Cubic Dresselhaus interaction parameter from quantum corrections to the conductivity in the presence of an in-plane magnetic field

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We evaluate the quantum corrections to the conductivity of a two-dimensional electron system with competing Rashba (R) and linear and cubic Dresselhaus (D) spin-orbit interactions in the presence of an in-plane magnetic field **B**. Within a perturbative approximation, we investigate the interplay between the spin-orbit coupling and the magnetic field in determining the transport regime in two different limiting scenarios: when only one of the linear terms, either Rashba or Dresselhaus, dominates, and at equal linear couplings, when the cubic Dresselhaus breaks the spin symmetry. In each instance, we find that for **B** higher than a critical value, the antilocalization correction is suppressed and the effective dephasing time saturates to a constant value determined only by the spin-orbit interaction. At equal R-D linear couplings, this value is directly proportional with the cubic Dresselhaus contribution. In the same regime, the magnetoconductivity is expressed as a simple logarithmic function dependent only on the cubic Dresselhaus constant.

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I. INTRODUCTION

Spin-orbit interaction induced effects in semiconductor structures with broken inversion symmetry have been intensively studied in the past decade for potential applications to spin control in electronic devices. In III-V semiconductors with zinc-blende structure, the origin of the interaction is either in the asymmetry of the quantum well, which determines the Rashba coupling (R) [1] or in the inversion asymmetry of the crystal which determines the Dresselhaus terms (D) [2]. While the Rashba interaction is linear in the electron momentum, with coupling constant α, the Dresselhaus interaction has both linear and cubic components with coupling constants β_1 and β_3, respectively.

The interest in considering all three terms simultaneously is motivated by the specific physics that arises when the two linear couplings α and β_1 are equal. Since they rotate the electron spin in opposite directions, it is possible to obtain a state in which, under their perfect cancellation, the electron spin becomes a good quantum number and avoids dephasing effects that change its direction. This particular situation has been flagged in numerous previous works as having interesting transport properties, such as very long spin-relaxation times [3–6], itinerant antiferromagnetic spin order [7,8], or the absence of the antilocalization correction to the conductivity [6,9,10]. The presence of the cubic Dresselhaus term disrupts this equilibrium and, by introducing an additional spin coupling, is bound to affect in a unique way some of the same transport properties.

In this paper, we investigate the influence of the cubic Dresselhaus interaction in the quantum expression of the conductivity of a III-V semiconductor quantum well grown along (0,0,1) in the presence of an in-plane magnetic field. In this geometry, the Zeeman coupling between the electron and the magnetic field introduces an additional term in the energy balance that is associated with impurity scattering that affects the propagation modes that involve spin flipping. The competition between the magnetic field that aligns the spins and the spin orbit that rotates them impacts directly the effective dephasing time, a measure of the inelasticity of the propagation, and the antilocalization correction, the constructive superposition of quantum states associated with a spin reversal during the scattering process. This phenomenology does not appear in a configuration where the magnetic field is perpendicular on the plane, when the electrons are scattered only within the same Landau level and any spin flipping is mediated by the spin-orbit coupling alone [9–12]. Here it is assumed that the magnetic field couples exclusively to the electron spin and any orbital effects that might develop across the width of the well are neglected, considering that the size of the Zeeman splitting is of the same order as the spin-orbit interaction.

Our discussion concerns the interplay between the linear spin-orbit interaction (SOI) terms and the cubic one in two different situations: a regime where a single linear SOI coupling, either Rashba or Dresselhaus, dominates and a transitory state toward the equal Rashba-Dresselhaus linear coupling regime, where the difference $\alpha - \beta_1$ is small and the cubic Dresselhaus interaction acts as a spin-symmetry breaking term. Experimentally, this situation is realized by taking advantage of the possibility of manipulating the α value through the application of an electric voltage across the well to become almost equal to β_1 which is a system parameter, as it is related to the degree of confinement of the electrons in the well [6]. Several experimental techniques for achieving this state were discussed in Refs. [4–6].

In each case, we find that at low magnetic field values, the effective dephasing time increases proportional to B^2, as the Zeeman energy favors spin flips in the scattering process. The saturation state obtained at high magnetic fields is the result of the spin alignment. In this regime, the dephasing occurs only as the result of the spin-orbit interaction that rotates the spins at a rate proportional with the coupling constants, independent of the values of the magnetic field. Further, the antilocalization correction is suppressed leading to a minimum in the conductivity. When the linear couplings are equal, both the dephasing rate and the minimum conductivity are simple functions only on the cubic Dresselhaus term offering an experimental opportunity to determine directly this parameter.

The analysis presented here adds to the large amount of literature on the weak localization corrections to the conductivity developed for the case of spin-orbit interaction in two-dimensional (2D) systems as it extends to larger magnetic fields some of the results derived in Ref. [13]. Other case studies were developed for either just the Dresselhaus interaction, in both linear and cubic form, acting alone and in the presence of perpendicular magnetic fields [11], or for both linear Rashba and Dresselhaus interactions considered directly in the regime of equal coupling strengths, without the inclusion of the cubic Dresselhaus term [9,10].

The paper starts with a description of the 2D electron system endowed with Rashba and Dresselhaus interactions, both linear and cubic in the electron momentum, in a rotated system of coordinates that showcases in a straightforward way the symmetry properties of the $\alpha = \beta_1$ state. For this Hamiltonian we develop the weak localization theory based on the Born approximation of scattering on impurities in the presence of an in-plane magnetic field, by solving self-consistently the impurity mediated equation satisfied by the Cooperon. The quantum corrections to the conductivity calculated through this algorithm are strongly influenced by the competition between the spin-orbit interactions and the magnetic field on the electron spins and the analytic results we derive reflect the dominant mechanism.

II. THE SINGLE-PARTICLE HAMILTONIAN

At the center of the quantum conductivity calculation is the single-particle Hamiltonian that describes an electron of effective mass m^*, momentum $\mathbf{p} = \{p_x, p_y, p_z\}$, and spin $\sigma = \{\sigma_x, \sigma_y, \sigma_z\}$, localized in the $\hat{x} - \hat{z}$ (the $\hat{y}$ axis is perpendicular on the plane), which experiences the Rashba α and Dresselhaus β_1 and β_3 spin-orbit interactions [14],

$$H_{\mathbf{p}} = \frac{p_x^2 + p_z^2}{2m^*} + \alpha(\sigma_z p_x - \sigma_x p_z) + \beta_1(\sigma_z p_z - \sigma_x p_x) - \beta_3\left(\sigma_z p_z p_x^2 - \sigma_x p_x p_z^2\right). \tag{1}$$

Under a change of coordinates, $p_x' = (-p_x + p_z)/\sqrt{2}$, $p_z' = (p_x + p_z)/\sqrt{2}$ along with the change of spin components, $\sigma_x' = (-\sigma_x + \sigma_z)/\sqrt{2}$, $\sigma_z' = (\sigma_x + \sigma_z)/\sqrt{2}$, the Hamiltonian becomes

$$\begin{aligned} H_{\mathbf{p}} &= \frac{p_x^2 + p_z^2}{2m^*} + (\alpha - \beta_1) p_z \sigma_x - (\alpha + \beta_1) p_x \sigma_z \\ &\quad - \frac{\beta_3}{2}\left(p_x^2 - p_z^2\right)(p_z\sigma_x - p_x\sigma_z) \\ &= \frac{p_x^2 + p_z^2}{2m^*} + \left[\alpha - \beta_1 - \frac{\beta_3}{2}\left(p_x^2 - p_z^2\right)\right] p_z \sigma_x \\ &\quad - \left[\alpha + \beta_1 - \frac{\beta_3}{2}\left(p_x^2 - p_z^2\right)\right] p_x \sigma_z. \end{aligned} \tag{2}$$

In the conductivity calculation, the Fermi liquid is assumed to be totally degenerate, the electrons engaged in transport having momenta $\mathbf{p}$ of equal magnitude, $p = m^* v$ corresponding to the Fermi velocity v. In terms of the angle made by the momentum with the $\hat{x}$ axis, $\varphi_{\mathbf{p}}$, the Hamiltonian becomes

$$\begin{aligned} H = \frac{p^2}{2m^*} &+ \left[p\left(\alpha - \beta_1 + \frac{\beta_3 p^2}{4}\right)\sin\varphi_{\mathbf{p}} - \frac{\beta_3 p^3}{4}\sin 3\varphi_{\mathbf{p}}\right]\sigma_x \\ &- \left[p\left(\alpha + \beta_1 - \frac{\beta_3 p^2}{4}\right)\cos\varphi_{\mathbf{p}} - \frac{\beta_3 p^3}{4}\cos 3\varphi_{\mathbf{p}}\right]\sigma_z. \end{aligned} \tag{3}$$

We follow Ref. [9] and introduce the notations

$$\begin{aligned} \hbar\Omega_1 &= \beta_1 p - \frac{p^3\beta_3}{4}, \\ \hbar\Omega_2 &= \alpha p, \\ \hbar\Omega_3 &= \frac{p^3\beta_3}{4}, \end{aligned} \tag{4}$$

such that

$$\begin{aligned} \Omega_{\mathbf{p}}^x &= (\Omega_2 + \Omega_1)\cos\varphi_{\mathbf{p}} - \Omega_3\cos 3\varphi_{\mathbf{p}}, \\ \Omega_{\mathbf{p}}^z &= (\Omega_2 - \Omega_1)\sin\varphi_{\mathbf{p}} - \Omega_3\sin 3\varphi_{\mathbf{p}}, \end{aligned} \tag{5}$$

and write the Hamiltonian in a compact form

$$H_{\mathbf{p}} = \frac{p^2}{2m^*} + \hbar(\Omega_{\mathbf{p}} \times \sigma)\cdot\hat{y}. \tag{6}$$

In this configuration, an in-plane magnetic field $\mathbf{B} = \{B_x, B_z\}$ is added along an arbitrary direction. Its coupling with the electron spin $\mathbf{S}$ through an effective coupling constant $\hbar\gamma$ adds the usual Zeeman term to Eq. (6), leading to the final form of the Hamiltonian that will be used throughout:

$$H_{\mathbf{p}} = \frac{p^2}{2m^*} + \hbar(\Omega_{\mathbf{p}} \times \sigma)\cdot\hat{y} + \hbar\gamma\sigma\cdot\mathbf{B}. \tag{7}$$

When diagonalized in the spin space, the Hamiltonian Eq. (7) generates two eigenvalues,

$$E_{\pm} = \frac{p^2}{2m^*} \pm \hbar\Delta_{\mathbf{p}}, \tag{8}$$

where the gap $\Delta_{\mathbf{p}}$ is given by

$$\Delta_{\mathbf{p}} = \sqrt{\left(\Omega_{\mathbf{p}}^z - \gamma B_x\right)^2 + \left(\Omega_{\mathbf{p}}^x + \gamma B_z\right)^2}. \tag{9}$$

The corresponding associate eigenstates are plane waves modulated by a spin function,

$$\begin{aligned} \psi_+ &= e^{i\mathbf{k}\cdot\mathbf{r}}[-\sin\theta_{\mathbf{p}}|\uparrow\rangle + \cos\theta_{\mathbf{p}}|\downarrow\rangle], \\ \psi_- &= e^{i\mathbf{k}\cdot\mathbf{r}}[\cos\theta_{\mathbf{p}}|\uparrow\rangle + \sin\theta_{\mathbf{p}}|\downarrow\rangle], \end{aligned} \tag{10}$$

with

$$\tan\theta_{\mathbf{p}} = \frac{\gamma B_x - \Omega_{\mathbf{p}}^z}{\gamma B_z + \Omega_{\mathbf{p}}^x}. \tag{11}$$

In the absence of the magnetic field, and when $\Omega_1 = \Omega_2$ and $\Omega_3 = 0$, $H_{\mathbf{p}}$ commutes with S_z, thus designating the spin projection along the $\hat{z}$ axis as a good quantum number. In this state, losing the spin orientation can occur only in the presence of an additional spin-orbit coupling interaction, here the cubic Dresselhaus.

Neglecting the product between the magnetic field and the SOI constants, as well as the product between the SOI

constants, the gap between the two Fermi surfaces $E_\pm$ is

$$\Delta_{\mathbf{p}} = \sqrt{(\Omega_1+\Omega_2)^2\cos^2\varphi_{\mathbf{p}} + (\Omega_1-\Omega_2)^2\sin^2\varphi_{\mathbf{p}} + \Omega_3^2 + (\gamma B)^2}. \tag{12}$$

At weak magnetic fields, the size of the gap is set by the spin-orbit coupling, while in the strong field regime it is determined by the Zeeman splitting, a behavior that strongly affects the quantum corrections to the conductivity.

III. WEAK LOCALIZATION FORMALISM

The quantum corrections to the conductivity result from the introduction in the Kubo formula of the Cooperon, a term that marks the renormalization of the scattering matrix element when the quantum nature of the electron states is considered. Impurity scattering is considered uncorrelated, elastic, and in this case, spin independent. The Fermi surface states involved, of a single spin density of states of $\nu_0 = m^*/2\pi\hbar^2$, generate a scattering matrix element $|V_{\mathbf{p},\mathbf{p}'}|^2$ dependent only on the angle between the directions of the incident and scattered momenta, $\mathbf{p}$ and $\mathbf{p}'$, φ. In the Born approximation, the scattering lifetime is τ_0,

$$\frac{\hbar}{\tau_0} = \nu_0 \int |V_{\mathbf{p},\mathbf{p}'}|^2(\varphi)d\varphi. \tag{13}$$

The quantum corrections to the conductivity are calculated from the general expression [15]

$$\Delta\sigma = -\frac{2e^2 D\tau_0^2\nu_0}{\hbar^2}\sum_{\mathbf{q},i} C_i(\mathbf{q}), \tag{14}$$

where $D = v^2\tau_1/2$ is the diffusion coefficient in two dimensions expressed as a function of the transport scattering time, τ_1. This is the first ($n = 1$) term in a series of transport times that describe the anisotropy of the scattering process,

$$\frac{\hbar}{\tau_n} = \nu_0 \int |V_{\mathbf{p},\mathbf{p}'}|^2(1-\cos n\varphi)d\varphi. \tag{15}$$

$C_i(\mathbf{q})$ are the eigenvalues of the Cooperon operator which represents an impurity averaged scattering amplitude for an electron state $\mathbf{p}$ that is almost perfectly backscattered into $\mathbf{p}' \approx -\mathbf{p}$, the deviation from this situation being represented by $\hbar\mathbf{q} = \mathbf{p} + \mathbf{p}'$, with $\hbar q \ll p$. The impurity mediated Cooperon satisfies a self-consistent equation

$$C_{\mathbf{p},\mathbf{p}'}(\mathbf{q}) = |V_{\mathbf{p},\mathbf{p}'}|^2 + \sum_{\mathbf{p}''} |V_{\mathbf{p},\mathbf{p}''}|^2 G^+_{-\mathbf{p}''+\hbar\mathbf{q},\epsilon+\hbar\omega} G^-_{\mathbf{p}'',\epsilon} C_{\mathbf{p}'',\mathbf{p}'}, \tag{16}$$

where the impurity averaged advanced (A) and retarded (R) Green's functions are given by

$$G^\pm(\mathbf{p},\epsilon) = \frac{1}{\epsilon - H_{\mathbf{p}} \pm i\frac{\hbar}{2\tau_0}}. \tag{17}$$

The kernel of the integral in Eq. (16) is evaluated by integrating first after the kinetic energy $p^2/2m^*$ in the complex plane, followed by an expansion in the scattering rate, $\hbar/\tau_0$, assumed to dominate the denominator. It is important to note that the directions of the electron spin, before and after the collision are uncorrelated and consequently have to carry different names, here σ and ρ. Thus, Eq. (16) becomes

$$\begin{aligned} C_{\mathbf{p},\mathbf{p}'}(\mathbf{q}) = |V_{\mathbf{p},\mathbf{p}''}|^2 + \nu_0 \int_0^{2\pi} d\varphi_{\mathbf{p}''} |V(\varphi_p - \varphi_{p''})|^2 \big\{ 1 + i\omega\tau_0 + i\gamma\mathbf{B}\cdot(\rho-\sigma)\tau_0 + i\mathbf{q}\cdot\mathbf{v}_{\mathbf{p}''}\tau_0 - (i\mathbf{q}\cdot\mathbf{v}_{\mathbf{p}''})^2\tau_0^2 \\ + [i\Omega_{\mathbf{p}''}\times(\sigma+\rho)\cdot\hat{y}]\tau_0 - [i\Omega_{\mathbf{p}''}\times(\sigma+\rho)\cdot\hat{y}]^2\tau_0^2 - 2(\mathbf{q}\cdot\mathbf{v}_{\mathbf{p}''})[\Omega_{\mathbf{p}''}\times(\sigma+\rho)\cdot\hat{y}]\tau_0^2 \big\} C_{\mathbf{p}'',\mathbf{p}'}. \end{aligned} \tag{18}$$

The Cooperon equation is linearized in an iterative approach where $C_{\mathbf{p},\mathbf{p}'}(q) = C^{(0)}_{\mathbf{p},\mathbf{p}'}(q) + C^{(1)}_{\mathbf{p},\mathbf{p}'}(q)\cos\varphi_{\mathbf{p}} + C^{(2)}_{\mathbf{p},\mathbf{p}'}(q)\cos 2\varphi_p + \cdots$, where φ_p is the angle between $\hbar\mathbf{q} = \mathbf{p} + \mathbf{p}'$ and $\mathbf{p}$. The first-order correction to the isotropic Cooperon is readily written in terms of the total spin $\mathbf{J} = (\sigma+\rho)/2$ (in $\hbar$ units) components,

$$\begin{aligned} C^{(1)} = {} & i(\tau_1-\tau_0)[\mathbf{v}_{\mathbf{p}''}\cdot\mathbf{q} + 2(\Omega_2-\Omega_1)J_x\sin\varphi_{\mathbf{p}''} \\ & - 2(\Omega_2+\Omega_1)J_z\cos\varphi_{\mathbf{p}''}] \\ & - i(\tau_3-\tau_0)(-2\Omega_3 J_x\sin 3\varphi_{\mathbf{p}''} + 2\Omega_3 J_z\cos 3\varphi_{\mathbf{p}''})C^{(0)}, \end{aligned} \tag{19}$$

where the magnitude of the velocity corresponds to the Fermi level, v.

Finally, a formal equation for the lowest order of the Cooperon operator can be obtained as

$$C^{(0)}_{\mathbf{p},\mathbf{p}'}(\mathbf{q}) = \frac{|V_{\mathbf{p},\mathbf{p}'}|^2}{\tau_0\mathcal{H}}, \tag{20}$$

where $\mathcal{H}$ is an operator in the 2×2 spin space,

$$\begin{aligned} \mathcal{H} = {} & Dq^2 + \frac{1}{\tau_\varphi} + 2\big[(\Omega_2+\Omega_1)^2\tau_1 + \Omega_3^2\tau_3\big]J_z^2 \\ & + 2\big[(\Omega_2-\Omega_1)^2\tau_1 + \Omega_3^2\tau_3\big]J_x^2 \\ & + 2(\Omega_2-\Omega_1)\tau_1 v q_z J_x - 2(\Omega_2+\Omega_1)\tau_1 v q_x J_z \\ & + 2i\gamma\mathbf{B}\cdot(\sigma-\rho). \end{aligned} \tag{21}$$

A measure of the inelasticity of the propagation, $-i\omega$ is replaced by $1/\tau_\varphi$, the dephasing time. We introduce the following notations that reflect in a straightforward way the superposition of the linear spin-orbit couplings in a symmetric

(S) and antisymmetric combination (A), as well as the cubic term, all normalized by the diffusion coefficient.

$$Q_S = \frac{2(\Omega_2+\Omega_1)}{v},$$
$$Q_A = \frac{2(\Omega_2-\Omega_1)}{v}, \tag{22}$$
$$Q_3 = \frac{2\Omega_3}{v}\sqrt{\frac{\tau_3}{\tau_1}},$$

and rewrite $\mathcal{H}$ as

$$\mathcal{H} = Dq^2 + \frac{1}{\tau_\varphi} + D\{[Q_S^2+Q_3^2]J_z^2 + [Q_A^2+Q_3^2]J_x^2 + 2Q_A q_z J_x - 2Q_S q_x J_z\} - i\gamma\mathbf{B}\cdot(\sigma-\rho). \tag{23}$$

The eigenvalues of $\mathcal{H}$ are employed to evaluate the quantum corrections to the conductivity, through Eqs. (20) and (14). The diagonalization of $\mathcal{H}$ is done in the state space of the total spin of the electron pair where the Zeeman splitting term will generate off-diagonal contributions. We note that the eigenvalue corresponding to $J^2=0$, $J_z=0$, the singlet Cooperon, enters the conductivity calculation with a negative sign as it corresponds to a spin state that is odd under a particle exchange. This is the antilocalization correction term.

IV. THE COOPERON EIGENVALUES

Equation (23) is solved in the basis of $\{J^2, J_z\}$ associated with total spin quantum numbers $J=0,1$. Analytic solutions can be obtained for the quartic characteristic equation in $\mathcal{E}$ in some simplifying limits set by the scales of the competing small factors that determine the characteristics of the quantum transport regime.

In the following considerations, we explore two different limits of the characteristic equation and its associated solutions. The first case corresponds to a dominant linear SOI coupling (either Ω_1 or Ω_2 can be considered since the result is symmetric with respect to these two values) and the cubic Dresselhaus term. The second instance occurs in the vicinity of the cancellation point of the two linear couplings, where $\alpha \simeq \beta_1$, and the only spin-orbit contribution is that of the cubic Dresselhaus, which is responsible for any remaining spin effects on transport.

In both situations, two terms in Eq. (23) can be set to zero. First, since we are interested in understanding the symmetry-breaking properties of the difference $\Omega_1-\Omega_2$ as well as of the cubic Dresselhaus, we neglect terms proportional to B_x which is essentially playing the same role, as all these fields drive the misalignment of the electron spins from the dominant direction, $\hat{z}$. Further, we set $q_z(\Omega_1-\Omega_2)=0$, an assumption justified by the fact that in both cases under consideration, one of the two terms of this product is approaching zero faster than the other quantities in the problem. When only one SOI is present, this term is small on account of $q_z\to 0$, while in the vicinity of the cancellation point of the two linear SOIs, both terms are small leading to higher order corrections.

With these simplifying assumptions, the eigenvalue equation for $\mathcal{H}$ is

$$\begin{vmatrix} \mathcal{H}_{11}-\mathcal{E} & 0 & D\frac{Q_A^2+Q_3^2}{2} & 0 \\ 0 & \mathcal{H}_{10}-\mathcal{E} & 0 & 2i\gamma B_z \\ D\frac{Q_A^2+\Omega_3^2}{2} & 0 & \mathcal{H}_{1-1}-\mathcal{E} & 0 \\ 0 & 2i\gamma B_z & 0 & \mathcal{H}_{00}-\mathcal{E} \end{vmatrix} = 0, \tag{24}$$

where

$$H_{11} = Dq^2 + 1/\tau_\varphi + D\left[Q_S^2+Q_3^2+\frac{Q_A^2+Q_3^2}{2}+2q_xQ_S\right],$$
$$H_{1-1} = Dq^2 + 1/\tau_\varphi + D\left[Q_S^2+\Omega_3^2+\frac{Q_A^2+Q_3^2}{2}-2q_xQ_S\right],$$
$$H_{10} = Dq^2 + 1/\tau_\varphi + D[Q_A^2+Q_3^2],$$
$$H_{00} = Dq^2 + 1/\tau_\varphi. \tag{25}$$

Several characteristic features of the eigenvalues of this equation can be assessed from general considerations based on the exact spectrum of the electron states involved in this problem, Eqs. (8) and (10). From this perspective, the states in the total spin momentum representation are linear combinations of the $\pm$ eigenstates, so in the energy balance of the scattering process one has to consider transitions between the two Fermi surfaces $E\pm$ that are separated by the gap $\Delta_{\mathbf{p}}$ twice. If the state is symmetric, i.e., the triplet Cooperon, then the gap function appears twice with opposite signs and generates no contribution in first order in the case of $J_z=0$ or does not appear at all when $J_z=\pm 1$. In contrast, the singlet state $J=0$, $J_z=0$ is antisymmetric and in its scattering energy balance the gap functions add, indicating that in this case the perturbing effect of the magnetic field is maximum.

A. $J_z = 0$ Cooperon modes and the dephasing time

The two Cooperon modes corresponding to total spin $J_z=0$, but total spin angular momentum $J=1$ and $J=0$, respectively, are coupled, as they satisfy

$$H_{00}[H_{00}+DQ_A^2+DQ_3^2]+\gamma^2B_z^2=0. \tag{26}$$

Consequently, we extract the eigenvalues associated with the triplet $J_z=0$, $\mathcal{E}_{10}$ and the singlet $J_z=0$, $\mathcal{E}_{00}$,

$$\mathcal{E}_{00}^{10} = Dq^2 + \frac{1}{\tau_\varphi} + D\frac{(Q_A^2+Q_3^2)}{2} \pm \mathrm{Re}\sqrt{D^2\left(\frac{Q_A^2+Q_3^2}{2}\right)^2 - 4\gamma^2B_z^2}, \tag{27}$$

where Re designates the real part of the expression and the plus sign corresponds to the upper index.

The behavior of $\mathcal{E}_{10}$ and $\mathcal{E}_{00}$ is determined by the relationship between two competing energies:

$$E_Z = 2\hbar\gamma B_z, \tag{28}$$

the energy required by an electron to flip spin and

$$E_{SOI} = \hbar D(Q_A^2+Q_3^2), \tag{29}$$

the energy associated with the spin-orbit interaction, proportional with the spin relaxation rate along the in-plane

directions,

$$\frac{1}{\tau_s} = \frac{1}{\tau_{xx}} = \frac{1}{\tau_{zz}} = D\left(Q_A^2 + Q_3^2\right). \tag{30}$$

Since $2E_Z$ is the energy required for the electron to flip spin twice in the scattering process and generate a constructive interference effect which impacts the conductivity corrections, the ratio $2E_Z/E_{SOI}$ is the parameter that is reflected by the solutions of Eq. (31),

$$\mathcal{E}_{00}^{10} = Dq^2 + \frac{1}{\tau_\varphi} + D\frac{\left(Q_A^2 + Q_3^2\right)}{2} \times \left[1 \pm \mathrm{Re}\sqrt{1 - \left(\frac{2E_Z}{E_{SOI}}\right)^2}\right]. \tag{31}$$

In the low field limit, when $2E_Z \leqslant E_{SOI}$, we obtain

$$\mathcal{E}_{00}^{10} = Dq^2 + \frac{1}{\tau_\varphi} + D\frac{\left(Q_A^2 + Q_3^2\right)}{2} \times \left\{1 \pm \left[1 - \frac{1}{2}\left(\frac{2E_Z}{E_{SOI}}\right)^2\right]\right\}. \tag{32}$$

Up to a small quadratic correction in E_Z/E_{SOI}, E_{10} is independent of the magnetic field, its magnitude being determined by the spin-orbit coupling interaction alone as anticipated from general considerations since E_{10} corresponds to a symmetric state. Thus,

$$\mathcal{E}_{10} = Dq^2 + \frac{1}{\tau_\varphi} + D\left(Q_A^2 + Q_3^2\right)\left[1 - \left(\frac{E_Z}{E_{SOI}}\right)^2\right]. \tag{33}$$

In contrast, the effect of the field on the singlet Cooperon is substantial:

$$\mathcal{E}_{10} = Dq^2 + \frac{1}{\tau_\varphi} + D\left(Q_A^2 + Q_3^2\right)\left(\frac{E_Z}{E_{SOI}}\right)^2. \tag{34}$$

It is customary to define an effective dephasing rate, such that

$$\frac{1}{\tau_\varphi(B)} = \frac{1}{\tau_\varphi} + D\left(Q_A^2 + Q_3^2\right)\left(\frac{E_Z}{E_{SOI}}\right)^2, \tag{35}$$

a result previously obtained in Ref. [13] which indicates that as the magnetic field increases, the dephasing rate increases proportionally as the probability of the scattering processes that lead to spin flips increases. This can be explained by recognizing that the additional Zeeman energy acquired by the electrons in the magnetic field favors transitions between the two Fermi surfaces $E_\pm$ which are now separated by a gap determined mostly by the spin-orbit coupling as indicated by Eq. (12). As the magnetic field increases, the rate of spin flips increases too with respect to the $B = 0$ case.

At high field values, when $2E_Z \geqslant E_{SOI}$, the two $J_z = 0$ modes of the Cooperon become equal, as the real part of the square root in Eq. (31) is zero. Then,

$$\mathcal{E}_{00}^{10} = Dq^2 + \frac{1}{\tau_\varphi} + D\frac{\left(Q_A^2 + Q_3^2\right)}{2}. \tag{36}$$

The saturation regime for the $J_z = 0$ is a result of the gap between the two Fermi surfaces $E_\pm$ being now set by the magnetic field. Hence scattering processes with spin flipping can be realized exclusively only as a result of the spin rotation imposed by SOI, which for a given system is a constant, independent of B.

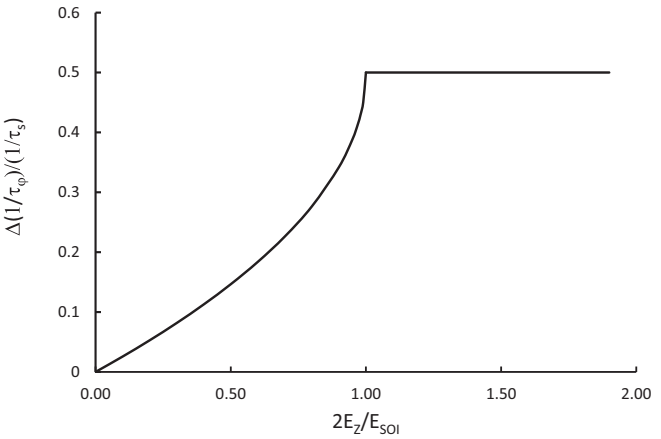

FIG. 1. The variation of the dephasing rate as a function of $2E_Z/E_{SOI}$ expressed in $1/\tau_s$ units. $\Delta(1/\tau_\varphi)$ saturates at a constant value equal to 1/2 of the in-plane spin-relaxation rate.

$\mathcal{E}_{00}$ and $\mathcal{E}_{10}$ have equal, but opposite signs, and their contributions to the conductivity cancel each other, signaling the disappearance of the antilocalization correction. The effective dephasing rate is also constant,

$$\frac{1}{\tau_\varphi(B)} = \frac{1}{\tau_\varphi} + \frac{D\left(Q_A^2 + Q_3^2\right)}{2} = \frac{1}{2\tau_s}, \tag{37}$$

a result which generalizes Ref. [16] where only linear Rashba was considered to the present case where both linear R-D and cubic D terms are considered. When only one linear spin-orbit coupling dominates, $Q_A \sim \Omega_1$, we note that the limit of the dephasing time is exactly half of the spin-relaxation rate along the in-plane direction, a result verified experimentally for a Rashba linear coupling term in Ref. [17]. The variation of the dephasing rate expressed in terms of the spin relaxation rate is represented as a function of the Zeeman splitting, measured in units of the spin-orbit energy, in Fig. 1. When $Q_A \approx 0$, the effective dephasing rate is determined only by Q_3, the term proportional with the cubic Dresselhaus interaction. Consequently, an experiment similar to that in Ref. [17] performed in the saturation regime close to the $Q_A = 0$ point, would allow a direct determination of Q_3 from the measurement of the dephasing rate. We note that $Q_A = 0$ when $\alpha = \beta_1 + \beta_3 p^2/4$, an offset from the strict equality of the linear couplings $\alpha = \beta_1$ determined by the cubic Dresselhaus parameter. At the $\alpha = \beta_1$ point, $Q_A = Q_3$ if $\tau_1 \simeq \tau_3$.

B. $J_z = \pm 1$ Cooperon modes and the conductivity corrections

The two remaining solutions of Eq. (24) correspond to the states that share $J_z = \pm 1$ and are obtained from

$$(H_{11} - \mathcal{E})(H_{1-1} - \mathcal{E}) - \left(D\frac{Q_A^2 + Q_3^2}{2}\right)^2 = 0. \tag{38}$$

The resulting eigenvalues are given by

$$\mathcal{E}_{1-1}^{11} = Dq^2 + 1/\tau_\varphi + D\left[Q_S^2 + Q_3^2 + \frac{Q_A^2 + Q_3^2}{2} \pm \sqrt{(2q_x Q_S)^2 + \left(\frac{Q_A^2 + Q_3^2}{2}\right)^2}\right]. \tag{39}$$

Equations (39) and (31) are derived under the general conditions discussed at the beginning of this section and they describe the complete effect of the SOI couplings on the eigenstates of the Cooperon.

When only one SOI coupling is considered, $Q_A = Q_S \sim \Omega_1$. Then in Eq. (39), $2q_x Q_S \to 0$, as, on account of q_x, it becomes much smaller than the spin-orbit term. Then, the $J_z = \pm 1$ eigenvalues of the Cooperon are

$$\mathcal{E}_{11} = Dq^2 + \frac{1}{\tau_\varphi} + 2D\left(Q_A^2 + Q_3^2\right),$$
$$\mathcal{E}_{1-1} = Dq^2 + \frac{1}{\tau_\varphi} + D\left(Q_A^2 + Q_3^2\right). \tag{40}$$

$\mathcal{E}_{11}$ and $\mathcal{E}_{1-1}$ are found to be independent of the magnetic field, their magnitude being determined by only the spin-orbit coupling. This is not surprising since the quantum states associated with these modes that have the same spin orientation before and after the collision are associated only with transitions between points on the same Fermi surface $E_\pm$ and their scattering energy balance does not involve the energy gap at all. It is customary to express these results in terms of the spin-relaxation rate along the in-plane axes τ_s^{-1} in Eq. (30), and the spin-relaxation rate along the axis perpendicular on the plane, $\tau_{yy}^{-1} = 2\tau_{zz}^{-1} = 2\tau_s^{-1}$ that enters $\mathcal{E}_{11}$. When the original spin-orbit coupling constants are introduced through Eqs. (22) the result of Ref. [11] is recovered.

From Eq. (14), the conductivity correction is obtained by integrating in the $\mathbf{q}$ space, with the radial integral extended only up to a maximum value $q_{\max}$, established by the condition $Dq_{\max}^2 \sim 1/\tau_1$, the transport time. With input from Eqs. (31), (37), (30), and (40), the general expression of the conductivity corrections in the limit of a dominant linear spin-orbit coupling at weak magnetic fields is

$$\Delta\sigma(B) = \frac{e^2}{4\pi\hbar^2}\left\{\ln\left[\frac{\tau_1}{\tau_\varphi} + 2\tau_1 D\left(Q_A^2 + Q_3^2\right)\right] + 2\ln\left[\frac{\tau_1}{\tau_\varphi} + \tau_1 D\left(Q_A^2 + Q_3^2\right)\right] - \ln\left(\frac{\tau_1}{\tau_\varphi(B)}\right)\right\}. \tag{41}$$

At high magnetic fields, such that $2E_Z \geqslant E_{SOI}$, the real part of the square root in the expressions of $\mathcal{E}_{10}$ and $\mathcal{E}_{00}$ is zero and the two solutions generate terms in the conductivity corrections that cancel each other. Thus, in this regime, the conductivity is constant, independent of the magnetic field,

$$\Delta\sigma = \frac{e^2}{4\pi\hbar^2}\left\{\ln\left[\frac{\tau_1}{\tau_\varphi} + 2\tau_1 D\left(Q_A^2 + Q_3^2\right)\right] + \ln\left[\frac{\tau_1}{\tau_\varphi} + \tau_1 D\left(Q_A^2 + Q_3^2\right)\right]\right\}. \tag{42}$$

The second limit of the Cooperon eigenvalues for $J_z = \pm 1$ is studied close to the point where the two linear spin-orbit couplings cancel. In our notations, this corresponds to $Q_S \gg Q_A, Q_3$, while $q_x Q_S$ remains finite. This implies that, from Eq. (39),

$$\mathcal{E}_{1-1}^{11} = D(q_x \pm Q_S)^2 + Dq_y^2 + 1/\tau_\varphi + D\left[Q_3^2 + \frac{Q_A^2 + Q_3^2}{2}\right]. \tag{43}$$

These values are independent of the magnetic field, since $J_z = \pm 1$ configuration corresponds to spin states whose orientation was left unchanged by the scattering.

In the limit of small magnetic fields, when $2E_Z \leqslant E_{SOI}$, with input from Eqs. (33), (34), and (43), the quantum conductivity correction is

$$\Delta\sigma(B) = \frac{e^2}{4\pi\hbar^2}\left\{2\ln\left[\frac{\tau_1}{\tau_\varphi} + \tau_1\frac{D\left(Q_A^2 + 3Q_3^2\right)}{2}\right] + \ln\left[\frac{\tau_1}{\tau_\varphi} + \tau_1 D\left(Q_A^2 + Q_3^2\right)\right] - \ln\left(\frac{\tau_1}{\tau_\varphi(B)}\right)\right\}. \tag{44}$$

When $2E_Z \geqslant E_{SOI}$, as before, $\mathcal{E}_{00} = \mathcal{E}_{10}$ saturate and become independent of the magnetic field at values given by Eq. (36) and the conductivity correction is constant:

$$\Delta\sigma(B) = \frac{e^2}{2\pi\hbar^2}\ln\left[\frac{\tau_1}{\tau_\varphi} + \tau_1\frac{D\left(Q_A^2 + 3Q_3^2\right)}{2}\right]. \tag{45}$$

When $Q_A = 0$, the conductivity correction depends only on Q_3, a term proportional with the cubic Dresselhaus interaction.

V. CONCLUSION

In conclusion, we calculated the quantum corrections to the conductivity in the presence of an in-plane magnetic field in a system with linear and cubic spin-orbit coupling. In two different limiting situations associated either with the dominance of one linear coupling or with the equality of the linear couplings, we explored the effect of an in-plane magnetic field on the dephasing time and the conductivity. We find that the characteristic features of the results are determined by the competing effects of the Zeeman coupling and the spin-orbit interaction. In each case, the larger of these energies determines the energy gap between the Fermi surfaces associated with the exact quantum states of the electrons, while the smallest determines the dephasing rate by favoring spin flipping in the scattering processes.

The most important result of this paper is that in the limit of the almost cancellation of the linear spin-orbit terms, at a magnetic field that is larger than the spin-orbit coupling, the effective dephasing rate as well as the quantum corrections are determined by the cubic Dresselhaus term alone. Experimentally, one would drive the system in the saturation phase by increasing the magnitude of the magnetic field until the conductivity becomes constant. Then, by varying the external field across the well, the Rashba coupling is modified until the conductivity reaches an absolute minimum which corresponds to the point where linear Rashba and Dresselhaus cancel each other. In this geometry the cubic Rashba coefficient is determined directly, offering some advantage over the other possible orientation of the magnetic field, perpendicular on the plane, where a more complicated dependence on β_3 is found in the magnetoconductance [12].

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