such small signals has already been demonstrated decades ago [33-38]. The proposed HBT experiment requires the phonons to propagate long distances and a crystal with a clean interface. This is however achievable given the low temperature of the experiment and the ability to shape and make good quality crystal interfaces [58]. Distortions to phonon propagation may occur at the interface, but we just need some of the phonons of the same frequency to persist after scattering and transmission through interfaces [59,60]. Last but not least, small nanocrystals can be used as local phonon detectors similar to Refs. [15,61] where sideband signals can be used to determine phonon coherences in optically inert materials of interest. Thus, it is foreseeable that the feasibility of our proposal is within experimental reach.

In conclusion, we propose a quantum theory of phonon detection using the sideband technique and found that distinct differences in sideband counting statistics between thermal and coherent phonons. We further propose a second-order correlation function unique to sideband signals that allows for rigorous distinction between thermal and coherent phonons. Our theory is relevant to correlation measurement with nontrivial response functions at the quantum level and potentially can bridge the gap of experimentally determining phonon coherence to be on par with that of photons.

- J. D. Joannopoulos, P. R. Villeneuve, and S. Fan, Nature (London) 386, 143 (1997).
- [2] R. H. Olsson III and I. El-Kady, Meas. Sci. Technol. 20, 012002 (2009).
- [3] W. Cai and V. Shalaev, *Optical Metamaterials* (Springer, New York, 2010).
- [4] G. Ma and P. Sheng, Sci. Adv. 2, e1501595 (2016).
- [5] M. Willander, J. Phys.: Conf. Ser. 486, 012030 (2014).
- [6] W. Chen, K. M. Beck, R. Bücker, M. Gullans, M. D. Lukin, H. Tanji-Suzuki, and V. Vuletic, Science 341, 768 (2013).
- [7] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Rev. Mod. Phys. 86, 1391 (2014).
- [8] N. Li, J. Ren, L. Wang, G. Zhang, P. Hanggi, and B. Li, Rev. Mod. Phys. 84, 1045 (2012).
- [9] H. Han, B. Li, S. Volz, and Y. A. Kosevich, Phys. Rev. Lett. 114, 145501 (2015).
- [10] M. Ikezawa, T. Okuno, Y. Masumoto, and A. A. Lipovskii, Phys. Rev. B 64, 201315 (2001).
- [11] N. D. Lanzillotti-Kimura, A. Fainstein, A. Huynh, B. Perrin, B. Jusserand, A. Miard, and A. Lemaitre, Phys. Rev. Lett. 99, 217405 (2007).

- [12] K. Vahala, M. Herrmann, S. Knünz, V. Batteiger, G. Saathoff, T. W. Hansch, and T. Udem, Nat. Phys. 5, 682 (2009).
- [13] T. J. Grimsley, F. Yang, S. Che, G. A. Antonelli, H. J. Maris, and A. V. Nurmikko, J. Phys.: Conf. Ser. 278, 012037 (2011).
- [14] S. Hong, M. S. Grinolds, P. Maletinsky, R. L. Walsworth, M. D. Lukin, and A. Yacoby, Nano Lett. 12, 3920 (2012).
- [15] Y. Tian, P. Navarro, and M. Orrit, Phys. Rev. Lett. 113, 135505 (2014).
- [16] L. Wang, S. Takeda, C. Liu, and N. Tamai, J. Phys. Chem. C 118, 1674 (2014).
- [17] S. Yoshino, G. Oohata, and K. Mizoguchi, Phys. Rev. Lett. 115, 157402 (2015).
- [18] S. Volz, J. Ordonez-Miranda, A. Shchepetov, M. Prunnila, J. Ahopelto, T. Pezeril, G. Vaudel, V. Gusev, P. Ruello, E. M. Weig, M. Schubert, M. Hettich, M. Grossman, T. Dekorsy, F. Alzina, B. Graczykowski, E. Chavez-Angel, J. S. Reparaz, M. R. Wagner, C. M. Sotomayor-Torres, S. Xiong, S. Neogi, and D. Donadio, Eur. Phys. J. B 89, 15 (2016).
- [19] K. Shinokita, K. Reimann, M. Woerner, T. Elsaesser, R. Hey, and C. Flytzanis, Phys. Rev. Lett. 116, 075504 (2016).
- [20] G. Chen, Nanoscale Energy Transport and Conversion: A Parallel Treatment of Electrons, Molecules, Phonons, and Photons (Oxford University Press, New York, 2005).
- [21] A. J. Kent, N. M. Stanton, L. J. Challis, and M. Henini, Appl. Phys. Lett. 81, 3497 (2002).
- [22] A. J. Kent, R. N. Kini, N. M. Stanton, M. Henini, B. A. Glavin, V. A. Kochelap, and T. L. Linnik, Phys. Rev. Lett. 96, 215504 (2006).
- [23] J. Cuffe, O. Ristow, E. Chávez, A. Shchepetov, P.-O. Chapuis, F. Alzina, M. Hettich, M. Prunnila, J. Ahopelto, T. Dekorsy, and C. M. Sotomayor Torres, Phys. Rev. Lett. **110**, 095503 (2013).
- [24] A. A. Maznev, F. Hofmann, A. Jandl, K. Esfarjani, M. T. Bulsara, E. A. Fitzgerald, G. Chen, and K. A. Nelson, Appl. Phys. Lett. 102, 041901 (2013).
- [25] M. N. Luckyanova, J. Garg, K. Esfarjani, A. Jandl, M. T. Bulsara, A. J. Schmidt, A. J. Minnich, S. Chen, M. S. Dresselhaus, Z. Ren, E. A. Fitzgerald, and G. Chen, Science 338, 936 (2012).
- [26] J. Ravichandran, A. K. Yadav, R. Cheaito, P. B. Rossen, A. Soukiassian, S. J. Suresha, J. C. Duda, B. M. Foley, C.-H. Lee, Y. Zhu, A. W. Lichtenberger, J. E. Moore, D. A. Muller, D. G. Schlom, P. E. Hopkins, A. Majumdar, R. Ramesh, and M. A. Zurbuchen, Nat. Mater. 13, 168 (2014).
- [27] B. Latour, S. Volz, and Y. Chalopin, Phys. Rev. B 90, 014307 (2014).
- [28] Y. Wang, H. Huang, and X. Ruan, Phys. Rev. B 90, 165406 (2014).
- [29] S. Alaie, D. F. Goettler, M. Su, Z. C. Leseman, C. M. Reinke, and I. El-Kady, Nat. Commun. 6, 7228 (2015).
- [30] X. Mu, L. Wang, X. Yang, P. Zhang, A. C. To, and T. Luo, Sci. Rep. 5, 16697 (2015).

PHYSICAL REVIEW B 97, 020302(R) (2018)

- [31] X. Mu, T. Zhang, D. B. Go, and T. Luo, Carbon 83, 208 (2015).
- [32] K. F. Renk and J. Deisenhofer, Phys. Rev. Lett. 26, 764 (1971).
- [33] K. F. Renk, in 1979 Ultrasonics Symposium, New Orleans, LA, USA (IEEE, New York, 1979), pp. 427–434.
- [34] W. E. Bron, Rep. Prog. Phys. 43, 301 (1980).
- [35] M. Mostoller, B. N. Ganguly, and R. F. Wood, Phys. Rev. B 4, 2015 (1971).
- [36] W. E. Bron and W. Grill, Phys. Rev. B 16, 5303 (1977).
- [37] H. Schwartz and K. F. Renk, Solid State Commun. 54, 925 (1985).
- [38] M. N. Wybourne and J. K. Wigmore, Rep. Prog. Phys. 51, 923 (1988).
- [39] A. Alkauskas, B. B. Buckley, D. D. Awschalom, and C. G. V. d. Walle, New J. Phys. 16, 073026 (2014).
- [40] D. H. Kühner, H. V. Lauer, and W. E. Bron, Phys. Rev. B 5, 4112 (1972).
- [41] A. Man and W. E. Bron, Phys. Rev. B 13, 5591 (1976).
- [42] F. T. Arecchi, Phys. Rev. Lett. 15, 912 (1965).
- [43] R. H. Brown and R. Q. Twiss, Nature (London) 177, 27 (1956).
- [44] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics*, 1st ed. (Cambridge University Press, New York, 1995).
- [45] P. Hänggi and H. Thomas, Z. Phys. B 26, 85 (1977).
- [46] Y. Toyozawa, *Optical Processes in Solids* (Cambridge University Press, Cambridge, UK, 2003).
- [47] T. H. Keil, Phys. Rev. 140, A601 (1965).
- [48] M. Lax, J. Chem. Phys. 20, 1752 (1952).
- [49] See Supplemental Material at http://link.aps.org/supplemental/ 10.1103/PhysRevB.97.020302 for details on derivation of Eqs. (3) and (4).
- [50] B. Henderson and G. F. Imbusch, *Optical Spectroscopy of Inorganic Solids* (Clarendon Press, Oxford, 1989).
- [51] M. Wagner, Z. Phys. A 214, 78 (2005).
- [52] M. Kozák, F. Trojánek, P. Galář, M. Varga, A. Kromka, and P. Malý, Opt. Express 21, 31521 (2013).
- [53] X. Hu and F. Nori, Phys. Rev. B 53, 2419 (1996).
- [54] X. Hu and F. Nori, Physica B 263, 16 (1999).
- [55] D. Hu, S.-Y. Huang, J.-Q. Liao, L. Tian, and H.-S. Goan, Phys. Rev. A 91, 013812 (2015).
- [56] K. S. Olsson, N. Klimovich, K. An, S. Sullivan, A. Weathers, L. Shi, and X. Li, Appl. Phys. Lett. **106**, 051906 (2015).
- [57] A. Albrecht, A. Retzker, F. Jelezko, and M. B. Plenio, New J. Phys. 15, 083014 (2013).
- [58] H. Ichinose, Sci. Technol. Adv. Mater. 1, 11 (2000).
- [59] C. Höss, J. P. Wolfe, and H. Kinder, Phys. Rev. Lett. 64, 1134 (1990).
- [60] J. P. Wolfe, Phys. Today **33**(12), 44 (2008).
- [61] A. Laraoui, H. Aycock-Rizzo, Y. Gao, X. Lu, E. Riedo, and C. A. Meriles, Nat. Commun. 6, 8954 (2015).