Theory of the spin Peltier effect

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A microscopic theory of the spin Peltier effect in a bilayer structure comprising a paramagnetic metal (PM) and a ferromagnetic insulator (FI) based on the nonequilibrium Green's function method is presented. Spin current and heat current driven by temperature gradient and spin accumulation are formulated as functions of spin susceptibilities in the PM and the FI, and are summarized by Onsager's reciprocal relations. By using the current formulas, we estimate heat generation and absorption at the interface driven by the heat-current injection mediated by spins from PM into FI.

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I. INTRODUCTION

In the field of spintronics, interconversion between heat and spin current has attracted considerable attention and has been studied actively since the discovery of the spin Seebeck effect [1–3]. The spin Seebeck effect refers to the spin-current generation from heat in magnetic materials [4,5]. The spin Seebeck effect has been observed in a variety of materials ranging from magnetic metals and semiconductors to insulators [1–3]. Recently, the spin Peltier effect which is the reciprocal phenomenon of the spin Seebeck effect, that is, heat generation from spin current, was reported experimentally [6,7]. While the spin Peltier effect has been studied using a phenomenological model [8–14], its microscopic theory is missing.

In this paper, we formulate a microscopic theory of the spin Peltier effect in paramagnetic metal (PM)/ferromagnetic insulator (FI) junction systems by using the nonequilibrium Green's function method. To reveal the microscopic mechanism of spin and heat transfer, we perform investigations using the setup shown in Fig. 1, where electron spins in PM, σ, are coupled with localized spins in FI, S, via the exchange interaction $J_{\rm sd}$.

Let us consider spin accumulation at the interface, $\delta\mu_S$, generated by the spin Hall effect [15] in PM. Owing to the exchange interaction, this spin accumulation excites the localized spins in FI, and then magnon flows are induced, accompanying both the spin and the heat.

The outline of this paper is as follows. In Sec. II, a brief review of the spin-current generation in PM/FI is given by using the nonequilibrium Green's function. In Sec. III, the heat-current generation in PM/FI is derived following the formalism shown in Sec. II. In Sec. IV, we estimate the temperature change at the PM/FI interface due to the spin Peltier effect. In Sec. V, we summarize our results.

II. SPIN-CURRENT GENERATION AT MAGNETIC INTERFACE

In this section, we briefly review spin-current generation in PM/FI by using the nonequilibrium Green's function. The PM/FI interface is modeled using the s-d exchange interaction:

$$H_{\rm sd} = J_{\rm sd}\sum_{i\in{\rm int}} \boldsymbol{\sigma}_i\cdot\mathbf{S}_i, \tag{1}$$

where $J_{\rm sd}$, $\boldsymbol{\sigma}_i$, and $\mathbf{S}_i$ represent the coupling constant of the exchange interaction, Pauli matrices, and localized spin of FI, respectively, and $\sum_{i\in{\rm int}}$ denotes the summation on the lattice sites at the interface.

The spin current $I^{\rm S}$ is defined by the time derivative of the z component of the conduction electron spin in PM, that is, $I^{\rm S}\equiv\sum_{i\in P}\langle\partial_t\sigma_i^z\rangle$, where $\langle\cdots\rangle$ denotes the statistical average [16]. The Heisenberg equation of motion for σ_i^z gives [5] $I^{\rm S}=(J_{\rm sd}/\hbar){\rm Re}[\sum_{i\in{\rm int}}(-i)\langle\sigma_i^+(t)S_i^-(t)\rangle]$, where $\sigma_i^\pm=\sigma_i^x\pm i\sigma_i^y$ and $S_i^\pm=S_i^x\pm iS_i^y$. After the perturbative calculation [17,18] of $\langle\sigma_i^+(t)S_i^-(t)\rangle$ up to the second order of $J_{\rm sd}$, the spin current is given by

$$I^{\rm S}=\frac{J_{\rm int}^2}{2}{\rm Re}\int_{\mathbf{qk}\omega}\left(\chi^R_{\mathbf{qr},\omega t}G^<_{\mathbf{kr}',\omega t}+\chi^<_{\mathbf{qr},\omega t}G^A_{\mathbf{kr}',\omega t}\right), \tag{2}$$

where, we have introduced the shorthand notation $\int_{\mathbf{qk}\omega}=\int d^3\mathbf{k}d^3\mathbf{q}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}$. $J_{\rm int}^2$ is given by $J_{\rm int}^2=(J_{\rm sd}/\hbar)^2N_{\rm int}$, with $N_{\rm int}$ being the number of sites at the interface. In Eq. (2), $\chi^{R(<)}_{\mathbf{qr},\omega t}$ is the retarded (lesser) component of the transverse spin susceptibility in PM given by $\chi^{R(<)}_{\mathbf{qr},\omega t}=\int_{\delta\mathbf{r}\delta t}\exp[-i\mathbf{q}\cdot\delta\mathbf{r}+i\omega\delta t]\chi^{R(<)}(\mathbf{r}+\delta\mathbf{r}/2,t+\delta t;\mathbf{r}-\delta\mathbf{r}/2,t-\delta t)$, where $\chi^R(\mathbf{r}+\delta\mathbf{r}/2,t+\delta t;\mathbf{r}-\delta\mathbf{r}/2,t-\delta t)$ and $\chi^<(\mathbf{r}+\delta\mathbf{r}/2,t+\delta t;\mathbf{r}-\delta\mathbf{r}/2,t-\delta t)$ are defined as $\chi^R(\mathbf{r}+\delta\mathbf{r}/2,t+\delta t;\mathbf{r}-\delta\mathbf{r}/2,t-\delta t)\equiv-i\theta(t_1-t_2)\langle[\sigma^+_{\mathbf{r}_1}(t_1),\sigma^-_{\mathbf{r}_2}(t_2)]\rangle$ and $\chi^<(\mathbf{r}+\delta\mathbf{r}/2,t+\delta t;\mathbf{r}-\delta\mathbf{r}/2,t-\delta t)\equiv-i\langle\sigma^+_{\mathbf{r}_1}(t_1)\sigma^-_{\mathbf{r}_2}(t_2)\rangle$, respectively, with $\mathbf{r}\equiv(\mathbf{r}_1+\mathbf{r}_2)/2$, $\delta\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$, $t\equiv(t_1+t_2)/2$ and $\delta t=t_1-t_2$. $G^{A(<)}_{\mathbf{kr}',\omega t}$ is the advanced (lesser) component of the transverse spin susceptibility in FI, and it is given by $G^{A(<)}_{\mathbf{kr}',\omega t}=\int_{\delta\mathbf{r}\delta t}\exp[-i\mathbf{k}\cdot\delta\mathbf{r}+i\omega\delta t]G^{A(<)}(\mathbf{r}'+\delta\mathbf{r}/2,t+\delta t;\mathbf{r}'-\delta\mathbf{r}/2,t-\delta t)$, where $G^A(\mathbf{r}'+\delta\mathbf{r}/2,t+\delta t;\mathbf{r}'-\delta\mathbf{r}/2,t-\delta t)$ and $G^<(\mathbf{r}'+\delta\mathbf{r}/2,t+\delta t;\mathbf{r}'-\delta\mathbf{r}/2,t-\delta t)$ are defined as $G^A(\mathbf{r}'+\delta\mathbf{r}/2,t+\delta t;\mathbf{r}'-\delta\mathbf{r}/2,t-\delta t)\equiv i\theta(t_2-t_1)\langle[S^+_{\mathbf{r}_1}(t_1),S^-_{\mathbf{r}_2}(t_2)]\rangle$ and $G^<(\mathbf{r}'+\delta\mathbf{r}/2,t+\delta t;\mathbf{r}'-\delta\mathbf{r}/2,t-\delta t)\equiv-i\langle S^+_{\mathbf{r}_1}(t_1)S^-_{\mathbf{r}_2}(t_2)\rangle$, respectively. Here, $\mathbf{r}$ is defined in FI, while $\mathbf{r}'$ is defined in PM. It is noted that $G^<_{\mathbf{kr}',\omega t}$ includes the effect of dynamics of the magnons in the ferromagnet.

Let us focus on the steady state in terms of time and spatially uniform interface, where $\chi^{R(<)}_{\mathbf{qr},\omega t}\to\chi^{R(<)}_{\mathbf{q}\omega}$ and $G^{A(<)}_{\mathbf{kr}',\omega t}\to G^{A(<)}_{\mathbf{k}\omega}$. By substituting the Kadanoff Baym ansatz [18] $\chi^<_{\mathbf{q}\omega}=2i{\rm Im}\chi^R_{\mathbf{q}\omega}f^{\rm P}_\omega$ and $G^<_{\mathbf{k}\omega}=2i{\rm Im}G^R_{\mathbf{k}\omega}f^{\rm F}_\omega$ into Eq. (2), with $f^{\rm P}_\omega=f(\omega/T_{\rm P})$ and $f^{\rm F}_\omega=f(\omega/T_{\rm F})$ being the

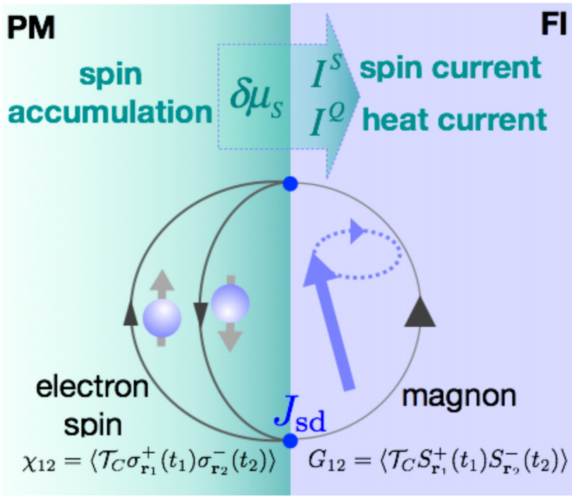

FIG. 1. Schematic view of the spin Peltier effect. We consider spin transport in a bilayer structure consisting of a paramagnetic metal (PM) and a ferromagnetic insulator (FI), where the electron spins in PI are coupled with the localized moments in FI via the exchange interaction $J_{\rm sd}$. The spin accumulation at the interface ($\delta\mu_s$) is found to be a driving force of spin and heat current (I^S and I^Q) by using the nonequilibrium Green's functions for electron spin χ and magnon G, where $\mathcal{T}_C$ denotes the time ordering on the Keldysh contour.

Bose-Einstein distribution functions in PM and FI, respectively, we obtain the general expression of spin current as follows:

$$I^{\rm S} = J_{\rm int}^2 \int_{\mathbf{q}\mathbf{k}\omega} {\rm Im}\chi^R_{\mathbf{q}\omega}{\rm Im}G^R_{\mathbf{k}\omega}\big(f^{\rm P}_\omega - f^{\rm F}_\omega\big). \tag{3}$$

Equation (3) is a spin-current version of the Meir-Wingreen formula [17], where $J_{\rm int}^2$ corresponds to the tunneling probability of the spin current at the interface. The integration of ${\rm Im}\chi^R_{\mathbf{q}\omega}$ over $\mathbf{q}$ and that of ${\rm Im}G^R_{\mathbf{k}\omega}$ over $\mathbf{k}$ represent the density of states of the transverse spin fluctuations in PM and FI, respectively. The difference $f^{\rm P}_\omega - f^{\rm F}_\omega$ plays a crucial role in spin-current generation and has a nonvanishing value only when the system is out of equilibrium. In the following, we investigate the effect of the temperature difference and the spin accumulation at the interface.

A. Spin current driven by spin Seebeck effect

First, let us consider the spin Seebeck effect [1–5] that spin current injection is driven by the temperature difference δT between PM and FI, given as $\delta T = T_{\rm P} - T_{\rm F}$. The difference between $f^{\rm P}_\omega$ and $f^{\rm F}_\omega$ is given by

$$f^{\rm P}_\omega - f^{\rm F}_\omega = \frac{\partial f}{\partial T}\delta T. \tag{4}$$

Substituting Eq. (4) into Eq. (3), we obtain the spin-current injection due to the spin Seebeck effect as follows [5]:

$$I^{\rm S} = J_{\rm int}^2 \int_{\mathbf{q}\mathbf{k}\omega} {\rm Im}\chi^R_{\mathbf{q}\omega}{\rm Im}G^R_{\mathbf{k}\omega}\frac{\partial f}{\partial T}\delta T. \tag{5}$$

B. Spin current driven by spin accumulation

Now, let us focus on the spin-current injection driven by the spin accumulation. The expression of spin accumulation at the interface is given by $\delta\mu_{\rm S} = 2e\alpha_{\rm SH}\rho_{\rm N}\lambda_{\rm N} j_{\rm c}\tanh(d_{\rm N}/2\lambda_{\rm N})$ [19,20], where $\alpha_{\rm SH}$, $\rho_{\rm N}$, $\lambda_{\rm N}$, $j_{\rm c}$, and $d_{\rm N}$ are the spin Hall angle, electrical resistivity, spin diffusion length, charge current, and thickness of metal, respectively. The retarded and the lesser components of the spin susceptibility in the metal, $\chi^R_{\mathbf{q}\omega}$ and $\chi^<_{\mathbf{q}\omega}$, are modified by the spin accumulation $\delta\mu_{\rm S}$ as $\chi^R_{\mathbf{q}\omega} \to \chi^R_{\mathbf{q},\omega+\delta\mu_{\rm S}}$ and $\chi^<_{\mathbf{q}\omega} \to \chi^<_{\mathbf{q},\omega+\delta\mu_{\rm S}} = 2i{\rm Im}\chi^R_{\mathbf{q},\omega+\delta\mu_{\rm S}} f^{\rm P}_{\omega+\delta\mu_{\rm S}}$, respectively.

The difference between $f^{\rm P}_{\omega+\delta\mu_{\rm S}}$ and $f^{\rm F}_\omega$ is as follows:

$$f^{\rm P}_{\omega+\delta\mu_{\rm S}} - f^{\rm F}_\omega = \frac{\partial f}{\partial\omega}\frac{\delta\mu_{\rm S}}{\hbar}. \tag{6}$$

Substituting Eq. (6) into Eq. (3), we obtain the spin-current injection driven by spin accumulation as follows:

$$I^{\rm S} = J_{\rm int}^2 \int_{\mathbf{q}\mathbf{k}\omega} {\rm Im}\chi^R_{\mathbf{q}\omega}{\rm Im}G^R_{\mathbf{k}\omega}\frac{\partial f}{\partial\omega}\frac{\delta\mu_{\rm S}}{\hbar}. \tag{7}$$

Note that Eq. (7) reduces to (S10) in Ref. [21] when we evaluate spin susceptibility in the metal $\chi^R_{\mathbf{q}\omega}$ for the noninteracting electrons.

III. HEAT TRANSPORT MEDIATED BY SPIN CURRENT

In this section, the heat-current generation in PM/FI is derived according to the formalism developed in Sec. II. Following Ref. [22], we define the heat current $I^{\rm Q}$ injected into the ferromagnet as the time derivative of the Hamiltonian of the ferromagnet $H_{\rm m}$, $I^{\rm Q} \equiv \sum_{i\in F}\langle\partial_t H_{\rm m}\rangle$, where $\langle\cdots\rangle$ denotes the statistical average. The Heisenberg equation of motion for $H_{\rm m}$ gives

$$\partial_t H_{\rm m} = \frac{1}{i\hbar}[H_{\rm m}, H_{\rm sd}]. \tag{8}$$

Substituting Eq. (1) into Eq. (8) and taking the statistical average gives the following heat current:

$$I^{\rm Q} = -J_{\rm sd}\sum_{i\in{\rm int}}\partial_{t'}\langle\boldsymbol{\sigma}_i(t)\cdot\mathbf{S}_i(t')\rangle_{t'\to t}, \tag{9}$$

where we use the Heisenberg equation of motion for localized spin at the interface $\partial_t\mathbf{S}_i = (i\hbar)^{-1}[\mathbf{S}_i, H_{\rm m}]$ to derive Eq. (9).

Now we consider the spin-wave approximation in the lowest order of $1/S_0$ expansion, with S_0 being the size of the localized spins. The time derivative of S^z_i vanishes because the z component of the localized spins S^z_i becomes constant. By performing the perturbative calculation up to the second order of the interfacial interaction $J_{\rm sd}$, we obtain the heat current as

$$I^{\rm Q} = \frac{J_{\rm int}^2}{2}{\rm Re}\int_{\mathbf{q}\mathbf{k}\omega}\hbar\omega\big[\chi^R_{\mathbf{q}\omega}G^<_{\mathbf{k}\omega} + \chi^<_{\mathbf{q}\omega}G^A_{\mathbf{k}\omega}\big]. \tag{10}$$

By substituting the Kadanoff Baym ansatz into Eq. (10), we can rewrite the heat current as

$$I^{\rm Q} = J_{\rm int}^2\int_{\mathbf{q}\mathbf{k}\omega}\hbar\omega{\rm Im}\chi^R_{\mathbf{q}\omega}{\rm Im}G^R_{\mathbf{k}\omega}\big(f^{\rm P}_\omega - f^{\rm F}_\omega\big). \tag{11}$$

Especially, substituting Eqs. (4) and (6) into Eq. (11), we obtain the interfacial heat current caused by the temperature difference:

$$I^{\mathrm{Q}} = J_{\mathrm{int}}^2 \int_{\mathbf{qk}\omega} \hbar\omega \mathrm{Im}\chi_{\mathbf{q}\omega}^R \mathrm{Im}G_{\mathbf{k}\omega}^R \frac{\partial f}{\partial T}\delta T, \tag{12}$$

and that caused by the spin accumulation:

$$I^{\mathrm{Q}} = J_{\mathrm{int}}^2 \int_{\mathbf{qk}\omega} \omega \mathrm{Im}\chi_{\mathbf{q}\omega}^R \mathrm{Im}G_{\mathbf{k}\omega}^R \frac{\partial f}{\partial \omega}\delta \mu_{\mathrm{S}}. \tag{13}$$

Equations (5), (7), (12), and (13) are summarized by Onsager's reciprocal relation [23]:

$$\begin{pmatrix} I^{\mathrm{S}} \\ I^{\mathrm{Q}} \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} \delta\mu_{\mathrm{S}} \\ -\delta T/T \end{pmatrix}, \tag{14}$$

where the transport coefficients are given by

$$L_{11} = J_{\mathrm{int}}^2 \int_{\mathbf{qk}\omega} \frac{1}{\hbar} \mathrm{Im}\chi_{\mathbf{q}\omega}^R \mathrm{Im}G_{\mathbf{k}\omega}^R \frac{\partial f}{\partial \omega}, \tag{15}$$

$$L_{12} = J_{\mathrm{int}}^2 \int_{\mathbf{qk}\omega} \mathrm{Im}\chi_{\mathbf{q}\omega}^R \mathrm{Im}G_{\mathbf{k}\omega}^R \left(-T\frac{\partial f}{\partial T}\right), \tag{16}$$

$$L_{21} = J_{\mathrm{int}}^2 \int_{\mathbf{qk}\omega} \omega \mathrm{Im}\chi_{\mathbf{q}\omega}^R \mathrm{Im}G_{\mathbf{k}\omega}^R \frac{\partial f}{\partial \omega}, \tag{17}$$

$$L_{22} = J_{\mathrm{int}}^2 \int_{\mathbf{qk}\omega} \hbar\omega \mathrm{Im}\chi_{\mathbf{q}\omega}^R \mathrm{Im}G_{\mathbf{k}\omega}^R \left(-T\frac{\partial f}{\partial T}\right). \tag{18}$$

Substituting the relation $\omega\partial f/\partial\omega = -T\partial f/\partial T$ into Eq. (17) yields the relation $L_{12} = L_{21}$.

IV. TEMPERATURE CHANGE AT THE INTERFACE

In this section, we estimate the temperature change ΔT due to the spin Peltier effect. At the interface, magnons are excited and accumulated by the spin Peltier effect. The energy change ΔE at the interface is generated by the accumulation of magnons. Then, the temperature change ΔT is obtained as $\Delta T = \Delta E/C_{\mathrm{FI}}$, with C_{FI} being the heat capacity of the ferromagnet.

Now, we formulate the energy change ΔE of the magnons with the lesser component of transverse spin susceptibility $G_{\mathbf{k}\omega}^<$. In the spin-wave approximation, the operators of localized spins are given by $S_i^+ \approx \sqrt{2S_0}a_i$, $S_i^- \approx \sqrt{2S_0}a_i^\dagger$, where a_i and $a_i^\dagger$ are the creation and the annihilation operators of the magnons. Substituting these relations into the lesser component of transverse spin susceptibility in FI, we obtain $G_{\mathbf{k}}^<(t_1,t_2) = -2iS_0\langle a_{\mathbf{k}}^\dagger(t_2)a_{\mathbf{k}}(t_1)\rangle$. Because the statistical average of $a_{\mathbf{k}}^\dagger(t_2)a_{\mathbf{k}}(t_1)$ can be interpreted as the number of the magnons when t_2 corresponds to t_1, the energy change ΔE is given by

$$\Delta E \equiv \frac{-1}{2S_0}\int_{\mathbf{k}\omega} \hbar\omega \mathrm{Im}\big(G_{\mathbf{k}\omega}^< - G_{\mathbf{k}\omega}^{0<}\big), \tag{19}$$

where $G_{\mathbf{k}\omega}^{0<} = 2i\mathrm{Im}G_{\mathbf{k}\omega}^R f_\omega^{\mathrm{F}}$ is the lesser Green's function of the free magnons.

Let us consider a bilayer system composed of the platinum (Pt) and the yittrium iron garnet (YIG). In spin-wave approximation, the retarded component of transverse spin susceptibility $G_{\mathbf{k}\omega}^R$ is given by $G_{\mathbf{k}\omega}^R = 2S_0(\omega - \omega_{\mathbf{k}} + i\alpha\omega)^{-1}$, where $\omega_{\mathbf{k}} = A\mathbf{k}^2 + \gamma H_0$ is the dispersion relation of magnons, with A, γ, and H_0 being the stiffness constant, gyromagnetic ratio, and static magnetic field in YIG, respectively. α is the Gilbert damping constant of the magnons. After perturbative calculation up to the second order of J_{sd}, we obtain the lesser Green's function of the magnons at the interface $G_{\mathbf{k}\omega}^<$ as follows:

$$G_{\mathbf{k}\omega}^< = G_{\mathbf{k}\omega}^{0<} - i\frac{J_{\mathrm{int}}^2 S_0}{\alpha\omega}\int_{\mathbf{q}} \mathrm{Im}\chi_{\mathbf{q}\omega}^R \mathrm{Im}G_{\mathbf{k}\omega}^R \frac{\partial f}{\partial\omega}\frac{\delta\mu_{\mathrm{S}}}{\hbar}. \tag{20}$$

Equation (20) shows the accumulation of magnons driven by spin-current injection. Let us consider the rate equation of the magnons at the interface. Since the number density of the excited magnons can be derived from the lesser component of the transverse spin susceptibility in FI, the rate equation of magnons is written as

$$\frac{\partial G_{\mathbf{k}\omega,t}^<}{\partial t} = -\frac{G_{\mathbf{k}\omega,t}^< - G_{\mathbf{k}\omega}^{0<}}{\tau_{\mathbf{k}\omega}} + I_{\mathbf{k}\omega}^{\mathrm{S}}, \tag{21}$$

where $\tau_{\mathbf{k}\omega} = (\alpha\omega)^{-1}$ is the lifetime of the magnons. In Eq. (21), the source term $I_{\mathbf{k}\omega}^{\mathrm{S}}$ is the spin current of a particular magnon with the wave number $\mathbf{k}$ and frequency ω, defined as $I_{\mathbf{k}\omega}^{\mathrm{S}} \equiv J_{\mathrm{int}}^2\int_{\mathbf{q}} \mathrm{Im}\chi_{\mathbf{q}\omega}^R \mathrm{Im}G_{\mathbf{k}\omega}^R(\partial f/\partial\omega)(\delta\mu_{\mathrm{S}}/\hbar)$. In the steady state, where the time derivative of $G_{\mathbf{k}\omega,t}^<$ vanishes ($\partial G_{\mathbf{k}\omega,t}^</\partial t \to 0$), Eq. (21) reduces to $G_{\mathbf{k}\omega}^< - G_{\mathbf{k}\omega}^{0<} = I_{\mathbf{k}\omega}^{\mathrm{S}}\tau_{\mathbf{k}\omega}$, corresponding to Eq. (20).

Substituting Eq. (20) into Eq. (19), we obtain the energy change of the magnons as

$$\Delta E = \frac{\hbar}{2\alpha}I^{\mathrm{S}}, \tag{22}$$

where I^{S} is shown in Eq. (7).

The spin susceptibility in Pt, $\chi_{\mathbf{q}\omega}^R$, is written as $\chi_{\mathbf{q}\omega}^R = \chi_{\mathrm{N}}(\tau_{\mathrm{sf}}^{-1} + D_{\mathrm{N}}\mathbf{q}^2 + i\omega)^{-1}$ [5], where τ_{sf} and D_{N} are the spin-flip time and the diffusion constant of Pt, respectively. By integrating I^{S} over ω in Eq. (22) by using the relation $\mathrm{Im}G_{\mathbf{k}\omega}^R \approx -\pi\delta(\omega - \omega_{\mathbf{k}})$, we have $\Delta E = -(N_{\mathrm{int}}g_{\mathrm{s}}/2)(k_{\mathrm{B}}T/\hbar\omega_{\mathrm{M}})^{3/2}(\gamma_1/\gamma_2)\delta\mu_{\mathrm{S}}$, where g_{s} is given as $g_{\mathrm{s}} = (J_{\mathrm{sd}}/\hbar)^2 S_0\int_{\mathbf{q}} \mathrm{Im}\chi_{\mathbf{q},\gamma H_0}^R/(\gamma H_0)$, with ω_{M} being the maximum energy of the magnons estimated from the Curie temperature T_{C} as $\omega_{\mathrm{M}} \equiv k_{\mathrm{B}}T_{\mathrm{C}}/\hbar$. The numerical factors γ_1 and γ_2 are defined by $\gamma_1 = \int_0^1 dx\int_{y_0}^{y_{\mathrm{M}}} dy y\sqrt{x(y-y_0)}\{4[(1+x)^2 + (yk_{\mathrm{B}}T\tau_{\mathrm{sf}}/\hbar)^2]\sinh^2(y/2)\}^{-1}$ and $\gamma_2 = \int_0^1 dx\sqrt{x}[(1+x)^2 + (\gamma H_0\tau_{\mathrm{sf}})^2]^{-1}$, respectively. In the factor γ_2, y_0 and y_{M} are given by $y_0 = \hbar\gamma H_0/k_{\mathrm{B}}T$ and $y_{\mathrm{M}} = \hbar\omega_{\mathrm{M}}/k_{\mathrm{B}}T$, respectively.

We examine the experiment in Ref. [7]. By using the parameters of Pt in Ref. [7] as $\rho_{\mathrm{N}} = 0.48\,\mu\Omega\cdot\mathrm{m}$, $\lambda_{\mathrm{N}} = 7.3$ nm [24], $j_{\mathrm{c}} = 1.0\times10^9$ A/m^2, $d_{\mathrm{N}} = 5$ nm, and $\alpha_{\mathrm{SH}} = 0.013$ [15], we obtain the spin accumulation at the interface as $\delta\mu_{\mathrm{S}} = 2.3\times10^{-8}$ eV. In the case of YIG, where $T_{\mathrm{C}} = 565$ K and $H_0 = 200$ Oe, we estimate $\gamma_1 = 0.215$ and $\gamma_2 = 0.285$ at room temperature. Combining the values of $\delta\mu_{\mathrm{S}}$, γ_1 and γ_2, and $\alpha = 10^{-5}$ and $g_{\mathrm{s}} = 0.1$ [24], we obtain the energy change normalized per site of localized spin at the interface $\Delta E/N_{\mathrm{int}}$ as $\Delta E/N_{\mathrm{int}} = -3.3\times10^{-5}$ eV. Taking $N_{\mathrm{int}} = 1.0\times10^{11}$ and $C_{\mathrm{FI}} = \tilde{c}_{\mathrm{FI}}\rho_{\mathrm{FI}}a_{\mathrm{FI}}^3 N_{\mathrm{int}}$, with the density $\rho_{\mathrm{FI}} = 5170$ (kg/m^3), the lattice constant $a_{\mathrm{FI}} = 1.24\times10^{-9}$ m, and the specific heat $\tilde{c}_{\mathrm{FI}} = 570$ J/(kg$\cdot$ K) of YIG, the temperature change is estimated to be $\Delta T = -1$ mK, which is consistent with the experimental result [7].

Finally, we mention the temperature change ΔT when the FI is replaced by a ferromagnetic metal. The Gilbert damping constant α in a ferromagnetic metal is, in general, much larger than that in a FI because of the interaction between the magnons and the conduction electrons [25]. According to Eq. (22), ΔT is inversely proportional to α. Therefore, it is expected that ΔT in the ferromagnetic metal is suppressed more than that in the FI.

V. CONCLUSION

In this study, a microscopic theory of the spin Peltier effect in a magnetic bilayer structure system consisting of PM and FI was formulated using the nonequilibrium Green's function method. We derived the spin and heat currents driven by temperature gradient as well as by spin accumulation at the interface in terms of spin susceptibility and the magnons' Green's function. These currents have been summarized using Onsager's reciprocal relation. In addition, we estimated heat generation and absorption at the interface due to spin injection from PM into FI. Our theory will provide a microscopic understanding of the conversion phenomena between spin and heat at the magnetic interface.

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[1] K. Uchida, S. Takahashi, K. Harii, J. Ieda, W. Koshibae, K. Ando, S. Maekawa, and E. Saitoh, Nature (London) **455**, 778 (2008).
[2] C. M. Jaworski, J. Yang, S. Mack, D. D. Awschalom, J. P. Heremans, and R. C. Myers, Nat. Mater. **9**, 898 (2010).
[3] K. Uchida, J. Xiao, H. Adachi, J. Ohe, S. Takahashi, J. Ieda, T. Ota, Y. Kajiwara, H. Umezawa, H. Kawai, G. E. W. Bauer, S. Maekawa, and E. Saitoh, Nat. Mater. **9**, 894 (2010).
[4] J. Xiao, G. E. W. Bauer, K. C. Uchida, E. Saitoh, and S. Maekawa, Phys. Rev. B **81**, 214418 (2010).
[5] H. Adachi, J. I. Ohe, S. Takahashi, and S. Maekawa, Phys. Rev B **83**, 094410 (2011).
[6] J. Flipse, F. K. Dejene, D. Wagenaar, G. E. W. Bauer, J. B. Youssef, and B. J. van Wees, Phys. Rev. Lett. **113**, 027601 (2014).
[7] S. Daimon, R. Iguchi, T. Hioki, E. Saitoh, and K. Uchida, Nat. Commun. **7**, 13754 (2016).
[8] L. Gravier, S. Serrano-Guisan, F. Reuse, and J. P. Ansermet, Phys. Rev. B **73**, 024419 (2006).
[9] M. Hatami, G. E. W. Bauer, Q. Zhang, and P. J. Kelly, Phys. Rev. B **79**, 174426 (2009).
[10] A. A. Kovalev and Y. Tserkovnyak, Phys. Rev. B **80**, 100408(R) (2009).
[11] G. E. W. Bauer, S. Bretzel, A. Brataas, and Y. Tserkovnyak, Phys. Rev. B **81**, 024427 (2010).
[12] J. Xiao and G. E. W. Bauer, arXiv:1508.02486v2.
[13] V. Basso, E. Ferraro, A. Magni, A. Sola, M. Kuepferling, and M. Pasquale, Phys. Rev. B **93**, 184421 (2016).
[14] L. J. Cornelissen, K. J. H. Peters, G. E. W. Bauer, R. A. Duine, and B. J. van Wees, Phys. Rev. B **94**, 014412 (2016).
[15] J. Sinova, S. O. Valenzuela, J. Wunderlich, C. H. Back, and T. Jungwirth, Rev. Mod. Phys. **87**, 1213 (2015).
[16] J. König and J. Martinek, Phys. Rev. Lett. **90**, 166602 (2003).
[17] Y. Meir and N. S. Wingreen, Phys. Rev. Lett. **68**, 2512 (1992).
[18] H. Haug and A.-P. Jauho, *Quantum Kinetics in Transport and Optics of Semiconductors* (Springer-Verlag, Berlin, 1996).
[19] S. Zhang, Phys. Rev. Lett. **85**, 393 (2000).
[20] S. Maekawa, S. O. Valenzuela, E. Saitoh, and T. Kimura, *Spin Current* (Oxford University Press, Oxford, 2012).
[21] Y. Kajiwara, K. Harii, S. Takahashi, J. Ohe, K. Uchida, M. Mizuguchi, H. Umezawa, H. Kawai, K. Ando, K. Takanashi, S. Maekawa, and E. Saitoh, Nature (London) **464**, 262 (2010).
[22] K. Maki and A. Griffin, Phys. Rev. Lett. **15**, 921 (1965).
[23] S. R. de Groot and P. Mazur, *Non-Equilibrium Thermodynamics* (Dover, New York, 1984).
[24] H. L. Wang, C. H. Du, Y. Pu, R. Adur, P. C. Hammel, and F. Y. Yang, Phys. Rev. Lett. **112**, 197201 (2014).
[25] Recently, ferromagnetic metals with low Gilbert damping have been discovered [M. A. W. Schoen, D. Thonig, M. L. Schneider, T. J. Silva, H. T. Nembach, O. Eriksson, O. Karis, and J. M. Shaw, Nat. Phys. **12**, 839 (2016)]; Since in the metals the interaction between magnons and conduction electrons is weak, the argument for the ferromagnetic insulators may be applied.