Criticality and phase diagram of quantum long-range O(N) models

Nicolò Defenu,[1] Andrea Trombettoni,[2,3,4] and Stefano Ruffo[3,4]
[1]*Institut für Theoretische Physik, Universität Heidelberg, D-69120 Heidelberg, Germany*
[2]*CNR-IOM DEMOCRITOS Simulation Center, Via Bonomea 265, I-34136 Trieste, Italy*
[3]*SISSA, Via Bonomea 265, I-34136 Trieste, Italy*
[4]*INFN, Sezione di Trieste, I-34151 Trieste, Italy*
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Several recent experiments in atomic, molecular, and optical systems motivated a huge interest in the study of quantum long-range systems. Our goal in this paper is to present a general description of their critical behavior and phases, devising a treatment valid in d dimensions, with an exponent $d+\sigma$ for the power-law decay of the couplings in the presence of an O(N) symmetry. By introducing a convenient ansatz for the effective action, we determine the phase diagram for the N-component quantum rotor model with long-range interactions, with $N=1$ corresponding to the Ising model. The phase diagram in the σ-d plane shows a nontrivial dependence on σ. As a consequence of the fact that the model is quantum, the correlation functions are anisotropic in the spatial and time coordinates for σ smaller than a critical value, and in this region the isotropy is not restored even at criticality. Results for the correlation length exponent ν, the dynamical critical exponent z, and a comparison with numerical findings for them are presented.

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I. INTRODUCTION

A series of remarkable advancements in the last decade led to a striking development of the experimental techniques for the control and manipulation of atomic, molecular, and optical systems such as Rydberg atoms [1], dipolar quantum gases [2], polar molecules [3], multimode cavities [4], and trapped ions [5]. Such progress paved the way for the simulation of a variety of quantum models [6], and in this direction one of the most active fields of research is the implementation and study of equilibrium and dynamical properties of quantum long-range (LR) models [7–18]. Ising and/or XY quantum spin chains with tunable LR interactions can currently be realized using Be ions stored in a Penning trap [7], neutral atoms coupled to photonic modes of a cavity [15,17,18], or with trapped ions coupled to motional degrees of freedom [12–14]. A key property in these studies is that the resulting interactions decay algebraically with the distance r and that the decay exponent can be experimentally tuned [12–14]. As an example, Rydberg gases have been used to observe and study spatially ordered structures [8] and correlated transport [16]. Dipolar spin-exchange interactions with lattice-confined polar molecules were observed as well [11]. The possibility of control LR interactions in spin chains simulated with trapped ions was also at the basis of the recent experimental simulation of the one-dimensional (1D) Schwinger model [19].

An important reason for the interest in the equilibrium and nonequilibrium properties of quantum LR systems is the connection with typical themes of classical LR physics. The traditional interest in the behavior of LR interacting statistical mechanics models has been largely due to the innumerable possible applications in condensed matter, plasma physics, astrophysics, and cosmology [20,21]. Therefore, the general question is how quantum fluctuations modify the traditional picture of LR interactions into complex systems. We focus on the study of the criticality in quantum LR systems and the development of a general renormalization group (RG) approach for their study.

From the point of view of critical behavior, the main difference between classical and quantum systems is due to the presence of unitary quantum dynamics in the latter case. The critical behavior in the time domain is usually described in terms of the dynamical critical exponent z, which is related to the critical scaling $\Delta \propto L^{-z}$ of the gap Δ in the thermodynamic limit $L \to \infty$, where L is the linear size of the system. This is different from classical systems, where the value of z depends on the chosen dynamics.

In several short-range (SR) systems the value of this exponent is strictly 1, leading to the well known equivalence between the universality class of a d dimensional quantum system and its classical equivalent in $d_{\mathrm{cl}} = d+1$ dimensions, where d_{cl} is the dimension of the corresponding classical system. For nonunitary values of the dynamical critical exponent z, which is the case of LR interacting quantum systems, the relation between classical and quantum critical behavior is more subtle. The general expectation is that classical and quantum universalities should be connected in general for $d_{\mathrm{cl}} = d+z$ [22].

The aim of our investigation is focused on the derivation of the critical exponent z and of its spatial counterpart, i.e., the correlation length critical exponent ν. Various relations, exact or not, which connect both classical and quantum, SR and LR systems, are also investigated. In order to accomplish this task we employ the functional renormalization group (FRG) formalism already developed for classical isotropic and anisotropic LR O(N) models [23,24].

The interplay between LR interactions and quantum effects is a traditional topic in condensed matter physics. A major example is provided by the $\frac{1}{r^2}$ Ising model [25], displaying a behavior related to the spin-$\frac{1}{2}$ Kondo problem with the occurrence of a topological phase transition of the Berezinskii-Kosterlitz-Thouless (BKT) type [26,27]. Moreover, the experimental realization of dipolar Ising spin glasses in the 1990s led to investigations of the behavior of quantum LR Ising and rotor models [28].

The application of functional RG techniques to classical LR spin systems produced a nice all-in-one picture of the phase transitions occurring in these models, contributing to clarify the behavior of the anomalous dimension of these systems [23,29–32]. Moreover functional RG formalism was successfully applied to the case of anisotropic LR interactions [24]. Good agreement with Monte Carlo (MC) simulations was found, not only in the two-dimensional Ising model [33–36], but also in one-dimensional LR bond percolation by means of effective dimension relations [37].

Comparing with traditional perturbative RG analysis, functional RG was able to produce, at least for SR O(N) models, numerical values for the critical exponents, whose accuracy remains stable as a function of all relevant system parameters, i.e., the dimension d and number of components N. In the classical SR O(N) models the accuracy never falls below 20%, while remaining well below 10% for all continuous symmetries $N \geqslant 2$ [38,39]. Such accuracy is expected to hold also in the LR case, where correlation effects should never be stronger than in the two-dimensional classical SR Ising model.

The issue we address is the general description of the criticality in quantum LR spin models and the development of a RG framework for this investigation. As a first step, one would naively think that the problem is reduced to the study of a LR classical problem, but in this paper it is shown explicitly that this is the case only for σ large enough. A main aim of this paper is therefore to widen and unify the theoretical picture for the phase transition quantum LR models and to construct the full landscape of their phase diagram within a single formalism. We also determine the dynamical critical exponent z, which characterizes the critical quantum dynamics.

The structure of the paper is the following. In Sec. II we introduce the models used in the rest of the paper. In Sec. III we discuss the field theoretical description and the RG methods, and we present our ansatz for the effective action. We also derive the mean field results and study the corrections to the phase diagram due to the effects of the presence of the anomalous dimension. Estimates for the critical exponents in $d = 1$ and $d = 2$ are presented in Sec. IV, together with a discussion of the comparison with some numerical results available in the literature. Our conclusions are presented in Sec. VI, together with a summary of the relevance and implications of our results.

II. LONG-RANGE MODELS

In this paper we consider two models, namely the Ising model in a transverse field and the quantum rotor model. The Ising Hamiltonian in a transverse field $\mathcal{H}$ reads

$$H_{\mathrm{I}} = -\sum_{ij} \frac{J_{ij}}{2} \sigma_i^z \sigma_j^z - \mathcal{H} \sum_i \sigma_i^x, \tag{1}$$

where the σ^z and σ^x are the Pauli matrices defined in the sites i,j of a d-dimensional hypercubic lattice. The quantum rotor model can be written as

$$H_{\mathrm{R}} = -\sum_{ij} \frac{J_{ij}}{2} \hat{\boldsymbol{n}}_i \cdot \hat{\boldsymbol{n}}_j + \frac{\lambda}{2} \sum_i \mathcal{L}_i^2, \tag{2}$$

where the $\hat{\boldsymbol{n}}_i$ are the N components' unit length vector operators ($\hat{\boldsymbol{n}}_i^2 = 1$), λ is a real constant, and $\mathcal{L}$ is the invariant operator formed from the asymmetric rotor space angular momentum tensor [22]. The interaction matrix in both cases is power-law decaying with the distance,

$$J_{ij} = \frac{J}{r_{ij}^{d+\sigma}}, \tag{3}$$

with J a positive constant, d the spatial dimension of the system, and σ any real number, while r_{ij} is the distance between the sites i and j.

The interaction potential in Eq. (3) induces in the model radically different behaviors depending on the value of the exponent σ. In particular, for $\sigma < 0$ the internal energy of the system may diverge in the thermodynamic limit, leading to an ill defined model. However, a suitable redefinition of the interaction strength [40] produces finite interaction energies, still preserving many interesting results typical of nonadditive systems [21].

For fast enough decaying interactions $\sigma > 0$ the system is always additive and thermodynamics is well defined. For σ lying into a certain range, the finite temperature system may present a phase transition with spontaneous symmetry breaking at a finite critical temperature T_c.

For a classical spin system, the phase diagram in the (d,σ) plane is therefore divided into three regions [29]. For $2\sigma \leqslant d$ the universal behavior is the one obtained at mean field level, while for σ larger than a critical value σ^* the system has the same critical behavior as its SR analog. Finally, for $d/2 < \sigma \leqslant \sigma^*$ LR interactions are relevant and the system has a phase transition with peculiar LR critical behavior.

The determination of the boundary σ^* where LR interactions become irrelevant with respect to the usual SR ones has been at the center of intense investigations in the last few years [23,32,34]; see as well the recent works [41,42]. The functional RG framework [43–45] proved to be useful to explore the effect of LR interactions [23,24] in a compact way. The accuracy of such results remains reliable for all values of the system parameters, i.e., the spatial dimension d, the number of components N, and the criticality index i.

Rotor models are the straightforward generalization to the quantum case of the classical O(N) models [22]. Their low energy behavior describes the physics of many relevant physical models, in particular the $N = 3$ case is in the same universality of antiferromagnetic quantum Heisenberg spin systems, while the $N = 2$ case is related to the quantum critical behavior of the Bose-Hubbard model and of stripe forming systems [46,47].

In this paper we only consider the case $\sigma > 0$ in order to have a well defined thermodynamics [20,21]. The system undergoes a paramagnetic to ferromagnetic quantum phase transition (QPT) at zero temperatures for all d and σ in the Ising case, Eq. (1), i.e., the one-component $N = 1$ case of the rotor model in Eq. (2). On the other hand, for $N \geqslant 2$, due to the Mermin-Wagner theorem (MWT) the system manifests spontaneous symmetry breaking (SSB) only for $\sigma < \sigma_*$ in $d = 1$, where σ_* is the threshold value above which short-range (SR) behavior is recovered.

At finite temperature $T > 0$ the system has a finite temperature classical phase transition (CPT), which lies in the same universality class of standard LR classical O(N) spin systems. The results for the finite temperature transition behavior were the subject of intense, long-lasting investigations in recent decades [23,29] and they will be used, but not further discussed, in this paper.

III. FIELD THEORETICAL REPRESENTATION AND PHASE DIAGRAM

The universal behavior of condensed matter and statistical mechanics models can be investigated by means of field theoretical techniques [48]. In order to employ the functional RG approach, one may consider the Wetterich equation [49]

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\frac{\partial_t R_k}{\Gamma^{(2)} + R_k}\right], \tag{4}$$

where R_k is an infrared regulator function, k a finite scale proportional to the inverse system size, $k \propto L^{-1}$, and $t = \ln(\frac{k}{k_0})$ the logarithmic scale. Γ is the effective action of the system and plays the role of an exact Ginzburg-Landau free energy, while $\Gamma^{(2)}$ is its second derivative with respect to the system order parameters.

In order to solve Eq. (4) it is useful to restrict ourselves to a functional space spanned by a finite number of functions and/or couplings. Using the field theory representation and the Trotter decomposition, it can be shown that the models defined in Eqs. (1) and (2) display the same universal behavior of a classical systems in $d + 1$ dimension where the interaction is LR in d directions and SR in the remaining Trotter dimension [28].

Thus a convenient ansatz for the effective action of an O(N) quantum rotor model is

$$\Gamma_k = \int d\tau \int d^d x\{K_k \partial_\tau \varphi_i \partial_\tau \varphi_i - Z_k \varphi_i \Delta^{\frac{\sigma}{2}} \varphi_i - Z_{2,k}\varphi_i \Delta \varphi_i + U_k(\rho)\}, \tag{5}$$

where Δ is the spatial Laplacian in d dimensions, τ is the "Trotter"/imaginary time direction, $\varphi_i(x)$ is the ith component ($i \in \{1,\ldots,N\}$) of the system magnetization density, $\rho \equiv \sum \frac{\varphi_i^2}{2}$ is the system order parameter, $U_k(\rho)$ is the scale dependent effective potential, and $(K_k, Z_k, Z_{2,k})$ are three scale dependent wave-function renormalization terms. In Eq. (5) the summation over repeated indexes is intended.

The effective action ansatz (5) is sufficient to investigate, at least at an approximate level, the low energy behavior of the models described in Hamiltonians (2) and (1). The presence of two kinetic terms in the d spatial directions is necessary to take into account for the competition between the LR nonanalytic momentum term q^σ in the propagator and the usual q^2 term in the $\sigma \simeq 2$ region.

The frequency and momentum dependence of the propagator at criticality are connected by the dynamical critical exponent z, defined by $\omega \propto q^z$. We also expect the momentum dependence of the propagator $G(q)$ for large wavelength to obey the scaling form

$$\lim_{q\to 0} G(q) \propto q^{2-\eta}. \tag{6}$$

Equation (6) defines the anomalous dimension η as the deviation of the long-wavelength behavior of the propagator from the standard SR mean field behavior q^2. Finally, as for the CPT, the correlation length of the quantum system diverges close to the critical point. Such divergence is power-law, at least for standard SSB, and the scaling behavior reads

$$\xi \propto (\lambda - \lambda_c)^{-\nu}, \tag{7}$$

where ξ is the correlation length, λ is the coupling appearing in Hamiltonian (2), and λ_c is the critical value of the coupling at which the system undergoes SSB. Then Eq. (7) can be considered the definition of the critical exponent ν. The universal behavior of any second-order QCP can be described only in terms of the three exponents (z,η,ν), with all the other exponents given by scaling relations [22].

A. Local potential approximation

The lowest-order approximation for the ansatz (5) is obtained considering only the actions parametrized by the effective potential $U_k(\rho)$. Imposing

$$\begin{aligned} Z_k &= K_k = 1,\\ Z_{2,k} &= 0, \end{aligned} \tag{8}$$

it is possible to derive the flow equation in the local potential approximation (LPA). The result is equivalent to the one obtained in the case of a classical anisotropic O(N) spin system [24]. One gets

$$\partial_t U_k(\rho) = (d+z)\,\tilde{U}_k(\tilde{\rho}) - (d+z-\sigma)\,\tilde{\rho}\,\tilde{U}^{(1)}(\tilde{\rho}) - \frac{1}{1+\tilde{U}^{(1)}(\tilde{\rho}) + 2\,\tilde{\rho}\tilde{U}^{(2)}(\tilde{\rho})} - \frac{N-1}{1+\tilde{U}^{(1)}(\tilde{\rho})}. \tag{9}$$

The $\sim$ indicates rescaled quantities, which are defined in order to ensure scale invariance $\Gamma = \tilde{\Gamma}$ of the effective action at the critical point:

$$U(\rho) = k^{D_U}\tilde{U}(\tilde{\rho}), \tag{10}$$

$$\varphi = k^{D_\varphi}\tilde{\varphi}, \tag{11}$$

with

$$D_U = d + z, \tag{12}$$

$$D_\varphi = d + z - \sigma. \tag{13}$$

At the LPA level the kinetic term is fixed and does not renormalize, thus anomalous dimension effects vanish in both the frequency and momentum sectors. Then both the anomalous dimension and the dynamical critical exponent attain their mean field values

$$\eta = 2 - \sigma, \tag{14}$$

$$z = \frac{\sigma}{2}. \tag{15}$$

Also, Eq. (9) is equivalent to the flow for the effective potential in the LPA approximation for a classical SR spin system

[38,39] in an effective fractional dimension

$$d''_{\rm SR} = \frac{2(d+z)}{\sigma}; \tag{16}$$

this equivalence does not account for anomalous dimension effects. Substituting the mean field result for the exponent z, one obtains

$$d'_{\rm SR} = \frac{2d}{\sigma} + 1, \tag{17}$$

which is exact in the vanishing anomalous dimension cases [50]. Similar effective dimension relations have already been introduced in the literature for the diluted model [51], where random disorder is introduced to simulate LR effects. The critical exponents of the LR model can be obtained from the ones of the SR model in dimension $d_{\rm SR}$ via some simple linear transformations, and they fulfill all the usual scaling relations included hyperscaling, as long as $d < d_{\rm uc}$. We refer the reader to [23] for a discussion on the reliability of the effective dimension; see as well [42].

We are then in position to predict the upper critical dimension, which is obtained from the condition $d_{\rm SR} \geqslant 4$, in such a way that

$$d_{\rm uc} = \tfrac{3}{2}\sigma, \tag{18}$$

in agreement with the one obtained by relevance arguments in [28]. For $d > d_{uc}$ the system undergoes SSB with mean field critical exponents given in Eqs. (14) and (15), with

$$\nu = \sigma^{-1}. \tag{19}$$

For continuous symmetries in the SR interacting case, the anomalous dimension vanishes also at the lower critical dimension as ensured by the MWT. Thus one can again employ the effective dimension approach to obtain

$$d_{\rm lc} = \frac{\sigma}{2}. \tag{20}$$

It is worth noting that, while relation (18) is valid for all N, including the $N = 1$ Ising case, the expression (20) is limited to continuous symmetries $N \geqslant 2$.

Summarizing, the final picture emerging from LPA approximation is rather simple. Equation (9) has, in general, two fixed-point solutions: One is the Gaussian fixed point, which represents the mean field universality, characterized by the critical exponents in Eqs. (14), (15), and (19). The other solution has finite renormalized mass $\tilde{U}^{(1)}(0) \neq 0$ and represents the interacting Wilson-Fisher (WF) universality.

For $d \geqslant d_{\rm uc}$ the Gaussian universality is the attractive one and the quantum LR system will display SSB with mean field exponents. On the other hand for $d < d_{\rm uc}$ the attractive fixed point is the WF one and no analytic expression for the critical exponents is known. Finally for $d \leqslant d_{\rm lc}$ no SSB is possible for continuous symmetries and the system will have a single phase in the $N \geqslant 2$ case.

At the border region $\sigma \simeq 2$ the LR and SR momentum terms are competing in the propagator, and anomalous dimension effects become increasingly important [39]. In this section we discarded anomalous dimension effects, and the threshold values at which LR interactions become irrelevant with respect to SR ones is $\sigma_* = 2$, as can be detected by mean field arguments [28]. However, it seems by now established that the actual boundary between LR and SR critical behavior is given by Sak's expression $\sigma_* = 2 - \eta_{\rm SR}$ [29], where $\eta_{\rm SR}$ is the anomalous dimension of the system in the $\sigma = 2$ case. In order to investigate this effect we shall rely on the complete ansatz (5).

B. Anomalous dimension effects

In order to introduce anomalous dimension effects, it is necessary to consider the flow of the kinetic sectors. In the simplified approach considered in this paper it is sufficient to compute the running of the coefficients Z_k, K_k, and $Z_{2,k}$. The flow for the LR wavefunction Z_k is given by

$$\partial_t Z_k = (2 - \sigma - \eta) Z_k, \tag{21}$$

where the anomalous dimension η is defined with respect to the SR term:

$$\eta = \frac{\partial_t Z_{2,k}}{Z_{2,k}}. \tag{22}$$

At the fixed point the running of the couplings should vanish, and thus we have only two possibilities: either

$$\eta = 2 - \sigma \tag{23}$$

or

$$\lim_{k\to 0} Z_k = 0. \tag{24}$$

The latter result will imply that LR interactions are irrelevant and do not influence the critical behavior. This analysis is consistent with Sak's result. Indeed for $\sigma < \sigma_*$ LR interactions are relevant and $\eta = 2 - \sigma$, while for $\sigma \geqslant \sigma_*$ the LR term vanishes at criticality and the system recovers SR universal behavior $\eta = \eta_{\rm SR}$.

Therefore the boundary value σ_* is obtained when the coefficient in the right-hand side of Eq. (21) vanishes, $2 - \sigma_* - \eta = 0$, and η attains its SR value $\eta_{\rm SR}$:

$$\sigma_* = 2 - \eta_{\rm SR}. \tag{25}$$

For $\sigma \geqslant \sigma_*$ the effective action of the system at the fixed point becomes isotropic and the dynamical critical exponent is $z = 1$. Then, according to quantum to classical correspondence [22,48] the QCP of the SR system is in the same universality of the CPT of its classical analog in $d_{\rm SR} = d + 1$.

The phase diagram of a quantum LR rotor model in the (d,σ) plane is reported in Fig. 1. In the discrete symmetry case, Fig. 1(a), the system has a mean field validity region, indicated by the cyan shaded area, where the exponents are the ones obtained by mean field approximation. Otherwise for $d < d_{\rm uc}$ the system has peculiar non-mean-field exponents. The boundary σ_* after which the system recovers SR behavior is indicated by the red solid line; the line has been computed using expression (25) with the expression for $\eta_{\rm SR}$ obtained by ansatz (5) without any nonanalytic terms. For discrete symmetries SSB appears also in $d = 1$.

For continuous symmetries, Fig. 1(b), the scenario is the same as for discrete symmetries, except that, due to the MWT, the σ_* boundary recovers its mean field value not only at $d = d_{\rm uc}$ but also for $d = d_{\rm lc} = 1$ at σ_*. For $d \leqslant d_{\rm lc}$, the gray shaded area, the system displays only a single phase.

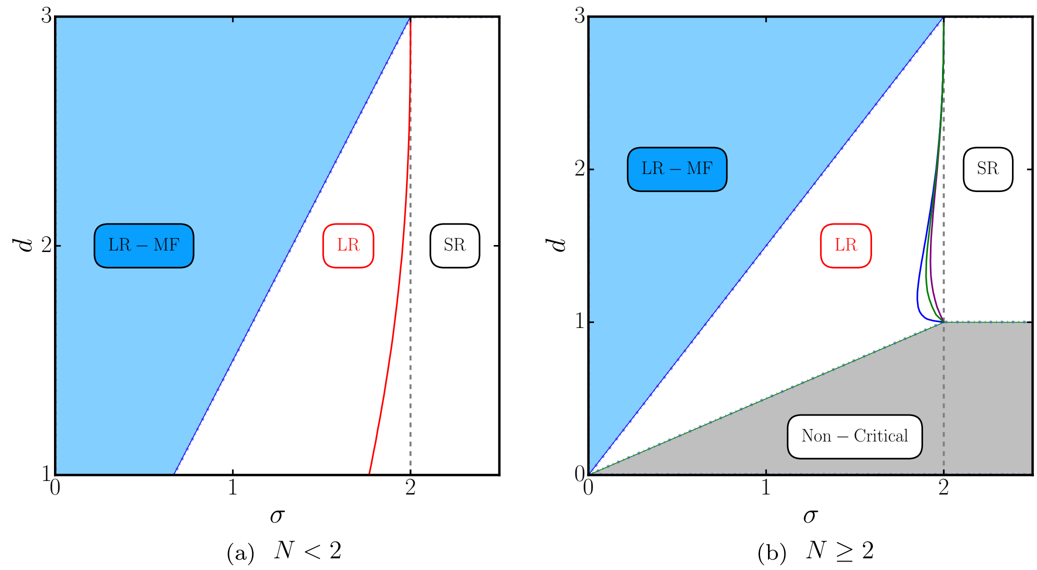


FIG. 1. Phase diagram of quantum LR rotor models for discrete (a) and continuous (b) symmetries. The cyan shaded area represents the mean field validity region, while the gray shaded area is the single-phase region. The mean field boundary $\sigma_* = 2$ is represented by a gray dashed line, and renormalized boundaries are solid lines in red, blue, green, and purple for $N = 1,2,3,4$ respectively.

IV. THE CRITICAL EXPONENTS

As shown in the previous section, the system has a phase diagram similar to the one already depicted for classical LR spin systems [23,34] with nontrivial LR behavior occurring only for $\sigma < \sigma_*$ and $d < d_{\mathrm{uc}}$. In this region no analytical expressions are possible for the critical exponents, and one must resort to numerical approximated techniques.

In order to compute the critical exponents in the nontrivial region, one can apply the same procedure as done in [24] for anisotropic LR spin systems. Again we consider the ansatz (5) in the case of vanishing analytic corrections $Z_{2,k} = 0$ in the momentum sector. Indeed the latter term is relevant only close to the boundary σ_*, and it has been shown in [23,24] to produce only minor corrections even for $\sigma \simeq \sigma_*$

The flow of the effective potential for vanishing analytic correction $Z_{2,k} = 0$ reads

$$\partial_t \bar{U}_k = (d+z)\bar{U}_k(\bar{\rho}) - (d+z-\sigma)\bar{\rho}\,\bar{U}'_k(\bar{\rho}) - \frac{\sigma}{2}(N-1)\frac{1-\frac{\eta_\tau z}{3\sigma+2d}}{1+\bar{U}'_k(\bar{\rho})} - \frac{\sigma}{2}\frac{1-\frac{\eta_\tau z}{3\sigma+2d}}{1+\bar{U}'_k(\bar{\rho})+2\bar{\rho}\,\bar{U}''_k(\bar{\rho})}, \tag{26}$$

where the anomalous dimension η_τ has been introduced. This quantity introduces a correction in the frequency dependence of the system propagator,

$$\lim_{\omega\to 0} G(\omega,1)^{-1} \propto \omega^{2-\eta_\tau}. \tag{27}$$

The expression for the frequency anomalous dimension is readily obtained by the flow of K_k,

$$\eta_\tau = \frac{f(\tilde{\rho}_0,\tilde{U}^{(2)}(\tilde{\rho}_0))(3\sigma+2d)}{d+(3\sigma+d)[1+f(\tilde{\rho}_0,\tilde{U}^{(2)}(\tilde{\rho}_0))]}, \tag{28}$$

where the function $f(\tilde{\rho}_0,\tilde{U}^{(2)}(\tilde{\rho}_0))$ is the expression for the spatial anomalous dimension of the correspondent SR O(N) model,

$$f(\tilde{\rho}_0,\tilde{U}^{(2)}(\tilde{\rho}_0)) = \frac{4\tilde{\rho}_0\tilde{U}^{(2)}(\tilde{\rho}_0)^2}{[1+2\tilde{\rho}_0\tilde{U}^{(2)}(\tilde{\rho}_0)]^2}, \tag{29}$$

as found in [38] after rescaling an unessential geometric coefficient.

Merging the definition of the dynamical critical exponent z with the definition of η_τ in Eq. (27), it is readily obtained that

$$z = \frac{\sigma}{2-\eta_\tau}, \tag{30}$$

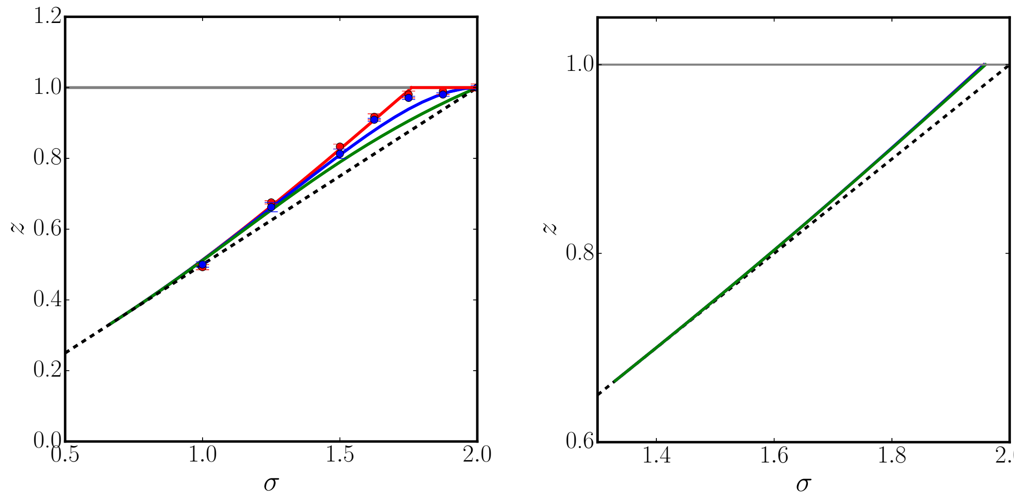
which has to be compared with the mean field expression (15).

Equation (26) allows for a higher order effective dimension relation which connects the universal behavior of a d-dimensional quantum LR system to its classical SR counterpart in d_{SR} dimensions with

$$d_{\mathrm{SR}} = (2-\eta_{\mathrm{SR}})\frac{(d+z)}{\sigma}, \tag{31}$$

where η_{SR} is the anomalous dimension of the SR system in dimension d_{SR}. Such a relation can be easily demonstrated using the procedure depicted in [23] to relate Eq. (26) to the potential flow shown in [39]. However, the exact demonstration only holds if we neglect the coefficients proportional to η_τ appearing in the second line of Eq. (26). These coefficients are small regulator-dependent quantities which produce $O(\eta_\tau^2)$ contributions to the universal quantities. It has been verified that neglecting such coefficients produces numerical errors below 1% in the solution of flow equations like (26) [23,45].

In Fig. 2 the dynamical critical exponent z as a function of the decay exponent σ is shown in $d = 1,2$, panels (a) and (b) respectively. The data have been obtained solving the expression for the fixed-point effective potential, Eq. (26) with the left-hand side posed to zero, and Eq. (28) in a self-consistent cycle. The dynamical critical exponent z attains its mean field value for $\sigma < \frac{2}{3}d$, while it becomes a nontrivial curve in the non-Gaussian region. It should be noted that



(a) $d = 1$ (b) $d = 2$

FIG. 2. The dynamical critical exponent z as a function of σ in $d = 1,2$, panels (a) and (b) respectively. The red, blue, and green curves are for $N = 1,2,3$ respectively; the mean field result is shown as a gray dashed line. The (blue and red) dots represent MC data for $d = 1$ derived for a spin system coupled to a bosonic bath with spectral density proportional to ω^σ (data taken from [52]), with the red dots for the Ising model (in good agreement with our predictions) and the blue dots for the O(2) model.

correlation effects always increase the value of the dynamical critical exponent with respect to the mean field prediction. However, in the continuos symmetry cases $N = 2,3$, for $d = 1$, due to the MWT the curves bend down at some finite value σ and the dynamical critical exponent z recovers its SR value only at $\sigma_* = 2$. On the other hand in the $d = 2$ case the Ising model curve (red solid line) is monotonically increasing and meets the SR line (gray solid line in figure 2) at the value $\sigma_* < 2$.

The dynamical critical exponent values obtained by the present approach for the Ising model, red solid line in 1(a), are in very good agreement with MC simulation on an Ising spin system coupled to an anomalous Bosonic bath, which lies in the same universality, red dots in 1(a). The agreement is poorer in the O(2) case: the blue solid denotes line theoretical values and blue dots MC simulation. The MC data are taken from Ref. [52].

The lower accuracy found for the $N = 2$ can be due to the presence of the BKT mechanism for this model. In passing we note that the MC points seems to provide a value $\sigma_* = 2 - \eta$ with $\eta = \frac{1}{4}$ even for $N = 2$, which is consistent with the BKT scenario.

The presented computation, despite having the merit of being valid for any d, σ, and N, is anyway not able to fully capture the effects of the topological phase transition, and no qualitative difference is found between $N > 2$ and $N = 2$ for $\sigma = 2$ in $d = 1$. However, close to $\sigma = 2$ for $N \geqslant 2$ it appears that MC simulations may be as well plagued by finite-size effects, as will become evident discussing the results of Fig. 3, since they do not reproduce the expected limiting behavior $\sigma \to 2$ for the correlation length exponent; see also the discussion in [52]. On the other hand our calculation of the quantity $(z\nu)^{-1}$, see Fig. 3, is fully consistent with the expected exact behavior coming from $d - 2$ expansions [53], and the discrepancy found between the theoretical prediction and the MC data for z in the $N = 2$ case is perhaps due to finite-size errors of the MC data rather than to unexpected BKT effects for $\sigma < 2$.

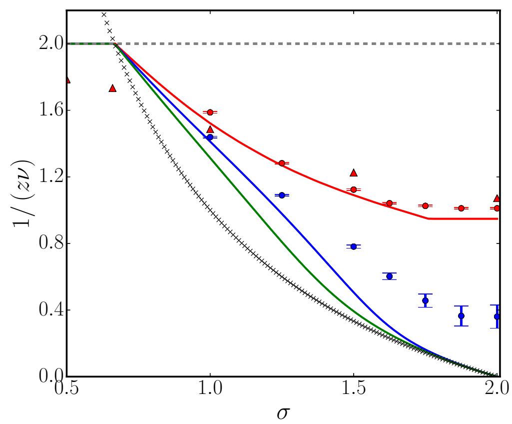
In order to completely characterize the critical behavior of quantum LR models, it is necessary to derive the values for the correlation length critical exponent in the nontrivial region. In order to accomplish this task it is sufficient to make a linear perturbation of the fixed point potential with the form $\tilde{U}(\tilde{\rho}) = \tilde{U}_*(\tilde{\rho}) + u(\tilde{\rho})e^{y_t t}$, where the $*$ denotes the any-scaling solution of Eq. (26).

Employing the technique already introduced in [39,55], it is possible to obtain the discrete spectrum of the eigenvalues y_t, which describe the scaling behavior of any physical perturbation of the fixed-point effective action. As is well known for standard second-order phase transitions, only one relevant perturbation of the critical Ginzburg-Landau free energy exists, which drives the system to either its high or low temperature phase. Then the eigenvalues' spectrum will contain only one positive solution, namely the inverse correlation exponent max $y_t = \nu^{-1}$.

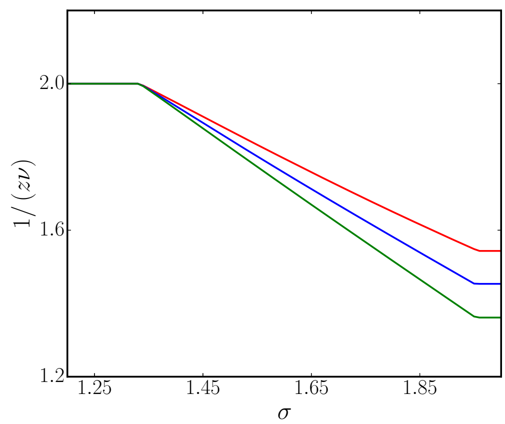
Another equivalent way to obtain the correlation length exponent ν is to take advantage of the effective dimension relation (31) in order to map the values of ν in the classical SR case to the quantum LR case via the relation

$$\nu = \frac{2 - \eta_{\mathrm{SR}}}{\sigma}\nu_{\mathrm{SR}}, \tag{32}$$

where ν_{SR} is the anomalous dimension of the SR system in dimension d_{SR}. This procedure is equivalent to solving the stability spectrum of Eq. (26) once the regulator dependent coefficients in the nonlinear part have been neglected [23]. In Fig. 3 the results for the exponent $(z\nu)^{-1}$ are reported in the case of a quantum LR rotor model in dimensions $d = 1$ and $d = 2$ in panels 3(a) and 3(b) respectively. The results for the Ising criticality (red solid line) are in good agreement with the numerical findings (red dots).



(a) $d = 1$



(b) $d = 2$

FIG. 3. $(z\nu)^{-1}$ as a function of σ in $d = 1,2$, panels (a) and (b) respectively. The red, blue, and green curves are for $N = 1,2,3$ respectively, the mean field result is shown as a gray dashed line. Again dots represent MC data for the Ising (red) and the O(2) models (blue) in $d = 1$; data are from Ref. [52]. The red triangles represent the estimation obtained with the perturbative continuous unitary transformation taken from [54]. These data, however, cannot reproduce the expected values in the mean field region, and seem unable to correctly account for the expected value of the SR boundary $\sigma_* = 2 - \eta_{\mathrm{SR}}$. Our results for the correlation length exponent ν have been obtained using the effective dimension relation (31) to map the values of ν for classical SR systems found in [39] to the quantum LR case; see the procedure outlined in [23]. It should be noted that for the SR Ising case it is necessary to employ two different definitions of anomalous dimensions in order to obtain reliable results; see [23,38,39]. In panel (a) leading order terms of $\varepsilon = (d_{uc} - d)$ and $\tilde{\varepsilon} = (d - d_{lc})$ expansions are shown as a gray dashed line and black crosses respectively.

On the other hand the $N = 2$ case [blue solid line in Fig. 3(a)] is, once again, in disagreement with numerical findings (blue dots). In this case, it appears that MC simulations are not completely reliable since they do not reproduce the exact behavior of the correlation length exponent in the $\sigma \to 2$ limit, where it is expected to diverge; see also the discussion in [52]. Such divergence is in agreement with the BKT behavior, where exponential divergence of the correlation length is found, as can be derived exactly by generalizing the $2 + \tilde{\varepsilon}$ expansion technique of the SR nonlinear σ model in Ref. [53] to the present case.

Such a generalization is more readily obtained by means of an effective dimension approach. One should consider equation (21) of Ref. [53] at lowest order in $\tilde{\varepsilon} = d - 2$,

$$\nu_{\mathrm{SR}}^{-1} = (d - 2) + O(\tilde{\varepsilon}^2), \tag{33}$$

employing the relation (31) with the effective dimension d_{SR} given by Eq. (17), in the limit of vanishing anomalous dimension. The result is

$$\nu \simeq 1 - \frac{\sigma}{2}, \tag{34}$$

which when multiplied by z^{-1} gives

$$\lim_{\sigma \to 2} (z\nu)^{-1} = \frac{2}{\sigma} - 1 \quad \forall\ N \geqslant 2. \tag{35}$$

It should be noted that the derivation above does not consider anomalous dimension effects, which we expect to be subleading with respect to the main contribution to ν. The latter result [black crosses in 3(a)] is in perfect agreement with our findings, but it cannot be reproduced by MC simulations, possibly due to finite-size effects close to the $\nu^{-1} \simeq 0$ region.

While we expect numerical simulations to be very accurate only for σ well below 2, it is also found that our results for $(z\nu)^{-1}$ should be slightly underestimated in the intermediate σ region, even if reproducing all the limiting behaviors. This is in agreement with previous investigation on classical LR and SR O(N) models [23,38,39], where the FRG results for the correlation length exponent are found to slightly overestimate high precision numerical estimation in the whole dimension range. Here the same effect is present as a function of the exponent σ. Thus it is fair to conclude that the exact correlation length exponent curve for the $N = 2$ case in the $d = 1$ case should be located in between the theoretical curve and the reported numerical estimates. Apart from the $N = 2$ case, which is rather special, exhibiting the BKT transition, we expect our findings to be very accurate for all $N > 2$, since for large components number ($N \to \infty$) the ansatz (5) becomes exact.

V. THE SPHERICAL MODEL CASE

In the infinite components limit ($N \to \infty$), the classical O(N) model lies in the same universality of the spherical model, which is exactly solvable. Within the classical analog of ansatz (5), the exact expressions for the critical exponents can be recovered [38,39]. Such a property also remains valid in the quantum case [56,57], leading to

$$z = \frac{\sigma}{2}, \tag{36}$$

$$\nu = \frac{2}{2d - \sigma}. \tag{37}$$

These expressions, obtained from the exact solution of the flow equations in the $N \to \infty$ limit, give the exact values of the critical exponents for the quantum spherical model [57].

It should be noted that the same expression for the critical exponents could be obtained via the effective dimension relations. Indeed in the classical SR O(N) models we have

$\nu = (d_{\rm SR} - 2)^{-1}$ and $\eta = 0$; then employing the effective dimension relation, Eq. (17), and multiplying by $2/\sigma$ we immediately get (37). Thus the effective dimension relation is exact in the $N \to \infty$ limit, as expected from the classical spherical model case [50].

VI. CONCLUSIONS

The results in Fig. 3 completes the necessary information to derive the scaling behavior of all the thermodynamic quantities of quantum LR rotor models. Indeed we determined both the dynamical critical exponent z and the inverse correlation length exponent ν^{-1} from which all the critical exponents can be obtained via the scaling relations. It should be noted that, in principle, one should also know the values for the anomalous dimension η, which however in the LR system is found to be always given by the mean field result $\eta = 2 - \sigma$. In Table I the values of the critical exponents for various values of N, d, and σ are reported.

Our motivations for the study presented in this paper were twofold: on one side the progress in the control of atomic, molecular, and optical systems made it possible to experimentally implement quantum long-range (LR) systems with tunable parameters, including the range of the interactions. These advancements paved the way for the study of the phase diagram and of the criticality quantum LR models, calling for the development of a unified treatment of their properties. On the other side, this motivates the search for the analogies and differences between quantum and classical LR systems, to understand what phenomena in classical systems typically due to the long-range character are also present in quantum systems and which are specifically connected to the presence of quantum fluctuations. Finally, a last motivation for the present work was to have a compact formalism to study different quantum LR systems at once, allowing us also to clarify the relation between quantum LR models in d dimensions and the corresponding anisotropic classical models in $d + 1$ dimensions.

We therefore developed a general description of quantum LR models based on renormalization group (RG) techniques and capable of working for different dimensions d, power-law decay exponents σ, and groups of symmetry. We focused on the derivation of universal exponents describing the critical behavior of power-law decaying interacting N-component quantum rotor models. The $N = 1$ case corresponds to a quantum LR Ising model, which is described by Hamiltonian (1), while the general $N \geqslant 2$ cases, described by Hamiltonian (2), are quantum generalizations of the celebrated O(N) symmetric models.

LR power decaying interactions deeply affect the critical behavior of quantum models and their critical dynamics. We remind that the usual short-range (SR) quantum model with local propagator $G^{-1}(q) \propto q^2$ has a quantum critical point at zero temperature which lies in the same universality class of the classical phase transition in $d + 1$ dimension [22]. As discussed in the text, in the LR interacting case the low-energy behavior of the propagator is not analytic in the momentum, being $G^{-1}(q) \propto q^{\sigma}$, and the quantum field theory obtained using Trotter decomposition is anisotropic in the extended $d + 1$-dimensional space. However, our analysis clearly shows that the problem is reduced to the study of a LR classical problem only for σ large enough, and that this is not the case for σ smaller than a critical value. For such small values of σ the correlation functions are genuinely strongly anisotropic in the spatial and time coordinates and the isotropy is not restored even at the criticality.

The mapping of a quantum SR universality into its $d + 1$ classical equivalent obtained by Trotter decomposition is exact, and so it is the relation between the universal behavior of a quantum LR model and its anisotropic classical equivalent. On the other hand, from the first line of Eq. (26) a correspondence emerges between the quantum LR system in dimension d and its classical analogous in dimension $d + z$.

Such correspondence naturally emerges in our framework, even if it appears not to be exact due to the difference in the kinetics sectors of the two models. This scenario is schematically drawn in Fig. 4, where the dashed lines stand for a nonexact mapping. Indeed the coefficients proportional to η_τ appearing in the second and third lines of Eq. (26) are substantially different from the ones of the classical case [23].

These coefficients are regulator dependent quantities whose precise value can be shown to not affect much the final result for the critical exponents; indeed both coefficients are proportional to η_τ, which is never larger than 0.25, and they can be shown to produce corrections to the critical exponents which are of order $O(\eta_\tau^2)$. We also explicitly solved the equations both

TABLE I. Critical exponents for quantum LR O(N) models for various values of N, d, and σ. The rational quantities are the exact results, in the mean-field validity region. The numerical values are obtained by self-consistent solutions of the flow equations with numerical precision ±0.002.

		$d = 2$		$d = 3$	
N	σ	z	$1/\nu$	z	$1/\nu$
1	2/3	1/3	2/3	1/3	2/3
2	2/3	1/3	2/3	1/3	2/3
3	2/3	1/3	2/3	1/3	2/3
1	4/5	0.402	0.720	2/5	4/5
2	4/5	0.402	0.703	2/5	4/5
3	4/5	0.402	0.689	2/5	4/5
1	1	0.513	0.780	1/2	1
2	1	0.513	0.706	1/2	1
3	1	0.512	0.657	1/2	1
1	6/5	0.633	0.832	3/5	6/5
2	6/5	0.630	0.641	3/5	6/5
3	6/5	0.626	0.545	3/5	6/5
1	7/5	0.761	0.878	0.700	1.365
2	7/5	0.751	0.487	0.700	1.357
3	7/5	0.737	0.376	0.700	1.351
1	8/5	0.892	0.921	0.803	1.442
2	8/5	0.866	0.269	0.803	1.405
3	8/5	0.839	0.224	0.803	1.377
1	9/5	1.000	0.948	0.911	1.504
2	9/5	0.959	0.110	0.912	1.422
3	9/5	0.926	0.105	0.911	1.364
1	2	1.000	0.948	1.000	1.543
2	2	1.000	0.000	1.000	1.417
3	2	1.000	0.000	1.000	1.327

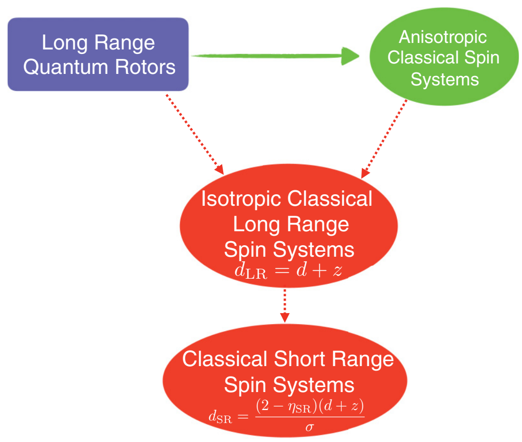


FIG. 4. Schematic representation of the relations between quantum and classical LR systems investigated in the paper. The green line indicates the exact mapping obtained via Trotter decomposition, while the red dotted lines indicate effective dimension relations, which, although not exact, produce very accurate results in the case of quantum LR models.

in presence and absence of the two coefficients in the classical case [23], showing that the error arising from neglecting these two quantities is well below 1% in all the σ range; in the quantum case such error is expected to be smaller.

The results depicted in Fig. 3 are thus consistent with the naive expectation of the equal universality class for quantum systems in dimension d and classical systems in dimension $d + z$. As discussed before, even at our simple approximation level such equivalence is spoiled by the presence of the regulator dependent coefficients multiplying the non linear term in (26). However, our results for the correlation length exponent can be exactly mapped into the ones of classical SR models via the effective dimension relation (31), since we disregarded the regulator dependents factors in the second line of Eq. (26). These results are in good agreement with the MC data taken from [52] (see Figs. 2 and 3), showing, once again, the reliability of the effective dimension approach [23].

The phase diagram of a LR system can be then represented in the (d,σ) plane, as in Fig. 1. For small enough values of the decay exponent σ the system undergoes spontaneous symmetry breaking with mean field exponents given by relations (14), (15), and (19) (cyan shaded region in Fig. 1). For intermediate values of σ the system has peculiar LR exponents which continuously merge with the SR values at the σ_* threshold.

In analogy with the classical case [23,34], such a threshold value is given by $\sigma^* = 2 - \eta_{\mathrm{SR}}$, where η_{SR} is the anomalous dimension found in the pure SR system.

It is worth noting that for continuous symmetries $N \geqslant 2$ it is also possible to identify a region [gray shaded area in Fig. 1(b)] where no phase transition is possible. Such identification does not have a counterpart in the discrete symmetry case of the Ising $N = 1$ model, where no exact lower critical dimension is known even in the traditional SR case.

This theoretical picture advocates for more detailed comparison with ongoing experiments on LR power decaying interactions in quantum systems. Their understanding can pave the way to comprehension of the role of dimension in quantum phase transitions and in the reliability of effective dimensional mappings.

Finally, we mention that a topic we did not address in detail in this paper is the emergence of the properties of the BKT phase transition in the $d = 1$, $N = 2$ limit. We think this is a deserving subject of investigation, also in connection with the properties of the one-dimensional XXZ model with LR couplings [58,59].

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