Decoherence of two coupled singlet-triplet spin qubits

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We study a pair of capacitively coupled singlet-triplet spin qubits. We characterize the two-qubit decoherence through two complementary measures, the decay time of coupled-qubit oscillations and the fidelity of entangled state preparation. We provide a quantitative map of their dependence on charge noise and field noise, and we highlight the magnetic field gradient across each singlet-triplet qubit as an effective tool to suppress decoherence due to charge noise.

I. INTRODUCTION

Localized electron spins in semiconductor quantum dots provide a promising architecture for quantum computation [1–4]. Compared with alternative platforms (e.g., trapped ions, trapped atoms, or superconducting circuits), semiconductor spin qubits have the threefold advantage of (1) fast single-qubit gate operations, (2) long coherence time [5], and (3) potential for scalability [6,7] because of advanced fabrication technology developed for semiconductor integrated circuits [8]. There has been significant progress in the experimental development of high-fidelity single spin qubit gate operations in different platforms using different materials. Unfortunately, despite considerable experimental efforts [9–13], it remains challenging to engineer high-fidelity two-qubit entangling gates for semiconductor spin qubits, as decoherence due to environmental noises is exacerbated by comparatively weak coupling between localized electron spins. In fact, the experimental progress in developing two-qubit gates in semiconductor spin quantum computing platforms has been disappointing so far when compared with the corresponding situation in superconducting and ion trap platforms. The relatively low entangling-gate fidelity of spin qubits (0.9 in GaAs as reported in Ref. [13]) is currently the main obstacle to unlocking their aforementioned advantage as a platform for large-scale quantum computing, and it calls for a better understanding of the effect of environmental noises on the dynamics of two coupled semiconductor spin qubits in order to enhance the coupled-qubit fidelity.

In this paper, we undertake this challenge and study the decoherence of two coupled singlet-triplet spin qubits [3,14], which are among the actively studied spin qubits with the added advantage that two-qubit gate operations have been demonstrated in GaAs-based singlet-triplet qubits [11,13]. Each singlet-triplet qubit consists of a pair of exchange-coupled electron spins localized in a double quantum dot, and the two qubits interact via an Ising-type capacitive coupling [11]. Compared with the exchange-coupled spin qubits [1] studied in a previous paper [15], where two localized electron spins are coupled through the Heisenberg coupling [16,17], the singlet-triplet system we consider here enjoys full two-axis control through purely electrical gating [18] and is protected from homogeneous magnetic field fluctuations in each double quantum dot [3]. It also operates in a larger active Hilbert space due to the lack of spin conservation, and has more complicated dynamics and richer physics. This makes it

harder to extract useful insights from analytical solutions [19]. Instead, we study the coupled singlet-triplet qubits through numerical calculations in order to provide quantitative insight into the detrimental role of (electric) charge and (magnetic) field noise on the two-qubit Ising gate operations. We mention that sophisticated dynamical decoupling schemes have already been developed for semiconductor singlet-triplet qubits enabling efficient and fault-tolerant gate operations [20–23], and substantial progress is likely in the near future once two-qubit entangling gates achieve higher fidelity, making our current theoretical analysis timely.

We consider coupled-qubit decoherence from two different types of environmental noises, charge noise from charge fluctuations in each qubit device [24], and field (Overhauser) noise due to nuclear spin dynamics in the semiconductor background [25]. We assume that the noises are slow relative to the qubit dynamics, and we model them in the quasistatic bath approximation by averaging observables over time-independent but randomly distributed disorder configurations. The quasistatic bath approximation has been used extensively in the semiconductor spin qubit studies and is generally considered to be quite valid in most situations [15].

The decoherence of the coupled qubits is examined quantitatively through a pair of complementary probes. First, we extract a characteristic time scale, the two-qubit coherence time, from the envelope decay of the coupled-qubit oscillations. This measures the persistence of the initial state information in the presence of environmental noises. It also provides a direct physical measure of the time duration of coherent gate oscillations. Second, we compute the fidelity of preparing an entangled state through time evolution from an unentangled product state. This quantifies the ability of the coupled qubits to carry out a precise unitary transformation despite the fluctuations in coupling parameters, and serves as a simple proxy for gate fidelity. We also note in this context the interesting possibility of singlet-triplet semiconductor qubits being effective quantum sensors because of their delicate dependence on charge and field noises. In particular, the type of theoretical analysis presented in the current work can be inverted and used for a quantitative determination of background charge and/or field fluctuations from the singlet-triplet entanglement information as described in our study.

We study the dependence of these two fidelity measures on charge noise, field noise, as well as the magnetic field gradient across each singlet-triplet qubit. When the average magnetic

field gradient is zero, we find that the coupled singlet-triplet qubits are significantly more susceptible to charge noise than field noise. This difference is manifested in both the two-qubit coherence time and the entanglement fidelity, although less pronounced in the latter. The situation changes dramatically when a strong magnetic field gradient is applied in each qubit. As the magnetic field gradient increases, the coupled-qubit system becomes more sensitive to field noise and less to charge noise. In the regime dominated by the magnetic field gradient, charge noise becomes relatively inconsequential and the decoherence of the coupled qubits is mainly driven by field noise, in sharp contrast to the situation without a strong magnetic field gradient. The change of noise sensitivity driven by the magnetic field gradient is a unique feature of the singlet-triplet system and has no direct counterpart in a system of two exchange-only qubits. We mention that the magnetic field gradient induced strong suppression of the charge noise effect on the two-qubit Ising gate operations for singlet-triplet qubits provides encouraging prospects for Si-based quantum computing platforms since isotopic purification enables the elimination of the nuclear field noise in Si systems [26].

The paper is organized as follows. In Sec. II, we describe the Ising model of two single-triplet spin qubits and discuss the two-qubit coherence time T_2^*. We provide an operational definition of T_2^* and examine its dependence on both charge noise and field noise, as well as the magnetic field gradient. In Sec. III, we introduce the fidelity of entangled state preparation and examine its parametric dependence. In Sec. IV, we summarize our numerical results and highlight the implications for future experiments.

II. TWO-QUBIT COHERENCE TIME

The system of two capacitively coupled singlet-triplet spin qubits is described by the following Ising-type Hamiltonian [11,13]:

$$H = \varepsilon J_1 J_2 \, \sigma_1^z \sigma_2^z + J_1 \sigma_1^z + J_2 \sigma_2^z + h_1 \sigma_1^x + h_2 \sigma_2^x. \quad (1)$$

Here we work in the singlet-triplet basis, with the $\sigma_i^z = +1$ (-1) eigenstate denoting the singlet (triplet) state of the two electrons in the double quantum dot constituting the ith spin qubit ($i = 1, 2$), respectively. For each spin qubit, the Zeeman $h_i \sigma_i^x$ term is controlled by the magnetic field gradient h_i across the corresponding double quantum dot, and the $J_i \sigma_i^z$ term is controlled by the intraqubit exchange coupling J_i between the two electrons in the qubit. The coupling $\varepsilon J_1 J_2 \sigma_1^z \sigma_2^z$ between the two qubits comes from the capacitive dipole-dipole interaction, with a strength approximately proportional to the product of intraqubit exchange couplings $J_1 J_2$ as argued empirically in Ref. [11].

We employ the quasistatic bath approximation and model the environmental noises by averaging observables over time-independent but randomly distributed model parameters. Specifically, the couplings J_1 and J_2 are drawn from a Gaussian distribution with mean J_0 and variance σ_J^2 but restricted to non-negative values, and the transverse fields h_1 and h_2 are drawn from a Gaussian distribution with mean h_0 and variance σ_h^2. Physically, the parameter J_0 is the (average) exchange coupling between the two electrons in each double quantum dot, and the parameter h_0 is the (average) magnetic

field gradient applied between the two electrons. In a typical experiment [13,27], the intraqubit exchange J_0 is on the order of 10^2 MHz, and the (quasistatic equivalent of) charge noise σ_J is on the order of $10^{-2} \sim 10^{-1} J_0$. The field noise σ_h may range from up to J_0 in GaAs to essentially negligible in isotopically purified ^{28}Si.

For our numerical calculations, we fix the interqubit coupling parameter ε to $0.1 J_0^{-1}$, and focus on the effect of the remaining dimensionless parameters σ_J / J_0, σ_h / J_0, and h_0 / J_0. For the disorder average, we typically use a sample size between 10^4 and 10^5 for each parameter set. We note that the dependence on h_0 is an important new element in the physics of Ising-coupled singlet-triplet qubits with no analog in the corresponding exchange coupled spin qubits studied in Ref. [15].

A. Decay of coupled-qubit oscillations

In the following we introduce an operational definition for the two-qubit coherence time T_2^* from the envelope decay of the coupled-qubit oscillations. Without loss of generality, we consider the product initial state $|\psi(0)\rangle = |\uparrow\rangle_x \otimes |\uparrow\rangle_x$ of the coupled qubits, and we compute the dynamics of the disorder-averaged return probability to the initial state

$$R(t) = [\![|\langle \psi(0)|\psi(t)\rangle|^2]\!]. \quad (2)$$

Here, the double bracket denotes the average over disorder realizations. As the initial state $|\psi(0)\rangle$ is not an eigenstate of the coupled qubits, the return probability is oscillatory in time. The oscillations have a typical frequency on the order of J_0, driven by the intraqubit exchange coupling $J_i \sigma_i^z$ terms in the Hamiltonian [Eq. (1)], and they are further modulated by beats with frequency around εJ_0^2 due to the interqubit coupling $\varepsilon J_1 J_2 \sigma_1^z \sigma_2^z$ term. The oscillations in $R(t)$ are damped by environmental noises through disorder averaging, in a fashion mathematically similar to (although physically distinct from) the decaying Rabi oscillations of a single qubit in the presence of environmental noises. Very loosely one can think of these oscillations as the two-qubit Rabi oscillations decaying due to charge and field noises. A few representative examples of the decaying $R(t)$ curves are shown in Fig. 1, using noise parameters approximately consistent with experimental situations.

We extract from $R(t)$ a characteristic time scale T_2^* associated with the decay of the oscillation envelope of the coupled qubits, and use it as a quantified measure of the coupled-qubit decoherence. Compared with the exchange-only case [15], the $R(t)$ oscillations here have more complicated wave forms, with extra beats in the decay envelope. Since these additional features are irrelevant to our main goal of characterizing the damping effect of environmental noises, we disregard them and adopt a simple fitting procedure that focuses only on the decay envelope. Operationally, we take the upper envelope of the $R(t)$ oscillations and perform on it a least-squares fit to an exponential decay of the form

$$R(\infty) + A \, e^{-t/T_2^*}, \quad (3)$$

where $R(\infty)$ is estimated from the asymptotic value of $R(t)$ and A is a nuisance parameter of no interest in the current work. The fitted upper envelope of $R(t)$ and the coherence time T_2^* are shown in Fig. 1 for a few representative examples.
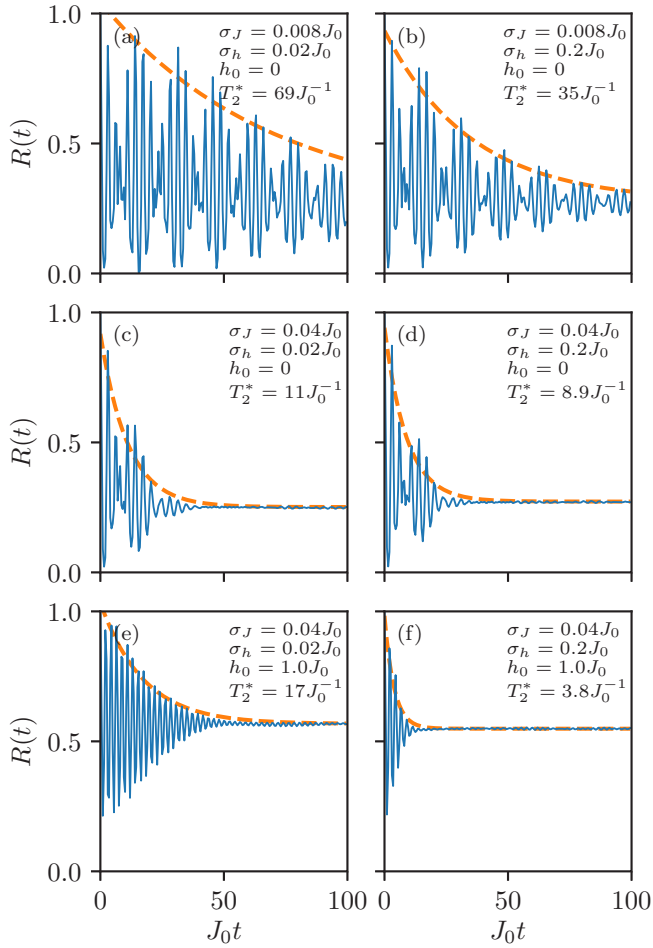
FIG. 1. Representative examples for the disorder-averaged return probability $R(t)$ (blue) for various system parameters. The dashed orange lines show the least-squares fit of the upper envelope of $R(t)$ to the exponential decay form in Eq. (3). The system parameters σ_J, σ_h, h_0 and the extracted coherence time T_2^* are listed in each panel.

Compared with the exchange-only case studied in Ref. [15], here we are using a slightly different operational definition for the two-qubit coherence time. This is necessitated by the irregular wave forms of the coupled-qubit oscillations allowed by a larger Hilbert space. We emphasize that this alternative choice only introduces moderate variations in the numerical value of T_2^* and does not affect our conclusions qualitatively. In addition, it is worth emphasizing that the coherence time in this paper measures the decay rate of the *oscillation envelope* of the disorder-averaged return probability $R(t)$, rather than the decay rate of $R(t)$ itself. As we noted in a previous paper [28], the latter definition is more appropriate for a large number of coupled qubits, whereas the definition adopted here provides a more precise measure of the decoherence process within a low-dimensional Hilbert space appropriate for just two coupled qubits.

B. Quality factor

The two-qubit coherence time T_2^* as defined above measures the time it takes for the envelope of the damped
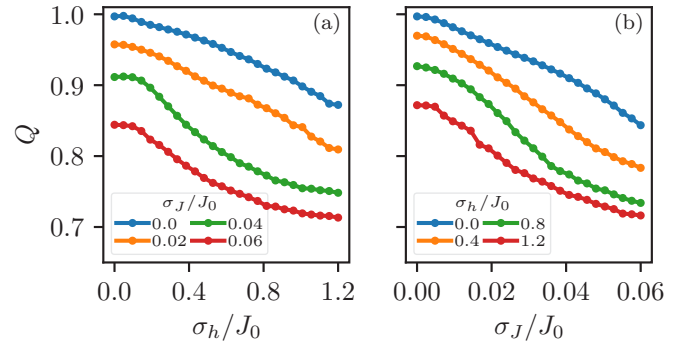

FIG. 2. Quality factor Q (a) as a function of the field noise σ_h for various values of the charge noise σ_J, and (b) as a function of σ_J for various values of σ_h. Both panels have magnetic field gradient $h_0 = 0$.

oscillations in $R(t)$ to decay to $1/e$ of its initial value. Hence, the dimensionless combination $J_0 T_2^*$ can be thought of as the number of appreciable oscillations in $R(t)$ before it saturates to the asymptotic value $R(\infty)$. In the results presented in Fig. 1, the dimensionless parameter $J_0 T_2^*$ varies from 69 [Fig. 1(a)] to 4 [Fig. 1(f)], with the results of Fig. 1(f) being the most representative of the current experimental state of the arts in GaAs singlet-triplet qubits [11,13] where only a few ($\lesssim 5$) two-qubit gate oscillations have so far been achieved experimentally. (We mention, however, that the experiments [11,13] are mostly in the $h_0 > 0$ regime more appropriate for the discussion in the next subsection of this paper.) To convert this into a number with a normalization comparable with other fidelity measures, we further define the quality factor [15]

$$Q = \exp\left(-\frac{1}{J_0 T_2^*}\right). \tag{4}$$

This quantity is essentially the exponential decay factor for the return probability oscillation envelope over $\Delta t = 1/J_0$, the intraqubit exchange coupling time scale.

In the rest of this section we present numerical results on the decoherence of two singlet-triplet qubits using the quality factor Q as a quantitative measure of coherence. We will make comparisons with the exchange-only case studied in Ref. [15] where appropriate, and explain how the additional tunability of the singlet-triplet system through the magnetic field gradient may be exploited to suppress decoherence due to charge noise.

C. Noise dependence

We first consider the case where the magnetic field gradient is zero on average, $h_0 = 0$, and examine the variation of the coherence time with respect to both the charge noise σ_J and the field noise σ_h. Within the $h_0 = 0$ parameter subspace, we find that the singlet-triplet qubits behave similarly to the exchange-only qubits as reported in Ref. [15].

Figures 2 and 3 show the dependence of the quality factor Q on the charge noise σ_J as well as the field noise σ_h. We find that the quality factor for the coupled qubits is suppressed when either type of noise increases, and the system is significantly more susceptible to charge noise than field noise. As marked by a contour line in Fig. 3, to achieve a quality factor Q higher
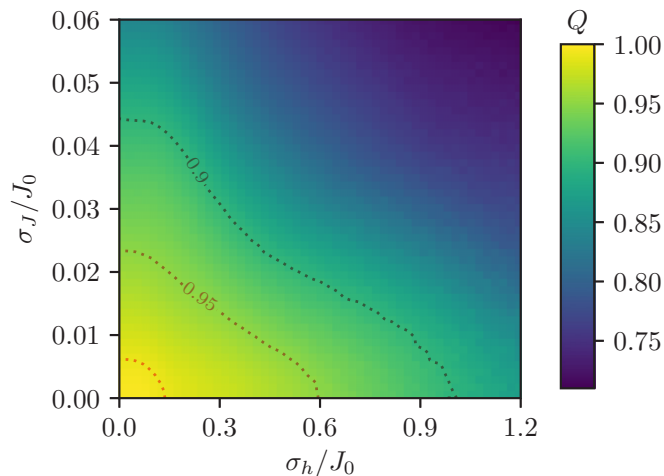
FIG. 3. Quality factor Q as a function of the charge noise σ_J and the field noise σ_h, for magnetic field gradient $h_0 = 0$. The three dotted contour lines mark the levels $Q = 0.9$, 0.95, and 0.99.

than 0.9 (corresponding to a coherence time $T_2^* \sim 10 J_0^{-1}$), the maximum allowed charge noise σ_J is around $0.045 J_0$, while the maximum allowed field noise σ_h is around $1.0 J_0$.

The order of magnitude difference between the sensitivity to charge noise and the sensitivity to field noise as measured by two-qubit coherence time is in agreement with the results previous reported on the exchange-only qubits [15]. Quantitatively, we find that the singlet-triplet system is about 3 (1.5) times more sensitive to the charge (field) noise compared with the exchange-only system in the regime with a quality factor $Q \geqslant 0.9$. This is consistent with the intuitive observation that the exchange-only system enjoys an additional protection due to the spin S_z conservation. The fact that charge noise is the dominant decohering mechanism for singlet-triplet qubits (and is even more detrimental here than for exchange-only qubits) is, however, only true for $h_0 = 0$ as we discuss next.

D. Effect of the magnetic field gradient

Experimentally, charge noise in GaAs-based spin-qubit devices is typically much weaker in absolute strength than field noise, due to the strong Overhauser nuclear spin fluctuations. In Si-based spin-qubit devices, however, the nuclear spin fluctuations can be significantly suppressed thanks to isotope purification of ^{28}Si [26]. In this case, the strong sensitivity to charge noise may pose a serious obstacle to the fidelity of coupled qubits, since there is no known way to systematically reduce the charge noise in semiconductor structures. From our numerical results, we find that this problem may be alleviated through the additional tunable parameter in the singlet-triplet system, namely, the average magnetic field gradient h_0 across each singlet-triplet qubit.

Figure 4 shows the effect of h_0 on the noise dependence of the quality factor Q. As the magnetic field gradient h_0 goes up, the coherence of the coupled-qubit dynamics becomes less sensitive to the charge noise σ_J, but more vulnerable to the field noise σ_h. When h_0 is higher than J_0, the coupled qubits become more susceptible to field noise than charge noise, in sharp contrast to the situation for $h_0 = 0$. The sensitivity to σ_h
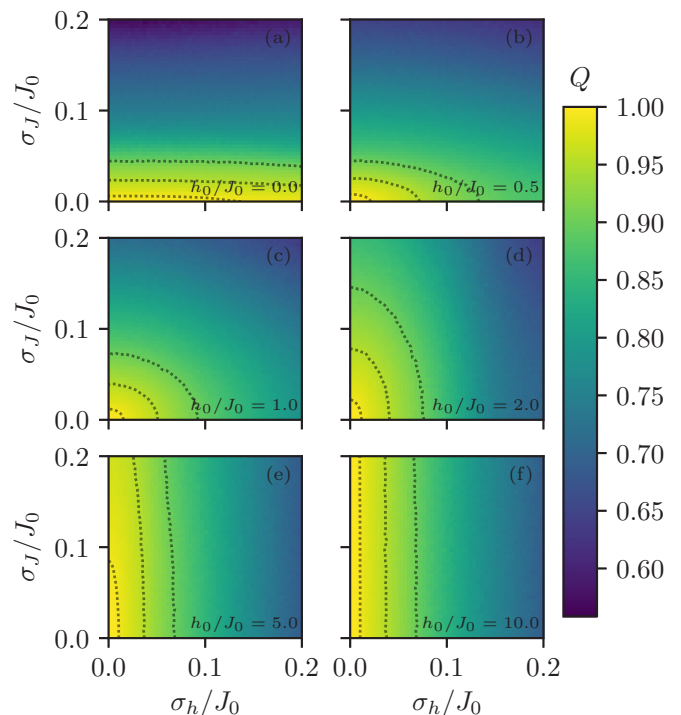

FIG. 4. Quality factor Q as a function of the charge noise σ_J and the field noise σ_h, for a range of magnetic field gradient h_0 from 0 to $10 J_0$. All panels share the same color map depicted on the right. In each panel, the three dotted contour lines mark the levels $Q = 0.99, 0.95, 0.90$ (from the bottom-left corner inwards).

saturates when h_0 is more than a few times stronger than J_0. For reference, we note that the entangling gate experiments reported in Ref. [13] were carried out at an effective $h_0 \sim 5 J_0$, albeit under a different setup with individual qubits driven by an oscillatory $J_i(t)$. Comparing the numerical results in Figs. 3 and 4(e), we find that the maximum allowed charge noise to achieve a high quality factor $Q \geqslant 0.99$ increases by more than 10 times as the magnetic field gradient h_0 is cranked up from zero to $5 J_0$. This enhanced stability against charge noise is consistent with the experimental observation in Ref. [13] that a magnetic field gradient $h_0 \sim 5 J_0$ increases the two-qubit coherence time by an order of magnitude in a device dominated by charge noise. We mention here that the GaAs system used in Ref. [13] obviously also has considerable field noise, arising from nuclear spin fluctuations in Ga and As, contributing to decoherence.

III. FIDELITY OF ENTANGLED STATE PREPARATION

The two-qubit coherence time T_2^* measures the persistence of two-qubit oscillations in the presence of environmental noises. This is a characterization of how well the system retains the initial non-eigenstate information. In this section, we study a different aspect of two-qubit fidelity, namely, the fidelity F_E of preparing an entangled state. We investigate how well the system produces an entangled state starting from an initial product state under the influence of environmental noises. This analysis is less sophisticated than a full-blown gate fidelity calculation using randomized benchmarking. Nevertheless, it

provides useful insights through a perspective complementary to the two-qubit coherence time T_2^*, and provides a single fidelity number (the entanglement fidelity, F_E) similar to the full-blown numerically intensive Clifford gate randomized benchmarking calculation which is beyond the scope of the current work.

A. Producing an entangled state

We choose the product initial state $|\phi(0)\rangle = |\uparrow\rangle_x \otimes |\downarrow\rangle_x$ and let the system evolve under the Hamiltonian H in Eq. (1) for a fixed amount of time t (to be specified). In the clean limit, the resulting state is

$$|E(t)\rangle = e^{-i[\varepsilon J_0^2 \sigma_1^z \sigma_2^z + J_0(\sigma_1^z + \sigma_2^z) + h_0(\sigma_1^x + \sigma_2^x)]t}|\phi(0)\rangle. \quad (5)$$

This is in general an entangled state, and as we discuss below, with a proper choice of the evolution time t, $|E(t)\rangle$ is in fact maximally entangled for both $h_0 = 0$ and $h_0 \gg J_0$. It should be noted that the Hamiltonian H is not always effective at generating entanglement starting from an arbitrary initial state. The particular initial state $|\phi(0)\rangle = |\uparrow\rangle_x \otimes |\downarrow\rangle_x$ chosen here provides a simple setup to discuss the effect of noise on entangled state preparation.

In the presence of environmental noises, the time-evolved state $e^{-iHt}|\phi(0)\rangle$ depends on the disorder realization and deviates from its clean limit $|E(t)\rangle$. Using the latter as a reference, we define the fidelity of entangled state preparation [29] F_E as the disorder-averaged overlap

$$F_E = [\![|\langle E(t)|e^{-iHt}|\phi(0)\rangle|^2]\!]. \quad (6)$$

Similarly to the two-qubit coherence time T_2^*, this is a function of the field noise σ_h, the charge noise σ_J, and the magnetic field gradient h_0.

We want to make sure that F_E indeed measures the fidelity associated with generating an *entangled* state. To this end, we now discuss the choice of the evolution time t that maximizes the entanglement between the two qubits in the reference state $|E(t)\rangle$. In the absence of a magnetic field gradient h_0, the reduced density matrix after tracing out one qubit in $|E(t)\rangle$ takes the simple form

$$\frac{1}{2}\begin{pmatrix} 1 & -e^{-2iJ_0 t}\cos\left(2\varepsilon J_0^2 t\right) \\ -e^{2iJ_0 t}\cos\left(2\varepsilon J_0^2 t\right) & 1 \end{pmatrix}. \quad (7)$$

This suggests setting the evolution time t in Eq. (6) to

$$t_0 = \frac{\pi}{4\varepsilon J_0^2}, \quad (8)$$

where εJ_0 measures the strength of the interqubit Ising coupling (set to 0.1 in this paper). This choice ensures that the reference state $|E(t_0)\rangle$ is maximally entangled between the two qubits for $h_0 = 0$, with entanglement entropy $S_E = \log 2$.

The situation for $h_0 \neq 0$ is less obvious. Figure 5(a) shows the dependence of the entanglement entropy S_E between the two qubits on the evolution time t, for various values of h_0. We find that for both $h_0 = 0$ and $h_0 \gg J_0$, the entanglement entropy of $|E(t)\rangle$ peaks at $t = t_0$, whereas for intermediate $h_0 \sim J_0$, the entanglement entropy has irregular dynamics but still reaches a moderate level at $t = t_0$. Figure 5(b) shows the entanglement entropy at $t = t_0$ as a function of the magnetic
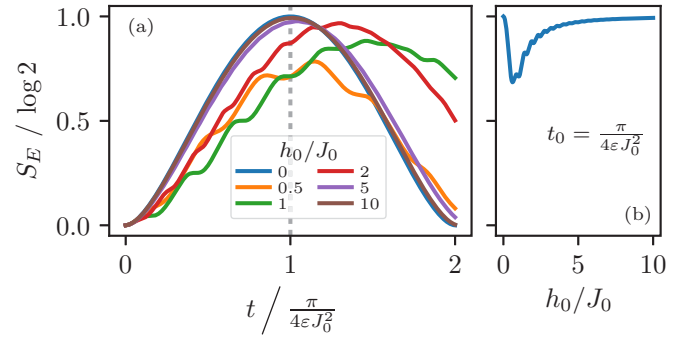

FIG. 5. (a) Entanglement entropy S_E of $|E(t)\rangle$ as a function of the evolution time t for various values of h_0. The evolution time t is shown in unit of $t_0 = \frac{\pi}{4\varepsilon J_0^2}$, whereas the entanglement entropy S_E is shown in units of its maximal value $\log 2$. (b) Entanglement entropy of $|E(t_0)\rangle$ for $t_0 = \frac{\pi}{4\varepsilon J_0^2}$, as a function of h_0/J_0. Data in both panels are computed in the clean limit $\sigma_J = \sigma_h = 0$.

field gradient h_0. We find that the reference state at $t = t_0$ is nearly maximally entangled for a wide range of h_0 except for a small window near J_0. This justifies our operational definition of the entanglement fidelity using the evolution time t_0 defined in Eq. (8).

B. Noise dependence of F_E

We now examine how the fidelity of entangled state preparation is affected by environmental noises. First we consider the case of zero magnetic gradient $h_0 = 0$. Figure 6 shows the dependence of the entanglement fidelity F_E as a function of the charge noise σ_J and the field noise σ_h. We find that F_E decays monotonically when either type of noise increases, and the system is more susceptible to the charge noise σ_J than the field noise σ_h. To reach F_E higher than 0.9, the maximum allowed charge noise σ_J is around $0.03 J_0$, while the maximum allowed field noise σ_h is around $0.18 J_0$. We observe that the fidelity of entangled state generation has
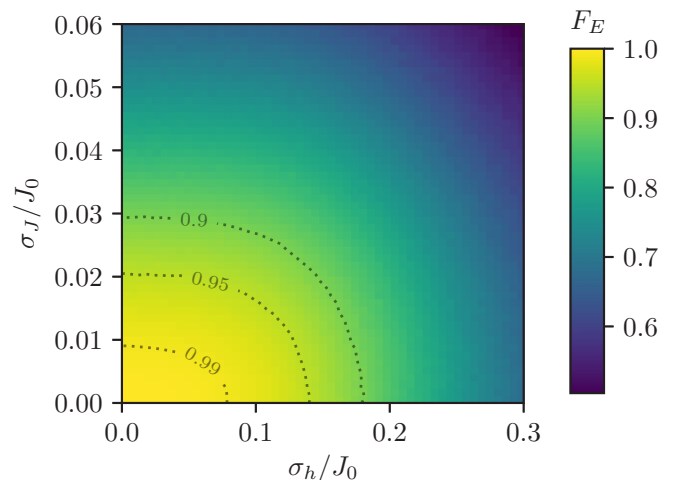

FIG. 6. Entanglement fidelity F_E as a function of the charge noise σ_J and the field noise σ_h at zero magnetic field gradient h_0. The three dotted contour lines mark the levels $F_E = 0.9$, 0.95, and 0.99.
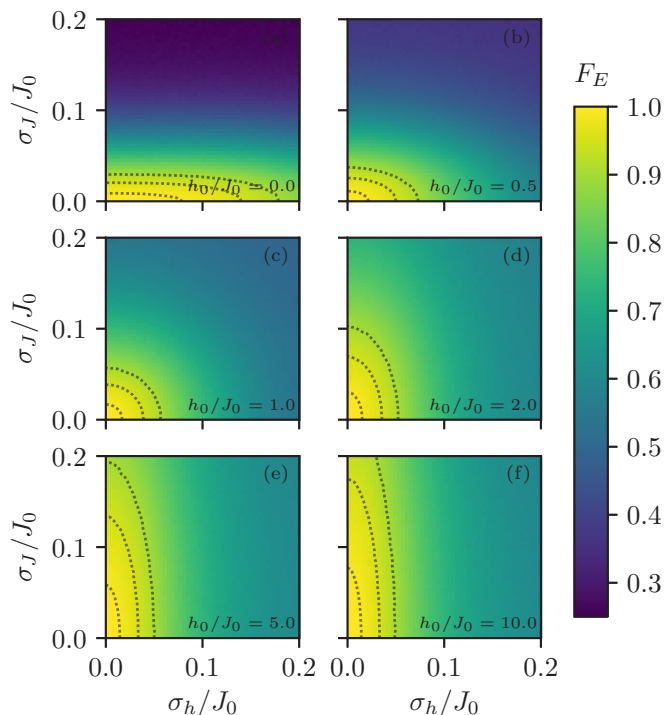
FIG. 7. Entanglement fidelity F_E as a function of the charge noise σ_J and the field noise σ_h, for a range of magnetic field gradient h_0 from 0 to $10J_0$. All panels share the same color map depicted on the right. In each panel, the dotted contour lines mark the levels $F_E = 0.99, 0.95, 0.90$ (from the bottom-left corner inwards).

a charge noise dependence comparable to that of the quality factor Q associated with the two-qubit coherence time T_2^*, but it has a field noise dependence about 5 times stronger than that of the quality factor Q. This suggests that field noise is more effective at disrupting the precise preparing of an entangled state than damping the coupled-qubit oscillations. This is germane for future progress in the subject since field noise can essentially be eliminated is Si qubits through isotopic purification.

Compared with the exchange-only qubits studied in Ref. [15], the entanglement fidelity F_E for the singlet-triplet qubits computed here is significantly more susceptible to charge noise. Intuitively, this is consistent with the fact that the Ising Hamiltonian of the singlet-triplet system has a weaker (by a factor of εJ_0) interqubit coupling and thus is less effective at entangling the two qubits than the Heisenberg Hamiltonian for the exchange-only system. The longer evolution time strengthens the effect of charge noise as it modifies the qubit precession frequency. This particular damaging aspect of charge noise can be partially rectified by having stronger interqubit coupling through careful qubit geometry engineering.

The noise dependence of F_E changes qualitatively when we turn on the magnetic field gradient h_0. The progression is shown in Fig. 7. As the magnetic field gradient h_0 increases, the entanglement fidelity F_E quickly develops more sensitivity to the field noise σ_h while becoming less susceptible to the charge noise σ_J. At the turning point $h_0 = J_0$, the noise dependence of F_E is approximately symmetric with respect to σ_J and σ_h. As the magnetic field gradient h_0 increases further, the sensitivity

to charge noise is quickly suppressed, while the sensitivity to field noise reaches a plateau. For $h_0 \gg J_0$, the entanglement fidelity F_E is limited mainly by the field noise σ_h (again implying a considerable advantage for isotopically purified Si qubits). Overall, we find that the fidelity of entangled state preparation has a noise dependence qualitatively similar to that of the coherence time quality factor.

IV. CONCLUSION

In this paper we have studied the decoherence of two singlet-triplet spin qubits with capacitive coupling under the influence of quasistatic environmental noises. We consider two complementary decoherence measures for coupled qubits, namely, the two-qubit coherence time characterizing the persistence of coupled-qubit oscillations, and the fidelity of entangled state preparation. Through numerical calculations, we provide a quantitative map of the dependence of each decoherence measure on charge noise, field noise, and the intraqubit magnetic field gradient.

We find that the noise dependence of the coupled-qubit coherence changes qualitatively as the magnetic field gradient increases. When the (average) magnetic field gradient vanishes, the coupled-qubit system is more susceptible to charge noise than field noise. For the two-qubit coherence time to be longer than $10J_0^{-1}$, the maximum allowed charge noise is an order of magnitude lower than the maximum allowed field noise. The fidelity of entangled state preparation has a similar (although less pronounced) bias in its noise sensitivity. In contrast, when the coupled-qubit system is dominated by a strong magnetic field gradient, the sensitivity to charge noise is strongly suppressed and becomes much weaker than the sensitivity to field noise, as visible in both the two-qubit coherence time and the entanglement fidelity.

Our results highlight the impact of the magnetic field gradient on the noise dependence of the coupled-qubit system. Increasing the magnetic field gradient h_0 proves to be an effective measure to protect against charge noise the coherence of coupled singlet-triplet qubits in terms of both the persistence of coupled-qubit oscillations and the precise preparation of entangled states. In addition, our work points to clear advantages for Si-based qubits over GaAs qubits since isotopic purification could eliminate field noise in Si (but not in GaAs). Elimination of field noise would enhance fidelity, and working in a large field gradient would suppress the charge noise, eventually leading to high-fidelity singlet-triplet semiconductor spin qubits suitable for quantum error correction protocols. Our work establishes, however, that even in the best possible circumstances (Si qubits with no field noise working at a large field gradient), the magnitude of the effective charge noise still must be reduced below 1%–2% of the basic intraqubit exchange coupling J_0 producing the singlet-triplet qubits, so that a quality factor and an entanglement fidelity surpassing 99% can be achieved for 2-qubit operations.

[1] D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998).

[2] D. P. DiVincenzo, D. Bacon, J. Kempe, G. Burkard, and K. B. Whaley, Nature (London) **408**, 339 (2000).

[3] J. Levy, Phys. Rev. Lett. **89**, 147902 (2002).

[4] F. H. L. Koppens, C. Buizert, K. J. Tielrooij, I. T. Vink, K. C. Nowack, T. Meunier, L. P. Kouwenhoven, and L. M. K. Vandersypen, Nature (London) **442**, 766 (2006).

[5] M. Veldhorst, J. C. C. Hwang, C. H. Yang, A. W. Leenstra, B. de Ronde, J. P. Dehollain, J. T. Muhonen, F. E. Hudson, K. M. Itoh, A. Morello, and A. S. Dzurak, Nat. Nanotechnol. **9**, 981 (2014).

[6] M. Friesen, P. Rugheimer, D. E. Savage, M. G. Lagally, D. W. van der Weide, R. Joynt, and M. A. Eriksson, Phys. Rev. B **67**, 121301 (2003).

[7] J. M. Taylor, H.-A. Engel, W. Dür, A. Yacoby, C. M. Marcus, P. Zoller, and M. D. Lukin, Nat. Phys. **1**, 177 (2005).

[8] F. A. Zwanenburg, A. S. Dzurak, A. Morello, M. Y. Simmons, L. C. L. Hollenberg, G. Klimeck, S. Rogge, S. N. Coppersmith, and M. A. Eriksson, Rev. Mod. Phys. **85**, 961 (2013).

[9] I. van Weperen, B. D. Armstrong, E. A. Laird, J. Medford, C. M. Marcus, M. P. Hanson, and A. C. Gossard, Phys. Rev. Lett. **107**, 030506 (2011).

[10] K. C. Nowack, M. Shafiei, M. Laforest, G. E. D. K. Prawiroatmodjo, L. R. Schreiber, C. Reichl, W. Wegscheider, and L. M. K. Vandersypen, Science **333**, 1269 (2011).

[11] M. D. Shulman, O. E. Dial, S. P. Harvey, H. Bluhm, V. Umansky, and A. Yacoby, Science **336**, 202 (2012).

[12] M. Veldhorst, C. H. Yang, J. C. C. Hwang, W. Huang, J. P. Dehollain, J. T. Muhonen, S. Simmons, A. Laucht, F. E. Hudson, K. M. Itoh, A. Morello, and A. S. Dzurak, Nature (London) **526**, 410 (2015).

[13] J. M. Nichol, L. A. Orona, S. P. Harvey, S. Fallahi, G. C. Gardner, M. J. Manfra, and A. Yacoby, npj Quantum Inf. **3**, 3 (2017).

[14] J. R. Petta, A. C. Johnson, J. M. Taylor, E. A. Laird, A. Yacoby, M. D. Lukin, C. M. Marcus, M. P. Hanson, and A. C. Gossard, Science **309**, 2180 (2005).

[15] R. E. Throckmorton, E. Barnes, and S. Das Sarma, Phys. Rev. B **95**, 085405 (2017).

[16] V. W. Scarola and S. Das Sarma, Phys. Rev. A **71**, 032340 (2005).

[17] X. Hu and S. Das Sarma, Phys. Rev. A **61**, 062301 (2000).

[18] X. Wu, D. R. Ward, J. R. Prance, D. Kim, J. K. Gamble, R. T. Mohr, Z. Shi, D. E. Savage, M. G. Lagally, M. Friesen, S. N. Coppersmith, and M. A. Eriksson, Proc. Natl. Acad. Sci. USA **111**, 11938 (2014).

[19] X. Wang, E. Barnes, and S. Das Sarma, npj Quantum Inf. **1**, 15003 (2015).

[20] X. Wang, L. S. Bishop, J. Kestner, E. Barnes, K. Sun, and S. Das Sarma, Nat. Commun. **3**, 997 (2012).

[21] X. Wang, L. S. Bishop, E. Barnes, J. P. Kestner, and S. Das Sarma, Phys. Rev. A **89**, 022310 (2014).

[22] C. Zhang, R. E. Throckmorton, X.-C. Yang, X. Wang, E. Barnes, and S. Das Sarma, Phys. Rev. Lett. **118**, 216802 (2017).

[23] R. E. Throckmorton, C. Zhang, X.-C. Yang, X. Wang, E. Barnes, and S. Das Sarma, arXiv:1709.02808.

[24] X. Hu and S. Das Sarma, Phys. Rev. Lett. **96**, 100501 (2006).

[25] R. de Sousa and S. Das Sarma, Phys. Rev. B **67**, 033301 (2003).

[26] W. M. Witzel, M. S. Carroll, A. Morello, L. Cywiński, and S. Das Sarma, Phys. Rev. Lett. **105**, 187602 (2010).

[27] F. Martins, F. K. Malinowski, P. D. Nissen, E. Barnes, S. Fallahi, G. C. Gardner, M. J. Manfra, C. M. Marcus, and F. Kuemmeth, Phys. Rev. Lett. **116**, 116801 (2016).

[28] Y.-L. Wu, D.-L. Deng, X. Li, and S. Das Sarma, Phys. Rev. B **95**, 014202 (2017).

[29] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).