

## Thermal decay of Coulomb blockade oscillations

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We study transport properties and the charge quantization phenomenon in a small metallic island connected to the leads through two quantum point contacts (QPCs). The linear conductance is calculated perturbatively with respect to weak tunneling and weak backscattering at QPCs as a function of the temperature  $T$  and gate voltage. The conductance shows Coulomb blockade (CB) oscillations as a function of the gate voltage that decay with the temperature as a result of thermally activated fluctuations of the charge in the island. The regimes of quantum  $T \ll E_C$  and thermal  $T \gg E_C$  fluctuations are considered, where  $E_C$  is the charging energy of an isolated island. Our predictions for CB oscillations in the quantum regime coincide with previous findings by Furusaki and Matveev [*Phys. Rev. B* **52**, 16676 (1995)]. In the thermal regime the visibility of Coulomb blockade oscillations decays with the temperature as  $\sqrt{T/E_C} \exp(-\pi^2 T/E_C)$ , where the exponential dependence originates from the thermal averaging over the instant charge fluctuations, while the prefactor has a quantum origin. This dependence does not depend on the strength of couplings to the leads. The differential capacitance, calculated in the case of a single tunnel junction, shows the same exponential decay, however the prefactor is linear in the temperature. This difference can be attributed to the nonlocality of the quantum effects. Our results agree with the recent experiment [*Nature (London)* **536**, 58 (2016)] in the whole range of the parameter  $T/E_C$ .

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### I. INTRODUCTION

The transport of electrons through small mesoscopic conductors, such as metallic and semiconductor quantum dots, has been extensively studied both experimentally and theoretically [1]. One of the most popular experimental systems in this field is the single electron transistor (SET), which consists of a quantum dot (typically, a small micrometer-scale metallic island) connected to two metallic leads by tunnel junctions and capacitively coupled to an additional gate electrode [2]. Such a three-terminal device has been theoretically proposed by Averin and Likharev [3], and fabricated and characterized by Fulton and Dolan [4]. The most important characteristic of these devices, which sets them apart from the conventional field effect transistors, is that they can be switched between insulating and conducting state by adding a small amount of charge to the gate electrode as the charge of an electron  $e$ .

Such strong charge sensitivity of SETs is a consequence of the fact that the charge of an isolated metallic island is quantized in units of the elementary charge  $e$ . Tunneling of an electron from a lead into the island changes its charge by  $e$  and increases the charging energy of the system by the amount of order  $E_C = e^2/2C$ , where  $C$  is a geometrical capacitance of the island. When the temperature is small,  $T \ll E_C$ , tunneling is strongly suppressed except at degeneracy points between states, where the number of electrons in the island differs by one. Therefore, the conductance of the SET exhibits a series of peaks as a function of the applied gate voltage. This well-known phenomenon, called the Coulomb blockade (CB) effect, is widely reviewed in the literature [1,2,5,6]. Recently, it has received renewing interest in the context of the quantum information processing, where the semiconductor-metal hybrid devices serve as crucial elements for quantum computing [7], and in the quest for topologically protected quantum bits [8].

The growing interest in single electron phenomena stimulated further research and recently resulted in the publication of the thorough experimental study of the decay of CB oscillations in mesoscopic circuits induced by thermal and quantum fluctuation of charge [9]. The experimental setup in Ref. [9] is schematically shown in Fig. 1. The experimentalists fabricated a hybrid SET based on a two-dimensional electron gas in the integer quantum Hall (QH) regime by attaching a central micrometer-scale metallic island to large electrodes with the help of edge channels. Unlike in earlier experiments [10–12], this new approach allows a precise control of coupling of the island to the leads by using two QPCs, which mix incoming and outgoing edge channels with the complex amplitudes  $\tau_L$  and  $\tau_R$ . The metallic island has a negligible level spacing, which implies that it can be considered a reservoir for electron-hole excitations. The experimentalists measured the visibility of CB oscillations in the linear conductance of the SET as a function of the tunneling amplitudes  $\tau_{L,R}$  and the temperature  $T$  in the whole range of parameters. The purpose of the present paper is to present the theory and interpretation of the observed effects in the regimes of weak tunneling and weak backscattering at QPCs, which are theoretically accessible by using the tunneling Hamiltonian approach.

Between a large number of previous theoretical works on the CB effect (see, e.g., Refs. [13–20]) the earlier theory [21] of Furusaki and Matveev deserves special attention in the present context, because they have addressed the CB oscillations with the help of the model described above (see Fig. 1). Using the standard bosonization technique, they have made quantitative predictions for the visibility of CB oscillations in the quantum regime  $T \ll E_C$  in the case of strong tunneling, where one or both QPCs are close to perfect transmission. Here we extend these predictions to arbitrary temperatures by using an alternative approach to bosonization [22,23], which is based on the scattering theory for bosons as well as the Langevin

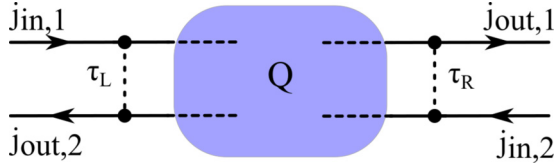


FIG. 1. The experimental system in Ref. [9] is schematically shown. In the integer quantum Hall regime, where only one spinless edge mode contributes to the transport, a metallic island (containing the charge  $Q$ ) is connected to the leads by two QPCs, characterized by the backscattering amplitudes  $\tau_L$  and  $\tau_R$ . The edge states are described by four bosonic operators, labeled by  $\alpha = \text{in, out}$  and  $j = 1, 2$ . The average current  $\langle I \rangle$  is calculated through the cross section immediately to the right of the right QPC.

equation method. Our quantitative predictions fully agree with the experimental data in the whole range of temperatures. In the thermal regime  $T \gg E_C$ , the amplitude of CB oscillations in the conductance scales as  $\delta G \propto \sqrt{T/E_C} \exp(-\pi^2 T/E_C)$ , independently of the geometry of the system. Here the exponential function results from the averaging of CB oscillations over the classical thermal fluctuations of the charge in the island, while the power-law prefactor has a quantum origin. Interestingly, the CB oscillations in the differential capacitance of the island acquire the linear in  $T$  prefactor, which points to the nonuniversality of this contribution and to its quantum origin.

The rest of the paper is organized as follows. In Sec. II we introduce the model of the system, starting with the Hamiltonian of all the constituting parts and the bosonization prescription, and formulate the method of quantum Langevin equations. In Sec. III we calculate the linear conductance and the visibility of CB oscillations for the case of the symmetric setup, i.e., for weak backscattering at both QPCs, using the perturbation approach in backscattering amplitudes. In Sec. IV we develop the perturbation theory for the case of the asymmetric setup with weak tunneling at one of the two QPCs and calculate the conductance and visibility in this regime. Section V is devoted to the derivation of the differential capacitance. We present our conclusions and the discussion in Sec. VI. Details of the calculations are given in the Appendixes. Throughout the paper we set  $e = \hbar = k_B = 1$ .

## II. THEORETICAL MODEL

### A. Hamiltonian

We start by introducing the Hamiltonian of the experimental system in Ref. [9], see Fig. 1. The relevant energy scales in this experiment are sufficiently small compared to the Fermi energy  $\epsilon_F$ , which suggests using the effective low-energy theory of QH edge states [24]. The advantage of this approach is that it allows us to take into account Coulomb interaction at the metallic island in a straightforward way [23]. According to the effective theory, edge states can be described as collective fluctuations of the charge densities  $\rho_{\alpha j}(x)$ , where indexes label the number of the channel  $j = 1, 2$  and the state  $\alpha = \text{in, out}$ . The charge density operators are expressed in terms of bosonic fields  $\phi_{\alpha j}(x)$ , namely,  $\rho_{\alpha j}(x) = (1/2\pi)\partial_x \phi_{\alpha j}(x)$ . These free bosonic fields satisfy canonical commutation relations

$$[\partial_x \phi_{\alpha j}(x), \phi_{\beta k}(y)] = (-1)^\alpha 2\pi i \delta_{\alpha\beta} \delta_{jk} \delta(x - y), \quad (1)$$

where the sign determines the propagation direction of the edge states.

The total Hamiltonian includes three terms

$$H = H_0 + H_{\text{int}} + H_T. \quad (2)$$

Here

$$H_0 = \frac{v_F}{4\pi} \sum_{\substack{\alpha = \text{in, out} \\ j = 1, 2}} \int dx [\partial_x \phi_{\alpha j}(x)]^2 \quad (3)$$

is the free part of the total Hamiltonian, and the Fermi velocity  $v_F$  is the same for each edge channel. The second term describes Coulomb interaction at the metallic island:

$$H_{\text{int}} = \frac{(Q - Q_0)^2}{2C}, \quad (4)$$

where

$$Q = \frac{1}{2\pi} \int_0^\infty dx [\partial_x \phi_{\text{in}1}(x) + \partial_x \phi_{\text{out}2}(x)] + \frac{1}{2\pi} \int_{-\infty}^0 dx [\partial_x \phi_{\text{in}2}(x) + \partial_x \phi_{\text{out}1}(x)] \quad (5)$$

is the operator of the total charge on the metallic island,  $C$  being its geometrical capacitance, and the parameter  $Q_0$  is proportional to the gate voltage and the capacitance  $C_g$  between the island and the gate, namely  $Q_0 = C_g V_g$ . Thus the role of the gate is to control the average charge of the metallic island.

The last term describes backscattering of electrons at the left and right QPC,

$$H_T = A_L + A_R + \text{H.c.},$$

$$A_L = \frac{\tau_L}{a} e^{i\phi_{\text{in}1}(0) - i\phi_{\text{out}2}(0)}, \quad (6)$$

$$A_R = \frac{\tau_R}{a} e^{i\phi_{\text{out}1}(0) - i\phi_{\text{in}2}(0)},$$

where  $\tau_i$  are the tunneling coupling constants, and  $a$  is the ultraviolet cutoff [23, 25–27]. Note that here we set the distance between the grain and QPCs to zero, because in the experiment it is much shorter than the characteristic wavelength of excitations  $\lambda \sim v_F/T$ .

The Hamiltonian (2) gives the complete description of our system. We note that the part  $H_0 + H_{\text{int}}$  is quadratic in bosonic operators, thus the dynamics associated with this Hamiltonian can be accounted for exactly by solving linear equations of motion. We follow Refs. [22, 23] and complement these equations with the boundary conditions in terms of dissipative currents originating from the reservoirs and the metallic island. Fluctuations of these currents are Gaussian, therefore, they can be considered Gaussian sources in the so-arising quantum Langevin equations. In contrast, the tunneling term (6) is nonlinear in bosonic operators. Therefore, in Secs. III and IV we develop the perturbation theory to leading order in backscattering and tunneling amplitudes.

### B. Quantum Langevin equations

As the first step, we consider the currents in the system shown in Fig. 1, when QPCs are fully open (symmetric setup), and write the equations of motion generated by the part

$H_0 + H_{\text{int}}$  of the total Hamiltonian in the following form [23]:

$$\begin{aligned} \frac{dQ(t)}{dt} &= \sum_{\alpha=1,2} j_{\text{in}\alpha}(t) - \sum_{\alpha=1,2} j_{\text{out}\alpha}(t), \\ j_{\text{out}\alpha}(t) &= \frac{Q(t) - Q_0}{R_q C} + j_{\alpha}^s(t), \end{aligned} \quad (7)$$

where  $R_q = 2\pi$  is a quantum resistance. Here the first equation expresses the conservation of charge. The second line is the Langevin equations, which have the following physical meaning. The outgoing currents acquire two contributions: the first one is the current induced by the time dependent potential  $Q(t)/C$ , and the second  $j_{\alpha}^s$  are the Langevin current sources originating from the metallic island.

Now we are ready to calculate the current  $\langle I \rangle_0$  through the system in absence of backscattering, i.e., in the case of fully open QPCs. We define it as the average current in the cross section immediately after the right QPC,

$$\langle I \rangle_0 = \langle j_{\text{out}1} \rangle - \langle j_{\text{in}2} \rangle. \quad (8)$$

Assuming that the first channel is voltage biased with  $\Delta\mu$ , while the second is grounded, we set  $\langle j_{\text{in}2} \rangle = 0$ . Averaging Eqs. (7) and taking into account that the average incoming current is equal to the average outgoing current, we obtain

$$\begin{aligned} \langle j_{\text{out}1} \rangle + \langle j_{\text{out}2} \rangle &= \Delta\mu/R_q, \\ \langle j_{\text{out}1} \rangle &= \langle j_{\text{out}2} \rangle = \frac{\langle Q \rangle - Q_0}{R_q C}. \end{aligned} \quad (9)$$

Next, we use these equations to calculate the average value for the total charge stored in the metallic island  $\langle Q \rangle = Q_0 + \Delta\mu C/2$ , and the average current through the system  $\langle I \rangle_0 = \langle j_{\text{out}1} \rangle = \Delta\mu/2R_q$ . Therefore, in the absence of backscattering the conductance acquires the value

$$G_0 = \frac{1}{2} G_q, \quad (10)$$

where  $G_q = 1/R_q = 1/2\pi$  is the conductance quantum, i.e., the system behaves as two quantum resistances connected in series.

According to Eqs. (7), the fluctuating currents satisfy the following equations:

$$\begin{aligned} \frac{d}{dt} \delta Q(t) &= \sum_{\alpha=1,2} \delta j_{\text{in}\alpha}(t) - \sum_{\alpha=1,2} \delta j_{\text{out}\alpha}(t), \\ \delta j_{\text{out}\alpha}(t) &= \frac{\delta Q(t)}{R_q C} + \delta j_{\alpha}^s(t). \end{aligned} \quad (11)$$

These equations can be easily solved in the frequency representation, and for the symmetric setup we obtain

$$\begin{pmatrix} \delta j_{\text{out}1}(\omega) \\ \delta j_{\text{out}2}(\omega) \end{pmatrix} = \begin{pmatrix} a(\omega) & b(\omega) & -b(\omega) & -b(\omega) \\ b(\omega) & a(\omega) & -b(\omega) & -b(\omega) \end{pmatrix} \begin{pmatrix} \delta j_1^s(\omega) \\ \delta j_2^s(\omega) \\ \delta j_{\text{in}1}(\omega) \\ \delta j_{\text{in}2}(\omega) \end{pmatrix}, \quad (12)$$

where  $a(\omega) = (i\omega R_q C - 1)/(i\omega R_q C - 2)$  and  $b(\omega) = 1/(i\omega R_q C - 2)$  are the scattering coefficients. Next, in order to address a particular experimental situation [9], we also

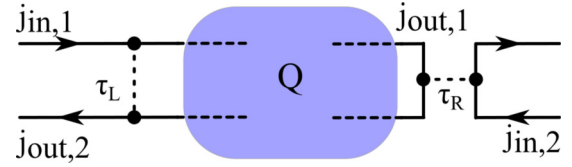


FIG. 2. The asymmetric setup, where the left QPC is almost fully open, while the right QPC is weakly transmitting, is schematically shown. We investigate CB oscillations to leading order in the backscattering amplitude  $\tau_L$  and tunneling amplitude  $\tau_R$ .

consider an asymmetric setup, i.e., where for instance the left QPC is fully open, while the right one is fully closed (see Fig. 2). We skip the details of the calculations, which are analogous to those in the case of the symmetric setup, and present the result

$$\begin{aligned} \delta j_{\text{out}1}(\omega) &= \delta j_1^s(\omega) + \frac{1}{i\omega R_q C - 1} [\delta j_2^s(\omega) - \delta j_{\text{in}1}(\omega)], \\ \delta j_{\text{out}2}(\omega) &= \frac{i\omega R_q C}{i\omega R_q C - 1} \delta j_1^s(\omega) - \frac{1}{i\omega R_q C - 1} \delta j_{\text{in}1}(\omega). \end{aligned} \quad (13)$$

Finally, the two-point correlation functions of the incoming currents and Langevin sources are given by the equilibrium spectral function [28]

$$\langle \delta j_{\alpha}^s(\omega) \delta j_{\beta}^s(\omega') \rangle = \langle \delta j_{\text{in}\alpha}(\omega) \delta j_{\text{in}\beta}(\omega') \rangle = \delta_{\alpha\beta} \delta(\omega + \omega') S(\omega), \quad (14)$$

where  $S(\omega) = 2\pi G_q \omega / (1 - e^{-\omega/T})$ . With the help of Eqs. (12), (13), and (14), one obtains two-point correlators of the currents at QPCs. In the following sections we use these correlators to derive the corrections to the bare conductance perturbatively in the cases of the symmetric and asymmetric setup. Below we will use that  $R_q = 2\pi$  and  $G_q = 1/2\pi$  explicitly. These quantities are easily restored in the final results for conductance.

### III. SYMMETRIC SETUP

We consider the regime of low bias  $\Delta\mu \ll T$  and evaluate the linear conductance through the metallic island in the symmetric setup shown in Fig. 1:

$$G = \left. \frac{d\langle I \rangle}{d\Delta\mu} \right|_{\Delta\mu=0}. \quad (15)$$

In the interaction representation the average current is given by the expression

$$\langle I \rangle = \text{Tr}[\rho_0 U^\dagger(t, -\infty) I(t) U(t, -\infty)], \quad (16)$$

where

$$U(t_1, t_2) = \hat{T} \exp \left[ -i \int_{t_2}^{t_1} dt H_T(t) \right] \quad (17)$$

is the evolution operator. The current operator is defined in the cross section immediately after the right QPC,

$$I(t) = -\frac{1}{2\pi} \partial_r [\phi_{\text{out}1}(x_r, t) - \phi_{\text{in}2}(x_r, t)], \quad (18)$$

and  $\rho_0 \propto \exp[-(H_0 + H_{\text{int}})/T]$  is the equilibrium density matrix.

We evaluate the average current perturbatively expanding the evolution operator to the lowest order in backscattering amplitudes,

$$\langle I \rangle = \langle I \rangle_0 + \delta \langle I \rangle, \quad (19)$$

where the nonperturbed current  $\langle I \rangle_0 = \Delta\mu/2R_q$  was derived in the previous section, and the second term can be written as (for details, see Appendix A)

$$\delta \langle I \rangle_0 = I_{\text{dir}} + I_{\text{osc}} = \sum_{l,l'=L,R} I_{ll'},$$

$$I_{ll'} = -\frac{1}{2} \int dt \langle [A_l^\dagger(t), A_{l'}(0)] \rangle_0, \quad (20)$$

where the average is taken with respect to the equilibrium nonperturbed state with the density matrix  $\rho_0$  and  $I_{\text{dir}} = I_{LL} + I_{RR}$  is the direct contribution to the current, while  $I_{\text{osc}} = I_{LR} + I_{RL}$  is the oscillating part. The later one oscillates as a function of the parameter  $Q_0$ , proportional to the gate voltage, as  $\cos(2\pi Q_0)$ , which is the manifestation of the CB effect. Finally, the prefactor 1/2 in this expression comes from the fact that the metallic island mixes the edge channels equally even in the absence of backscattering.

### A. Quantum regime

We evaluate the correction  $\delta G$  to the linear conductance by using the Kubo formula (20) and split it in a direct and oscillating part:  $\delta G = \delta \langle I \rangle_0 / \Delta\mu = G_{\text{dir}} + G_{\text{osc}}$ . In the low-temperature quantum regime  $T \ll E_C$  we obtain (see Appendix B for details)

$$G_{\text{dir}} = -\frac{|\tau_L|^2 + |\tau_R|^2}{v_F^2} \frac{e^\gamma E_C}{8\pi^2 T}, \quad (21)$$

where  $\gamma \approx 0.5772$  is Euler's constant. For the oscillating contribution in this regime we obtain the expression

$$G_{\text{osc}} = -\frac{2|\tau_L||\tau_R|}{v_F^2} \frac{e^\gamma E_C}{8\pi^2 T} \cos(2\pi Q_0). \quad (22)$$

Taking into account that  $G_q = 1/2\pi$  and combining Eqs. (10), (21), and (22), we present the total linear conductance in the following form:

$$G = \frac{G_q}{2} \left( 1 - \frac{\Gamma(Q_0)}{T} \right), \quad (23)$$

where

$$\Gamma(Q_0) = \frac{e^\gamma E_C}{2\pi v_F^2} [|\tau_L|^2 + |\tau_R|^2 + 2|\tau_L||\tau_R| \cos(2\pi Q_0)]. \quad (24)$$

It is important to mention that in Eq. (23) we kept only leading order terms in the parameter  $E_C/T$ , and according to our perturbation approach in weak backscattering  $T \gg \Gamma(Q_0)$ . In this limit our results fully agree with the earlier theory of Furusaki and Matveev [21]. Interestingly, the same expressions were derived for the conductance of a one-dimensional (1D) system with two defects [29].

### B. Thermal regime

In the thermal regime  $T \gg E_C$  we derive (see Appendix B for details) the following expressions for direct:

$$G_{\text{dir}} = -\frac{1}{8\pi} \frac{|\tau_L|^2 + |\tau_R|^2}{v_F^2} \quad (25)$$

and the oscillating term

$$G_{\text{osc}} = -\frac{|\tau_L||\tau_R|}{2v_F^2} \sqrt{\frac{\pi T}{E_C}} \exp\left[-\frac{\pi^2 T}{E_C}\right] \cos(2\pi Q_0). \quad (26)$$

Taking into account that  $G_q = 1/2\pi$  and combining Eqs. (10), (25), and (26), we obtain the following expression for the total linear conductance:

$$G = \frac{G_q}{2} \left[ 1 - \frac{|\tau_L|^2 + |\tau_R|^2 + 2|\tau_L||\tau_R| F(T) \cos(2\pi Q_0)}{2v_F^2} \right], \quad (27)$$

where the temperature dependent factor is given by

$$F(T) = 2\pi \sqrt{\frac{\pi T}{E_C}} \exp\left[-\frac{\pi^2 T}{E_C}\right]. \quad (28)$$

In contrast to the quantum regime, here the temperature influences only CB oscillations.

We note that these results have an interesting interpretation. While at high temperatures the bare currents at the left and right QPC can be considered being independent 1D currents, at low temperatures  $T \ll E_C$  (and consequently at low frequencies) the metallic island splits them equally. This leads to the phenomenon of charge fractionalization (half of an electron is reflected by the island), and thus reduces the exponent of the power-law correlators in (20) by the factor of 2. As a result, the direct conductance in (21) acquires the singular temperature dependence  $1/T$ .

### C. Visibility

The strength of the CB oscillations is described by the visibility function

$$V = \frac{G_{\text{max}} - G_{\text{min}}}{G_{\text{max}} + G_{\text{min}}}, \quad (29)$$

where  $G_{\text{max(min)}}$  is the maximum (minimum) SET conductance over one gate voltage period.

In the quantum regime the visibility is given by

$$V = \frac{|\tau_L||\tau_R| e^\gamma E_C}{\pi v_F^2 T}, \quad T \ll E_C, \quad (30)$$

and according to the perturbation approach in weak backscattering,  $V \ll 1$ . In the thermal regime we obtain the following result:

$$V = \frac{2|\tau_L||\tau_R|}{v_F^2} \pi \sqrt{\frac{\pi T}{E_C}} \exp\left[-\frac{\pi^2 T}{E_C}\right], \quad T \gg E_C. \quad (31)$$

The exact dependence of the visibility on the temperature in whole range of temperatures, from quantum to thermal regime, is found by calculating time integrals in Appendix B numerically. The results are presented in Fig. 3 together with

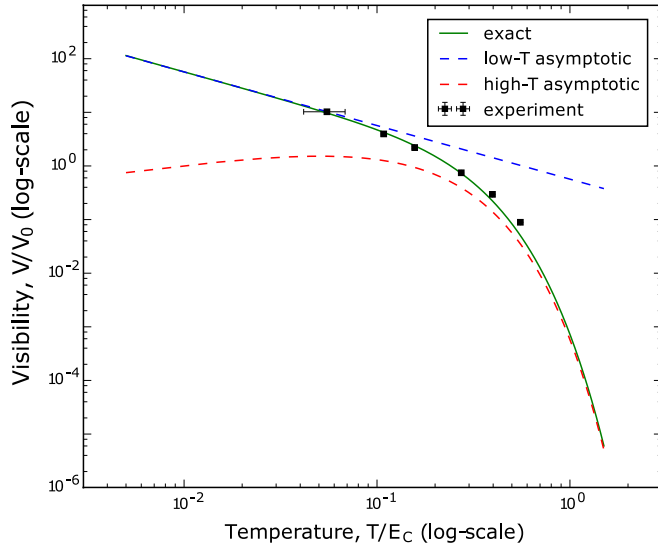


FIG. 3. The exact temperature dependence of the visibility (green line) is shown for the case of the symmetric setup together with the asymptotic low- and high-temperature solutions (dashed lines) and experimental results (black squares). The visibility is normalized to the value  $V_0 = |\tau_L||\tau_R|/v_F^2$  in order to get rid of the nonuniversal backscattering amplitudes. The experimental results are obtained for various transmissions at the left QPC [9]:  $G_L/G_q = 0.983, 0.974, 0.974, 0.75, 0.75, 0.75$  for  $T/E_C = 0.055, 0.108, 0.157, 0.273, 0.397, 0.553$ , respectively. The transmission at the right QPC,  $G_R/G_q$ , is kept very close to 1.

asymptotic solutions (30) and (31), and the results of the experiment [9]. It is worth mentioning a good agreement with the experiment.

#### IV. ASYMMETRIC SETUP

In this section we study the case of the asymmetric setup, namely, we assume that the right QPC is almost closed (pinched-off), while the left one is almost open (see Fig. 2). To find the conductance of the system we apply the perturbation expansion in two steps: we first calculate the linear conductance perturbatively in the tunneling at the right QPC, and then consider corrections to it taking into account weak backscattering at the left QPC.

The electron current is defined as a rate of change of the electron number  $N_R$  in the right arm, namely

$$I_R = i[H_T, N_R] = i(A_R - A_R^\dagger). \quad (32)$$

We evaluate the average current to the lowest order in the tunneling amplitude  $\tau_R$ :

$$\langle I_R \rangle = \int dt \langle [A_R^\dagger(t), A_R(0)] \rangle. \quad (33)$$

Here the average is taken with respect to the density matrix

$$\rho = U(0, -\infty)\rho_0 U^\dagger(0, -\infty), \quad (34)$$

perturbed by weak backscattering at the left QPC. To the lowest order in the backscattering amplitude  $\tau_L$ , the evolution operator

is given by the expression

$$U(t_1, t_2) = 1 - i \int_{t_2}^{t_1} dt (A_L + A_L^\dagger). \quad (35)$$

Using Eq. (33), we present the linear conductance  $G = \langle I_R \rangle / \Delta\mu$  in the following form:

$$G = \frac{i|\tau_R|^2}{a^2} \int dt t [K_0(t)G_1(t) - K_0^*(t)G_2(t)], \quad (36)$$

where

$$K_0(t) = \frac{-iT}{2v_F \sinh[\pi T(t - i0)]} \quad (37)$$

is the two-point correlation function of free fermions in the right arm of the system in Fig. 2. The correlation functions

$$G_1(t) = \langle U^\dagger(t, -\infty) e^{-i\phi_{\text{out}}(t)} \times U(t, 0) e^{i\phi_{\text{out}}(0)} U(0, -\infty) \rangle_0, \quad (38)$$

$$G_2(t) = \langle U^\dagger(0, -\infty) e^{i\phi_{\text{out}}(0)} \times U(0, t) e^{-i\phi_{\text{out}}(t)} U(t, -\infty) \rangle_0 \quad (39)$$

are calculated perturbatively to the lowest order in the backscattering amplitude  $\tau_L$ , i.e., by using Eq. (35). After expanding Eqs. (38) and (39) and substituting them into Eq. (36), one can split the conductance  $G$  in the sum of two terms

$$G = G_{\text{dir}} + G_{\text{osc}}. \quad (40)$$

Here the term  $G_{\text{dir}}$  is proportional to  $|\tau_R|^2$ , and does not depend on the gate parameter  $Q_0$ . In contrast, the contribution  $G_{\text{osc}}$  is proportional to  $|\tau_L||\tau_R|^2$  and demonstrates CB oscillations. Detailed calculations of these two terms are presented in Appendix C.

##### A. Quantum regime

Repeating the same steps as in the Sec. III, we obtain the following expression for the direct and oscillating contributions to the conductance in the limit of low temperatures  $T \ll E_C$  (see Appendix C):

$$G_{\text{dir}}(T) = \frac{2\pi^4 T^2 G_R}{3e^{2\gamma} E_C^2}, \quad (41)$$

where  $G_R = |\tau_R|^2 / 2\pi v_F^2$  is the bare conductance of the right QPC. The oscillating contribution acquires the form

$$G_{\text{osc}} = -\frac{2\pi^4 T^2 G_R}{3e^{2\gamma} E_C^2} \xi \frac{|\tau_L|}{v_F} \cos(2\pi Q_0). \quad (42)$$

Substituting Eqs. (41) and (42) into Eq. (40), we arrive at the following expression for the total linear conductance:

$$G = \frac{2\pi^4 T^2 G_R}{3e^{2\gamma} E_C^2} \left[ 1 - \xi \frac{|\tau_L|}{v_F} \cos(2\pi Q_0) \right], \quad (43)$$

where  $\xi = 4e^\gamma$  is the dimensionless numeric constant [21]. Note that even in the absence of backscattering at the left QPC the conductance scales as  $T^2$ .

### B. Thermal regime

In this case,  $T \gg E_C$ , we get the following expressions for the direct and oscillating contributions to the total conductance (details of calculations are presented in Appendix C):

$$G_{\text{dir}} = \frac{|\tau_R|^2}{2\pi v_F^2} = G_R \quad (44)$$

and

$$G_{\text{osc}} = -G_R \frac{|\tau_L|}{v_F} M(T) \cos(2\pi Q_0), \quad (45)$$

where the temperature dependent factor is given by

$$M(T) = 2\pi \sqrt{\frac{\pi T}{E_C}} \exp\left[-\frac{\pi^2 T}{E_C}\right]. \quad (46)$$

Combining two terms, we present the total conductance as

$$G = G_R \left[ 1 - \frac{|\tau_L|}{v_F} M(T) \cos(2\pi Q_0) \right]. \quad (47)$$

Note that the temperature affects only the oscillating contribution, in contrast to the quantum regime.

### C. Visibility

As it follows from Eq. (43), the visibility in the quantum regime is constant:

$$V = \xi \frac{|\tau_L|}{v_F}, \quad T \ll E_C. \quad (48)$$

In the thermal regime [see Eq. (47)], the visibility acquires the following form:

$$V = \frac{2\pi |\tau_L|}{v_F} \sqrt{\frac{\pi T}{E_C}} \exp\left[-\frac{\pi^2 T}{E_C}\right], \quad T \gg E_C. \quad (49)$$

The results of the exact numerical calculation of time integrals for the visibility in Appendix C are shown in Fig. 4 together with the asymptotic forms and the experimental data [9]. Our results agree quite well with the experiment.

### V. DIFFERENTIAL CAPACITANCE

Without loss of generality, we consider here an asymmetric case, namely, we assume that the right QPC is pinched-off with  $\tau_R = 0$  (see Fig. 2), while the left QPC is almost open with the weak backscattering amplitude  $\tau_L$ . In this case, the CB effect manifests itself in the oscillations of the equilibrium characteristics of the system, such as its ground state energy or the average charge of the island. The single-electron capacitance spectroscopy of quantum dots [30,31] can be used to measure experimentally the differential capacitance between the gate and the lead. The differential capacitance is defined, up to a nonuniversal constant prefactor, as

$$C_{\text{diff}} = -\frac{\partial^2 F}{\partial Q_0^2}, \quad (50)$$

where  $F$  is the free energy of the system

$$F = -T \log(Z), \quad (51)$$

and  $Z = \text{Tr}(e^{-H/T})$  is the partition function.

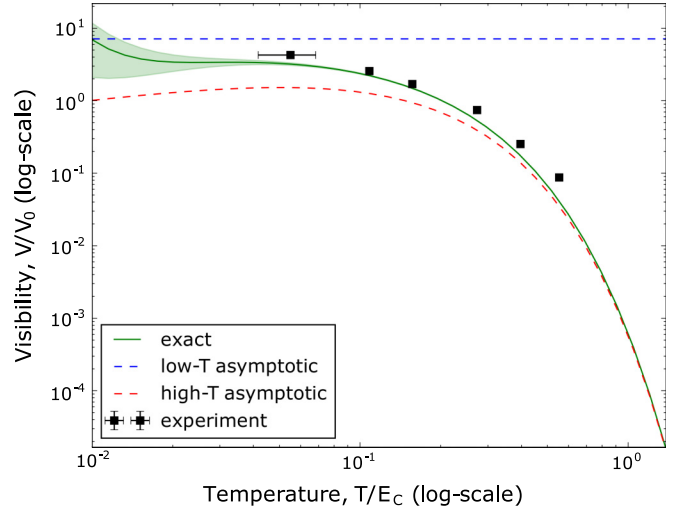


FIG. 4. The exact temperature dependence of the visibility (green line) is shown for the case of the asymmetric setup together with the asymptotic low- and high-temperature solutions (dashed lines) and experimental results (black squares). The visibility is normalized to the value  $V_0 = |\tau_L|/v_F$  in order to get rid of the nonuniversal backscattering amplitude. The experimental results are obtained for transmission at the left QPC [9]:  $G_L/G_q = 0.075$  for  $T/E_C = 0.055, 0.108, 0.157, 0.273, 0.397, 0.553$  and the transmission at the right QPC,  $G_R/G_q$ , is kept very close to 1. The green shadowed region shows the error bars of the numerical evaluation.

The fully open island does not exhibit the QB effect, therefore we evaluate the partition function perturbatively in the backscattering amplitude  $\tau_L$  and concentrate on the correction to the capacitance. We write

$$Z = Z_0 + \delta Z, \quad (52)$$

where  $Z_0 = \text{Tr}[e^{-(H_0+H_{\text{int}})/T}]$  is the bare part, while the correction is given by

$$\delta Z = -\text{Tr}\left[e^{-(H_0+H_{\text{int}})/T} \int_0^{1/T} d\tau H_{\text{T}}(\tau)\right], \quad (53)$$

where  $\tau$  is the imaginary time. Consequently, the free energy can be rewritten as

$$F = F_0 + \delta F, \quad (54)$$

where  $F_0 = -T \log(Z_0)$ , and the correction has the following form:

$$\delta F = -T \frac{\delta Z}{Z_0} = \text{Tr}(\rho_0 H_{\text{T}}), \quad (55)$$

where  $H_{\text{T}} = A_L + A_L^\dagger$ .

Next, we write the right-hand side of Eq. (55) in bosonic operators using the definition (6), and evaluate the correlation function  $\langle e^{i\phi_{\text{out}}(0)} e^{-i\phi_{\text{in}}(0)} \rangle_0$  by using the scattering matrix (13) for bosons in the asymmetric setup and the Gaussian character of the theory. Thus, we obtain

$$\delta F = \frac{2|\tau_L|}{a} \cos(2\pi Q_0) e^{\gamma(T)}, \quad (56)$$

where

$$y(T) = - \int \frac{dx x}{1+x^2} \frac{1}{1 - e^{-x/2\pi TC}}. \quad (57)$$

We evaluate  $y(T)$  in the limit of high temperatures  $T \gg E_C$ :

$$y(T) = \log(a/4\pi^2 v_F C) - 2\pi^2 TC + \log(4\pi^2 TC), \quad (58)$$

and find the correction to the free energy

$$\delta F = \frac{2|\tau_L|}{v_F} T e^{-\pi^2 T/E_C} \cos(2\pi Q_0). \quad (59)$$

Consequently, the correction to the differential capacitance in the thermal regime is given by

$$\delta C_{\text{diff}} = 8\pi^2 \frac{|\tau_L|}{v_F} T \exp\left[-\frac{\pi^2 T}{E_C}\right] \cos(2\pi Q_0). \quad (60)$$

Interestingly, the differential capacitance contains the same exponential factor as in the linear conductance for both setups. This can be easily explained by the fact that the exponential dependence on the temperature simply follows from averaging CB oscillations over instant fluctuations of the charge in the island, which are distributed with the equilibrium Gibbs weights  $\rho_G \propto \exp[-Q^2 E_C/T]$ . However, the power-law prefactor in the differential capacitance is different from the one in the linear conductances [see Eqs. (28) and (46)], which can be understood taking into account its quantum character, since one of the contributions to it comes from high-energy modes.

## VI. CONCLUSION

The charge of an isolated metallic system is quantized in units of the elementary electron charge. This phenomenon manifests itself in oscillations of the conductance, if a system is attached to metallic leads, and in the oscillations of the differential capacitance. The degree of the charge quantization is described by the dimensionless visibility  $V$  of such oscillations. In the recent experiment conducted in the group of Pierre [9], the transport through a small metallic island connected to two leads by QPCs (as schematically shown in Fig. 1) has been thoroughly studied in the regime, where the QPCs are pinched-off to allow only one mode to partially propagate. In such systems quantum and thermal fluctuations of the current at QPC gradually reduce the quantization of charge as the tunneling coupling strength at QPCs is increased, and the CB oscillations vanish for fully open contacts. The work [9] has reported measurements of the visibility of CB oscillations as a function of temperature as well as of the tunneling coupling strength at QPCs.

Motivated by this experiment, we have developed a quantitative theory of the linear conductance and differential capacitance of the metallic island in the whole range of temperatures, from below to above the charging energy  $E_C$ , in the cases of symmetric and asymmetric tunneling coupling at QPCs. To do so, we have used the quantum Langevin equation approach to account for the dynamics and fluctuations of the collective charge fluctuations in a fully open system, and the tunneling Hamiltonian approach to account for weak backscattering and weak tunneling of electrons at QPCs [23,25]. This method is based on the fact that the electrical circuit elements typically create only Gaussian fluctuations and therefore the fully open

system in the bosonic picture remains Gaussian, because the interaction part of Hamiltonian is quadratic in bosonic fields.

We have found that in the low-temperature quantum regime  $T \ll E_C$  the temperature dependence of the linear conductance coincides with the results of Furusaki and Matveev [21] in both cases of symmetric and asymmetric setup. For instance, in the symmetric case (see Fig. 1) the direct and oscillating part of the weak backscattering contribution to the conductance of the system both acquire the power-law temperature dependence  $\delta G \propto E_C/T$ . On the other hand, in the case of asymmetric setup (see Fig. 2) the total conductance including the oscillating correction acquires the temperature dependence  $G \propto (T/E_C)^2$ , thus the visibility of CB oscillations stays constant. Different power-law scaling of the conductance with the temperature depending on the geometry of the system indicates that an important role is played by the character of mixing of the collective modes in the metallic island. For instance, in the case of the symmetric setup the metallic island splits equally incoming currents, and thus the scaling  $E_C/T$  originates from the charge fractionalization induced by such a current splitter, which is known to affect the power-law exponents.

In the high-temperature limit  $T \gg E_C$  the temperature dependence of the oscillating correction to the conductance is entirely different: it is given by the product of a power-law prefactor and exponentially decaying function  $\delta G \propto \sqrt{T/E_C} \exp(-\pi^2 T/E_C)$ , both for the symmetric and asymmetric setup. In order to compare our theory to the experiment [9], we have found the visibility of CB oscillations exactly in the entire range of temperatures by calculating time integrals numerically. Our results are shown in Figs. 3 and 4 together with the asymptotic forms and the results of measurements. This comparison shows a good agreement of our theory with the experiment.

Finally, we have investigated CB oscillations in a partially closed system, where the right QPC is disconnected from the circuit ( $\tau_L = 0$ , see Fig. 2) by evaluating the differential capacitance of the metallic island. Interestingly, the oscillating correction to the differential conductance in the high-temperature limit acquires the same exponential decay with the temperature as the one for the linear conductance. Such universality can be explained by the fact that this effect is completely classical and originates from the thermal averaging of the CB oscillations over instant configurations of the charge in the island with the equilibrium Gibbs weights. On the contrary, the power-law prefactor is different [see Eq. (60)], which can be attributed to its quantum origin.

## ACKNOWLEDGMENTS

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## APPENDIX A: DERIVATION OF THE KUBO FORMULA FOR SYMMETRIC SETUP

In this Appendix we derive Eq. (20). The operator of outgoing current to the right from the right QPC at point  $x_r > 0$  (see Fig. 1) expressed in bosonic fields reads

$$I(t) = -\frac{1}{2\pi} \partial_t [\phi_{\text{out}1}(x_r, t) - \phi_{\text{in}2}(x_r, t)]. \quad (\text{A1})$$

Then the average current is given by the following expression:

$$\langle I \rangle = \langle U^\dagger(t, -\infty) I(t) U(t, -\infty) \rangle_0, \quad (\text{A2})$$

where the average is taken with respect to the unperturbed state  $\rho_0 \propto \exp[-(H_0 + H_{\text{int}})/T]$ , and the evolution operator in interaction picture is expanded to second order in tunneling Hamiltonian Eq. (6):

$$U(t, -\infty) = 1 - i \int_{-\infty}^t dt_1 H_T(t_1) - \int_{-\infty}^t dt_1 \int_{-\infty}^{t_1} dt_2 H_T(t_1) H_T(t_2). \quad (\text{A3})$$

By substituting (A3) into (A2) and doing simple algebra, one arrives at the expression (19).

The first term in (19) has been evaluated in Sec. II with the result (8). Here we concentrate on the second term in (19) and rewrite it in the following form:

$$\delta \langle I \rangle = - \int_{-\infty}^t dt_1 \int_{-\infty}^{t_1} dt_2 \langle [[I(t), H_T(t_1)], H_T(t_2)] \rangle_0. \quad (\text{A4})$$

The integrand contains four nonzero terms,

$$\langle [[I(t), H_T(t_1)], H_T(t_2)] \rangle_0 = \Pi_1 + \Pi_2 + \text{c.c.}, \quad (\text{A5})$$

where the first two terms read

$$\begin{aligned} \Pi_1 &= \langle [[I(t), A_L(t_1)], A^\dagger(t_2)] \rangle_0, \\ \Pi_2 &= \langle [[I(t), A_R(t_1)], A^\dagger(t_2)] \rangle_0. \end{aligned} \quad (\text{A6})$$

Next, we concentrate on the first term  $\Pi_1$  (the second term  $\Pi_2$  can be evaluated in the same way) and simplify the commutator in  $\Pi_1$ :

$$[I(t), A_L(t_1)] \propto [\partial_t \phi_{\text{out}1}(x_r, t) - \partial_t \phi_{\text{in}2}(x_r, t), e^{i[\phi_{\text{in}1}(t_1) - \phi_{\text{out}2}(t_1)]}] \quad (\text{A7})$$

by using the following properties of operators. If an operator  $D = [B, C]$  satisfies  $[B, D] = [C, D] = 0$ , then the following relations hold:  $[B, \exp(C)] = [B, C] \exp(C)$  and  $\exp(B) \exp(C) = \exp(B + C) \exp([B, C]/2)$ . Thus, we obtain

$$\Pi_1 = R(t - t_1) K_L(t_1 - t_2), \quad (\text{A8})$$

where  $K_L(t_1 - t_2) = \langle [A_L(t_1), A^\dagger(t_2)] \rangle_0$  and  $R(t - t_1) = -\frac{1}{2\pi} \{ \partial_t \phi_{\text{out}1}(x_r, t) - \partial_t \phi_{\text{in}2}(x_r, t), i[\phi_{\text{in}1}(t_1) - \phi_{\text{out}2}(t_1)] \}$ . Then, using the Langevin equations (12) and the commutator of incoming currents  $[j_{\text{in}\alpha}(\omega), j_{\text{in}\beta}(\omega')] = \omega \delta(\omega + \omega') \delta_{\alpha\beta}$ , we obtain the relation

$$R(t - t_1) = \frac{1}{4\pi} \int \frac{d\omega \omega^2}{\omega^2 + \eta^2} \left( \frac{e^{i\omega x_r/v_F}}{i\pi\omega C - 1} - \frac{e^{-i\omega x_r/v_F}}{i\pi\omega C + 1} \right) \times e^{-i\omega(t-t_1)}. \quad (\text{A9})$$

It can be shown similarly that

$$\Pi_2 = F(t - t_1) K_R(t_1 - t_2), \quad (\text{A10})$$

where  $K_R(t_1 - t_2) = \langle [A_R(t_1), A^\dagger(t_2)] \rangle_0$  and  $F(t - t_1) = -\delta(t - x_r/v_F - t_1) - \delta(t + x_r/v_F - t_1) - R(t - t_1)$ .

Finally, substituting expressions (A8) and (A10) into Eq. (A5), and then into Eq. (A4), we arrive at the following

expression:

$$\delta \langle I \rangle = -\frac{1}{4\pi} \int d\omega \left( \frac{K(\omega)}{i\omega + \alpha} + \frac{K^*(\omega)}{-i\omega + \alpha} \right), \quad (\text{A11})$$

where  $\alpha \rightarrow +0$ , and  $K(\omega) = K_L(\omega) + K_R(\omega)$ . We use the property  $K(\omega) = K^*(\omega)$ , which follows from  $K^*(t) = \langle [A(t), A^\dagger(0)] \rangle^* = \langle [A(0), A^\dagger(t)] \rangle = K(-t)$ . Thus, only  $\omega = 0$  contributes in the integral, and we obtain

$$\delta \langle I \rangle = -\frac{1}{2} \int dt \langle [A^\dagger(t), A(0)] \rangle_0, \quad (\text{A12})$$

and eventually, Eq. (20) in the main text.

## APPENDIX B: PERTURBATION THEORY FOR SYMMETRIC SETUP

In this Appendix we derive the leading order corrections to the conductance of the system in the symmetric setup. We use the Gaussian character of the theory to present the average of four vertex operators in Eq. (20) in the following form:

$$\begin{aligned} & \langle e^{i\lambda_1 \phi_1} e^{i\lambda_2 \phi_2} e^{i\lambda_3 \phi_3} e^{i\lambda_4 \phi_4} \rangle_0 \\ &= \exp \left( -\frac{1}{2} \sum_{i=1}^4 \lambda_i^2 \langle \phi_i^2 \rangle_0 - \sum_{i<j}^4 \lambda_i \lambda_j \langle \phi_i \phi_j \rangle_0 \right), \\ & \lambda_i = \pm 1, \end{aligned} \quad (\text{B1})$$

where the average is taken with respect to the equilibrium density matrix  $\rho_0 \propto \exp[-(H_0 + H_{\text{int}})/T]$ . We evaluate two-point correlation functions on the right-hand side of this expression using the quantum Langevin equation approach (12). According to Eq. (20), the correction to the average current consists of the direct and oscillating part  $\delta \langle I \rangle_0 = I_{\text{dir}} + I_{\text{osc}}$ . Here we consider them separately.

The direct term has the following form:

$$I_{\text{dir}} = -\frac{|\tau_L|^2 + |\tau_R|^2}{2} \int dt e^{i\Delta\mu t/2} [K_{\text{ff}}(t) e^{-\beta(t)} - \text{c.c.}], \quad (\text{B2})$$

where  $\beta(t)$  is given below, and

$$\begin{aligned} K_{\text{ff}}(t) &= \frac{1}{a^2} \exp \left( 2 \int \frac{d\omega \omega}{\omega^2 + \eta^2} \frac{e^{-i\omega t} - 1}{1 - e^{-\omega/T}} \right) \\ &= \frac{-T^2}{4v_F^2 \sinh^2[\pi T(t - i0)]} \end{aligned} \quad (\text{B3})$$

is the square of the free fermion correlation function. The oscillating term is given by the following expression:

$$\begin{aligned} I_{\text{osc}} &= -2 |\tau_L| |\tau_R| \\ &\times \text{Re} \left[ \frac{e^{-2\pi i Q_0 + \alpha_S(T)}}{2a^2} \int dt e^{i\Delta\mu t/2} (e^{\beta(t)} - \text{c.c.}) \right]. \end{aligned} \quad (\text{B4})$$

Here

$$\begin{aligned} \alpha_S(T) &= -2 \int dx \frac{x}{1+x^2} \frac{1}{1 - e^{-x/\pi TC}} \\ &= 2\gamma + 2 \log \left( \frac{a}{2\pi^2 v_F C} \right) + 2\pi^2 TC \\ &\quad + 2 \log(2\pi^2 TC) + 2\Psi(1/2\pi^2 TC), \end{aligned} \quad (\text{B5})$$



$\Psi(x) = d \log[\Gamma(x)]/dx$  is the digamma function of real variable  $x$ ,  $\Gamma(x)$  is the gamma function, and  $\beta(t) = \beta_1(t) + \text{Re}\beta_2(t) + i\text{Im}\beta_2(t)$ .

Below we need the expressions for  $\beta_1(t)$  and  $\beta_2(t)$  only for positive times  $t > 0$ , where they take the following form:

$$\beta_1(t) = -2 \int_0^\infty \frac{d\omega}{\omega^2 + \eta^2} \frac{\omega}{1 + (\pi\omega C)^2} \frac{\cos(\omega t) - 1}{1 - e^{\omega/T}}, \quad (\text{B6})$$

$$\begin{aligned} \text{Re}\beta_2(t) &= \int_0^\infty d\omega \frac{\omega}{\omega^2 + \eta^2} \frac{\cos(\omega t) - 1}{1 + (\pi\omega C)^2} \\ &= -\gamma - \log(t/\pi C) + \frac{e^{-t/\pi C}}{2} \text{Ei}(t/\pi C) \\ &\quad + \frac{e^{t/\pi C}}{2} \text{Ei}(-t/\pi C), \end{aligned} \quad (\text{B7})$$

$$\begin{aligned} \text{Im}\beta_2(t) &= - \int_0^\infty d\omega \frac{\omega}{\omega^2 + \eta^2} \frac{\sin(\omega t)}{1 + (\pi\omega C)^2} \\ &= -\frac{\pi}{2} (1 - e^{-t/\pi C}). \end{aligned} \quad (\text{B8})$$

Here  $\text{Ei}(x) = - \int_{-x}^\infty dy e^{-y}/y$  is the exponential integral for real nonzero values of  $x$ . Below we use derived in this Appendix integral representations for the current, take the derivative with respect to  $\Delta\mu$  to obtain the conductance, and find analytic expressions for the thermal  $T \gg E_C$  and quantum  $T \ll E_C$  regimes.

### 1. Quantum regime

We first consider the direct term (B2). The contribution from the poles to the time integral at small times  $t \sim 1/\epsilon_F$  is a constant of temperature, while the dominant contribution to (B2) scales as  $E_C/T$ . Therefore, we neglect the contributions from poles and present the direct part of the conductance as an integral over positive times, because the integrand is an even function of variable  $t$  in this case:

$$\begin{aligned} G_{\text{dir}} &= - \frac{|\tau_L|^2 + |\tau_R|^2}{v_F^2} \int_0^\infty dt t \frac{T^2}{4 \sinh^2(\pi T t)} e^{-\beta_1(t) - \text{Re}\beta_2(t)} \\ &\quad \times \sin \left[ \frac{\pi}{2} (1 - e^{-t/\pi C}) \right]. \end{aligned} \quad (\text{B9})$$

Next, taking into account that  $T \ll E_C$ , we set  $C = 0$  in  $\beta_1(t)$  and obtain  $\beta_1(t) = -\log(\frac{\sinh(\pi T t)}{\pi T t})$ . Then, the main contribution to integral for  $G_{\text{dir}}$  comes from large times, namely  $t/\pi C \gg 1$ . In this case, we can simplify  $\text{Re}\beta_2(t) = -\gamma - \log(t/\pi C)$  and use  $\sin[\frac{\pi}{2}(1 - e^{-t/\pi C})] \sim 1$ . After substitution these expressions in Eq. (B9), we obtain

$$\begin{aligned} G_{\text{dir}} &= - \frac{|\tau_L|^2 + |\tau_R|^2}{v_F^2} \frac{e^\gamma}{4\pi^4 T C} \int_0^\infty dx \frac{x}{\sinh(x)} \\ &= - \frac{|\tau_L|^2 + |\tau_R|^2}{v_F^2} \frac{e^\gamma E_C}{8\pi^2 T}. \end{aligned} \quad (\text{B10})$$

In the oscillating term, at small temperatures  $T \ll E_C$ , the factor  $\alpha_S(T)$  simplifies as  $\alpha_S(T) \approx 2\gamma + 2 \log(a/2\pi^2 v_F C)$ . Then, using the same arguments as for the direct term, we

obtain

$$\begin{aligned} G_{\text{osc}} &= - \frac{2 |\tau_L| |\tau_R|}{v_F^2} \cos(2\pi Q_0) \frac{e^\gamma}{4\pi^4 T C} \int_0^\infty dx \frac{x}{\sinh(x)} \\ &= - \frac{2 |\tau_L| |\tau_R|}{v_F^2} \frac{e^\gamma E_C}{8\pi^2 T} \cos(2\pi Q_0). \end{aligned} \quad (\text{B11})$$

Combining (B10), (B11), and (10) we obtain Eq. (23) in the main text.

### 2. Thermal regime

In this regime  $T \gg E_C$ , the prefactor  $\alpha_S(T)$  has the following form:

$$\alpha_S(T) \approx 2 \log(a/2\pi^2 v_F C) - 2\pi^2 T C + 2 \log(2\pi^2 T C). \quad (\text{B12})$$

We evaluate  $\beta(t)$  by expanding the distribution function  $1/(1 - e^{-\omega/T})$  at large temperatures:

$$\begin{aligned} \beta(t) &= \int d\omega \frac{\omega}{\omega^2 + \eta^2} \frac{e^{-i\omega t} - 1}{1 + (\pi\omega C)^2} \left[ \frac{T}{\omega} + \frac{1}{2} \right] \\ &= \pi^2 T C (1 - e^{-|t/\pi C|} - |t/\pi C|) \\ &\quad - \frac{i\pi}{2} (1 - e^{-|t/\pi C|}) \text{sgn}(t), \end{aligned} \quad (\text{B13})$$

and then expanding the right-hand side of this equation in series of  $t/\pi C \ll 1$ , taking into account the fact that the main contribution to the time integral comes from small times, and that  $T \gg E_C$ . Thus, we obtain the following expression:

$$\beta(t) = - \frac{T t^2}{2C} - \frac{i t}{2C}. \quad (\text{B14})$$

Now we are ready to write the correction for the conductance.

The direct term reads

$$\begin{aligned} G_{\text{dir}} &= \frac{|\tau_L|^2 + |\tau_R|^2}{16v_F^2} \int dt t T^2 e^{T t^2/2C} \\ &\quad \times \left( \frac{e^{it/2C}}{\sinh^2[\pi T(t - i0)]} - \text{c.c.} \right). \end{aligned} \quad (\text{B15})$$

In this integral the main contribution comes from small times  $t \sim 1/\epsilon_F$ , and we can neglect exponential factors in the integrand. Introducing the dimensionless variable  $x = \pi T t$ , we write

$$\begin{aligned} G_{\text{dir}} &= \frac{|\tau_L|^2 + |\tau_R|^2}{16\pi^2 v_F^2} \\ &\quad \times \int dx i x \left( \frac{1}{\sinh^2[x - i0]} - \frac{1}{\sinh^2[x + i0]} \right) \\ &= - \frac{1}{8\pi} \frac{|\tau_L|^2 + |\tau_R|^2}{v_F^2}. \end{aligned} \quad (\text{B16})$$

Next, we take derivative with respect to  $\Delta\mu$  in Eq. (B4) to get the following expression for the oscillating part:

$$\begin{aligned} G_{\text{osc}} &= - \frac{4 |\tau_L| |\tau_R|}{v_F^2} T^2 C^2 e^{-2\pi^2 T C} \cos(2\pi Q_0) \\ &\quad \times \int dy y \sin y \exp(-2TCy^2), \end{aligned} \quad (\text{B17})$$

where we introduced the dimensionless variable  $y = t/2C$ . Evaluating this integral, we obtain

$$G_{\text{osc}} = -\frac{|\tau_L||\tau_R|}{2v_F^2} \sqrt{\frac{\pi T}{E_C}} \exp\left[-\frac{\pi^2 T}{E_C}\right] \cos(2\pi Q_0). \quad (\text{B18})$$

Here we neglected the factor  $\exp(-1/4\pi^2 TC)$ , because  $T \ll E_C$ . Combining (B16), (B18), and (10), we arrive at the expression for conductance (27) in the main text.

### APPENDIX C: PERTURBATION THEORY FOR ASYMMETRIC SETUP

In this Appendix we use Eqs. (36)–(40) to derive the leading order contribution to the conductance in the case of the asymmetric setup. We recall that the direct contribution  $G_{\text{dir}}$  is proportional to  $|\tau_R|^2$ , while the oscillating term  $G_{\text{osc}}$  comes as a correction proportional to  $|\tau_L||\tau_R|^2$  due to weak backscattering at the left QPC, and thus acquires oscillations as a function of the parameter  $Q_0$ . We derive analytical expressions in the quantum  $T \ll E_C$  and classical  $T \gg E_C$  regimes.

#### 1. Quantum regime

We first concentrate on the direct contribution. We use the solution Eq. (13) of quantum Langevin equations for the asymmetric setup to evaluate bosonic correlators that arise in the perturbative expansion for the conductance and arrive at the following expression:

$$G_{\text{dir}} = -\frac{|\tau_R|^2}{4v_F^2} \int dt it \left( \frac{T^2}{\sinh^2[\pi T(t-i0)]} e^{2\beta(t)} - \text{c.c.} \right), \quad (\text{C1})$$

where  $\beta(t)$  is given by the same expressions as in Appendix B with  $C$  replaced by  $2C$ , simply because in the asymmetric setup the conductance is twice as small. The expression (C1) holds for arbitrary temperatures and serves as a starting point for evaluating low- and high-temperature limits. The contribution from the poles at small times  $t \sim 1/\epsilon_F$  takes the constant value  $|\tau_R|^2/2\pi v_F^2$ . Using the same arguments as in case of the symmetric setup (see Appendix B), we obtain the expression for the direct part in the form:

$$G_{\text{dir}} = \frac{|\tau_R|^2}{2\pi v_F^2} - \frac{|\tau_R|^2}{v_F^2} \int_0^\infty dt \frac{\pi^2 T^4 t^3}{\sinh^4(\pi T t)} e^{2\text{Re}\beta_2(t)} \times \sin[\pi(1 - e^{-t/2\pi C})]. \quad (\text{C2})$$

In the low-temperature limit  $T \ll E_C$ , the main contribution to the integral in  $G_{\text{dir}}$  comes from times much smaller than the inverse temperature  $t \ll 1/\pi T$ . Thus, we can expand the temperature dependent part of the integrand in small  $Tt$ :

$$G_{\text{dir}} = \frac{|\tau_R|^2}{2\pi v_F^2} - \frac{4|\tau_R|^2}{v_F^2} \int_0^\infty dt \pi^2 T^4 t^3 \times \left[ \frac{1}{\pi^4 T^4 t^4} - \frac{2}{3\pi^2 T^2 t^2} \right] e^{2\text{Re}\beta_2(t)} \sin[\pi(1 - e^{-t/2\pi C})]. \quad (\text{C3})$$

From the last equation it is obvious that  $G_{\text{dir}}$  consists of a constant part and a temperature dependent part. The former

can be presented as

$$G_{\text{dir}}(T=0) = \frac{|\tau_R|^2}{2\pi v_F^2} - \frac{|\tau_R|^2}{2\pi v_F^2} C_1. \quad (\text{C4})$$

It turns out that the constant  $C_1$  is exactly equal to 1, and  $G_{\text{dir}}(T=0)$  vanishes. However, the easiest way to prove this fact is to rewrite equation (C1) for  $T=0$  in the form

$$G_{\text{dir}}(T=0) \propto \int ds s \left[ \frac{e^{M(s)}}{(s-i0)^2} - \frac{e^{M^*(s)}}{(s+i0)^2} \right], \quad (\text{C5})$$

where  $M(s) = 2 \int_0^\infty \frac{dx(e^{-ix}-1)}{x(1+x^2)}$ , and  $s$  and  $x$  are dimensionless variables. Here we note that  $M(s)$  is an analytical function in the low half plane. Moreover, it behaves as  $-2 \log|s|$  at  $\text{Im}(s) \rightarrow -\infty$ . Therefore, in the first term of the integral (C5) the contour may be deformed to  $\text{Im}(s) \rightarrow -\infty$ . Next, at  $\text{Im}(s) \rightarrow -\infty$  the integrand behaves as  $1/|s|^3$ , and thus the integral vanishes. The same arguments are applied to the second term, where  $M^*(s)$  is analytical in the upper half plane. Thus, we have proven that  $G_{\text{dir}}(T=0) = 0$ .

Now we consider the temperature dependent term. To integrate over  $t$  in Eq. (C3), we use the following integral identity  $\int_0^\infty dx x e^{2\text{Re}\beta_2(x)} \sin[\pi(1 - e^{-x})] = \frac{\pi}{2e^{2\gamma}}$ , where  $x = t/2\pi C$  is the dimensionless variable. Introducing the bare conductance  $G_R = |\tau_R|^2/2\pi v_F^2$ , we present the final result

$$G_{\text{dir}}(T) = G_R \frac{2\pi^4 T^2}{3e^{2\gamma} E_C^2}. \quad (\text{C6})$$

Similar arguments may be used for the oscillating part of the conductance to show that it is proportional to  $(T/E_C)^2$  as well. We confirm this fact by the exact numerical evaluation of the time integrals (see Fig. 4). Therefore, the final result in the case of an asymmetric setup at small temperatures can be written as (43).

#### 2. Thermal regime

In the limit  $T \gg E_C$  the exponential factor in the integrand can be neglected and the main contribution to the integral comes from poles, i.e., from small times  $t \sim 1/\epsilon_F$ :

$$G_{\text{dir}} = -\frac{|\tau_R|^2}{4v_F^2} \int dt it \left[ \frac{1}{\sinh^2(t-i0)} - \frac{1}{\sinh^2(t+i0)} \right] = \frac{|\tau_R|^2}{2\pi v_F^2} = G_R. \quad (\text{C7})$$

Next, the oscillating term has the following form:

$$G_{\text{osc}} = -2 \frac{|\tau_R|^2}{a} \exp[\alpha_A(T)] \text{Re}(\tau_L e^{-2\pi i Q_0} G_1), \quad (\text{C8})$$

where

$$\begin{aligned} \alpha_A(T) &= - \int \frac{dx x}{1+x^2} \frac{1}{1 - e^{-x/2\pi TC}} \\ &= \gamma + \log\left(\frac{a}{4\pi^2 v_F C}\right) + 2\pi^2 TC \\ &\quad + \log(4\pi^2 TC) + \Psi(1/4\pi^2 TC), \end{aligned} \quad (\text{C9})$$

$\gamma$  being the Euler gamma constant,  $\Psi(x)$  is the digamma function, and

$$G_1 = \int_{-\infty}^{\infty} dt it [K(t)A_1(t) - K^*(t)R_1(t)]. \quad (\text{C10})$$

Here we introduced  $K(t) = K_{\text{ff}}(t)e^{2\beta(t)}$ ,  $K_{\text{ff}}(t)$  is given by (B3) and expressions for  $A_1(t)$ ,  $R_1(t)$  read

$$\begin{aligned} A_1(t) &= \int_0^{-\infty} dt_1 [e^{F(t-t_1)-F(-t_1)} - 1] \\ &+ \int_t^0 dt_1 [e^{F(t-t_1)+F(t_1)} - 1] \\ &+ \int_{-\infty}^t dt_1 [e^{-F(t_1-t)+F(t_1)} - 1], \\ R_1(t) &= \int_t^{-\infty} dt_1 [e^{F(t-t_1)-F(-t_1)} - 1] \\ &+ \int_0^t dt_1 [e^{-F(t_1-t)-F(-t_1)} - 1] \\ &+ \int_{-\infty}^0 dt_1 [e^{-F(t_1-t)+F(t_1)} - 1], \end{aligned} \quad (\text{C11})$$

where

$$F(t_1) = 4\pi i C \int d\omega \frac{1}{1 + 4(\pi\omega C)^2} \frac{e^{-i\omega t_1} - 1}{1 - e^{-\omega/T}}. \quad (\text{C12})$$

One should note that  $F^*(t_1) = -F(-t_1)$ , which implies that  $R_1(t) = -A_1^*(t)$ , and thus we need to calculate only the function  $A_1(t)$ . At large temperatures  $T \gg E_C$  we expand  $1/(1 - e^{-\omega/T}) \approx T/\omega + 1/2$  in the integrand of (C12), and after integrating over  $\omega$  we obtain

$$\begin{aligned} F(t_1) &= -i\pi(1 - e^{-|t_1/2\pi C|}) \\ &+ 4\pi^2 T C (1 - e^{-|t_1/2\pi C|}) \text{sgn}(t_1). \end{aligned} \quad (\text{C13})$$

Next, using again  $T \gg E_C$  we conclude that the main contribution to the integral (C10) comes from times smaller than  $2\pi C$ , because  $T \gg E_C$ , therefore one can expand (C13):  $A_1(t) = R_1(t) \approx i(e^{2\pi T t} - 1)/T$ .

Substituting  $A_1(t)$  and  $R_1(t)$  into Eq. (C10), we arrive at the following expression:

$$G_1 = \frac{-iT}{4v_F^2 C} \int dt t^2 e^{-Tt^2/2C} \frac{e^{2\pi T t} - 1}{\sinh^2(\pi T t)}, \quad (\text{C14})$$

where we have simplified  $K(t) = K_{\text{ff}}(t)e^{-Tt^2/2C - it/2C}$ , because the integral comes from  $t/C \ll 1$ . Moreover, the main contribution comes from times  $t \gg 1/T$ , so that  $(e^{2\pi T t} - 1)/\sinh^2(\pi T t) \approx 4$ . After integrating over  $t$ , we get  $G_1 = -i\sqrt{2\pi C}/2v_F^2 \sqrt{T}$ . Finally, combining this with Eqs. (C7) and (C8), and absorbing the phase factor  $e^{-i\pi/2}$  into  $\tau_L$ , we arrive at the result (47) in the main text.

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- [1] H. Grabert and M. H. Devoret, Eds., *Single Charge Tunneling* (Plenum, New York, 1992).
- [2] M. A. Kastner, *Rev. Mod. Phys.* **64**, 849 (1992).
- [3] D. V. Averin and K. K. Likharev, in *Mesoscopic Phenomena in Solids*, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (Elsevier, Amsterdam, 1991).
- [4] T. A. Fulton and G. J. Dolan, *Phys. Rev. Lett.* **59**, 109 (1987).
- [5] P. Hawrylak, L. Jacak, and A. Wojs, *Quantum Dots* (Springer, Berlin, 1998).
- [6] T. Ando, Y. Arakawa, K. Furuya, S. Komiyama, and H. Nakashima, Eds., *Mesoscopic Physics and Electronics* (Springer, Berlin, 1998).
- [7] T. W. Larsen, K. D. Petersson, F. Kuemmeth, T. S. Jespersen, P. Krogstrup, J. Nyg ard, and C. M. Marcus, *Phys. Rev. Lett.* **115**, 127001 (2015).
- [8] S. M. Albrecht, A. P. Higginbotham, M. Madsen, F. Kuemmeth, T. S. Jespersen, J. Nyg ard, P. Krogstrup, and C. M. Marcus, *Nature (London)* **531**, 206 (2016).
- [9] S. Jezouin, Z. Iftikhar, A. Anthore, F. D. Parmentier, U. Gennser, A. Cavanna, A. Ouerqhi, I. P. Levkivskiy, E. Idrisov, E. V. Sukhorukov, L. I. Glazman, and F. Pierre, *Nature (London)* **536**, 58 (2016).
- [10] L. P. Kouwenhoven, N. C. van der Vaart, A. T. Johnson, W. Kool, C. J. P. M. Harmans, J. G. Williamson, A. A. M. Staring, and C. T. Foxon, *Z. Phys. B* **85**, 367 (1991).
- [11] A. A. M. Staring, J. G. Williamson, H. van Houten, C. W. J. Beenakker, L. P. Kouwenhoven, and C. T. Foxon, *Physica B (Amsterdam)* **175**, 226 (1991).
- [12] N. C. van der Vaart, A. T. Johnson, L. P. Kouwenhoven, D. J. Maas, W. de Jong, M. P. de Ruyter van Steveninck, A. van der Enden, C. J. P. M. Harmans, and C. T. Foxon, *Physica B (Amsterdam)* **189**, 99 (1993).
- [13] S. E. Korshunov, *JETP Lett.* **45**, 434 (1987).
- [14] G. Sch on and A. D. Zaikin, *Phys. Rep.* **198**, 237 (1990).
- [15] K. Flensberg, *Phys. Rev. B* **48**, 11156 (1993).
- [16] Y. V. Nazarov, *Phys. Rev. Lett.* **82**, 1245 (1999).
- [17] D. Chouvaev, L. S. Kuzmin, D. S. Golubev, and A. D. Zaikin, *Phys. Rev. B* **59**, 10599 (1999).
- [18] D. S. Golubev, J. K onig, H. Schoeller, G. Sch on, and A. D. Zaikin, *Phys. Rev. B* **56**, 15782 (1997).
- [19] M. Titov and D. B. Gutman, *Phys. Rev. B* **93**, 155428 (2016).
- [20] I. S. Burmistrov, *Low Temp. Phys.* **43**, 95 (2017).
- [21] A. Furusaki and K. A. Matveev, *Phys. Rev. B* **52**, 16676 (1995).
- [22] E. V. Sukhorukov, *Physica E (Amsterdam)* **77**, 191 (2016).
- [23] A. O. Slobodeniuk, I. P. Levkivskiy, and E. V. Sukhorukov, *Phys. Rev. B* **88**, 165307 (2013).
- [24] X. G. Wen, *Phys. Rev. B* **41**, 12838 (1990).
- [25] E. V. Sukhorukov and V. V. Cheianov, *Phys. Rev. Lett.* **99**, 156801 (2007).
- [26] I. P. Levkivskiy and E. V. Sukhorukov, *Phys. Rev. B* **78**, 045322 (2008).
- [27] A. O. Slobodeniuk, E. G. Idrisov, and E. V. Sukhorukov, *Phys. Rev. B* **93**, 035421 (2016).
- [28] E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics, Part 2*, Landau and Lifshitz Course of Theoretical Physics Vol. 9 (Butterworth/Heinemann, Oxford, 1980).
- [29] A. Furusaki and N. Nagaosa, *Phys. Rev. B* **47**, 3827 (1993).
- [30] R. C. Ashoori, H. L. Stormer, J. S. Weiner, L. N. Pfeiffer, S. J. Pearton, K. W. Baldwin, and K. W. West, *Phys. Rev. Lett.* **68**, 3088 (1992).
- [31] R. C. Ashoori, H. L. Stormer, J. S. Weiner, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, *Phys. Rev. Lett.* **71**, 613 (1993).