Dissipatively driven hardcore bosons steered by a gauge field

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The interplay between dissipation, interactions, and gauge fields opens the possibility to rich emerging physics. Here we focus on a setup in which the system is coupled at its extremities to two different baths which impose a current. We then study the system's response to a gauge field depending on the filling. We show that the current induced by the baths has a marked dependence on the magnetic field at low fillings which is significantly reduced at larger fillings. We explain the interplay between interactions, gauge field, and dissipation by studying the system's energy spectrum at the different fillings. This interplay also results in the emergence of negative differential conductivity. For this study we have developed a number-conserving treatment which allows a numerical exact treatment of fairly large system sizes, and which can be extended to a large class of systems.

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I. INTRODUCTION

A deeper understanding of the far-from-equilibrium transport properties of complex quantum systems would lead to fascinating progress for future nanotechnologies. A particularly interesting challenge is that of characterizing and controlling the transport properties of quantum systems. In such systems, many-body effects can induce, remove, or shift phase transition lines. This significantly affects the properties of a system and our ability to control it. Another salient tool used to control transport and induce new phases of matter is a gauge field. The quantum Hall effect is a paramount example of the role of gauge fields on transport properties [1,2]. In a type-II superconductor the increase of the magnitude of the magnetic field can drive a transition from a diamagnetic Meissner phase to a superconductor with an Abrikosov vortex lattice [3]. Adding interactions to this system can lead to even more exotic phases of matter with topological order [4–6].

The minimal setup in which such rich phenomenology can be explored is that of coupled chains (a ladder) with a gauge field such as the one depicted in Fig. 1. This system has attracted intense theoretical scrutiny [7–23] and it has been experimentally studied both with Josephson-junction arrays [24–27] and with ultracold gases with bosons and fermions [28–30], thanks to the use of synthetic gauge fields [31,32].

Here we will consider a quantum system in the presence of a gauge field and connected at its extremities to two different baths which would impose a current through it (so-called dissipatively boundary-driven systems). Ion trap experiments promise to be an ideal setup for a clean realization and study of such transport problems [33–35]. The setup could be realized using ion microcavity arrays with a nonlinear local potential [35–39], for which the gauge field is generated by Raman coupling [40]. It should be noted that magnetic fields have already been used to effectively modify heat transport in Josephson junctions [41].

Recently, a boundary-driven coupled chain of free bosons under the effect of a gauge field was studied [42]. There it was shown that, depending on how the baths were coupled to the ladder, the chiral current changes abruptly on two phase transition lines, implying the emergence of a previously unpredicted nonequilibrium phase transition. At this transition line, coinciding with the opening of a gap in the bulk spectrum, the total current through the system also starts to change significantly; hence the gauge field can be used to strongly control the current flow.

However it is necessary to gain a deeper understanding of how the interplay between the gauge fields and interactions between the bosons will affect the transport. Hence in this paper we are going to study the steady state transport properties of hardcore bosons driven out of equilibrium by dissipative boundary driving. We will show that the controllability of the current via a gauge field (i.e., the ability to alter the current) is significantly reduced as the average filling is increased. We will also show that the nonlinear dependence of the current on the density also results, depending on the gauge field, in the emergence of negative differential conductance. To this end we mention that although in one-dimension there is an exact mapping between hardcore bosons and free fermions, for a ladder this is not possible. This lack of mapping is central for the nonlinear behavior, which would not exist for free fermions following Ref. [42].

Importantly, in order to analyze this system, we introduce an exact numerical approach with conserved quantum numbers to compute the steady state. With this method we are able to readily study the exact density matrix of a system with up to 14 sites. Moreover this method also allows us to gain a much deeper understanding of the system.

The paper is composed of the following sections: In Sec. II we present the model, then in Sec. III we describe the current as a function of filling and gauge field. In Sec. IV we describe the numerical method introduced in this work and the insight that it allows us to gain. In Sec. V we show the emergence of negative differential conductance and how this can be tuned via the gauge field, and finally we draw our conclusions in Sec. VI.

II. MODEL

We study a ladder made of two coupled chains (or legs) each composed of L sites. As we focus on the role of strong interactions, we consider hardcore bosons, for which at most a single boson can occupy one site. Two sites which are coupled and belong to different chains form a rung. The ladder is coupled at its extremities to two baths which can inject or remove bosons at different rates. The setup is depicted in Fig. 1.

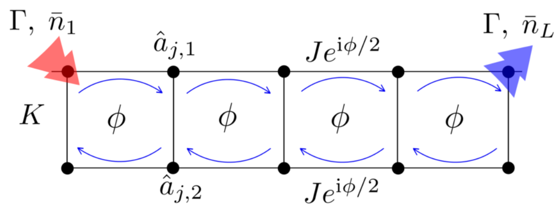

FIG. 1. Ladder made of two coupled linear chains, referred to as legs, with local bosonic excitations described by the annihilation operators at site j, $\hat{a}_{j,p}$, where $p = 1,2$ respectively for the upper and the lower leg. K is the tunneling amplitude between the legs, on what are referred to as rungs of the ladder, while J is the amplitude of tunneling between sites in the legs. A gauge field imposes a phase ϕ. The coupling to the baths is represented by the thick arrows. Each bath is characterized by the average density of bosons $\bar{n}_j$ the bath itself imposes on the rung j and the strength of the coupling Γ.

The evolution of the density operator $\hat{\rho}$ is given by a master equation with a Lindbladian $\mathcal{L}$,

$$\frac{d\hat{\rho}}{dt} = \mathcal{L}(\hat{\rho}) = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] + \mathcal{D}(\hat{\rho}), \tag{1}$$

where the Hamiltonian $\hat{H}$ is given by

$$\hat{H} = -J\sum_{p,j} e^{i\Phi_p}\hat{a}^\dagger_{j,p}\hat{a}_{j+1,p} - K\sum_{j}\hat{a}^\dagger_{j,1}\hat{a}_{j,2} + \text{H.c.} \tag{2}$$

Here K is the tunneling constant in the rungs, J for the legs, and the phase of the tunneling in the legs $\Phi_p = (-1)^{(p-1)}\phi/2$ such that a particle doing a loop around one plaquette acquires a phase ϕ. The operator $\hat{a}_{j,p}$ ($\hat{a}^\dagger_{j,p}$) annihilates (creates) a boson in the upper ($p = 1$) or lower ($p = 2$) chain at the jth rung of the ladder. And the hardcore constraint means $\hat{a}^\dagger_{j,p}\hat{a}^\dagger_{j,p} = 0$. The dissipator in Lindblad form [43,44] is given by

$$\mathcal{D}(\hat{\rho}) = \sum_{j=1,L}\Gamma[(1-\bar{n}_j)(\hat{a}_{j,1}\hat{\rho}\hat{a}^\dagger_{j,1} - \hat{a}_{j,1}\hat{a}^\dagger_{j,1}\hat{\rho})$$

$$+\bar{n}_j(\hat{a}^\dagger_{j,1}\hat{\rho}\hat{a}_{j,1} - \hat{a}^\dagger_{j,1}\hat{a}_{j,1}\hat{\rho}) + \text{H.c.}], \tag{3}$$

where $j = 1$ or L, and Γ is the overall coupling constant. The dissipator tends to set the local density at site $(j,1)$ to the value $\bar{n}_j$ if decoupled from the others [45]. The baths will thus induce a particle current when $\Delta\bar{n} = \bar{n}_1 - \bar{n}_L \neq 0$.

III. TOTAL CURRENT

We focus our attention on the particle current at steady state [46]. The total current through the system, $\mathcal{J}$, is given by the sum of the current in each leg $\mathcal{J} = \sum_p \mathcal{J}^L_{j,p}$, where the current in the p leg is $\mathcal{J}^L_{j,p} = \langle iJe^{i(-1)^{p+1}\phi/2}\hat{a}^\dagger_{j,p}\hat{a}_{j+1,p} + \text{H.c.}\rangle/\hbar$ and $\langle\dots\rangle$ means the expectation value for the steady state. The current in the rungs is instead given by $\mathcal{J}^R_{j,1\to2} = \langle iK\hat{a}^\dagger_{j,1}\hat{a}_{j,2} + \text{H.c.}\rangle/\hbar$. The leg and rung currents are associated with the continuity equations $\frac{\partial\langle\hat{n}_{j,1}\rangle}{\partial t} = \mathcal{J}^L_{j-1,1} - \mathcal{J}^L_{j,1} - \mathcal{J}^R_{j,1\to2}$ and $\frac{\partial\langle\hat{n}_{j,2}\rangle}{\partial t} = \mathcal{J}^L_{j-1,2} - \mathcal{J}^L_{j,2} + \mathcal{J}^R_{j,1\to2}$ for $1 < j < l$. We also define the average density as $\bar{n}_{\text{av}} = (\bar{n}_1 + \bar{n}_L)/2$.

In Fig. 2(a) we show the total current in the system as a function of the gauge field ϕ for a small value of $\Delta\bar{n}$ in panel (a) ($\Delta\bar{n} = 0.1$) and a larger value in panel (b) ($\Delta\bar{n} = 0.4$). As

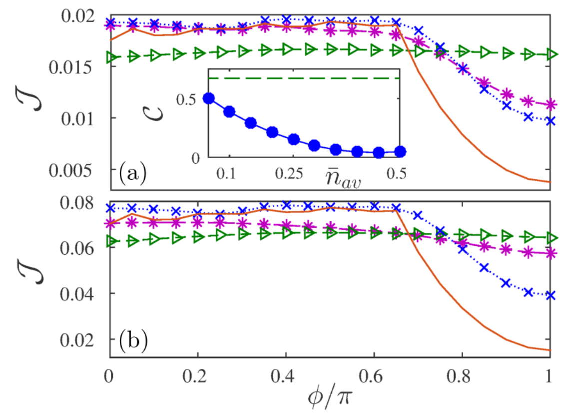

FIG. 2. Total current $\mathcal{J}$ versus phase ϕ for different values of average density $\bar{n}$. As the system approaches $\bar{n}_{\text{av}} = 0.5$ the current becomes much less sensitive to the phase. (a), (b) The continuous red lines are for free bosons and a ladder of length $L = 200$. The other three lines are for $L = 6$ and free bosons (blue dotted line with ×), $\bar{n}_{\text{av}} = 0.1$ (purple dashed line with ∗), and $\bar{n}_{\text{av}} = 0.5$ (green dashed line with triangles). In panel (a) $\Delta\bar{n} = 0.1$, while in panel (b) $\Delta\bar{n} = 0.4$. The inset of panel (a) shows the controllability $\mathcal{C}$ versus $\bar{n}_{\text{av}}$ for $\Delta\bar{n} = 0.1$ (circles) and for free bosons (dashed green line). The controllability is strongly reduced as the average filling increases.

a reference, the red continuous thin line shows the current for a large ladder ($L = 200$) of noninteracting bosons. This was computed in Ref. [42] and it shows a significant change in the total current at the critical gauge field $\phi_c = 2\pi/3$. However, because of the computational complexity of a ladder of hardcore bosons, in this work we are limited to short ladders, which, as shown later, already manifest remarkable effects. In order to fairly compare the strongly interacting bosons to the noninteracting ones, we show, with the blue dotted line with crosses, the total current versus ϕ for a ladder of 6 rungs of free bosons. The curve is smooth and has larger oscillations; however it still shows a marked dependence on the phase ϕ, and it shows a strong current suppression for ϕ approaching π. We now consider the case of hardcore bosons, from low average filling, to around half fillings. At low $\bar{n}_{\text{av}}$, the behavior of the current as a function of the phase naturally resembles that of noninteracting bosons (see pink dashed line with ∗). However, as we increase $\bar{n}_{\text{av}}$ such that the local occupation reaches half filling, the difference between the response of the free bosons compared to the hardcore bosons is striking. In particular, close to half filling the dependence of the current on the gauge field is highly reduced, as shown by the bold green continuous line. One direct consequence is that, at larger fillings, the current can be much larger than for free bosons because the gauge field cannot significantly reduce it (see the region for ϕ close to π). This may seem surprising from an analysis of the ground state because, near half filling, the spectrum of the hardcore boson ladder is gapped. In the following we will explain the mechanisms behind this behavior. Figure 2(b) shows a similar behavior but with less marked difference because in this panel $\Delta\bar{n} = 0.4$.

In the inset of Fig. 2(a) we show the controllability $\mathcal{C}$, which describes how well the gauge field can tune the current in the system. For given dissipative boundary driving and tunneling parameters, the controllability is given by

$\mathcal{C} = [\max_\phi \mathcal{J} - \min_\phi \mathcal{J}]/\{[\max_\phi \mathcal{J} + \min_\phi(\mathcal{J})]/2\}$. The inset shows that, for $\Delta\bar{n} = 0.1$, the controllability $\mathcal{C}$ is suppressed by one order of magnitude as the average filling increases.

IV. EXACT NUMERICAL APPROACH WITH QUANTUM NUMBERS

To gather a deeper understanding and to be able to analyze in a numerically exact way this system, we have developed an approach which takes into account the total quantum number. We first write the density operator as

$$\hat{\rho} = \sum_{\vec{m}_N,\vec{m}'_{N'},N,N'} \rho^{\vec{m}_N,N}_{\vec{m}'_{N'},N'} |\vec{m}_N,N\rangle\langle\vec{m}'_{N'},N'|, \tag{4}$$

where N (N') is the total number of particles respectively for the ket (bra), while $\vec{m}_N$ $(\vec{m'}_{N'})$ describes the distribution of the N (N') atoms between the $2L$ sites. It should be noted that the Hamiltonian $\hat{H}$ in Eq. (2) conserves the total number of atoms either in the bra or in the ket. Moreover the dissipator $\mathcal{D}$ in Eq. (3) only couples an element $\rho^{\vec{m}_N,N}_{\vec{m}'_{N'},N'}$ with another element $\rho^{\vec{n}_{N\pm1},N\pm1}_{\vec{n}'_{N'\pm1},N'\pm1}$ where the total number of particles in the ket and bra is either increased or decreased by one particle. Last it should be pointed out that since the steady state is unique and the initial condition can be chosen to be in a pure state, the steady state is exactly described by a much simpler ansatz of the form

$$\hat{\rho}_{ss} = \sum_N \hat{\rho}^N_{ss} = \sum_{\vec{m}_N,\vec{m}'_N,N} \rho^{\vec{m}_N,N}_{\vec{m}'_N,N} |\vec{m}_N,N\rangle\langle\vec{m}'_N,N|, \tag{5}$$

where the total number of particles in the bra or in the ket is the same. Hence, to find the steady state we can use a state within one number block as an initial condition and evolve it using the ansatz in Eq. (5) with the master equation (1). For chains up to 7 rungs (14 sites), we compute the steady state of the Lindbladian $\mathcal{L}$ by directly solving the linear equation $\mathcal{L}(\hat{\rho}) = 0$ with ARPACK.

The exact ansatz (5) also allows us to gain a deeper understanding of the system. In fact it is now possible to compute the current for each number sector and thus realize which sectors contribute most to the current [47]. We thus fix $\Delta\bar{n}$ as in Fig. 2(a) and we compute the current in each number sector N, $\mathcal{J}_N$, for various values of $\bar{n}_1$. More precisely $\mathcal{J}_N = \sum_p \langle iJe^{i(-1)^{p+1}\phi/2}\hat{a}^\dagger_{j,p}\hat{a}_{j+1,p} + \text{H.c.}\rangle_N/\hbar$ and $\langle\ldots\rangle_N$ means expectation over $\hat{\rho}^N_{ss}$. In Fig. 3 we show $\mathcal{J}_N$ as a function of the sector's particle number N, for different values of the gauge field ϕ. We observe that for low fillings the current is mostly due to the sector with 1 particle and also that the total current strongly varies as the gauge field changes. In particular for $\phi = \pi$ (blue dashed line with $\times$), the current is significantly lower than for $\phi = 0,\pi/2$ (blue dashed lines with respectively $\circ$ and $+$). For larger $\bar{n}_{\text{av}}$ the particle number sectors which contribute most to the current are those of larger particle number and, for them, the current is much less affected by a change in the gauge field.

The repartition of the density matrix in different number sectors, as in Eq. (5), can give even further insight. Since the different number sectors are only coupled by the dissipator, and since the effectiveness of the coupling is strongly dependent on the spectrum in each number sector, by analyzing the spectrum of the Hamiltonian in each sector we can foresee whether the change of the phase ϕ would significantly affect the steady state and hence the current. In the left panels of Fig. 4 [panels (a), (b), (c)] we show the energy spectrum for total particle numbers 2 and 3 for a ladder of 10 rungs. The spectrum changes significantly, especially for $N = 2$; hence we expect a great change in the steady state and in the current as ϕ changes. In the right panels instead, (d), (e), (f), we show the spectrum for a ladder of 7 rungs and either 7 or 8 hardcore bosons, corresponding to half filling and half filling plus one atom,

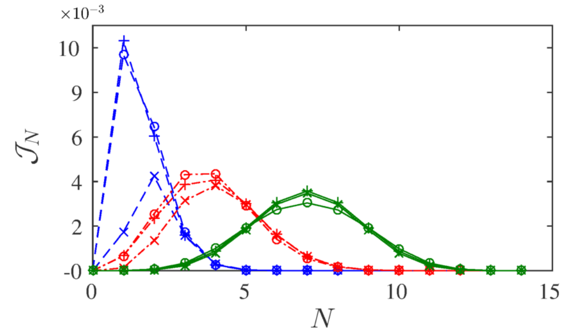

FIG. 3. Particle current per number sector $\mathcal{J}_N$ for average filling $\bar{n}_{\text{av}} = 0.05$ (blue dashed lines), $\bar{n}_{\text{av}} = 0.25$ (red dot-dashed lines), and $\bar{n}_{\text{av}} = 0.5$ (green continuous lines). For each $\bar{n}_{\text{av}}$ we show the current for different gauge fields: $\phi = 0$ ($\circ$), $\phi = \pi/2$ ($+$), and $\phi = \pi$ ($\times$). Other parameters are $L = 7$ and $K = J$.

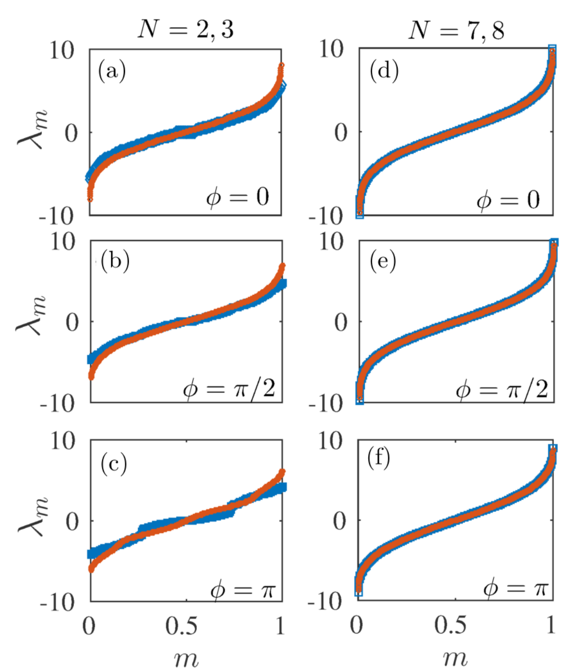

FIG. 4. : Energy spectra for different particle filling and gauge fields ϕ. For (a)–(c) the ladder has 10 rungs and total particle number 2 (blue squares) or 3 (red circles), i.e., close to 1/10 filling, while for (d)–(f) the ladder has 7 rungs and total particle number 7 (blue squares) or 8 (red circles), i.e., close to half filling. For panels (a), (d) $\phi = 0$; (b), (e) $\phi = \pi/2$; and (c), (f) $\phi = \pi$. For all these panels $K = J$.

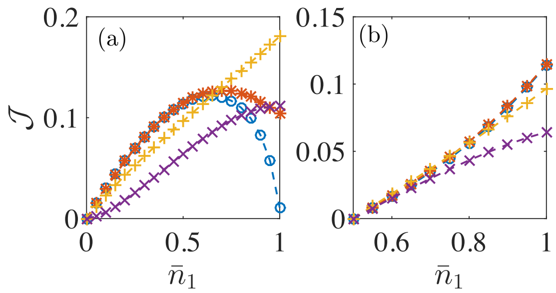

FIG. 5. Current vs $\bar{n}_1$ for phases $\phi = 0$ ($\circ$), $\phi = \pi/10$ ($*$), $\phi = \pi/2$ ($+$), and $\phi = \pi$ ($\times$). In panel (a) $\bar{n}_L = 0$ while in panel (b) $\bar{n}_L = 0.5$. Other parameters are $L = 7$ and $K = 1.5J$.

two number sectors which would also be directly coupled by the dissipator. In this case, in contrast to the low-filling case represented in the left panels, the energy spectrum does not change so significantly and the curves of the spectrum are always close to each other [48]. This is why at larger fillings a much lower variation of the current as a function of the gauge field ϕ is expected, which justifies the results in Figs. 2 and 3. It should be stressed that for nonequilibrium steady states, unlike in (zero temperature) quantum phase transitions, it is in general important to consider the full spectrum and not just the low-energy part.

V. NEGATIVE DIFFERENTIAL CONDUCTANCE

Because of the presence of strong interactions, it is possible for negative differential conductance to emerge [49,50]. However the conductance can also be affected by the gauge field ϕ thus possibly altering its character. This is shown in Fig. 5 where the current is depicted as a function of $\bar{n}_1$ for different values of $\bar{n}_L$. In particular we show in panels (a) and (b) respectively the cases for $\bar{n}_L = 0$ and $\bar{n}_L = 0.5$. In both panels we observe a strong nonlinear response with $\Delta\bar{n}$ which is very different for different values of the phase ϕ.

Interestingly, in panel (a) we observe a strong signature of negative differential conductance for $\phi = 0$. Increasing the gauge field the response changes, see red ($*$) for $\phi = \pi/10$, and becomes roughly linear, yellow ($+$) for $\phi = \pi/2$, and then, at $\phi = \pi$, purple ($\times$), the response can be superlinear; i.e., the current increases more than linearly when $\Delta\bar{n}$ increases.

In panel (b), for $\bar{n}_L = 0.5$, the superlinear behavior is even clearer, and this time it occurs for $\phi = 0$, the case for which, at lower $\bar{n}_L$, negative differential conductance occurred. This behavior could have been anticipated from the results in Fig. 2. There, for small values of the phase ϕ, the current is higher for lower fillings while at large ϕ the current is in general lower at lower fillings.

VI. CONCLUSIONS

The interplay between gauge fields and dissipation can induce nonequilibrium phase transitions and markedly change the properties of a system. Also the interplay between a gauge field and interactions can induce quantum phase transitions. In this work we have studied the interplay of dissipation, gauge field, and interactions. In particular, we have shown how interactions can strongly alter the ability to tune the transport properties of a system using, for example, a gauge field. Previous works [42] had shown that a gauge field can be used to strongly vary the current flowing through two coupled chains, and phase transitions could emerge. Here we have shown that because of strong interactions, as the filling increases, the sensibility of the system to the gauge field is significantly reduced. Due to the interplay between the gauge field and the filling, the conductance has a strong nonlinear behavior as a function of the system parameters, resulting also in a negative differential conductance which can be tuned with the gauge field. Our calculations are exact and greatly simplified thanks to the use of quantum number conservation for the steady state density matrix (a method which can be readily implemented in many setups and which allows great insight into the systems). In the future it would be important to extend the current work to include the role of finite interactions and longer chains. It would be particularly interesting to understand the fate of the nonequilibrium phase transitions predicted for free bosons as the interaction is smoothly changed from 0 to a finite large value. A different nature of the particles (e.g., interacting fermions) or of the baths (e.g., non-Markovian thermal baths) should also be analyzed to understand deeply the transport properties of dissipatively boundary-driven many-body quantum systems.

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[45] To ensure that the filling at the edges is exactly $\bar{n}_j$ we set $\hbar\Gamma = 10J$.
[46] For the parameters studied the steady state is unique.
[47] Note that the current operator conserves the number of particles; hence only the blocks with the same total number of particles in the bra and in the ket contribute to the current.
[48] Variations in the spectrum are present. For instance, at half filling, the ground state undergoes a phase transition between a chiral and a vortex Mott insulator, one which has a neutral gap while the other does not [20].
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