

Heat currents in electronic junctions driven by telegraph noiseO. Entin-Wohlman,^{1,2,*} D. Chowdhury,^{2,†} A. Aharony,^{1,2} and S. Dattagupta^{3,4}¹*Raymond and Beverly Sackler School of Physics and Astronomy, Tel Aviv University, Tel Aviv 69978, Israel*²*Physics Department, Ben-Gurion University, Beer Sheva 84105, Israel*³*Jawaharlal Nehru Centre for Advanced Scientific Research, Bangalore 560064, India*⁴*Bose Institute, Kolkata 700054, India*

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The energy and charge fluxes carried by electrons in a two-terminal junction subjected to a random telegraph noise, produced by a single electronic defect, are analyzed. The telegraph processes are imitated by the action of a stochastic electric field that acts on the electrons in the junction. Upon averaging over all random events of the telegraph process, it is found that this electric field supplies, on the average, energy to the electronic reservoirs, which is distributed unequally between them: the stronger is the coupling of the reservoir with the junction, the more energy it gains. Thus the noisy environment can lead to a temperature gradient across an unbiased junction.

DOI: [10.1103/PhysRevB.96.195435](https://doi.org/10.1103/PhysRevB.96.195435)**I. INTRODUCTION**

Nanoscale electronic devices driven by time-dependent fields (besides being subjected to stationary voltages and/or temperature gradients) are currently attracting considerable attention due to the possibility to control quantum-coherent charge and heat dynamics in the time domain. Experimental endeavors aimed to manipulate quantum capacitors [1], flying single electrons [2] and other charge excitations [3], or to perform fast thermometry [4,5] are conspicuous examples. These systems are also the topic of numerous theoretical studies, in which the time-dependent sources are oscillating electric fields [6–10], periodic temperature variations [11], and periodic time-dependent hybridizations of the mesoscopic system and its reservoirs [12,13]. (See Ref. [14] for an extensive review.)

The precise definition of energy and heat currents in the time domain in such junctions, which comprise nanoscale devices that accommodate a relatively small number of electrons coupled to macroscopic reservoirs, is a fascinating theoretical subject [15,16]. A meticulous analysis can be carried out when the external force operating on the mesoscopic system is slowly varying in time [16,17], particularly in the regime of adiabatic quantum pumping [6,18,19], where proper response coefficients can be derived via the Kubo formula [20].

An intriguing feature concerning heat transport in the time domain is the role played by the energy flux associated with the term in the Hamiltonian that couples the nanostructure with the bulky reservoirs [13,15]. While the coupling part does not contribute directly to the particle flux, it does store energy momentarily, even when the hybridization between the nanosystem and the reservoirs does not vary with time. The generic Hamiltonian in which the time dependence is confined to the nanosystem (see Fig. 1) is

$$\mathcal{H} = \mathcal{H}_{\text{leads}} + \mathcal{H}_{\text{sys}}(t) + \mathcal{H}_{\text{tun}}, \quad (1)$$

where \mathcal{H}_{tun} pertains to the coupling between the nanosystem [whose Hamiltonian is $\mathcal{H}_{\text{sys}}(t)$] and the reservoirs (described

by $\mathcal{H}_{\text{leads}}$). The operator of the particles' current, say of the left lead, is simply the time derivative of the particles' number operator in that lead. On the other hand, the operator of the total energy flux is the time derivative of the total Hamiltonian. It comprises the energy flux associated with the particles in each of the leads: the one associated with the coupling and the one arising from the time derivative of \mathcal{H}_{sys} . The latter consists of the energy flux of the particles residing on the nanosystem and the explicit time derivative of the applied force, which amounts to the power supplied to the junction by the time-dependent field. Thus, each of these fluxes has an intuitive meaning except perhaps for the flux arising from the tunneling term. While being a negligible contribution in macroscopic systems, it becomes comparable to the other energy fluxes in the nanoscale, where the surface-to-volume ratio is not vanishingly small. The proposal to regard this energy flux as part of the time-dependent heat current of the corresponding reservoir [9,21,22], based on a comparison between the Green's function and the scattering approaches for calculating averaged quantities of noninteracting electrons [23], is still debated [12,17,24,25].

The lion's share of the theoretical literature on thermoelectric phenomena in mesoscopic junctions in the time domain focuses on periodic ac fields, preferably of frequencies that are low compared to the inverse time it takes the carriers to traverse the sample [18,26]. Here we consider charge and energy currents in a nanodevice subjected to a time-dependent source which is a noisy environment, described by the so-called telegraph process, or telegraph noise [27,28]. Telegraph noise is believed to result from the (almost unavoidable) presence of defects with internal degrees of freedom (coined "elementary fluctuators") that have two (or more) metastable configurations and can switch between them due to their interaction with a thermal bath (of their own). In many cases, the fluctuations are due to the dynamics of charge carriers trapped at the defects [29–33]. This picture of a noisy environment was widely exploited at the time to study decoherence and dephasing of qubits [34–39].

Charge fluctuations such as the ones that give rise to telegraph noise result in fluctuating electric potentials [40]. The transport of spinless electrons through a junction subjected to a stochastic potential (or field) due to telegraph noise has

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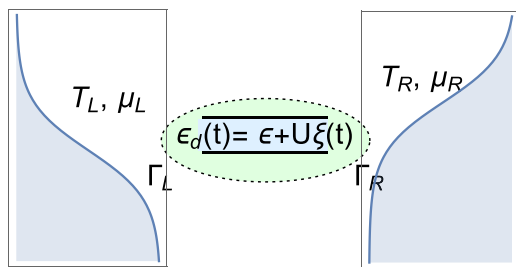


FIG. 1. Schematic picture of the model junction: a localized electronic level is coupled to two electronic reservoirs, held at chemical potentials μ_L and μ_R , at temperatures T_L and T_R , respectively. An electron residing on this level is subjected to a stochastic electric field. This is imitated by a stochastic time dependence of the level energy, $\epsilon_d(t)$. The coupling with the reservoirs causes the level ϵ to become a resonance, of width $\Gamma = \Gamma_L + \Gamma_R$, where Γ_L, Γ_R are the partial widths.

been analyzed in various regimes of the noise characteristics [41,42], but, to our best knowledge, energy fluxes in such junctions were not discussed. Exceptions occur in Ref. [43], which discusses the thermal transport, and in Ref. [44], which focuses on the thermopower in a junction subjected to electromagnetic environment.

Here we present a detailed study of the electrons' energy fluxes in this configuration, carried out for the simplest (but realizable [45]) junction: that of a single localized level (referred to below as a "quantum dot") attached to two reservoirs of spinless electrons. In addition to the possible relevance of this study to the influence of environments on electronic and thermoelectric transport, we also aim at a comparison of these effects with those of periodically oscillating fields.

The effect of the telegraph processes on the transport is embedded in the time dependence of the energy on the dot, $\epsilon_d(t)$, which fluctuates randomly in time (see Fig. 1). It turns out that, within the Keldysh formalism for time-dependent nonequilibrium Green's functions, this simple geometry is amenable to an analytical solution for the particle and the energy fluxes *in the time domain*. This happens when it can be assumed that the densities of states in the electronic reservoirs that supply the electrons are faithfully represented by their value at the respective chemical potentials, an approximation coined "the wideband limit" [46,47]. We show that, once the time-dependent particle and energy fluxes are averaged over the processes of the telegraph noise, they lose their dependence on time and become stationary. In particular, by examining the energy conservation conditions of the junction we obtain the electric power supplied to the junction from the source of the telegraph process. We find that this power is not equally distributed between the two electronic reservoirs: rather the bigger part of the supplied power is absorbed by the reservoir which is connected more strongly to the dot. Thus, even when the electronic reservoirs are at identical temperatures and chemical potentials, the telegraph noise "attempts" to create a temperature difference between them. It is interesting to invoke in this context Ref. [48], which calculates the charge current at zero bias and equal electronic temperatures under the effect of a local ac field. Averaged over a period, the resulting current is

nonzero, as long as the partial resonance widths (Γ_L and Γ_R) on the dot are energy dependent (that is, once the wideband approximation is abandoned).

The calculation of the electronic properties in the time domain, by now rather standard, is well documented in the literature (mainly with the aim of investigating the currents under the effect of oscillatory fields; see, e.g., Ref. [14]). It is summarized briefly in Sec. II and in Appendix A in a way which is particularly suitable for carrying out the average over the telegraph processes. The averaging procedure itself is explained in Sec. III and in Appendix B. After this somewhat technical detour, we return to the fluxes in our junction and present the results for their stationary limits in Sec. IV. Our conclusions are summarized in Sec. V.

II. ENERGY AND PARTICLE FLUXES IN THE TIME DOMAIN

As mentioned, our model system consists of a localized level; hence,

$$\mathcal{H}_{\text{sys}} = \epsilon_d(t) d^\dagger d, \quad (2)$$

where d (d^\dagger) annihilates (creates) an electron on the level, whose energy depends on time. This time dependence need not be specified in this section. The dot is coupled to two electronic reservoirs of spinless electrons by tunneling amplitudes $V_{\mathbf{k}}$ and $V_{\mathbf{p}}$,

$$\mathcal{H}_{\text{tun}} = \sum_{\mathbf{k}} (V_{\mathbf{k}} c_{\mathbf{k}}^\dagger d + \text{H.c.}) + \sum_{\mathbf{p}} (V_{\mathbf{p}} c_{\mathbf{p}}^\dagger d + \text{H.c.}), \quad (3)$$

where $c_{\mathbf{k}(\mathbf{p})}$ ($c_{\mathbf{k}(\mathbf{p})}^\dagger$) are the annihilation (creation) operators for the electrons in the leads; the latter are modeled as free electron gases:

$$\mathcal{H}_{\text{leads}} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}}^\dagger c_{\mathbf{k}} + \sum_{\mathbf{p}} \epsilon_{\mathbf{p}} c_{\mathbf{p}}^\dagger c_{\mathbf{p}}. \quad (4)$$

[We use the wave vector \mathbf{k} (\mathbf{p}) for the states on the left (right) lead.]

The various fluxes in this simple junction, in terms of the Keldysh Green's functions in the time domain, are as follows. The particle flux of the left lead, i.e., the rate of change of the number of particles there, is

$$I_L(t) = \left\langle \frac{d}{dt} \sum_{\mathbf{k}} c_{\mathbf{k}}^\dagger c_{\mathbf{k}} \right\rangle = \sum_{\mathbf{k}} [V_{\mathbf{k}}^* G_{\mathbf{k}d}^<(t,t) - V_{\mathbf{k}} G_{d\mathbf{k}}^<(t,t)]. \quad (5)$$

Here $G_{\mathbf{k}d}^<(t,t')$ is the lesser Green's function; the angular brackets indicate the quantum average. (This notation of the Green's function pertains to all others, e.g, G_{dd} and $G_{\mathbf{k}\mathbf{p}}$.) As usual, one expresses $I_L(t)$ in terms of the Green's functions on the dot [46],

$$I_L(t) = \int dt_1 [\Sigma_L(t,t_1) G_{dd}(t_1,t) - G_{dd}(t,t_1) \Sigma_L(t_1,t)]^<, \quad (6)$$

where $\Sigma_L(t, t')$ is the self-energy due to the tunnel coupling with the left lead,

$$\Sigma_L(t, t') = \sum_{\mathbf{k}} |V_{\mathbf{k}}|^2 g_{\mathbf{k}}(t, t'), \quad (7)$$

and $g_{\mathbf{k}}(t, t')$ is the Green's function of the decoupled left lead. The lesser Green's function of the product in Eq. (6) is found by using the Langreth rules [46]. The particle flux of the right lead is derived from Eqs. (6) and (7) by interchanging $L \leftrightarrow R$ and $\mathbf{k} \leftrightarrow \mathbf{p}$. The flux of particles of the dot itself is then compensated by the sum of the two,

$$I_d(t) = \left\langle \frac{d}{dt} d^\dagger d \right\rangle = -[I_L(t) + I_R(t)]; \quad (8)$$

that is, particle number in the junction is conserved.

Explicitly, within the wideband approximation, the particle current Eq. (6) in the time domain is

$$I_L(t) = 2i\Gamma_L \left(\int \frac{d\omega}{2\pi} f_L(\omega) [G_{dd}^a(\omega, t) - G_{dd}^r(\omega, t)] - G_{dd}^<(\omega, t) \right), \quad (9)$$

where $\Gamma_{L(R)} = \pi |V_{k_F}|^2 \mathcal{N}_{L(R)}$, $\Gamma = \Gamma_L + \Gamma_R$, $\mathcal{N}_{L(R)}$ is the density of states of the left (right) lead at the respective Fermi energy, and

$$f_{L(R)}(\omega) = [e^{(\omega - \mu_{L(R)})/(k_B T_{L(R)})} + 1]^{-1} \quad (10)$$

is the Fermi distribution there. The retarded and advanced Green's functions on the dot are

$$G_{dd}^{r(a)}(\omega, t) = \mp i \int dt' e^{\pm i(t-t')(\omega \pm i\Gamma) \mp i \int_{t'}^t d\tau \epsilon_d(\tau)}. \quad (11)$$

This result is derived in Appendix A. As shown there,

$$G_{dd}^<(\omega, t) = \int \frac{d\omega'}{2\pi} \Sigma^<(\omega') \int dt' e^{-2\Gamma(t-t')} \times [iG_{dd}^r(\omega, t') - iG_{dd}^a(\omega, t')], \quad (12)$$

where

$$\Sigma^<(\omega) = 2i[\Gamma_L f_L(\omega) + \Gamma_R f_R(\omega)]. \quad (13)$$

Equations (9)–(13) completely describe the particle current in the time domain.

We next turn to the energy fluxes that flow in the time domain. First, there is the usual energy flux of the left lead,

$$I_L^E(t) = \left\langle \frac{d}{dt} \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}}^\dagger c_{\mathbf{k}} \right\rangle = \sum_{\mathbf{k}} (\epsilon_{\mathbf{k}} [V_{\mathbf{k}}^* G_{d\mathbf{k}}^<(t, t) - V_{\mathbf{k}} G_{d\mathbf{k}}^<(t, t)]), \quad (14)$$

and the analogous energy flux associated with the right reservoir, derived from Eq. (14) by interchanging $L \leftrightarrow R$ and $\mathbf{k} \leftrightarrow \mathbf{p}$. In terms of the Green's functions on the dot, this energy flux reads

$$I_L^E(t) = \int dt_1 [\Sigma_L^E(t, t_1) G_{dd}(t_1, t) - G_{dd}(t, t_1) \Sigma_L^E(t_1, t)]^<, \quad (15)$$

where $\Sigma_L^E(t, t')$ is

$$\Sigma_L^E(t, t') = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} |V_{\mathbf{k}}|^2 g_{\mathbf{k}}(t, t'). \quad (16)$$

Explicitly, the rate of change of the energy in the left lead [see Eq. (14)], is

$$I_L^E(t) = 2i\Gamma_L \left(\int \frac{d\omega}{2\pi} \omega f_L(\omega) [G_{dd}^a(\omega, t) - G_{dd}^r(\omega, t)] - \epsilon_d(t) G_{dd}^<(t, t) \right) - i\Gamma_L \int \frac{d\omega}{2\pi} \Sigma^<(\omega) [G_{dd}^a(\omega, t) + G_{dd}^r(\omega, t)]. \quad (17)$$

Details of the derivation of this expression, in particular the last term on the right-hand side, are given in Appendix A. The particle and energy fluxes associated with the leads, i.e., Eqs. (6) and (15), are all that is needed to investigate thermoelectric effects in stationary two-terminal electronic junctions [49].

In the time domain, however, there are two additional energy fluxes. The first, which results from the temporal variation of the (left and right) tunneling Hamiltonians, Eq. (3), reads

$$I_{\text{tun},L}^E(t) = \left\langle \frac{d}{dt} \sum_{\mathbf{k}} (V_{\mathbf{k}} c_{\mathbf{k}}^\dagger d + \text{H.c.}) \right\rangle = \epsilon_d(t) I_L(t) - I_L^E(t) + \sum_{\mathbf{k}, \mathbf{p}} [V_{\mathbf{k}}^* V_{\mathbf{p}} G_{\mathbf{k}\mathbf{p}}^<(t, t) - V_{\mathbf{k}} V_{\mathbf{p}}^* G_{\mathbf{p}\mathbf{k}}^<(t, t)], \quad (18)$$

(with an analogous expression for $I_{\text{tun},R}^E$). The calculation of the last term on the right-hand side is carried out in Appendix A. From Eq. (A16) one finds

$$I_{\text{tun},L}^E(t) = \epsilon_d(t) I_L(t) - I_L^E(t) + 2\Gamma_L \Gamma_R \int \frac{d\omega}{2\pi} [f_R(\omega) - f_L(\omega)] \times [G_{dd}^a(\omega, t) + G_{dd}^r(\omega, t)]. \quad (19)$$

The second “new” energy flux is the one that comes from the temporal variation of the dot's Hamiltonian, Eq. (2):

$$I_d^E(t) = \left\langle \frac{d}{dt} \epsilon_d(t) d^\dagger d \right\rangle = -i \frac{d\epsilon_d(t)}{dt} G_{dd}^<(t, t) + \epsilon_d(t) I_d(t). \quad (20)$$

The first term on the right-hand side of Eq. (20) results from the explicit time dependence of the localized energy; i.e., it is due to a time-dependent electric potential acting on the dot. Since the electronic occupation on the dot [46], $Q_d(t)$, is

$$Q_d(t) = -i G_{dd}^<(t, t), \quad (21)$$

this term expresses the power supplied to the system by the field. We denote this power by $P_d(t)$, with

$$P_d(t) = Q_d(t) \frac{d\epsilon_d(t)}{dt}. \quad (22)$$

Combining Eqs. (14), (18), and (20), one finds

$$I_L^E(t) + I_R^E(t) + I_{\text{tun},L}^E(t) + I_{\text{tun},R}^E(t) + I_d^E(t) = P_d(t), \quad (23)$$

which expresses the energy conservation in the junction.

III. AVERAGE OVER THE RANDOM TELEGRAPH PROCESS

The localized energy on the dot in our junction fluctuates with time,

$$\epsilon_d(t) = \epsilon + U\xi(t), \quad (24)$$

where $\xi(t)$ describes a random telegraph process. Telegraph noise is an example of a dichotomous, stationary, and discrete process. Beginning at an initial time t_0 , the function $\xi(t)$ jumps instantaneously between the values $+1$ and -1 at random instants $t_0 < t_1 < t_2 < t_3 < \dots < t$. Each history of the system involves a certain sequence of times at which the jumps occur. In our calculation, we average the time-dependent fluxes (at time t) over all histories; this procedure amounts to averaging over all the time sequences, and over the two possible initial values of the random variable $\xi(t_0)$. The average contains the case with no jump, the case with one jump at any intermediate time between the initial time t_0 and t , the case with two jumps at any intermediate times $t_0 < t_1 < t_2 < t$, etc. The average value of any physical quantity depends only on the time difference, $t - t_0$; due to this property, the averages of the time-dependent fluxes derived in Sec. II become stationary.

In the simplest example, the telegraph noise is characterized by the *a priori* probabilities of the occurrence of $\xi = +1$ (or -1). In the example of the elementary charge fluctuators, each fluctuator is assumed to be in one of two states; these states, which occur with probabilities p_+ or p_- , generate an electrostatic potential $+U$ or $-U$ on the electron which occupies the quantum dot. If the two states of the fluctuator have energies 0 or $E_0 > 0$, and when their occupation is determined by their interaction with a separate heat bath of temperature T , then the probabilities are given by the (normalized) Boltzmann factors [41],

$$p_{\pm}(T) = \frac{\exp[\pm E_0/(2k_B T)]}{2\cosh[E_0/(2k_B T)]}. \quad (25)$$

The ‘‘telegraph noise temperature’’ T is an *effective* temperature that models the probabilities p_{\pm} . Since the model Hamiltonian does not include the interaction between the fluctuator and the dot, nor does it include the back-action from the electrons to the fluctuator, the system is not in equilibrium and there is no meaning in a comparison of T with the temperatures of the electronic reservoirs. At zero telegraph-noise temperature, $T = 0$, this example yields $p_+ = 1$ and hence no fluctuations. The effect of the fluctuations then increases as T is raised. Other models of the telegraph noise give similar results. With these probabilities, the average of $\xi(t)$ is independent of the time:

$$\bar{\xi} = p_+(T) - p_-(T) = \tanh[E_0/(2k_B T)]. \quad (26)$$

(We denote an average over the telegraph process by an overbar to distinguish it from the quantum average, which is indicated by angular brackets.) As $T \rightarrow \infty$, the average $\bar{\xi}$ tends to zero.

In addition to the probabilities p_{\pm} , one also needs to specify the mean rate γ at which the instantaneous jumps occur. Assuming detailed balance, the probability per unit time to jump from $\xi = a (= \pm 1)$ to $\xi = b (= \mp 1)$ is $W_{ab} = \gamma p_b$, and the inverse lifetime of the state a is $-W_{aa} = \sum_{b \neq a} W_{ab} = \gamma p_{-a}$. The total rate for any jump is $\gamma = W_{1,-1} + W_{-1,1}$. It

is expedient to present the four possible values of W_{ab} in a matrix form:

$$\mathbf{W} = \gamma \begin{bmatrix} -p_- & p_- \\ p_+ & -p_+ \end{bmatrix}. \quad (27)$$

To average over the telegraph noise histories, it is convenient to define conditional averages: the average of the function $F(t, t')$ under the assumption that $\xi(t') = a$ and $\xi(t) = b$ (with $t > t'$) is expected to depend only on $t - t'$ and is hence denoted by $F(t - t')_{ab}$. The average over all histories and all initial and final values of ξ is

$$\overline{F(t, t')} = \sum_{a,b} p_a F(t - t')_{ab}. \quad (28)$$

In particular, the conditional average probability that $\xi(t) = b$, given that $\xi(t') = a$, is the 2×2 matrix $\mathbf{P}(t, t')$, which solves the differential equation

$$\frac{d}{dt} \mathbf{P}(t, t') = \mathbf{W} \mathbf{P}(t, t'), \quad (29)$$

with the initial condition $\mathbf{P}(t, t') = \mathbf{I}$, the 2×2 unit matrix. The solution depends only on $t - t'$,

$$\begin{aligned} \mathbf{P}(t, t') &\equiv \mathbf{P}(t - t') = e^{\mathbf{W}(t-t')} \\ &= \mathbf{I} + \mathbf{W}/\gamma - \mathbf{W}e^{-\gamma(t-t')}/\gamma. \end{aligned} \quad (30)$$

We now demonstrate the averaging over the telegraph noise by considering the particle flux on the left lead, $I_L(t)$. As seen from Eq. (9), this average requires the averages over $G_{dd}^a(\omega, t) = [G_{dd}^r(\omega, t)]^*$, which also determine the average over $G_{dd}^<(\omega, t)$, Eq. (12). To this end, we rewrite Eq. (11) as

$$G_{dd}^a(\omega, t) = i \int_{t'}^t dt' e^{-i(t-t')(\omega - \epsilon - U\bar{\xi} - i\Gamma)} X(t, t'), \quad (31)$$

with the random part

$$X(t, t') = e^{iU \int_{t'}^t d\tau [\xi(\tau) - \bar{\xi}]}. \quad (32)$$

Since $dX(t, t')/dt = iU[\xi(t) - \bar{\xi}]X(t, t')$, integration yields [27]

$$X(t, t') = 1 + iU \int_{t'}^t d\tau [\xi(\tau) - \bar{\xi}] X(\tau, t'). \quad (33)$$

The conditional average of this equation is expected to depend only on $t - t'$, and it obeys the equation

$$\begin{aligned} X(t - t')_{ab} &= P(t - t')_{ab} + iU \sum_c \int_{t'}^t d\tau X(\tau - t')_{ac} \\ &\quad \times [c - \bar{\xi}] P(t - \tau)_{cb}. \end{aligned} \quad (34)$$

In matrix form,

$$\begin{aligned} \mathbf{X}(t - t') &= \mathbf{P}(t - t') + iU \int_{t'}^t d\tau \mathbf{X}(\tau - t') [\sigma_z - \bar{\xi} \mathbf{I}] \mathbf{P}(t - \tau), \end{aligned} \quad (35)$$

where σ_z is the Pauli matrix. Equation (35) is conveniently solved by Laplace transforming it [27],

$$\tilde{\mathbf{X}}(s) = \tilde{\mathbf{P}}(s) + iU \tilde{\mathbf{X}}(s) (\sigma_z - \bar{\xi} \mathbf{I}) \tilde{\mathbf{P}}(s), \quad (36)$$

where $\tilde{f}(s) = \int_0^\infty d\tau e^{-s\tau} f(\tau)$. In particular, $\tilde{\mathbf{P}}(s) = [s\mathbf{I} - \mathbf{W}]^{-1}$, which results from Eq. (30). The solution for the matrix $\tilde{\mathbf{X}}(s)$ is

$$\begin{aligned} \tilde{\mathbf{X}}(s) &= [\tilde{\mathbf{P}}(s)^{-1} - iU(\sigma_z - \bar{\xi}\mathbf{I})]^{-1} \\ &= \frac{1}{D} \begin{bmatrix} s + iU(\bar{\xi} + 1) + \gamma p_+ & \gamma p_- \\ \gamma p_+ & s + iU(\bar{\xi} - 1) + \gamma p_- \end{bmatrix}, \end{aligned} \quad (37)$$

with

$$D = (s + iU\bar{\xi})(s + iU\bar{\xi} + \gamma) + U^2 - iU\gamma\bar{\xi}. \quad (38)$$

It follows from Eq. (28) that [50]

$$\bar{X}(s) = [s + \gamma + 2iU\bar{\xi}]/D. \quad (39)$$

Inspection of the definition of the advanced Green's function, Eq. (31), reveals that its telegraph-process average is independent of the time t and is given by

$$\overline{G_{dd}^a(\omega)} = i\bar{X}(s = \Gamma + i[\omega - \epsilon - U\bar{\xi}]). \quad (40)$$

The telegraph-process averaging over the energy fluxes involves averages of products of Green's functions with $\epsilon_d(t)$, Eq. (24), and its derivative. There are essentially two quantities to average, $\epsilon_d(t)[G_{dd}^a(\omega, t) - G_{dd}^r(\omega, t)]$ and $-i\epsilon_d(t)G_{dd}^<(t, t)$. Their calculation is quite similar to the one represented in detail above, and is relegated to Appendix B.

IV. THE STATIONARY FLUXES

Once all histories of the telegraph-process events are averaged upon, the physical expressions become independent of time. It is illuminating to begin the analysis of this stationary limit by inspecting the averages of the occupation on the dot, Q_d [Eq. (21)], and of the power absorbed from the telegraph-noise source, P_d [Eq. (22)]. Using Eq. (40) in conjunction with Eq. (12) gives

$$\bar{Q}_d = 2 \int \frac{d\omega}{2\pi} \frac{\Gamma_L f_L(\omega) + \Gamma_R f_R(\omega)}{\Gamma} \text{Im}[G_{dd}^a(\omega)]. \quad (41)$$

This is in fact the usual expression for the stationary occupation of a dot coupled to two reservoirs that supply electrons. The first factor is the weighted average of the two Fermi distributions [which becomes simply $f_L(\omega) = f_R(\omega) \equiv f(\omega)$ for an unbiased dot]. The second factor is the density of states on the dot, with

$$\begin{aligned} \overline{G_{dd}^a(\omega)} &= [G_{dd}^r(\omega)]^* \\ &= \frac{\omega - \epsilon_- - i(\Gamma + \gamma)}{[\omega - \epsilon_+ - i\Gamma][\omega - \epsilon_- - i(\Gamma + \gamma)] - \bar{\mathcal{E}}^2}. \end{aligned} \quad (42)$$

As seen, the telegraph noise turns our single resonance on the dot (centered around ϵ in the absence of it) into two resonances [42], centered essentially around ϵ_+ and ϵ_- ,

$$\epsilon_\pm = \epsilon \pm U\bar{\xi}, \quad (43)$$

with disparate widths, essentially (i.e., small values of U) Γ and $\Gamma + \gamma$.

The more interesting factor is the “effective electric field” (squared), $\bar{\mathcal{E}}^2$:

$$\bar{\mathcal{E}}^2 = U^2(1 - \bar{\xi}^2) = 4U^2 p_+(T) p_-(T). \quad (44)$$

This quantity, which vanishes when $T = 0$ [see Eq. (26)], determines the power absorbed by the junction. Indeed, using Eq. (B12) in Eq. (22) yields

$$\bar{P}_d = -\bar{\mathcal{E}}^2 \frac{4\Gamma\gamma}{2\Gamma + \gamma} \int \frac{d\omega}{2\pi} \frac{\Gamma_L f_L(\omega) + \Gamma_R f_R(\omega)}{\Gamma} \text{Im}[\mathcal{F}^a(\omega)], \quad (45)$$

where the function \mathcal{F}^a , given in Eq. (B7) and reproduced here for clarity, is

$$\begin{aligned} \mathcal{F}^a(\omega) &= [\mathcal{F}^r(\omega)]^* \\ &= \frac{1}{[\omega - \epsilon_+ - i\Gamma][\omega - \epsilon_- - i(\Gamma + \gamma)] - \bar{\mathcal{E}}^2}. \end{aligned} \quad (46)$$

The expression for the power \bar{P}_d possesses several remarkable properties. First, Eq. (45) yields a finite *positive* power, as long as the temperature of the noise source is finite, i.e., $|\bar{\xi}| \neq 1$. This implies that the junction absorbs energy from the source responsible for the telegraph noise. For instance, when one confines oneself to the lowest order in $\bar{\mathcal{E}}^2$ (or, equivalently, to order U^2), then

$$\lim_{\bar{\mathcal{E}}^2 \rightarrow 0} \text{Im}[\mathcal{F}^a(\omega)] = \frac{1}{2\gamma} \frac{\partial}{\partial \omega} \ln \left[\frac{(\omega - \epsilon)^2 + \Gamma^2}{(\omega - \epsilon)^2 + (\Gamma + \gamma)^2} \right]. \quad (47)$$

Inserting this expression into Eq. (45) and integrating by parts, one obtains

$$\begin{aligned} \bar{P}_d &= \bar{\mathcal{E}}^2 \frac{2}{2\Gamma + \gamma} \int \frac{d\omega}{2\pi} \left(-\frac{\partial}{\partial \omega} [\Gamma_L f_L(\omega) + \Gamma_R f_R(\omega)] \right) \\ &\quad \times \ln \left[\frac{(\omega - \epsilon)^2 + (\Gamma + \gamma)^2}{(\omega - \epsilon)^2 + \Gamma^2} \right], \end{aligned} \quad (48)$$

which is obviously positive. In the more general case, but assuming for simplicity [51] that the two electronic reservoirs are identical, i.e., $f_L(\omega) = f_R(\omega) \equiv f(\omega)$, one finds

$$\begin{aligned} &\int \frac{d\omega}{2\pi} f(\omega) \mathcal{F}^a(\omega) \\ &= \frac{1}{\omega_+ - \omega_-} \int \frac{d\omega}{2\pi} \left(-\frac{\partial f(\omega)}{\partial \omega} \right) \ln \frac{\omega - \omega_+}{\omega - \omega_-}, \end{aligned} \quad (49)$$

where

$$\omega_\pm = \epsilon + i(\Gamma + \gamma/2) \pm \sqrt{U^2 - (\gamma/2)^2 - i\gamma U\bar{\xi}}. \quad (50)$$

Using this expression, Fig. 2 shows $\bar{P}_d/\bar{\mathcal{E}}^2$ as a function of U/Γ for $\bar{\xi} = 0.5$ and three values of $\epsilon - \mu$, the bare energy on the dot relative to the common chemical potential in the leads, $\mu_L = \mu_R = \mu$ (the resonance width Γ scales all energies). At intermediate values of U such that $U > \gamma$, one observes a fast increase and a peak in $\bar{P}_d/\bar{\mathcal{E}}^2$ near $U p_+ = \epsilon - \mu$. This peak arises as the chemical potential crosses the resonance at $\epsilon - U p_+$. At large values of U , the integral in Eq. (49) is well approximated by $-i\pi/U$, making the power *positive and linear* in U , while it is *quadratic* in U at small values. The curves in Fig. 2 are computed for zero electronic temperatures,

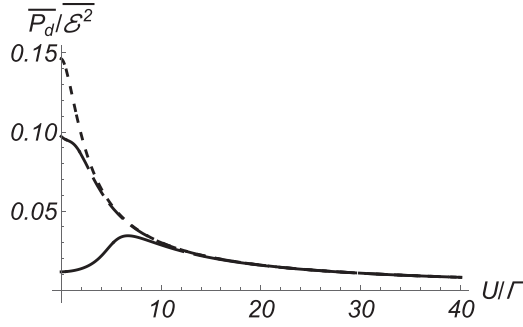


FIG. 2. The power absorbed by the junction (in units of the effective electric field squared, $\overline{\mathcal{E}^2}$), for identical electronic reservoirs (see text), $\overline{\xi} = 0.5$ and $\gamma = 1$. The different curves with increasing dashes are for $\epsilon - \mu = 0.1, 1, 5$. (Energies are in units of Γ .)

i.e., for $[-\partial f(\omega)/\partial\omega] \rightarrow \delta(\omega)$. However, this approximation is also good at finite low temperatures, since the logarithmic function in Eq. (49) varies slowly with ω . Indeed, numerical integration at higher electronic temperatures gives similar qualitative results. [The temperature of the noise source is determined by $\overline{\xi}$; see Eq. (26).]

Second, the power absorbed by the junction necessitates tunnel coupling with at least one lead. The reason for this is quite clear: an empty dot (which is the situation assumed for the decoupled junction) cannot absorb energy from a fluctuating electric field. With two leads, Eq. (48) shows that the absorbed power is a weighted average over the absorption of the two terminals (note that $\Gamma = \Gamma_L + \Gamma_R$). Third, as noted, power is absorbed also when the junction is not biased, i.e., when $f_L(\omega) = f_R(\omega) \equiv f(\omega)$. Finally, we note that the absorbed power vanishes when the mean rate γ of instantaneous jumps vanishes. This is particularly interesting in view of the fact that the telegraph noise does affect the density of states on the dot, $\text{Im}[G_{dd}^a(\omega)]$ [see Eq. (42)], even when $\gamma = 0$ [52].

How is this flux of energy distributed between the two electronic reservoirs? To answer this question we examine the various electronic currents. The average of the particle current over the telegraph processes [using Eq. (42) in Eq. (9)] takes the usual form of stationary particle current in a two-terminal junction [49],

$$\overline{I}_L = \int \frac{d\omega}{2\pi} [f_R(\omega) - f_L(\omega)]T(\omega), \quad (51)$$

where

$$T(\omega) = 4\Gamma_L\Gamma_R \text{Im}[G_{dd}^a(\omega)]/\Gamma \quad (52)$$

is the transmission through the junction; it vanishes unless the junction is biased and/or there is a temperature difference across it. The telegraph noise affects the density of states on the dot, and so modifies the transmission [42].

The electronic energy fluxes, however, differ substantially from their “canonical” forms in two-terminal junctions. We find that the average of the energy flux associated with the left reservoir [see Eq. (17) and the technical details given in Appendix B] is

$$\overline{I}_L^E = \int \frac{d\omega}{2\pi} [f_R(\omega) - f_L(\omega)]\omega T(\omega) + \frac{\Gamma_L}{\Gamma} \overline{P}_d. \quad (53)$$

The first term, which vanishes unless the junction is biased (by a voltage and/or temperature difference), is indeed the usual energy current in a two-terminal electronic junction, with the transmission $T(\omega)$ given in Eq. (52). The second term comes from the power supplied by the source of the telegraph processes. The sign of the first term depends on the bias and/or on the temperature difference across the junction; this means that energy flux represented by the first term can flow out of or into the left reservoir [49]. As opposed, \overline{P}_d is positive, which means that energy flows into the left reservoir from the source of the telegraph noise. Since \overline{I}_R^E is given by Eq. (53) with $L \leftrightarrow R$, it follows that energy flows also into the right reservoir. Thus, the telegraph noise supplies energies to both reservoirs, with the larger portion going into the more strongly coupled one.

Interestingly enough, the averages over the energy fluxes associated with the tunneling terms in the Hamiltonian, $I_{\text{tun},L(R)}^E$ [Eq. (19)], vanish (as is also the case when the junction is subjected to an oscillatory field; this point is elaborated upon further in Sec. V). As a result, one finds that

$$\overline{I}_L^E + \overline{I}_R^E + \overline{I}_d^E = \overline{P}_d, \quad (54)$$

which is the stationary form of energy conservation in the junction [\overline{I}_d^E is the total average energy flux of the electrons on the dot, Eq. (20)].

V. DISCUSSION

Let us begin by summarizing our results and comparing them with the electronic fluxes derived in the presence of an ac electric field that acts on the dot. We considered a single-level quantum dot coupled to two leads, on which the electrons are subjected to a stochastic electric field that imitates telegraph-noise processes. Expressions for the electronic properties of this junction in the time domain can be found analytically within the Keldysh formalism for nonequilibrium Green’s functions. This calculation is carried out without specifying the explicit time dependence of electric field; it thus pertains also for a junction in which a periodic ac electric field acts on electrons residing on the dot [21,22]. We then focused on the specific time dependence that characterizes telegraph-noise processes, and averaged the fluxes over the history of those processes, to obtain their stationary limits. In studies devoted to the effect of oscillatory fields, this step is replaced by an integration over a single period of the ac field. It turns out that the energy currents associated with the tunneling terms in the Hamiltonian vanish when integrated over a period of the ac field; as found above, this is also the fate of these currents in the stationary limit of the telegraph noise.

Whereas the telegraph noise affects only modestly the particle current by changing the density of states on the dot without inducing any dramatic modifications, this is not the case with the energy currents. In contrast with the charge (or particle) current that is established only when the junction is biased (either by voltage or by temperature gradient), disparate energy currents do flow from the dot to the two leads even when the two reservoirs are identical.

We believe that investigations of the correlations of the particle and energy currents will shed further light on this

interesting problem. This is because the average currents, either over the telegraph processes or, in the case of an ac field, over a period, produce qualitatively similar results. It may well be that the difference in the time dependence of the stochastic electric field and that of the oscillatory one will manifest itself in the correlations.

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APPENDIX A: THE GREEN'S FUNCTIONS

In the time domain, the Dyson equations for the Green's functions depend on two time arguments. For instance [46],

$$G_{\mathbf{k}d}(t, t') = \int dt_1 g_{\mathbf{k}}(t, t_1) V_{\mathbf{k}} G_{dd}(t_1, t'). \quad (\text{A1})$$

(The lowercase Green's functions pertain to the decoupled junction.) The Green's function of the decoupled left reservoir is

$$g_{\mathbf{k}}^{r(a)}(t - t') = \mp i \Theta(\pm t \mp t') \langle \{c_{\mathbf{k}}(t), c_{\mathbf{k}}^\dagger(t')\} \rangle = \mp i \Theta(\pm t (\mp) t') e^{-i\epsilon_{\mathbf{k}}(t-t')} \quad (\text{A2})$$

and

$$g_{\mathbf{k}}^<(t - t') = i f(\epsilon_{\mathbf{k}}) e^{-i\epsilon_{\mathbf{k}}(t-t')}, \quad f(\epsilon_{\mathbf{k}}) = \langle c_{\mathbf{k}}^\dagger c_{\mathbf{k}} \rangle. \quad (\text{A3})$$

[The superscript $r(a)$ indicates the retarded (advanced) Green's function and corresponds to the upper (lower) signs on the right-hand side.] The Green's functions Eqs. (A2) and (A3) determine the self-energies Σ_L and Σ_L^E , Eqs. (7) and (16), respectively. When the densities of states in the reservoirs are assumed to be independent of the energy, i.e., in the wide-band limit [46], then

$$\sum_{\mathbf{k}} |V_{\mathbf{k}(p)}|^2 g_{k(p)}^{r(a)}(\omega) \approx \mp i \Gamma_{L(R)}, \quad (\text{A4})$$

where $\Gamma_{L(R)} = \pi |V_{\mathbf{k}f}|^2 \mathcal{N}_{L(R)}$, and $\mathcal{N}_{L(R)}$ is the constant density of states of the left (right) lead. As a result,

$$\Sigma_{L(R)}^{r(a)}(\omega) = \mp i \Gamma_{L(R)}, \quad \Sigma_{L(R)}^<(\omega) = 2i \Gamma_{L(R)} f_{L(R)}(\omega), \quad \Sigma_{L(R)}^{Er(a)}(\omega) = \mp i \omega \Gamma_{L(R)}, \quad \Sigma_{L(R)}^{E<}(\omega) = 2i \omega \Gamma_{L(R)} f_{L(R)}(\omega), \quad (\text{A5})$$

where $f_{L(R)}(\omega)$ [Eq. (10)] is the Fermi distribution there. (Note that the self-energies depend only on the time difference and therefore are conveniently represented by their Fourier transforms.)

The Dyson equation for the Green's function on the dot reads

$$G_{dd}(t, t') = g_d(t, t') + \int dt_1 dt_2 g_d(t, t_1) \Sigma(t_1, t_2) G_{dd}(t_2, t'), \quad (\text{A6})$$

where

$$\Sigma(t, t') = \Sigma_L(t, t') + \Sigma_R(t, t'), \quad \Gamma = \Gamma_L + \Gamma_R. \quad (\text{A7})$$

The Green's function of the decoupled dot is

$$g_d^{r(a)}(t, t') = \mp i \Theta(\pm t \mp t') e^{-i \int_{t'}^t dt_1 \epsilon_d(t_1)}, \quad (\text{A8})$$

and $g_d^< = 0$, since it is assumed that the dot is empty in the decoupled junction. Solving Eq. (A6) for the retarded (advanced) Green's function gives [47]

$$G_{dd}^{r(a)}(t, t') = \mp i \Theta(\pm t \mp t') e^{-i \int_{t'}^t dt_1 \epsilon_d(t_1) \mp \Gamma(t-t')}. \quad (\text{A9})$$

The Fourier transforms of these functions,

$$G_{dd}^r(\omega, t) = \int dt' e^{i\omega(t-t')} G_{dd}^r(t, t'), \quad G_{dd}^a(\omega, t) = \int dt' e^{-i\omega(t-t')} G_{dd}^a(t', t), \quad (\text{A10})$$

lead to Eq. (11) in the main text. Inserting these solutions into the Dyson equation (A6) for the lesser Green's function on the dot yields

$$G_{dd}^<(t, t) = \int \frac{d\omega}{2\pi} \Sigma^<(\omega) \int^t dt_1 \int^{t_1} dt_2 e^{\Gamma(t_1+t_2-2t)} e^{i\omega(t_2-t_1)} e^{i \int_{t_2}^{t_1} d\tau \epsilon_d(\tau)}. \quad (\text{A11})$$

By changing the double integration,

$$\int^t dt_1 \int^{t_1} dt_2 F(t_1, t_2) = \int^t dt_1 \int^{t_1} dt_2 [F(t_1, t_2) + F(t_2, t_1)], \quad (\text{A12})$$

and using Eq. (11), one derives Eq. (12).

The rate of energy associated with the left reservoir [see Eqs. (14) and (15)] involves the expression

$$\int dt' [\Sigma_L^{Er}(t, t') G_{dd}^<(t', t) - G_{dd}^<(t, t') \Sigma_L^{Ea}(t', t)] = -\Gamma_L \int d\tau \delta(\tau) \frac{\partial}{\partial \tau} [G_{dd}^<(t, t - \tau) - G_{dd}^<(t - \tau, t)], \quad (\text{A13})$$

where we have used Eqs. (A5). Inserting here

$$G_{dd}^<(t, t') = \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')} G_{dd}^r(\omega, t) \Sigma^<(\omega) G^a(\omega, t') \quad (\text{A14})$$

[\$\Sigma^<(\omega)\$ is given in Eq. (13)] and carrying out the derivatives exploiting Eqs. (A9) and (11) results in the last term on the right-hand side of Eq. (17).

The last term in Eq. (18) for the energy rate associated with the tunneling between the dot and the left reservoir is calculated from the Dyson equation,

$$G_{\mathbf{k}\mathbf{p}}(t, t') = V_{\mathbf{k}} V_{\mathbf{p}}^* \int dt_1 \int dt_2 g_{\mathbf{k}}(t, t_1) G_{dd}(t_1, t_2) g_{\mathbf{p}}(t_2, t'). \quad (\text{A15})$$

With the definition of the self-energy, Eq. (7) (and the analogous one for \$\Sigma_R\$), this term in Eq. (18) becomes

$$\int dt' \int dt'' [\Sigma_L(t, t') G_{dd}(t', t'') \Sigma_R(t'', t) - \Sigma_R(t, t') G_{dd}(t', t'') \Sigma_L(t'', t)]^<. \quad (\text{A16})$$

It remains to apply the Langreth rules to the product of Green's functions; the result is the last term in Eq. (19).

APPENDIX B: DETAILS OF THE AVERAGE OVER THE TELEGRAPH PROCESS

As mentioned in Sec. III, the averages of the energy fluxes are determined by averaging over products of Green's functions with \$\epsilon_d(t)\$, Eq. (24), and its derivative. Using Eq. (32) and the definition

$$Y(t, t', t'') = \xi(t) X(t', t'') \quad \text{for } t > t' > t'', \quad (\text{B1})$$

one can write

$$\epsilon_d(t) G_{dd}^a(\omega, t) = i \int dt' (e^{-i(\omega - \epsilon_+ - i\Gamma)(t-t')} [\epsilon X(t, t') + U Y(t, t', t')]). \quad (\text{B2})$$

The inverse Laplace transform of Eq. (39) yields

$$\overline{X(t, t')} = \overline{X(t - t')} = \frac{1}{2} [e^{s_+(t-t')} + e^{s_-(t-t')}] + \frac{\gamma/2 + iU\bar{\xi}}{2u} [e^{s_+(t-t')} - e^{s_-(t-t')}], \quad (\text{B3})$$

with

$$u = \sqrt{\left(\frac{\gamma}{2} + iU\bar{\xi}\right)^2 - \mathcal{E}^2} \quad \text{and} \quad s_{\pm} = -\frac{\gamma}{2} \pm u. \quad (\text{B4})$$

[\$\mathcal{E}^2\$ is defined in Eq. (44).] Similarly,

$$\overline{Y(t, t', t'')} \equiv \overline{Y(t - t', t - t'')} = \bar{\xi} \overline{X(t' - t'')} + \frac{iU(1 - \bar{\xi}^2)}{2u} [e^{s_+(t'-t'')} - e^{s_-(t'-t'')}] e^{-\gamma(t-t')}. \quad (\text{B5})$$

Inserting Eqs. (B3) and (B5) into the average of Eq. (B2) yields

$$\overline{\epsilon_d(t) G_{dd}^a(\omega, t)} = \overline{\epsilon_d(t) [G_{dd}^r(\omega, t)]^*} = \epsilon_+ \overline{G_{dd}^a(\omega)} + \mathcal{E}^2 \mathcal{F}^a(\omega), \quad (\text{B6})$$

where the function \$\mathcal{F}^a(\omega)\$ is

$$\mathcal{F}^a(\omega) = [\mathcal{F}^r(\omega)]^* = [(\omega - \epsilon_+ - i\Gamma)(\omega - \epsilon_- - i(\Gamma + \gamma)) - \mathcal{E}^2]^{-1}, \quad (\text{B7})$$

with \$\epsilon_{\pm}\$ given in Eq. (43).

The second quantity to average is \$-i\epsilon_d(t)G_{dd}^<(t, t)\$. Inserting the definitions (32) and (B1) into Eq. (12) gives

$$i\epsilon_d(t) G_{dd}^<(t, t) = \int \frac{d\omega}{2\pi} \Sigma^<(\omega) \int dt' e^{-2\Gamma(t-t')} \int^{t'} dt'' (ie^{-i(\omega - \epsilon_+ - i\Gamma)(t'-t'')} [\epsilon X(t' - t'') + U Y(t', t', t'')] - \text{c.c.}). \quad (\text{B8})$$

The average over the telegraph process of this expression is obtained upon using Eqs. (B3) and (B5),

$$\overline{i\epsilon_d(t) G_{dd}^<(t, t)} = \int \frac{d\omega}{2\pi} \Sigma^<(\omega) \left(\frac{\omega - i\Gamma}{2\Gamma} \overline{G_{dd}^a(\omega)} - \mathcal{E}^2 \frac{\gamma}{2\Gamma(2\Gamma + \gamma)} \mathcal{F}^a(\omega) - \text{c.c.} \right). \quad (\text{B9})$$

Note the identity

$$(\omega - i\Gamma)\overline{G_{dd}^a(\omega)} - \text{c.c.} = \epsilon_+ \overline{G_{dd}^a(\omega)} + \overline{\mathcal{E}^2} \mathcal{F}^a(\omega) - \text{c.c.}, \quad (\text{B10})$$

which turns the average Eq. (B9) into

$$i\overline{\epsilon_d(t)G_{dd}^<(t,t)} = \int \frac{d\omega}{2\pi} \Sigma^<(\omega) \left[\frac{\epsilon_+}{2\Gamma} \overline{G_{dd}^a(\omega)} + \frac{\overline{\mathcal{E}^2}}{2\Gamma + \gamma} \mathcal{F}^a(\omega) - \text{c.c.} \right]. \quad (\text{B11})$$

Finally, the average of the product of $d\epsilon_d(t)/dt$ with $G^<$ is

$$\begin{aligned} i\overline{\frac{d\epsilon_d(t)}{dt}G_{dd}^<(t,t)} &= \int \frac{d\omega}{2\pi} \Sigma^<(\omega) \int^t dt' e^{-2\Gamma(t-t')} \int^{t'} dt'' \left(i e^{-i(\omega - \epsilon - i\Gamma)(t'-t'')} U \frac{dY(t',t'')}{dt} - \text{c.c.} \right) \\ &= - \int \frac{d\omega}{2\pi} \Sigma^<(\omega) \left[\frac{\gamma \overline{\mathcal{E}^2}}{2\Gamma + \gamma} \mathcal{F}^a(\omega) - \text{c.c.} \right]. \end{aligned} \quad (\text{B12})$$

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- [50] Equation (39) and its inverse Laplace transform take a particularly simple form when $\bar{\xi} = 1$ (or -1), $\overline{X(t-t')} = \exp[iU(t-t')]$ (or $\overline{X(t-t')} = \exp[-iU(t-t')]$). Unfortunately, Eq. (38) of the first of Refs. [41] fails to reproduce these limits.
- [51] This is not a drastic approximation: in the general case, the derivative $\partial f(\omega)/\partial \omega$ is replaced by $\partial[\Gamma_L f_L(\omega) + \Gamma_R f_R(\omega)]/(\Gamma \partial \omega)$.
- [52] $\gamma = 0$ is the static limit of the telegraph noise, in which the site energy increases or decreases by U , depending on the initial value of ξ : $\omega_{\pm} = \epsilon \pm U + i\Gamma$. However, our procedure averages over the initial conditions, and therefore the final result contains the two resonances, independent of the initial conditions.