Pairing instability near a lattice-influenced nematic quantum critical point

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We study how superconducting T_c is affected as an electronic system in a tetragonal environment is tuned to a nematic quantum critical point (QCP). Including coupling of the electronic nematic variable to the relevant lattice strain restricts criticality only to certain high symmetry directions. This allows a weak-coupling treatment, even at the QCP. We develop a criterion distinguishing weak and strong T_c enhancements upon approaching the QCP. We show that negligible T_c enhancement occurs only if pairing is dominated by a non-nematic interaction away from the QCP, and simultaneously if the electron-strain coupling is sufficiently strong. We argue this is the case of the iron superconductors.

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I. INTRODUCTION

The origin of high superconducting transition temperature T_c of the copper and iron based systems remains to be well understood [1–5]. Among the various possibilities as likely causes of high T_c, one is that of the presence of a quantum critical point (QCP) in the vicinity of which the effective pairing interaction is strong, leading to enhanced T_c. In fact, T_c boosted by an antiferromagnetic QCP remains among the more promising scenarios for these systems [6]. A related question, addressed here, is whether one expects similar enhancement of T_c close to a nematic QCP, where the ground state is poised to break discrete rotational symmetry.

Several recent theoretical works on this issue have concluded that, indeed, the superconducting T_c is boosted near a nematic QCP [7–10]. The motivation for this conclusion is intuitively clear, at least in the regime where weak-coupling theory of pairing is applicable. It is well known that, for small momentum transfer, the effective electron-electron interaction mediated by the long-wavelength nematic fluctuations leads to attractive interaction both in the s- and d-wave channels. This interaction increases as the system approaches a nematic QCP, leading to larger T_c. References [7,9] have studied the problem beyond weak coupling, and both have concluded that the intuitive picture stays intact.

In the phase diagram of the cuprates the location of a nematic QCP, if present, remains to be well established. Consequently, at present the Fe-based systems are better suited to study the issue of T_c enhancement from an experimental point of view. However, while for most iron based superconductors (FeSC) the nematic transition line and the QCP is well identified [11–16], the presence of a magnetic transition and its associated QCP complicates the matter [5], since it is hard to disentangle the effects of the two. In this respect an ideal system is $FeSe_{1-x}S_x$, which has a nematic QCP but not a magnetic one. Interestingly, in contradiction with the above theoretical expectations, in this model system the T_c is hardly affected by the nematic QCP [17–19]. Thus clearly there is a missing element in the above theories.

In this work we identify the missing element to be a symmetry-allowed coupling between the electronic nematic degree of freedom and a lattice shear strain mode. We show that, once this coupling is included in the theory of nematic criticality, the presence of a nematic QCP does not necessarily lead to significant enhancement of T_c. We identify the conditions under which the enhancement is negligible near the QCP. This occurs when the following two conditions are simultaneously satisfied. Namely, (i) the pairing is dominated by a non-nematic interaction *away* from the QCP and (ii) if the electron-strain coupling is sufficiently large. Note, condition (i) does not trivialize the problem since, by itself, it does not preclude the nematic term to dominate *near* the QCP and provide significant T_c enhancement. Our result provides a route to understand qualitatively why T_c is unaffected by the nematic QCP in $FeSe_{1-x}S_x$.

The main physics ingredient of our work is enshrined in the standard theory of elasticity for an acoustic instability involving Ising-nematic symmetry, such as a second order tetragonal-orthorhombic transition. It is well known that, in this case, the divergence of the correlation length, which manifests as vanishing acoustic phonon velocity, is restricted to two high-symmetry directions [20–25]. This is because along the generic directions the noncritical shear strains, that are invariably present in a solid, come into play and cut off criticality. The physics of this cutoff can be also understood as follows.

Consider a translation symmetry preserving second order phase transition involving a local variable $\mathcal{X}(\mathbf{r}) = \mathcal{X}_0 + \sum_{\mathbf{q}\neq 0} \mathcal{X}_q e^{i\mathbf{q}\cdot\mathbf{r}}$, where $\mathcal{X}_0$ is the order parameter. Within the Landau paradigm the free energy has mean field and fluctuation contributions. The former has the structure $F_{\rm MF} = (a/2)\mathcal{X}_0^2 + (A/4)\mathcal{X}_0^4$, while the latter, to Gaussian order, is related to the action $S_{\rm fluc} = \sum_{\mathbf{q}\neq 0}(b+q^2)|\mathcal{X}_q|^2$. While in usual theories $b = a$, in those involving crystalline strains $b(\hat{q})$ is no longer a parameter but, rather, a function of the Brillouin zone angles $\hat{q}$ containing information about the crystalline anisotropy [21,22]. In other words, the concept of correlation length becomes angle dependent. In this situation, the condition $b(\hat{q}) = a$ is satisfied only for certain high symmetry directions, and only along these directions the correlation length diverges at the transition defined by $a = 0$. Along other directions $b(\hat{q}) > 0$, and correlation length stays finite at the transition.

The above property is inherited by the electronic nematic subsystem once its coupling with the strain is included [26–28]. This leads to two important conclusions concerning Cooper pairing, which are the main results of this paper. (i) Under certain standard assumptions, the weak coupling BCS analysis

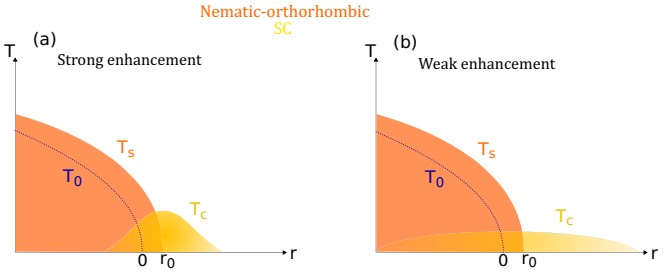

FIG. 1. Schematic phase diagrams with nematic/orthorhombic (orange) and superconducting (yellow) phases below temperatures $T_s(r)$ and $T_c(r)$, respectively. r is a dimensionless control parameter. $T_0(r)$ (dotted lines) is the nominal electron-nematic transition in the absence of nematoelastic coupling. The coupling shifts the nematic quantum critical point (QCP) from $r = 0$ to $r = r_0$. r_0 is a measure of the strength of the coupling (see text). Panels (a) and (b) are two possible scenarios. In (a) there is "strong" enhancement of $T_c(r)$ at the QCP. In (b) the enhancement is "weak," as in $FeSe_{1-x}S_x$ [17–19]. We show that (b) occurs if the pairing is dominated by a non-nematic potential away from the QCP, and simultaneously if the nematoelastic coupling is sufficiently strong [see Eq. (5)].

remains valid arbitrarily close to the nematic QCP. (ii) We identify the criterion that distinguishes between strong and weak T_c enhancements upon tuning the system to the nematic QCP (see Fig. 1). The latter occurs only if the pairing is dominated by a non-nematic interaction away from the QCP, and simultaneously if the electron-phonon interaction is sufficiently strong. We argue that this is the case of the FeSC.

II. MODEL

We consider a system of itinerant electrons in a tetragonal lattice, with negligible dispersion along the z axis, which is close to a nematic/structural QCP that is driven by electronic correlations. Ignoring electron-lattice interaction for the moment, the long wavelength fluctuations of the nematic variable $\phi_{\mathbf{q}}$, which is a collective mode of the electrons, is described by a susceptibility of the standard Ornstein-Zernike form $\chi_0^{-1}(\mathbf{q}) = r + q_{2d}^2/(2k_F)^2$, where $\mathbf{q}_{2d} \equiv (q_x, q_y)$, and r is a dimensionless tuning parameter of the theory that governs closeness to the QCP. Without the lattice coupling, the bare QCP is at $r = 0$. In what follows the frequency dependence of the susceptibility can be ignored.

A crucial ingredient in the model is the symmetry-allowed nematoelastic term linking $\phi(\mathbf{r})$ with the local orthorhombic strain $\varepsilon(\mathbf{r}) = \varepsilon + i \sum_{\mathbf{q}\neq 0} [q_x u_x(\mathbf{q}) - q_y u_y(\mathbf{q})] e^{i\mathbf{q}\cdot\mathbf{r}}$, where ε is the uniform macroscopic strain and $\vec{u}(\mathbf{r})$ is the atomic displacement. ε is nonzero only in the symmetry-broken nematic/orthorhombic phase. This coupling can be written as $g \int d\mathbf{r}\, \phi(\mathbf{r})\varepsilon(\mathbf{r})$, where g has dimension of energy.

The effect of the nematoelastic term on criticality has been discussed earlier [26–28]. Here, for the sake of completeness, we recapitulate the main points. (i) It shifts the QCP to $r = r_0 \equiv g^2\nu/C_0$ (see Fig. 1), where ν has dimension of density of states and C_0 is the bare orthorhombic elastic constant. Thus r_0 is a dimensionless parameter that measures the strength of the nematoelastic coupling. In the following we take $r_0 \leqslant r \leqslant 1$. (ii) The nematoelastic coupling leads to hybridization of $\phi_{\mathbf{q}}$ with the acoustic phonons [see Fig. 2(a)], which renormalizes the nematic susceptibility to $\chi^{-1} = \chi_0^{-1} - \Pi$, with $\Pi(\hat{q}) = (g^2/\rho) \sum_\mu (\mathbf{a}_{\mathbf{q}} \cdot \hat{u}_{\mathbf{q},\mu})^2/\omega^2_{\mathbf{q},\mu}$. Here ρ is the density, μ is the polarization index, $\mathbf{a}_{\mathbf{q}} \equiv (q_x, -q_y, 0)$, and $\hat{u}_{\mathbf{q},\mu}$ is the polarization vector for the bare acoustic phonons with angle-dependent velocity $\mathbf{v}^{(0)}_{\hat{q},\mu}$ and dispersion $\omega_{\mathbf{q},\mu} = \mathbf{v}^{(0)}_{\hat{q},\mu} \cdot \mathbf{q}$. The above follows simply from integrating out the lattice variables. Evidently, $\Pi(\hat{q})$ is independent of the magnitude q and has fourfold symmetry of the tetragonal unit cell in the non-nematic phase. Thus the nematoelastic term makes the mass of $\phi_{\mathbf{q}}$ angle dependent with $r \to r(\hat{q}) \equiv r + \Pi(\hat{q})$, and criticality, or divergence of correlation length, is restricted to the high symmetry directions $\hat{q}_{1,2} \equiv (\hat{q}_x \pm \hat{q}_y)/\sqrt{2}$, for which $r(\hat{q}_{1,2}) = 0$ at the QCP (see Fig. 3). Along the remaining directions $r(\hat{q}) > 0$ at the QCP. Note, since divergence of χ also implies vanishing of the sound velocity *renormalized* by the coupling g, the above direction dependence is consistent with the fact that only along $\hat{q}_{1,2}$ the

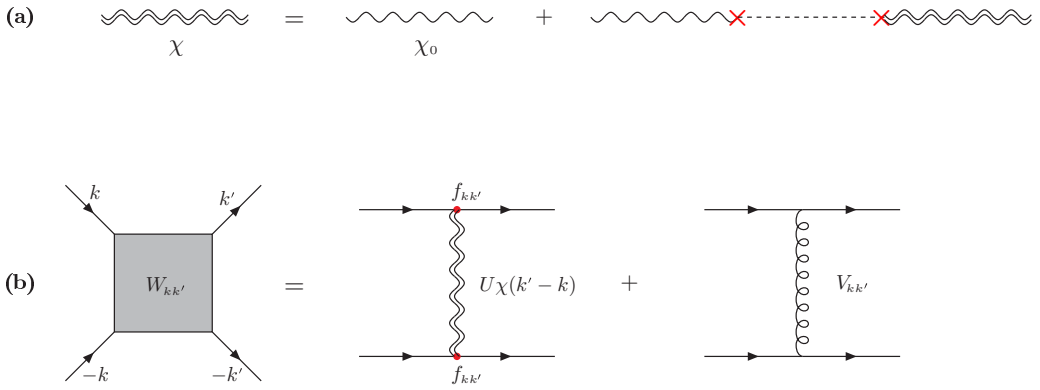

FIG. 2. Diagrammatic representation of the relevant microscopic processes. (a) The bare electron-nematic susceptibility (single wavy line) is dressed (double wavy line) by the nematoelastic coupling (red crosses). The dashed line is an acoustic phonon. (b) The pairing potential $W_{\mathbf{k},\mathbf{k}'}$ consists of the dressed nematic interaction of strength U, and a non-nematic interaction $V_{\mathbf{k},\mathbf{k}'}$ (gluon line) of strength V. The former interaction vertex is accompanied by a form factor $f_{\mathbf{k},\mathbf{k}'}$. The interesting regime is $V > U$, i.e., when pairing is dominated by the non-nematic interaction away from the nematic QCP.

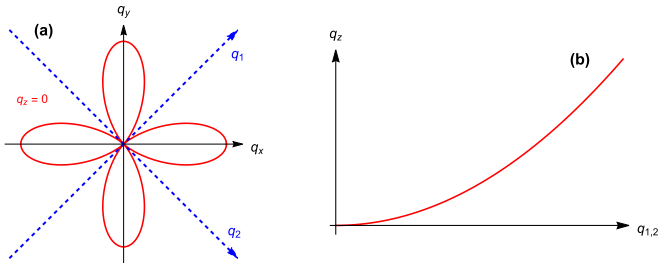

FIG. 3. Mass of the nematic susceptibility χ, defined as $r(\hat{q}) \equiv \lim_{\mathbf{q}\to 0} \chi^{-1}(\mathbf{q},\omega = 0)$, becomes anisotropic in the presence of the nematoelastic coupling. In the non-nematic phase the angular dependence of $r(\hat{q})$ has tetragonal symmetry. (a) Variation of $r(\hat{q})$ on the $q_z = 0$ plane (red line) at the nematic QCP. It is zero only along the high-symmetry directions $\hat{q}_{1,2} \equiv (\hat{q}_x \pm \hat{q}_y)/\sqrt{2}$ (blue arrows). (b) For finite q_z, $r(\hat{q}) \propto q_z^2$.

sound velocity vanishes at this nematic/structural transition [29,30]. (iii) Since the coupling g cuts off divergence along the generic directions, the effect of the quantum fluctuations is weak. Therefore, below the temperature scale $T_{\rm FL} \sim r_0^{3/2} T_F$ the system behaves as a Fermi liquid for thermodynamic and single-electron properties [28]. Here T_F is the Fermi temperature. Above $T_{\rm FL}$ the nematoelastic coupling can be neglected.

The effective electron-electron interaction mediated by the nematic variable $\phi_{\mathbf{q}}$ has the form

$$\mathcal{H}_{\rm nem} = -U \sum_{\mathbf{q}} \chi(\mathbf{q}) \mathcal{N}_{\mathbf{q}} \mathcal{N}_{-\mathbf{q}}, \tag{1}$$

where $\mathcal{N}_{\mathbf{q}} = \sum_{\mathbf{k},\sigma} f_{\mathbf{k},\mathbf{q}} c^{\dagger}_{\mathbf{k}+\mathbf{q},\sigma} c_{\mathbf{k},\sigma}$, in terms of the creation and annihilation operators of electron with spin σ. The form factor $f_{\mathbf{k},\mathbf{q}} = (h_{\mathbf{k}} + h_{\mathbf{k}+\mathbf{q}})/2$, where $h_{\mathbf{k}}$ transforms as $(k_x^2 - k_y^2)$. The parameter U, with dimension of energy, sets the scale of the nematic interaction.

In the following we solve the linearized BCS equations for superconducting gap $\Delta_{\mathbf{k}}$, assuming singlet pairing. This involves calculating the largest eigenvalue λ satisfying the relation

$$\lambda \Delta_{\mathbf{k}} = \nu_{\rm FS} \oint_{\rm FS'} W_{\mathbf{k},\mathbf{k}'} \Delta_{\mathbf{k}'}, \tag{2}$$

where $\nu_{\rm FS}$ is the density of states at the Fermi surface,

$$W_{\mathbf{k},\mathbf{k}'} = V^{(s,d)}_{\mathbf{k},\mathbf{k}'} + U f^2_{\mathbf{k},\mathbf{k}'} \chi(\mathbf{k} - \mathbf{k}'), \tag{3}$$

and FS$'$ implies the $\mathbf{k}'$ integral is restricted to the Fermi surface [see Fig. 2(b)].

In the above we added a second interaction $-V^{(s,d)}$ of non-nematic origin that stays constant as a function of r. Depending on the context, it favors s- and d-wave pairing, respectively. The addition of $V^{(s,d)}$ can be motivated as follows. In all likelihood, the pairing in the FeSC and the cuprates is mediated not just by the nematic fluctuations. In addition, there is, e.g., short wavelength spin/charge fluctuations [5] or Mott correlations [3,4] that mediate pairing. Consequently, it is physical to expect that, close to a nematic QCP, the pairing potential has a nematic component which is a strong function of r [included in $\chi(\mathbf{q})$], and it has a non-nematic component which does not vary with r (represented by $V^{(s,d)}$). Note, in what follows the precise microscopic origin and structure of $V^{(s,d)}$ is not relevant. Besides this physical relevance, as we show below, the inclusion of $V^{(s,d)}$ is crucial to distinguish the two limiting cases of "strong" and "weak" enhancement of T_c upon tuning the system to the nematic QCP with $r \to r_0$.

Our goal is to study how $\lambda(r)$ changes as the system is tuned to the QCP, from which we can deduce the variation of $T_c \sim \Lambda\, e^{-1/\lambda}$, where $\Lambda \ll E_F$ is the high-energy cutoff of the pairing problem. Note, an *important consequence* of the coupling g is that it is now possible to consider the case where $T_c(r_0) < T_{\rm FL}$, the Fermi liquid scale. For $T < T_{\rm FL}$ the dynamics of the nematic pairing potential is irrelevant, and the problem can be treated within BCS formalism.

In the above model $\lambda(r)$ increases monotonically as the system approaches the QCP, since the nematic interaction itself is attractive and monotonic. However, the crucial question is whether this increment is significant. To address this issue quantitatively, we define $\delta\lambda \equiv \lambda(r = r_0) - \lambda(r = 1)$, and we distinguish between strong and weak enhancements of T_c, depending on whether $\delta\lambda \gg \lambda(r = 1)$ or not, respectively. Qualitatively, this criteria distinguishes between whether pairing is dominated by long wavelength nematic fluctuations or by a non-nematic pairing interaction at the QCP.

III. RESULTS

The momentum anisotropy of the susceptibility $\chi(\mathbf{q})$ due to the coupling g can be modeled as follows. (a) For $q_z \leqslant q_{2d}$, we get $\chi^{-1}(\mathbf{q}) \approx r(\hat{q}) + q^2_{2d}/(2k_F)^2$. The anisotropic mass $r(\hat{q})$ has tetragonal symmetry, and satisfies $r(\hat{q}_{1,2}) = 0$ at the QCP. The simplest function consistent with these requirements is $r(\hat{q}) = (r - r_0) + r_0 (q_z/q_{2d})^2 + r_0 \cos^2 2\phi_{\mathbf{q}}$, where $\phi_{\mathbf{q}}$ is the azimuthal angle of $\mathbf{q}$ (see Fig. 3). This region of $\mathbf{q}$ space also contains the critical modes. (b) For $q_z \geqslant q_{2d}$, the nematoelastic coupling can be neglected and $\chi(\mathbf{q}) \approx \chi_0^{-1}(\mathbf{q}) = r + q^2_{2d}/(2k_F)^2$. However, this does not imply singular susceptibility at the QCP, since its location is shifted from $r = 0$ to $r = r_0$. In this region of $\mathbf{q}$ space the modes are, thus, noncritical.

The main qualitative physics can be already illustrated by considering the simplest case of a single band with a cylindrical Fermi surface around the Brillouin zone center, and where the noncritical pairing term supports s-wave gap with $V^{(s)}_{\mathbf{k},\mathbf{k}'} = V > 0$. The details of the calculation are given in the Appendix. For a uniform gap the leading r dependence of the eigenvalue $\lambda(r \geqslant r_0)$ is given by

$$\lambda/\nu_{\rm FS} = V + \frac{U}{2\sqrt{r}} - \frac{U}{\pi}(\ln \max[r - r_0, r_0] + c_1), \tag{4}$$

where $c_1 = 8/3 - 2\ln 2 \approx 1.28$ is nonuniversal. In the right-hand side (RHS) of the above the second term comes from the momentum space (b), as discussed above, where the fluctuations are massive and noncritical, while the third term comes from the region (a) which includes the critical modes. However, since the critical momentum space is rather restricted (equivalently, the critical theory can be mapped to an isotropic model in effective space dimension $d_{\rm eff} = 5$ [24]), its contribution to the eigenvalue is subleading. Thus

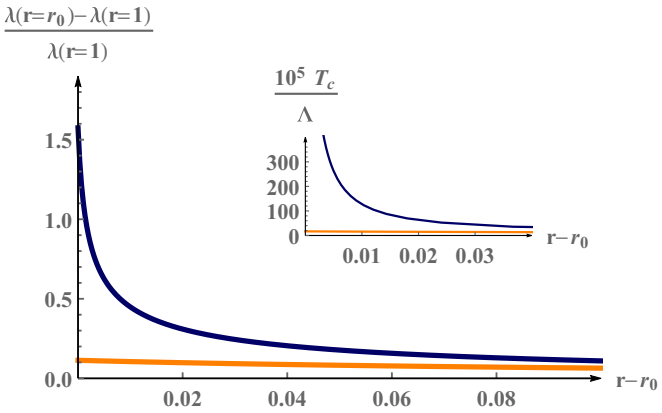

FIG. 4. Calculated variation of the BCS eigenvalue $\lambda(r)$ (main panel) and the associated superconducting transition temperature $T_c(r)$ (inset) upon tuning the system to the nematic QCP at $r = r_0$. The pairing kernel is shown graphically in Fig. 2. Large enhancement (blue curves) of λ and T_c is observed only when the nematoelastic coupling strength r_0 is sufficiently weak [see Eq. (5)]. Note, in this limit, at the QCP the $T_c \propto e^{-\sqrt{r_0}/(v_{FS}U)}$ (not shown) is still a small fraction of the ultraviolet cutoff Λ. In the opposite limit of strong nematoelastic coupling (orange curves) the enhancement is negligible. This limit is relevant for the Fe-based superconductors such as $FeSe_{1-x}S_x$ [17–19].

the leading nematic contribution to λ is from the noncritical region (b).

Importantly, from the above we deduce that, upon tuning the system to the QCP by $r \to r_0$, the T_c enhancement will be "significant," i.e., $\delta\lambda \gg \lambda(r = 1)$, provided the electron-lattice coupling is weak enough such that

$$r_0 \ll (U/V)^2. \tag{5}$$

Note, if the nematoelastic coupling is ignored we get $r_0 = 0$ and we recover the result of Refs. [7–10], namely, the presence of a nematic QCP necessarily implies strong T_c increase. Note also the complete absence of a non-nematic pairing term ($V = 0$) leads to strong T_c enhancement as well.

As importantly, the opposite limit of weak T_c enhancement, which is relevant for understanding the phase diagram of $FeSe_{1-x}S_x$, occurs if $1 \gg r_0 \gg (U/V)^2$ (see Fig. 4). Physically, this implies a situation where the following two conditions are simultaneously satisfied. (i) The pairing is dominated by a non-nematic interaction *away* from the QCP (since $V \gg U$) and (ii) the electron-strain coupling is sufficiently large [since $r_0 \gg (U/V)^2$]. Note, condition (i) does not trivialize the issue since, by itself, it does not preclude the nematic term to dominate *near* the QCP and provide significant T_c enhancement. In fact, this is why condition (ii) comes into play. The origin of condition (i) lies in the physical expectation that the energy scale generated by the electron-lattice interaction is well below the Fermi energy, i.e., $r_0 \ll 1$.

Besides the case of the isotropic s-wave gap, we also study the following situations in the Appendix. (i) An extended s-wave gap, since the lattice-renormalized nematic interaction is intrinsically anisotropic, and it can give rise to angular variations of the gap. (ii) Motivated by the cuprates, we consider the case where the non-nematic interaction $V^{(d)}_{\mathbf{k},\mathbf{k}'} = V \cos 2\phi_{\mathbf{k}} \cos 2\phi_{\mathbf{k}'}$ favors a d-wave gap with $\Delta_{\mathbf{k}} = \Delta_0 \cos 2\phi_{\mathbf{k}}$. (iii) Motivated by the FeSC we study a system with Fermi pockets at (0,0), $(\pm\pi,0)$, and $(0, \pm\pi)$, with a form of $V^{(s)}_{\mathbf{k},\mathbf{k}'}$ that leads to $s_\pm$ gap. In all these cases we find that qualitatively $\lambda(r)$ is described by Eq. (4), except with different numerical prefactors. We conclude that the above criterion for T_c enhancement in Eq. (5) is robust.

IV. CONCLUSION

The strength of the nematoelastic coupling can be estimated as $r_0 \sim (T_s - T_0)/T_F$, where T_0 is the nominal nematic transition temperature of the electron subsystem in the absence of this coupling (see Fig. 1), accessible from, say, electronic Raman scattering [15]. In FeSe we get $T_0 \sim 10$ K and $T_s \sim 90$ K [31]. We estimate the Fermi temperature from the bottom of the smallest electron pocket as measured by photoemission above T_s, which is around 25 meV in FeSe [17,32]. Thus, for $FeSe_{1-x}S_x$ we estimate $r_0 \sim 0.3$ and $T_{FL} \sim 40$ K. Note, the condition $T_c < T_{FL}$, needed for a weak-coupling theory, is well respected in this case. A similar estimate for $Ba(Fe_{1-x}Co_x)_2As_2$ yields $r_0 \sim 0.05$ and $T_{FL} \sim 10$ K [28]. Since, in this system the maximum $T_c \sim 25$ K is comparable to T_{FL}, a more careful quantitative analysis is needed.

The estimation of U and V requires a full microscopic theory of pairing that is currently unavailable. Consequently, a quantitative application of the theory to real systems is not possible at present. However, experimentally it is clear that $FeSe_{1-x}S_x$ has a nematic QCP around $x \approx 0.16$, but the superconducting $T_c(x)$ remains remarkably flat around this doping [17–19]. This can be due to a strong nematoelastic effect violating the condition in Eq. (5). In turn, this would imply that the pairing interaction in $FeSe_{1-x}S_x$ is mostly non-nematic in origin. A similar case can also be made for $Ba(Fe_{1-x}Co_x)_2As_2$, where quantum critical nematic fluctuations have been detected only over a narrow doping range of $x = 0.65$–0.75 in the low-T superconducting phase [14]. It is remarkable that over the same doping range $T_c(x)$ hardly varies, implying that even here the lattice cutoff is operational. Thus the domelike structure of $T_c(x)$ over a wider doping range in $Ba(Fe_{1-x}Co_x)_2As_2$ is likely due to the antiferromagnetic QCP, while the absence of a magnetic QCP in $FeSe_{1-x}S_x$ results in a flat $T_c(x)$.

To summarize, we argued that nematoelastic coupling can play a crucial role in determining if superconducting T_c is strongly enhanced in the vicinity of a nematic quantum critical point. We showed that, in the presence of a significant non-nematic pairing interaction, strong nematoelastic coupling implies that the nematic fluctuations do not boost T_c significantly. Based on existing experiments on $FeSe_{1-x}S_x$ and on $Ba(Fe_{1-x}Co_x)_2As_2$ we argued that this is likely the case of the iron-based superconductors. This would imply that the main pairing interaction is non-nematic in origin in these materials. More generally, from the perspective of material design for high temperature superconductivity, we conclude that (a) hard crystals are better suited for boosting T_c near a nematic quantum critical point and that (b) the lattice cutoff can be also avoided provided the non-nematic pairing potential

is strong enough to guarantee $T_c(r=1) \gg T_{FL}$, in which case the physics of Refs. [7–10] will be operational.

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APPENDIX

In this appendix we provide the technical details for the calculation of the BCS eigenvalue λ defined in Eq. (2) of the main text which is

$$\lambda\Delta_{\mathbf{k}} = \nu_{\rm FS}\oint_{\rm FS'} W_{\mathbf{k},\mathbf{k}'}\Delta_{\mathbf{k}'}.$$

The interaction potential is $W_{\mathbf{k},\mathbf{k}'} = V^{s,d}_{\mathbf{k},\mathbf{k}'} + U f^2_{\mathbf{k},\mathbf{k}'}\chi(\mathbf{k}'-\mathbf{k})$. The form factor $f_{\mathbf{k},\mathbf{k}'} = (h_{\mathbf{k}} + h_{\mathbf{k}'})/2$, where $h_{\mathbf{k}}$ transforms as $k_x^2 - k_y^2$ in the $k_x - k_y$ plane. A simple choice is $h_k = \cos(2\phi_{\mathbf{k}})$, where $\phi_{\mathbf{k}}$ is the azimuthal angle of $\mathbf{k}$. Note that the nematic pairing potential is intrinsically anisotropic in the presence of the nematoelastic coupling. This anisotropy can be taken into account by dividing the momentum space into two regions, (a) $q_z \geqslant q_{2d}$ and (b) $q_z \leqslant q_{2d}$, and by working with asymptotic forms of χ in these two regions. Therefore, the pairing potential can be broken in three parts:

$$W_{\mathbf{k},\mathbf{k}'} = V^{s,d}_{\mathbf{k},\mathbf{k}'} + U f^2_{\mathbf{k},\mathbf{k}'}\chi(\mathbf{k}'-\mathbf{k})|_{q_z\geqslant q_{2d}} + U f^2_{\mathbf{k},\mathbf{k}'}\chi(\mathbf{k}'-\mathbf{k})|_{q_z\leqslant q_{2d}} \quad \text{(A1)}$$

The asymptotic forms of χ in the two regions are described in the main text. Note, the critical manifold is contained in the third term above, while the second term above involves noncritical modes. Furthermore, we will assume that $V > U$, where V is the strength of the non-nematic pairing potential $V^{s,d}_{\mathbf{k},\mathbf{k}'}$. Physically this implies that sufficiently far from the nematic QCP the pairing is dominated by the non-nematic term. As noted in the main text, the opposite limit of $U > V$ is trivial, since if the nematic potential already dominates pairing far away from the QCP, then, irrespective of the strength of the nematoelastic coupling, it will invariably lead to large T_c enhancement because the dominant pairing potential grows (even if it stays finite) as the QCP is approached.

(a) s-wave superconductivity with a uniform gap and a cylindrical Fermi surface (FS). We will assume that the non-nematic pairing potential is a constant with $V^s_{\mathbf{k},\mathbf{k}'} = V$. Since Eq. (2) of the main text is restricted to the Fermi surface, $q_{2D} = 2k_F|\sin(\frac{\phi_{\mathbf{k}}-\phi'_{\mathbf{k}}}{2})|$ and $\cos^2(2\phi_{\mathbf{q}}) = \cos^2(\phi_{\mathbf{k}} + \phi'_{\mathbf{k}})$. The FS integral turns into angular integrals $\lambda = \langle W_{\mathbf{k},\mathbf{k}'}\rangle$, where

$$\langle f\rangle = \int_0^{2\pi}\frac{du\,dv}{(2\pi)^2} f(u,v), \quad \text{(A2)}$$

and $u = \phi_{\mathbf{k}} + \phi'_{\mathbf{k}}$ and $v = \phi_{\mathbf{k}} - \phi'_{\mathbf{k}}$. This mean value is to be estimated to lowest order in the parameter $r \leqslant 1$ which governs the nearness to the QCP (see Fig. 1, main text).

The contribution from the second term of Eq. (A1) is given by

$$U\left\langle \cos^2 u\,\cos^2 v\frac{1}{r+(1-\cos v)/2}\right\rangle \approx \frac{U}{2\sqrt{r}} \quad \text{(A3)}$$

Note, in the above estimation, typical $q_z \sim \pi/c$, where c is unit cell length along the z direction, while typical $q_{2d} \sim r^{1/2}$. Therefore, to leading order in r the constraint $q_z \geqslant q_{2d}$ is automatically satisfied in the above estimation, even though the angular integrals are performed freely.

The third term of Eq. (A1) involves momentum dependence along the z direction and, therefore, the estimation of its contribution to $\lambda(r)$ involves averaging along the length of the cylindrical Fermi surface. Anticipating that the typical momentum transfer along z is small compared to Fermi wave vector k_F we can write $\oint_{\rm FS} \to \int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^1 d(k_z/k_F)$. This implies that the contribution from the third term of Eq. (A1) is given by

$$2U\langle\cos^2 u\,\cos^2 v|\sin\frac{v}{2}\left|\int_0^1 dx\frac{1}{r-r_0+r_0\cos^2 u+\sin^2(v/2)+r_0x^2}\right\rangle \approx -\frac{U}{\pi}(\ln\max[r-r_0,r_0]+c_1), \quad \text{(A4)}$$

where $c_1 = 8/3 - 2\ln 2 \approx 1.28$. This leads to the equation

$$\lambda(r\geqslant r_0)/\nu_{\rm FS} = V + \frac{U}{2\sqrt{r}} - \frac{U}{\pi}(\ln\max[r-r_0,r_0]+c_1),$$

which is Eq. (4) of the main text. Note, the leading r dependence comes from the noncritical modes, rather than the critical ones which have a rather limited volume in momentum space. The critical contribution gives only to a weak logarithmic dependence which can be ignored to leading order in r.

It is clear from the above that there will be considerable T_c enhancement close to the nematic QCP defined by $r = r_0$ only if in this regime the nematic contribution dominates. This, in turn, is possible only if the nematoelastic coupling is weak enough such that

$$r_0 < (U/V)^2$$

This is the condition mentioned in Eq. (5) of the main text.

In the following we consider few other cases and we show explicitly that the structure of Eq. (4) remains the same; only numerical prefactors change. This implies that the conclusion obtained in Eq. (5) is robust.

(b) s-wave superconductivity with higher order gap harmonics. Keeping s-wave symmetry we can introduce anisotropy in the gap function by considering higher order harmonics as $\Delta(\mathbf{k}) = \Delta_0 + \sqrt{2}\Delta_4\cos(4\phi_{\mathbf{k}})$. The second term of the RHS is the normalized first higher order s-wave harmonic. We proceed to project the gap equation onto each

orthogonal polynomial to get the secular equation

$$\lambda\Delta_0 = \lambda_{00}\Delta_0 + \lambda_{40}\Delta_4,$$
$$\lambda\Delta_4 = \lambda_{40}\Delta_0 + \lambda_{44}\Delta_4, \tag{A5}$$

where we have defined $\lambda_{nn'} = \langle W_{kk'} g_n g_{n'} \rangle$, with g_n the nth orthogonal cosine polynomial. The secular system implies that the physical T_c is to be given by the largest value of the matrix $(\lambda_{nn'})$. With the above ansatz for the gap we get

$$\lambda = \frac{\lambda_{00} + \lambda_{44}}{2} + \sqrt{\left(\frac{\lambda_{00} - \lambda_{44}}{2}\right)^2 + \lambda_{40}^2} \tag{A6}$$

The calculation is then identical to case (a). Ignoring the log corrections from the critical manifold, we find to lowest order in r,

$$\lambda(r \geqslant r_0)/\nu_{\rm FS} = V + \left(1 + \frac{1}{\sqrt{2}}\right)\frac{U}{2\sqrt{r}}, \tag{A7}$$

which is the same as in case (a) except for a numerical prefactor. It is also possible to do the calculation in the limit of an infinite number of s-wave harmonics, and we find that the superconducting eigenvalue goes as $\lambda(r) = V + U/\sqrt{r}$.

(c) d-wave superconductivity. Motivated by the cuprates, we take a non-nematic pairing interaction which promotes d-wave superconductivity $V^d_{\mathbf{k},\mathbf{k}'} = 2V\cos(2\phi_{\mathbf{k}})\cos(2\phi'_{\mathbf{k}})$ on top of the nematic pairing potential. With a d-wave gap ansatz $\Delta(\mathbf{k}) = \Delta_0\cos(2\phi_{\mathbf{k}})$, we find

$$\lambda(r \geqslant r_0)/\nu_{\rm FS} = V + \frac{3U}{4\sqrt{r}} \tag{A8}$$

Thus once again the BCS eigenvalue is the same as in Eq. (4) except for a numerical prefactor.

(d) The multiband case of Fe-based superconductors. Motivated by the physics of the Fe-based superconductors, we now consider a three band model with one hole band centered around the (0,0) point of the Brillouin zone and two electron pockets located at $(\pi,0)$ and $(0,\pi)$, respectively, in the one-Fe/unit cell representation. The non-nematic pairing potential is now a matrix in the band space, and we take it to be

$$V^s_{\mathbf{k},\mathbf{k}'} = -V\begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & -1/2 \\ 1/2 & -1/2 & 0 \end{pmatrix}. \tag{A9}$$

Note, Eq. (2) is written with the convention that repulsive interactions have negative sign and attractive ones have positive sign. Thus the above interaction implies that the non-nematic pairing potential is only interband, and that it is repulsive for the electron-hole pairing term, while it is attractive for the electron-electron pairing term. This invariably leads to a $s_\pm$ gap, which in the three-band language has the form $\Delta_0(1,-1,-1)$, which is the most discussed gap structure for these systems. Note, the nematic pairing potential is attractive and is, by definition, intraband. For circular Fermi surfaces, and assuming that the gaps on each of the pockets are constant, we get, following case (a),

$$W_{\mathbf{k},\mathbf{k}'} = V\begin{pmatrix} x & -1/2 & -1/2 \\ -1/2 & 2x & 1/2 \\ -1/2 & 1/2 & 2x \end{pmatrix}, \tag{A10}$$

where $x = U/(2\sqrt{r}V)$. This leads to a BCS eigenvalue:

$$\lambda(r \geqslant r_0)/\nu_{\rm FS} = \frac{1}{4}\left(V + \frac{3U}{\sqrt{r}} + \sqrt{9V^2 + \frac{U^2}{r} + \frac{2UV}{\sqrt{r}}}\right) \tag{A11}$$

Since the only energy scales here are V and $U/\sqrt{r}$, it is simple to check that significant T_c enhancement is only possible if the condition in Eq. (5) holds. Note also that, while the magnitudes of the gaps become different on the hole and the electron pockets with the inclusion of the nematic pairing, both for small and for large x there is a change in the sign of the gap between the hole and the electron surfaces.

We conclude that in all the above cases the condition for significant T_c enhancement is given by Eq. (5), while in the opposite limit there is hardly any impact of the QCP on the T_c.

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