Loss of adiabaticity with increasing tunneling gap in nonintegrable multistate Landau-Zener models

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We consider the simplest nonintegrable model of the multistate Landau-Zener transition. In this model, two pairs of levels in two tunnel-coupled quantum dots are swept past each other by the gate voltage. Although this 2×2 model is nonintegrable, it can be solved analytically in the limit when the interlevel energy distance is much smaller than their tunnel splitting. The result is contrasted to the similar 2×1 model, in which one of the dots contains only one level. The latter model does not allow interference of the virtual transition amplitudes, and it is exactly solvable. In the 2×1 model, the probability for a particle, residing at time $t \to -\infty$ in one dot, to remain in the same dot at $t \to \infty$, falls off exponentially with tunnel coupling. By contrast, in the 2×2 model, this probability *grows* rapidly with tunnel coupling. The physical origin of this growth is the formation of the tunneling-induced collective states in the system of two dots. This can be viewed as a manifestation of the Dicke effect.

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I. INTRODUCTION

Motivations for the study of the transition probabilities between multiple intersecting levels (multistate Landau-Zener transitions) were different over different periods of time. For example, Ref. [1], in which the scattering matrix was found for a particular variant of crossing of the large number of levels, was motivated by research [2–5] on inelastic atomic collisions. A multilevel description of the electron transfer in the course of the collision is required when the crossing levels are dense, so that the tunnel splitting exceeds the level spacing. In this situation, the conventional Landau-Zener (LZ) theory [6–9] developed for a single crossing is inapplicable.

Later, the physics of multiple level crossings emerged in quantum optics [10]; in particular, in the problem of two optical transitions having a common level in an atom driven by two laser beams. Theoretical works of this period [11–19] broadened the class of exactly solvable models. Also, for general multistate models, the exact results for certain elements of the scattering matrix had been established.

Finally, the motivation for the very recent studies of the multilevel LZ transitions [20–29] was the ongoing experimental research on qubit manipulation by time-dependent fields in relation to information processing. In these studies [20–29] a number of exactly solvable models were identified, although the conclusion about their solvability was drawn on the basis of numerics.

The simplification, which allowed the authors of Ref. [1] to find the scattering matrix exactly, stemmed from the assumption about the time evolution of the energy levels. Namely, it was assumed that $N-1$ out of N levels evolved with the same velocity, and only one level evolved with differing velocity. Thus, the number of crossings was $N-1$. The behavior of the amplitudes to stay on a given level at $t \to -\infty$, i.e., far from all crossings, can be found semiclassically. The contour integral method employed in Ref. [1] allows us to establish the relations between these amplitudes at $t \to -\infty$ and $t \to \infty$. With $N-1$ crossings, these conditions are sufficient to fix all $\frac{1}{2}(N^2+3N-2)$ nonzero transition probabilities [30]. The above approach, along with others, was employed in later theoretical works. In particular, in Refs. [14] and [31], the transition probabilities were derived upon summation of the perturbation expansion in powers of the interlevel coupling strengths.

The fact that a given multistate LZ problem with a finite number of intersecting levels can be solved exactly implies that the elements of the scattering matrix can be constructed from the partial LZ probabilities P_{LZ} for individual pairs of intersecting levels. In other words, the time intervals between the successive intersections do not enter in the result even when these intervals are much smaller than the characteristic time of the LZ transition. Yet another way to express this remarkable fact is that the independent-crossing approximation, valid for small tunneling gaps, remains applicable even when the gaps are much bigger than the energy separation of the neighboring crossing points.

Note that, for sufficiently slow drive velocities or for sufficiently big LZ gaps, when the individual P_{LZ} values approach 1, the "survival" probability for a particle to stay on the initial level is exponentially small. This immediately suggests that, for exactly solvable (integrable) models, the survival probabilities fall off exponentially with increasing gap. Then the question arises as to whether the above conclusion is valid for nonintegrable models. This question is addressed in the present paper. We focus on a simple example of the electron transfer between two multilevel quantum dots. Our main finding is that the survival probability can, actually, *increase* with increasing tunneling gap. We relate this finding to the Dicke effect [32]. The reason that the nonintegrable model can be solved analytically is that, for a very slow drive, the semiclassical approach for the time-dependent amplitudes applies even in the vicinity of the LZ transition [33,34]. By "semiclassical approach" we mean the solution of the Schrödinger equation for the amplitudes in the form of a slow prefactors multiplied by the fast phase factor, common for all amplitudes.

II. THE MODEL

We start by illustrating the difference between integrable and nonintegrable models by using the simplest example of two quantum dots, depicted in Fig. 1. In Fig. 1(a) there are two levels in the left dot separated by 2Δ and one level in the right dot. The left-dot levels are driven, say, by the gate voltage, with velocity $v/2$, while the right-dot level is driven in the opposite direction with the same velocity (if the velocities

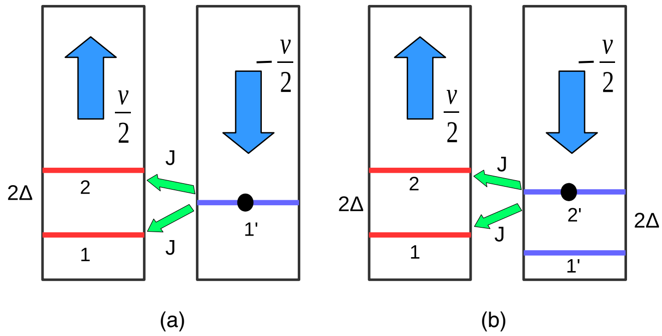

FIG. 1. Two elementary multistate LZ models are illustrated: (a) A 2×1 model of tunnel-coupled dots: a single level in the right dot is swept by two levels in the left dot with relative velocity v. (b) A 2×2 model: the energy spacing 2Δ between the levels in both dots is the same. Each level in one dot is coupled with both levels in the other dot with coupling constant J.

are different, v is measured from the average velocity). Both left-dot levels are coupled to the right-dot level by the same coupling constant J. The matrix form of the Hamiltonian is the following:

$$\hat{H}_{2,1} = \begin{pmatrix} -\Delta - \frac{vt}{2} & 0 & J \\ 0 & \Delta - \frac{vt}{2} & J \\ J & J & \frac{vt}{2} \end{pmatrix}. \quad (1)$$

The evolution of the amplitudes $a_1(t)$, $a_2(t)$, and $b_1(t)$ [see Fig. 1(a)], is governed by the Schrödinger equation

$$i \begin{pmatrix} \dot{a}_1 \\ \dot{a}_2 \\ \dot{b}_1 \end{pmatrix} = \hat{H}_{2,1} \begin{pmatrix} a_1 \\ a_2 \\ b_1 \end{pmatrix}. \quad (2)$$

To find the semiclassical eigenvalues we assume that all the amplitudes $a_1(t)$, $a_2(t)$, $b_1(t)$ are proportional to the fast phase factor $\exp[i\int_C^t dt'\Lambda(t')]$, while the prefactors are the slow functions of time. Neglecting the time derivatives of these prefactors, we arrive at the following cubic equation for $\Lambda(t)$:

$$\Lambda^3 + vt\Lambda^2 - (\Delta^2 + v^2t^2 + 2J^2)\Lambda = vt(-\Delta^2 + v^2t^2 + 2J^2). \quad (3)$$

It is easy to see that the behavior of $\Lambda(t)$ (in the units of J) as a function of the dimensionless time vt/J is governed by a single dimensionless parameter Δ/J. Upon changing this parameter, the semiclassical levels evolve as shown in Fig. 2. For small gap, $J \ll \Delta$, the levels exhibit two LZ transitions. At critical $\Delta = 2^{1/2}J$, the slope of the middle level changes sign. Finally, for large coupling $J \gg \Delta$, the asymptotic solutions of Eq. (3) are

$$\Lambda \approx vt, \quad \Lambda \approx \pm(v^2t^2 + 2J^2)^{1/2}. \quad (4)$$

Equation (4) implies that, in the limit $\Delta \ll J$, the middle semiclassical level decouples from the upper and lower levels, which are given by the conventional LZ expressions with J replaced by $2^{1/2}J$.

The power of the integrability can be now illustrated as follows: Suppose that at $t = -\infty$ the electron is in the right dot. For large Δ, in order to remain in the right dot at $t \to \infty$, it should survive two LZ transitions. Then the survival

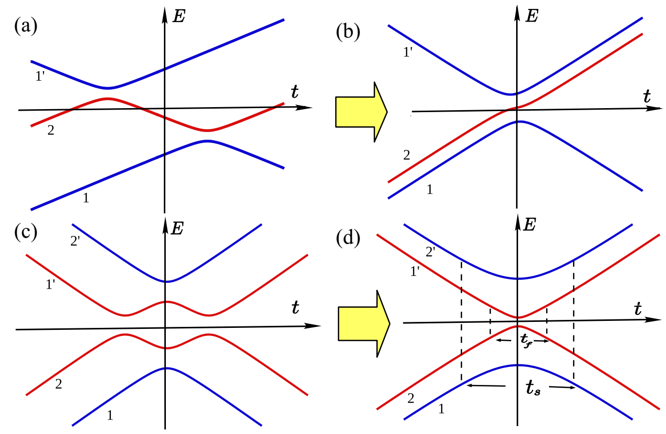

FIG. 2. The evolution of the semiclassical levels in (a), (b) the 2×1 model and (c), (d) in the 2×2 model upon increasing the tunnel coupling. In the 2×1 model the two individual LZ transitions evolve into a single transition at big J, while the middle branch (red) gets decoupled. In the 2×2 model the four individual LZ transitions evolve into the fast transition (red) and the slow transition (blue). The levels are plotted in units of J from the solutions of (a), (b) Eqs. (3) and (c), (d) (14) for parameters (a), (c) $\Delta/J = 10$ and (b), (d) $\Delta/J = 0.5$. The vertical scale is set by the gap at $t = 0$: $2(2 + \Delta^2/J^2)^{1/2}$ in panels (a) and (b), and $2[1 + (1 + \Delta^2/J^2)^{1/2}]$ in panels (c) and (d).

probability of each transition is given by

$$Q_{\rm LZ}\Big|_{\Delta \gg J} = \exp\left(-2\pi\frac{J^2}{v}\right). \quad (5)$$

In the opposite limit of strong coupling the electron undergoes a single LZ transition. Integrability suggests that the survival probability in this limit is given by the same formula as for weak coupling, i.e., one should have

$$Q_{\rm LZ}\Big|_{\Delta \ll J} = \left(Q_{\rm LZ}\Big|_{\Delta \gg J}\right)^2. \quad (6)$$

Indeed, substituting $2^{1/2}J$ into Eq. (5), we realize that the relation (6) holds.

We now turn to the nonintegrable four-level model with the Hamiltonian

$$\hat{H}_{2,2} = \begin{pmatrix} -\Delta - \frac{vt}{2} & 0 & J & J \\ 0 & \Delta - \frac{vt}{2} & J & J \\ J & J & -\Delta + \frac{vt}{2} & 0 \\ J & J & 0 & \Delta + \frac{vt}{2} \end{pmatrix}. \quad (7)$$

In this model, there are two levels in the right dot, which are also split by 2Δ; see Fig. 1(b). Instead of the amplitudes a_1, a_2, b_1, b_2, it is convenient to introduce the combinations

$$A_1 = a_1 + a_2, \quad A_2 = a_1 - a_2, \quad (8)$$

$$B_1 = b_1 + b_2, \quad B_2 = b_1 - b_2. \quad (9)$$

The time evolution of A_1, A_2, B_1, B_2 is governed by the system

$$i\dot{A}_2 - \frac{vt}{2}A_2 - \Delta A_1 = 0, \quad (10)$$

$$i\dot{B}_2 + \frac{vt}{2}B_2 - \Delta B_1 = 0, \quad (11)$$

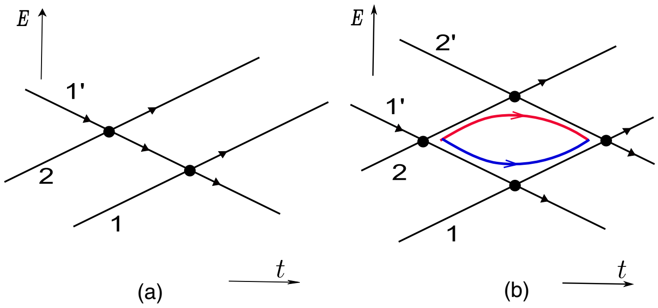

FIG. 3. Different paths of the multilevel LZ transition are illustrated for (a) 2×1 and (b) 2×2 models. Red and blue arrows illustrate the two interfering paths.

$$i\dot{A}_1 - \frac{vt}{2}A_1 - 2JB_1 = \Delta A_2, \tag{12}$$

$$i\dot{B}_1 + \frac{vt}{2}B_1 - 2JA_1 = \Delta B_2. \tag{13}$$

The equation for the semiclassical levels similar to Eq. (3) takes the form

$$\left[\left(\Lambda - \frac{vt}{2}\right)^2 - \Delta^2\right]\left[\left(\Lambda + \frac{vt}{2}\right)^2 - \Delta^2\right] = 4J^2\left[\Lambda^2 - \left(\frac{vt}{2}\right)^2\right]. \tag{14}$$

The solutions of this equation are given by

$$\Lambda^2 = 2J^2 + \left(\frac{vt}{2}\right)^2 + \Delta^2 \pm 2\left[J^4 + J^2\Delta^2 + \Delta^2\left(\frac{vt}{2}\right)^2\right]^{1/2}. \tag{15}$$

Our main point is that, in the limit of strong coupling $J \gg \Delta$, the solutions Eq. (15) can be classified into "slow" and "fast"; namely,

$$\Lambda_s \approx \pm\left[4J^2 + \left(\frac{vt}{2}\right)^2\right]^{1/2}, \tag{16}$$

$$\Lambda_f \approx \pm\left[\frac{\Delta^4}{4J^2} + \left(\frac{vt}{2}\right)^2\right]^{1/2}. \tag{17}$$

We see that, while the characteristic time for the slow solution is the conventional LZ time, $t_s \sim J/v$, the characteristic time for the fast solutions is $t_f \sim \Delta^2/Jv$, i.e., it is much shorter [see also Fig. 2(d)]. This is in striking contrast with the integrable model. Unlike the integrable model, the splitting enters the result even if this splitting is very small. Such a sensitivity to the times of the level crossings can be viewed as an indication that it is interference of the scattering paths which makes the model nonintegrable. This interference is illustrated in Fig. 3. In fact, the signs $+$ and $-$ in Eq. (15) describe the constructive and destructive interference, respectively.

Note in passing that, in the opposite limit $\Delta \gg J$, the interference is also important. As illustrated in Fig. 3(b), it affects the survival probability, but only if the system starts in the excited state. Then the difference of Λ in Eq. (15) corresponding to the sign $+$ and to the sign $-$ determines the phase difference between the red and blue tunneling paths.

It is believed that, in nonintegrable models, the two-level description is not applicable. In fact, Eq. (17) suggests that the scattering process decouples into two two-level LZ transitions with modified gaps. From Eqs. (16) and (17) we can readily infer the survival probabilities of the slow and fast transitions:

$$Q_{\rm LZ}^{\rm slow} = \exp\left[-2\pi\left(\frac{4J^2}{v}\right)\right], \tag{18}$$

$$Q_{\rm LZ}^{\rm fast} = \exp\left[-2\pi\left(\frac{\Delta^4}{4J^2v}\right)\right]. \tag{19}$$

We see that, due to smallness of the "fast" gap, $Q_{\rm LZ}^{\rm fast}$ is much bigger than $Q_{\rm LZ}^{\rm slow}$, i.e., there is an anomalous survival of electrons in a given dot. In other words, due to the interference, the adiabaticity of the transition between the two dots is lifted.

The above consideration was purely semiclassical. Thus, it applies when the probability $Q_{\rm LZ}^{\rm fast}$ is small. This requires that the splitting 2Δ, while smaller than J, exceeds $J(v/J^2)^{1/4}$, as follows from Eq. (19). In the next section we go beyond the semiclassics and demonstrate that the condition of strong coupling, $J \gg \Delta$, is sufficient for Eq. (19) to apply.

III. ANOMALOUS SURVIVAL PROBABILITY

Our goal is to find the asymptotic solution of the system Eqs. (10)–(13) by using the small parameter Δ/J. It is seen that, in the zeroth order, $\Delta \to 0$, the systems for A_1, B_1 and for A_2, B_2 are completely decoupled from each other. In this order, A_1 is still coupled to B_1 via a big coupling constant $2J$. For finite Δ, an indirect coupling between the amplitudes A_2 and B_2 via A_1 and B_1 emerges. We are interested to capture the fast LZ transition described by the amplitudes A_2 and B_2. For this reason, we start with Eqs. (12) and (13) and express A_1, B_1 via A_2 and B_2 in the following way:

$$\begin{pmatrix} A_1(t) \\ B_1(t) \end{pmatrix} = c_s^+(t)\begin{pmatrix} X_s^+(t) \\ Y_s^+(t) \end{pmatrix} + c_s^-(t)\begin{pmatrix} X_s^-(t) \\ Y_s^-(t) \end{pmatrix}, \tag{20}$$

where $X_s^{\pm}(t)$ and $Y_s^{\pm}(t)$ are the pairs of the linear-independent solutions of Eqs. (12) and (13) without the right-hand sides. In the presence of the right-hand side, in order to satisfy the system, the functions c_s^+ and c_s^- should obey the following conditions:

$$i\dot{c}_s^+ X_s^+ + i\dot{c}_s^- X_s^- = \Delta A_2, \tag{21}$$

$$i\dot{c}_s^+ Y_s^+ + i\dot{c}_s^- Y_s^- = \Delta B_2. \tag{22}$$

Solving the system (21) and (22), we find

$$i\dot{c}_s^+ = \frac{\Delta}{JW_s}(A_2Y_s^- - B_2X_s^-), \tag{23}$$

$$i\dot{c}_s^- = \frac{\Delta}{JW_s}(B_2X_s^+ - A_2Y_s^+), \tag{24}$$

where we have introduced the notation

$$JW_s = X_s^+Y_s^- - Y_s^+X_s^- \tag{25}$$

so that W_s has the meaning of the Wronskian, which is time independent. Substituting Eqs. (23) and (24) into Eq. (20), and then Eq. (20) into Eqs. (10) and (11), we arrive at the closed system of integral-differential equations for $A_2(t)$ and $B_2(t)$:

$$i\dot{A}_2 - \frac{vt}{2}A_2 + i\frac{\Delta^2}{JW_s}\int_{-\infty}^{t} dt' K_{xy}(t,t')A_2(t') = i\frac{\Delta^2}{JW_s}\int_{-\infty}^{t} dt' K_{xx}(t,t')B_2(t'), \tag{26}$$

$$i\dot{B}_2 + \frac{vt}{2}B_2 - i\frac{\Delta^2}{JW_s}\int_{-\infty}^{t} dt' K_{xy}(t',t)B_2(t') = i\frac{\Delta^2}{JW_s}\int_{-\infty}^{t} dt' K_{yy}(t',t)A_2(t'), \tag{27}$$

where the three kernels are defined as

$$K_{xx}(t,t') = X_s^+(t)X_s^-(t') - X_s^-(t)X_s^+(t'), \tag{28}$$

$$K_{yy}(t,t') = Y_s^+(t)Y_s^-(t') - Y_s^-(t)Y_s^+(t'), \tag{29}$$

$$K_{xy}(t,t') = X_s^+(t)Y_s^-(t') - X_s^-(t)Y_s^+(t'). \tag{30}$$

Up to now, we did not make use of the smallness of Δ. As we found above [see Eq. (17)], the characteristic time of the fast LZ transition is $t_f \sim \Delta^2/Jv$, so that $vt_f \ll J$. This allows us to neglect the terms $\pm vt/2$ in the equations for X_s and Y_s, which, in turn, leads to the following solutions:

$$X_s^+(t) = \exp(2iJt), \quad Y_s^+(t) = -\exp(2iJt), \tag{31}$$

$$X_s^-(t) = \exp(-2iJt), \quad Y_s^-(t) = \exp(-2iJt). \tag{32}$$

In fact, the true asymptotic behavior of the solutions (31) and (32) contains corrections originating from the $vt/2$ terms. For example, the asymptote for X_s^+ has the form

$$X_s^+(t) = \exp(2iJt) + \exp\left(-\pi\frac{4J^2}{v}\right)\exp(-2iJt). \tag{33}$$

The second term can be neglected due to the condition that the slow LZ transition is adiabatic. Under this condition, the kernels also get greatly simplified and acquire the form

$$K_{xx}(t,t') = 2i\sin[2J(t-t')], \tag{34}$$

$$K_{yy}(t,t') = -2i\sin[2J(t-t')], \tag{35}$$

$$K_{xy}(t,t') = 2\cos[2J(t-t')], \tag{36}$$

while the Wronskian assumes the value $JW_s = 2$. The above expressions for X and Y apply at short times $t \ll t_s \sim J/v$, i.e., at times shorter than the time of the slow LZ transition. Still, t_s is much bigger than t_f, which allows us to use the kernels (28)–(30) in the system (26) and (27). The substitution yields

$$i\dot{A}_2 - \frac{vt}{2}A_2 + i\Delta^2\int_{-\infty}^{t} dt'\cos[2J(t-t')]A_2(t') = \Delta^2\int_{-\infty}^{t} dt'\sin[2J(t-t')]B_2(t'), \tag{37}$$

$$i\dot{B}_2 + \frac{vt}{2}B_2 - i\Delta^2\int_{-\infty}^{t} dt'\cos[2J(t-t')]B_2(t') = \Delta^2\int_{-\infty}^{t} dt'\sin[2J(t-t')]A_2(t'). \tag{38}$$

As a next step, we argue that the kernels are rapidly oscillating functions, while $A_2(t')$ and $B_2(t')$ are slow functions of time. If we take them out of the integrals at $t' = t$, then the integral on the left-hand side will turn to zero, while the integral in the right-hand side will assume the value $1/2J$. As a result, the system (26) and (27) will simplify to

$$i\dot{A}_2 - \frac{vt}{2}A_2 = \frac{\Delta^2}{2J}B_2, \tag{39}$$

$$i\dot{B}_2 + \frac{vt}{2}B_2 = \frac{\Delta^2}{2J}A_2. \tag{40}$$

The above system describes the conventional LZ transition with coupling $\frac{\Delta^2}{2J}$, so that the corresponding survival probability will be given by Eq. (19).

In our derivation we did not assume that the fast LZ transition is adiabatic. In fact, Q_{LZ}^{fast} can be comparable to 1. Certainly, the simplification of the integrals in Eqs. (37) and (38) requires justification. In the Appendix we consider this simplification in detail.

IV. DISCUSSION

To illuminate our main message, let us compare the theoretical predictions for the 2×2 model in two limits: $\Delta \gg J$ and $\Delta \ll J$. In the first limit the smallness of the LZ gap allows us to obtain the transition probabilities from simple reasoning. Suppose that at $t = -\infty$ the electron is in the state 1 in the left dot; see Fig. 1. In this situation, the survival implies that at $t \to \infty$ the electron remains in the state 1, i.e., it survives two LZ transitions. The probability for this is $Q_{1\to1} = Q_{LZ}^2$. If at $t = -\infty$ the electron is in the state 2, then the survival probability is the sum of probabilities to remain either in state 2 or in state 1. The first probability is $Q_{2\to2} = Q_{LZ}^2$. With regard to the second probability, it should be taken into account that there are two paths from 2 to 1, as illustrated in Fig. 3. Corresponding amplitudes interfere with each other. If the big phase difference, accumulated during the time $2\Delta/v$, is treated as random, one can add the corresponding probabilities, so that $Q_{2\to1} = 2Q(1-Q)^2$. The average (with respect to the initial states) survival probability reads

$$\mathcal{Q}_L = \frac{Q_{1\to1} + Q_{2\to2} + Q_{2\to1}}{2} = Q_{LZ}\left(Q_{LZ}^2 - Q_{LZ} + 1\right). \tag{41}$$

Consider now the limit $J \gg \Delta$. If initially the electron is in the ground state, then at $t \to -\infty$ we have $A_1 = \frac{1}{\sqrt{2}}$ and $A_2 = \frac{1}{\sqrt{2}}$. If the electron starts in the excited state, then the initial conditions are $A_1 = \frac{1}{\sqrt{2}}$ and $A_2 = -\frac{1}{\sqrt{2}}$. For both initial conditions the outcome of the LZ transition is the same; namely, A_2 survives with high probability, while A_1 survives with low probability. Then for the net survival probability we have

$$\mathcal{Q}_L = \frac{1}{2}\left\{\exp\left[-\frac{2\pi}{v}\frac{\Delta^4}{4J^2}\right] + \exp\left[-2\pi\frac{4J^2}{v}\right]\right\}. \quad (42)$$

We see that, for $J \gg \Delta$, the probability (42) is much bigger than (41), which seems counterintuitive. Moreover, for $J \gg \Delta$, $\mathcal{Q}_L$ increases with increasing tunneling, i.e., the adiabaticity of the multilevel LZ transition gets suppressed.

In this paper we have focused on the simplest example of the nonintegrable model: the crossing of two pairs of levels in the left and right dots. It would certainly be interesting to establish how general our conclusion is about the anomalous survival of electron in a given dot. We can go one step further and generalize the model to the case when two groups of N levels in the left and right dots cross each other. Two assumptions [13], (i) all N^2 couplings are the same, and (ii) the levels are aligned at $t = 0$, greatly simplify the analysis. Namely, instead of Eq. (14) we get the following generalized equation:

$$\left[\sum_{k=1}^{N}\frac{1}{\Lambda + \varepsilon_k - \frac{vt}{2}}\right]\left[\sum_{p=1}^{N}\frac{1}{\Lambda + \varepsilon_p + \frac{vt}{2}}\right] = \frac{1}{J^2}. \quad (43)$$

In the limit $J \gg \varepsilon_k$, which we assumed throughout the paper, the structure of the solutions is the following. One solution describes the fast transition. Neglecting ε_k in the denominators, we find

$$\Lambda_N^{\text{slow}} = \pm\left[\left(\frac{vt}{2}\right)^2 + N^2J^2\right]^{1/2}. \quad (44)$$

The fact that Λ_N^{slow} is much bigger than ε_k justifies neglecting ε_k in the denominators. The corresponding survival probability is

$$Q_{\text{LZ}}^{\text{slow}}(N) = \exp\left[-\frac{2\pi N^2J^2}{v}\right] = \left(Q_{\text{LZ}}^{\text{slow}}\right)^{N^2}. \quad (45)$$

This result should be contrasted to

$$Q_{\text{LZ}}^{\text{slow}}(N) = \exp\left[-\frac{2\pi NJ^2}{v}\right] = \left(Q_{\text{LZ}}^{\text{slow}}\right)^{N}, \quad (46)$$

which emerges within the independent crossing approach and also applies to the integrable models. Indeed, to enforce integrability in a multilevel model (see e.g., Ref. [27]), a portion of tunnel couplings should be set to be zero.

The other $N-1$ solutions of Eq. (43) describe the fast LZ transitions. The values of Λ for these solutions are close to the values $\tilde{\Lambda}_N$ for which the sum $\sum_{k=1}^{N}(\Lambda + \varepsilon_k)^{-1}$ passes through zero. This emphasizes the role of interference in the formation of the fast transitions. Indeed, the eigenvector, corresponding to a given $\tilde{\Lambda}_n$, is composed of many levels. If all ε_k reside in the interval Δ, then the estimate for $\tilde{\Lambda}_n$ is also Δ, which is much smaller than J. To find the corresponding survival probabilities we expand Eq. (43) near $\tilde{\Lambda}_n$. The linear terms proportional to $vt/2$ get canceled out and we obtain

$$(\Lambda - \tilde{\Lambda}_n)^2 - \left(\frac{vt}{2}\right)^2 = \frac{1}{J^2\left[\sum_{k=1}^{N}\frac{1}{(\tilde{\Lambda}_n+\varepsilon_k)^2}\right]^2}. \quad (47)$$

From here we find that the survival probability corresponding to a given $\tilde{\Lambda}_n$

$$Q_{\text{LZ}}^{\text{fast}}(N) = \exp\left\{-\frac{2\pi}{v}\frac{1}{J^2\left[\sum_{k=1}^{N}\frac{1}{(\tilde{\Lambda}_n+\varepsilon_k)^2}\right]^2}\right\}. \quad (48)$$

All the terms in the sum $\sum_k(\tilde{\Lambda}_n + \varepsilon_k)^{-2}$ are positive, and the value of the sum is determined only by the levels ε_k closest to $-\tilde{\Lambda}_n$. The distance between these levels is $\sim(\Delta/N)$. Thus, the sum can be estimated as $(\frac{N}{\Delta})^2$. Finally, within a numerical factor in the exponent, we have

$$Q_{\text{LZ}}^{\text{fast}}(N) = \exp\left\{-\frac{2\pi}{v}\frac{\Delta^4}{N^4J^2}\right\}. \quad (49)$$

We conclude that, for the fast transitions, the survival probability grows rapidly with N. Note also that the result (48) is straightforward generalization of the result (19) to the case of N intersecting levels. We derived it by using the semiclassical approach. However, we have proven above that Eq. (19) applies beyond semiclassics. This proof can be generalized to demonstrate that Eq. (48) is valid beyond semiclassics.

To explain qualitatively the loss of adiabaticity with increasing tunneling gap we draw the analogy between this effect and the Dicke effect [32], which is well known in optics. If two emitters are separated by a distance much smaller than the emitted wavelength, the radiation lifetime of the pair increases drastically. This is because the two eigenmodes of the oscillating emitters are the symmetric and antisymmetric combinations of the individual oscillations. The antisymmetric mode weakly overlaps with the emission field. Hence the long lifetime. In the model we considered, due to tunneling, the correct eigenstates of, say, the left dot are also $A_1 = \frac{1}{\sqrt{2}}(a_1 + a_2)$ and $A_2 = \frac{1}{\sqrt{2}}(a_1 - a_2)$. The gap for A_1 is twice the gap in the individual LZ transition, while the gap for A_2 is suppressed and decreases with J. This is the origin of the anomalous survival. The bigger is the number of levels in each dot, the less strict is the requirement that all tunnel couplings are the same [35].

V. CONCLUDING REMARKS

(i) It is common to judge on whether the system with many degrees of freedom is integrable based on numerically generated level statistics in a limited spectral interval; see, e.g., Ref. [36]. If the statistics is Poissonian, the system can be decoupled into individual "blocks" which do not interact with each other. This is an indication that the system is integrable. If, alternatively, the level statistics is Wigner-Dyson, different energy levels repel each other, suggesting that the corresponding eigenstates "know" about the entire system.

Such a system is nonintegrable. With regard to multistate LZ models, a similar approach has been employed in Refs. [26,27]. If the time evolution of the semiclassical levels exhibited avoided crossings, the model was judged to be nonintegrable. Certainly, it is the interference of many partial amplitudes that is responsible for the level repulsion in many-body systems. Similarly, in nonintegrable multilevel LZ models the time evolution between two distant level crossings allows more than one path.

(ii) Although the integrable models in Refs. [21,22] contain interfering paths, the parameters of these models are fine tuned in order to enforce the destructive interference.

(iii) It might seem that if we modify the 2×2 model by making the two spacings uneven and sending one of the spacings to zero, the 2×2 model will cross over to the 2×1 model. We would like to emphasize that this is not the case. Even if the levels in the right dot are separated by $2\Delta_1 \ll 2\Delta$, we will not emulate the 2×1 situation. The formal reason for this is that the level degeneracy in the right dot will be lifted due to coupling of the degenerate levels via the left dot. With asymmetry, the width of the gap corresponding to the fast LZ transition is modified from $\Delta^2/2J$ to $\Delta\Delta_1/2J$.

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APPENDIX

In this Appendix we explore the assumptions leading from the system (37), (38) to the system (39), (40). We first assume that the system (39), (40) applies and use it to trace the above assumptions.

Consider the integral on the right-hand side of Eq. (38). Upon performing the integration by parts twice, it can be cast into the form

$$\int_{-\infty}^{t} dt' \sin[2J(t-t')]A_2(t') = -\frac{1}{2J}A_2(t) - \frac{1}{4J^2}\int_{-\infty}^{t} dt' \sin[2J(t-t')]\frac{\partial^2 A_2(t')}{\partial t'^2}. \tag{A1}$$

It is now convenient to combine the left-hand side with the term containing the second derivative on the right-hand side:

$$\int_{-\infty}^{t} dt' \sin[2J(t-t')]\left[A_2(t') + \frac{1}{4J^2}\frac{\partial^2 A_2(t')}{\partial t'^2}\right] = -\frac{1}{2J}A_2(t). \tag{A2}$$

If the system (39), (40) applies, $\partial^2 A_2/\partial t'^2$ can be expressed through A_2. Substituting this expression into Eq. (A2), we get

$$\int_{-\infty}^{t} dt' \sin[2J(t-t')]\left\{A_2(t')\left[1 - \frac{\Delta^4}{16J^4} + i\frac{v}{4J^2} - \frac{v^2t^2}{16J^2}\right]\right\} = \frac{1}{2J}A_2(t). \tag{A3}$$

Now we see that taking $A_2(t)$ out of the integral amounts to keeping only the first term in the square brackets. Indeed, the second term is much smaller than 1 by virtue of the condition $\Delta \ll J$. The third term is much smaller than 1 since the slow transition is adiabatic. With regard to the fourth term, the characteristic t' is of the order of the time of the fast LZ transition. If the fast transition is adiabatic, then t' is of the order of Δ^2/Jv, so that the fourth term is of order of the second term. If the fast transition is nonadiabatic, then t' is of the order of $v^{-1/2}$. In this limit the fourth term is of the order of the third term. In both cases the terms which we neglected are small. A similar consideration justifies the simplification of the other integrals.

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[30] The origin of this number can be understood from the following argument: If we change the number of levels moving with the same velocity from $N - 1$ to N by adding a new level on top of the others, then the number of new transition probabilities will be $N + 2$. This is because the no-go theorem will allow the transitions from all $N - 1$ parallel moving levels into a new one. Also the transition from one level, crossing all $N - 1$ levels, to the new level will be allowed. Finally, if the system starts in the new level, it can either stay on this new level or go to the level which crosses all other levels.

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