Quantum efficiency bound for continuous heat engines coupled to noncanonical reservoirs

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(Received 19 June 2017; revised manuscript received 26 August 2017; published 15 September 2017)

We derive an efficiency bound for continuous quantum heat engines absorbing heat from squeezed thermal reservoirs. Our approach relies on a full-counting statistics description of nonequilibrium transport and it is not limited to the framework of irreversible thermodynamics. Our result, a generalized Carnot efficiency bound, is valid beyond the small-squeezing and high-temperature limit. Our findings are embodied in a prototype three-terminal quantum photoelectric engine where a qubit converts heat absorbed from a squeezed thermal reservoir into electrical power. We demonstrate that in the quantum regime, the efficiency can be greatly amplified by squeezing. From the fluctuation relation, we further receive other operational measures in linear response, for example, the universal maximum power efficiency bound.

DOI: 10.1103/PhysRevB.96.104304

I. INTRODUCTION

The efficiency of heat engines, defined by the ratio of the extracted work to the absorbed heat, is fundamentally restricted by the second law of thermodynamics to the Carnot limit. This canonical bound is being challenged nowadays by quantum and classical effects [1,2]. For example, quantum phenomena such as steady-state coherence [3–5] and quantum correlations [6], which persist in multilevel quantum systems, are suggested as a resource for the design of more efficient engines.

In addition, nonequilibrium, stationary reservoirs that are characterized by additional parameters besides their temperature are exploited to construct devices with efficiency beyond the Carnot bound [7–14]. In particular, a four-stroke Otto heat engine, operating between two reservoirs, a hot *squeezed* thermal bath, and a cold thermal bath, was examined in Refs. [7,8,10], reaching a unit value in the asymptotic, high-squeezing limit.

Beyond the analysis of the averaged efficiency, a quantum mechanical, full-counting statistics derivation provides the ultimate, fundamental description of out-of-equilibrium quantum statistical phenomena. Such an approach hands over symmetries, bounds, and noise terms (cumulants) to characterize, e.g., particle and energy transport. It is unclear, however, whether the steady-state fluctuation symmetry [15–17] holds for transport phenomena between noncanonical reservoirs. Another fundamental question is whether quantum principles impose new bounds on energy-conversion efficiency in such systems, to extend the second law of thermodynamics.

In this paper, we fill these gaps by employing a full-counting statistics approach to study energy conversion in quantum engines absorbing heat from a noncanonical reservoir. Our device consists of a single qubit coupled to a hot squeezed photon bath and two cold electronic reservoirs (the source and drain); see Fig. 1. We show that the nonequilibrium fluctuation relation (FR) for entropy production can be recovered once an effective temperature for the squeezed thermal bath is identified. From the fluctuation symmetry, we derive a generalized, quantum efficiency bound for the heat engine, surpassing the Carnot limit. Since the FR encompasses linear-response thermodynamics, we immediately receive other operational measures of heat engines in the linear response: the universal maximum power efficiency bound [18,19] and properties of fluctuations statistics [20]. Our theory is exemplified with a quantum mechanical, full-counting statistics description of a nanoscale photoelectric device.

We begin with a quick review of the fundamentals of the entropy production fluctuation theorem [15–17]. Based on the microreversibility of the Hamiltonian dynamics and the canonical form of the initial condition, one can prove a universal relation in the steady state,

$$\ln\left[\frac{P_t(\Delta S)}{P_t(-\Delta S)}\right] = \Delta S. \tag{1}$$

Here, $P_t(\Delta S)$ is the probability distribution for entropy production ΔS during a time interval t. It is convenient to define the characteristic function

$$\mathcal{Z}(\lambda) \equiv \int d\Delta S\, e^{i\lambda\Delta S}\, P_t(\Delta S), \tag{2}$$

with λ the so-called counting parameter. One can immediately prove the Gallavotti-Cohen fluctuation symmetry from the fluctuation relation (1), $\mathcal{Z}(\lambda) = \mathcal{Z}(-\lambda + i)$ [15–17]. Moreover, by using $\lambda = 0$ in Eq. (2), it is easy to prove that $1 = \langle e^{-\Delta S}\rangle$. This equality immediately leads to the second law of thermodynamics, $\langle \Delta S\rangle \geqslant 0$, by using Jensen's inequality for convex functions.

II. THREE-TERMINAL PHOTOELECTRIC DEVICES

We now apply these considerations onto a quantum heat engine consisting of three terminals. In our construction (see Fig. 1), a qubit is coupled to a photonic heat source (ph), which may be canonical (equilibrium) or squeezed (out of equilibrium). In addition, the qubit is exchanging energy with an electronic circuit with two metal leads, L and R, which can be set out of equilibrium by the application of a finite

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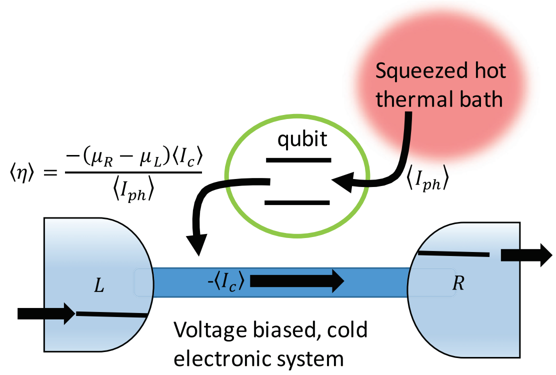

FIG. 1. Photoelectric quantum heat engine made of a qubit as the "working fluid." Energy absorbed by the qubit from a hot squeezed thermal reservoir is converted to electrical power in the cold source-drain junction.

voltage bias $\Delta\mu = \mu_R - \mu_L$ and a temperature difference. For simplicity, we assume that the two electrodes are maintained at the same temperature, $\beta_{\rm el} = \beta_\alpha$; $\alpha = L,R$, and that the photon bath is hotter than the electronic system, $\beta_{\rm ph} < \beta_{\rm el}$. Our interest here is in the conversion of photon energy into electrical work.

In order to describe the system quantum mechanically, we use the two-time measurement protocol [15,16] and define the characteristic function as

$$\mathcal{Z}(\lambda_c,\lambda_e,\lambda_{\rm ph}) = \langle e^{i\lambda_c\hat{A}_c + i\lambda_e\hat{A}_e + i\lambda_{\rm ph}\hat{A}_{\rm ph}} e^{-i\lambda_c\hat{A}_c(t) - i\lambda_e\hat{A}_e(t) - i\lambda_{\rm ph}\hat{A}_{\rm ph}(t)}\rangle.$$

Here, $\lambda_{c,e,{\rm ph}}$ are counting parameters for charge, electronic energy, and photonic energy, respectively. $\hat{A}_c$, $\hat{A}_e$, and $\hat{A}_{\rm ph}$ are the respective operators: $\hat{A}_c$ is the number operator corresponding to the total charge in, e.g., the R lead. $\hat{A}_e$ is the Hamiltonian operator for the R electrode and $\hat{A}_{\rm ph}$ is the Hamiltonian operator for the photon bath. Time evolution corresponds to the Heisenberg representation, and $\langle\cdot\rangle$ represents an average with respect to the total initial density matrix, which takes a factorized form with respect to the system (s) and (L, R, and ph) baths, $\rho_T(0) = \rho_s(0)\otimes\rho_L\otimes\rho_R\otimes\rho_{\rm ph}$. The state of the metal leads is described by a grand canonical distribution, $\rho_\alpha = \exp[-\beta_{\rm el}(\hat{H}_\alpha - \mu_\alpha\hat{N}_\alpha)]/Z_\alpha$, with $Z_\alpha = {\rm Tr}\{\exp[-\beta_{\rm el}(\hat{H}_\alpha - \mu_\alpha\hat{N}_\alpha)]\}$ as the partition function.

A. Equilibrium thermal photon bath

Let us begin by assuming that the state of the photon bath is canonical, $\rho_{\rm ph} = \exp[-\beta_{\rm ph}\hat{H}_{\rm ph}]/Z_{\rm ph}$, with $Z_{\rm ph} = {\rm Tr}[\exp(-\beta_{\rm ph}\hat{H}_{\rm ph})]$. The fluctuation relation (1) translates to

$$\frac{P_t(N,E_e,Q_{\rm ph})}{P_t(-N,-E_e,-Q_{\rm ph})} = e^{\beta_{\rm el}\Delta\mu N + (\beta_{\rm el}-\beta_{\rm ph})Q_{\rm ph}}. \tag{3}$$

Here, N denotes the number of electrons transferred from R to L during the time interval t. Similarly, E_e is the electronic energy and $Q_{\rm ph}$ is the photonic heat that are exchanged between the baths during the time interval t. The characteristics function thus satisfies

$$\mathcal{Z}(\lambda_c,\lambda_e,\lambda_{\rm ph}) = \mathcal{Z}[-\lambda_c + i\beta_{\rm el}(\mu_R-\mu_L), -\lambda_e, -\lambda_{\rm ph} - i(\beta_{\rm ph}-\beta_{\rm el})]. \tag{4}$$

This relation immediately implies that

$$1 = \langle e^{-\beta_{\rm el}\Delta\mu N + (\beta_{\rm ph}-\beta_{\rm el})Q_{\rm ph}}\rangle. \tag{5}$$

Using Jensen's inequality, we receive $[-\beta_{\rm el}\Delta\mu\langle N\rangle + (\beta_{\rm ph} - \beta_{\rm el})\langle Q_{\rm ph}\rangle] \leqslant 0$. The efficiency, $\langle\eta\rangle \equiv -\frac{\Delta\mu\langle N\rangle}{\langle Q_{\rm ph}\rangle}$ [21], thus obeys the Carnot bound ($T = 1/\beta$),

$$\langle\eta\rangle \leqslant \frac{\beta_{\rm el}-\beta_{\rm ph}}{\beta_{\rm el}} = 1 - \frac{T_{\rm el}}{T_{\rm ph}}. \tag{6}$$

B. Noncanonical photon bath

We now repeat this exercise—with a squeezed, hot thermal reservoir. The electric field of a single-mode wave can be written as a combination of orthogonal (quadrature) components, which oscillate as $\cos\omega t$ and $\sin\omega t$ [22]. Squeezed states have reduced fluctuations in one of the quadratures—but enhanced noise in the other quadrature—so as to satisfy the bosonic commutation relation. Such states are defined by two parameters: the squeezing factor r and phase ϕ [22].

For simplicity, the quantum "working fluid" system includes a single qubit with an energy gap $\hbar\omega_0$. The squeezed bath can excite and deexcite the qubit, with rate constants $k_u^{\rm ph}$ and $k_d^{\rm ph}$, satisfying [7]

$$\frac{k_d^{\rm ph}}{k_u^{\rm ph}} = \frac{N(\omega_0)+1}{N(\omega_0)}. \tag{7}$$

Here [23], $N(\omega_0) = N_{\rm th}(\omega_0)(\cosh^2 r + \sinh^2 r) + \sinh^2 r$, with the squeezing parameter r reflecting the nonequilibrium nature of the bath. The phase ϕ does not appear in this expression, as it only affects transients. For a canonical thermal bath ($r = 0$), the occupation number reduces to the Bose-Einstein distribution function, $N(\omega_0) \to N_{\rm th}(\omega_0) = 1/[e^{\beta_{\rm ph}\hbar\omega_0} - 1]$, and the rate constants satisfy the detailed balance relation with respect to the photon bath, $k_d^{\rm ph}(\omega_0)/k_u^{\rm ph}(\omega_0) = e^{\beta_{\rm ph}\hbar\omega_0}$. To restore the detailed balance relation for the $r \neq 0$ case, one can identify an effective temperature, which is unique in the present model [7],

$$\beta_{\rm eff}(\beta_{\rm ph},r,\omega_0) = \frac{1}{\hbar\omega_0}\ln\frac{1+N(\omega_0)}{N(\omega_0)}. \tag{8}$$

Simple manipulations provide

$$\beta_{\rm eff} = \beta_{\rm ph} + \frac{1}{\hbar\omega_0}\ln\left[\frac{1+(1+e^{-\beta_{\rm ph}\hbar\omega_0})\sinh^2 r}{1+(1+e^{\beta_{\rm ph}\hbar\omega_0})\sinh^2 r}\right]. \tag{9}$$

It is important to note the following: (i) $\beta_{\rm eff} \leqslant \beta_{\rm ph}$. This observation implies that more work can be extracted from a squeezed bath than the case with $r = 0$. (ii) The effective temperature (9) may depend on system parameters, i.e., the energy gap ω_0 in the present case. However, in the small-r and high-temperature limit, one recovers a proper "thermodynamical" temperature,

$$\beta_{\rm eff} \to \frac{\beta_{\rm ph}}{1+2\sinh^2 r}, \tag{10}$$

which is solely described in terms of bath parameters. Therefore, in this limit, universal relations of traditional linear irreversible thermodynamics hold.

Identifying the entropy production associated with the photon energy flow by $\langle\Delta S\rangle = (\beta_{\rm el} - \beta_{\rm eff})\langle Q_{\rm ph}\rangle$, one performs a quantum mechanical, counting statistics analysis, similarly to the canonical case [24], and confirms the symmetry given by Eq. (4), only replacing $\beta_{\rm ph}$ by $\beta_{\rm eff}$,

$$\mathcal{Z}(\lambda_c,\lambda_e,\lambda_{\rm ph}) = \mathcal{Z}[-\lambda_c + i\beta_{\rm el}(\mu_R - \mu_L), -\lambda_e, -\lambda_{\rm ph} - i(\beta_{\rm eff} - \beta_{\rm el})]. \tag{11}$$

The FR implies that $1 = \langle e^{-\beta_{\rm el}\Delta\mu N + (\beta_{\rm eff}-\beta_{\rm el})Q_{\rm ph}}\rangle$, and thus the averaged efficiency, $\langle\eta\rangle \equiv -\Delta\mu\langle N\rangle/\langle Q_{\rm ph}\rangle$, is bounded by

$$\langle\eta\rangle \leqslant 1 - \frac{\beta_{\rm eff}}{\beta_{\rm el}}. \tag{12}$$

This bound is universal, holding beyond the squeezed-bath case. It is valid for any nonequilibrium thermal bath that can be characterized by a unique, stationary, effective temperature; see Ref. [11] for some examples. Explicitly, the efficiency bound for our photoelectric engine is given by

$$\langle\eta\rangle \leqslant 1 - \frac{T_{\rm el}}{T_{\rm ph}} + \frac{1}{\beta_{\rm el}\hbar\omega_0}\ln\left[\frac{1+(1+e^{\beta_{\rm ph}\hbar\omega_0})\sinh^2 r}{1+(1+e^{-\beta_{\rm ph}\hbar\omega_0})\sinh^2 r}\right], \tag{13}$$

which is the main result of our work. It was derived from the fluctuation theorem and is valid to describe continuous quantum heat engines, unlike earlier studies which were focused on four-stroke engines; see, e.g., Ref. [11]. Since the third term in this expression is positive for nonzero r, squeezing of a thermal bath always increases the heat-to-work efficiency bound.

We now discuss several interesting limits of Eq. (13). First, we expand it close to thermal equilibrium assuming $\sinh^2 r$ is a small parameter. In addition, we assume that the temperature of the photon bath is high, $\beta_{\rm ph}\hbar\omega_0 \ll 1$. The expression in the square brackets reduces to

$$\ln\left[1+\frac{(e^{\beta_{\rm ph}\hbar\omega_0} - e^{-\beta_{\rm ph}\hbar\omega_0})\sinh^2 r}{1+(1+e^{-\beta_{\rm ph}\hbar\omega_0})\sinh^2 r}\right] \to \frac{\beta_{\rm ph}\hbar\omega_0 \times 2\sinh^2 r}{1+2\sinh^2 r}, \tag{14}$$

and Eq. (13) becomes

$$\langle\eta\rangle \leqslant 1 - \frac{T_{\rm el}}{T_{\rm ph}(1+2\sinh^2 r)}. \tag{15}$$

Remarkably, this agrees with Refs. [8,10]. Recall that our derivation concerns continuous heat engines; Refs. [8,10], in contrast, received this limit by constructing a four-stroke cycle. This agreement can be rationalized by noting that Eq. (15) should be regarded as a linear-response limit for r, which is a resource to drive energy current between equal-temperature baths [7]. The general form of the bound received in Ref. [10] for the discrete (stroke) Otto-like heat engine is different than the one obtained in our continuous heat engine case, given by Eq. (13). This is because the quantum limit is *nonuniversal* and depends on model parameters. In the proper thermodynamic limit (small r and high temperature), the different efficiency bounds reduce to the universal form given by Eq. (15).

Another interesting case is the deep quantum regime, $\beta_{\rm ph}\hbar\omega_0 \gg 1$. Assuming small r, we receive from Eq. (13) an exponential quantum enhancement in comparison to the classical case,

$$\langle\eta\rangle \leqslant 1 - \frac{T_{\rm el}}{T_{\rm ph}} + \frac{1}{\beta_{\rm el}\hbar\omega_0}\left[\frac{\sinh^2 r}{1+\sinh^2 r} \times e^{\beta_{\rm ph}\hbar\omega_0}\right]. \tag{16}$$

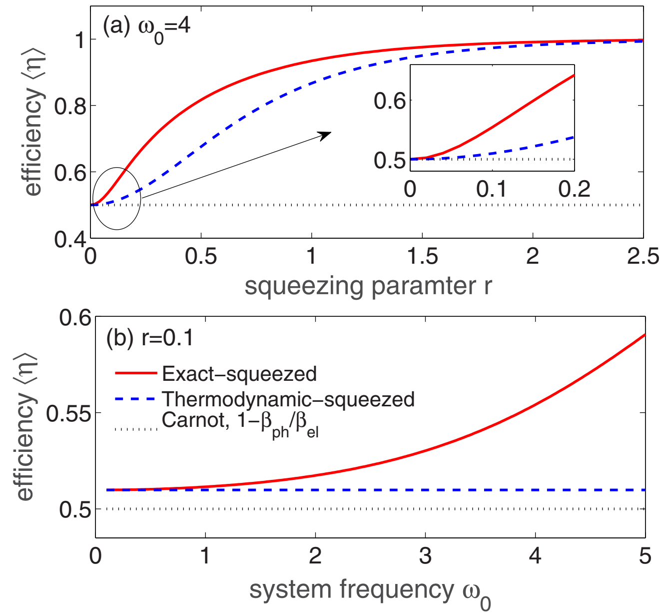

FIG. 2. Efficiency bound as a function of (a) squeezing parameter and (b) subsystem frequency. Exact result from Eq. (13) (solid line), thermodynamical limit from Eq. (15) (dashed line), and Carnot bound (dotted line). We use $\beta_{\rm el} = 2$ and $\beta_{\rm ph} = 1$.

Note that the expansion assumes that the term inside the square brackets is kept below 1. Finally, at large r, the natural logarithm term in (13) cancels out the second contribution for both high and low $T_{\rm ph}$. The efficiency bound then saturates to a unit value, $\langle\eta\rangle \to 1$, realizing a complete conversion of heat to work. We display these results in Fig. 2: Squeezing enhances the efficiency beyond the Carnot limit. In the quantum regime, $\beta_{\rm ph}\omega_0 > 1$, the bound is greatly reinforced beyond the "thermodynamical" value, given by Eq. (15).

A squeezed bath coupled to a qubit can be described by a single, unique effective temperature in the thermodynamical limit of high $T_{\rm ph}$ and small r. Since the fluctuation theorem embodies linear irreversible thermodynamics, all linear-response operational results immediately follow. In particular, the averaged maximum power efficiency (MPE) satisfies the universal linear-response result [18] $\langle\eta^*\rangle = \langle\eta_M\rangle/2$, with $\langle\eta_M\rangle$ the upper bound in Eq. (15). For a four-stroke Otto engine, the MPE is given by the Curzon-Ahlborn bound (beyond linear response), $\langle\eta^*\rangle = 1 - \sqrt{\frac{T_{\rm cold}}{T_{\rm eff}}}$, with the identification of the thermodynamic temperature (10). This agrees with Ref. [8].

Our approach can be further generalized to the case with the metals prepared at different temperatures. In particular, in the Appendix we analyze the operation of a continuous quantum absorption refrigerator with three thermal reservoirs: L (hot), R (cold), and ph (termed "work"). We assume that $T_{\rm ph} > T_L > T_R$, with the work reservoir prepared in a squeezed thermal state, and thus characterized by a the effective temperature $T_{\rm eff}$. Using the fluctuation symmetry, we receive a generalized Carnot bound for absorption refrigeration, $\langle\eta_{\rm ref}\rangle \leqslant \frac{\beta_L-\beta_{\rm eff}}{\beta_R-\beta_L}$. This result agrees with a previous study [25], but moreover generalizes it beyond the weak system-bath coupling limit.

III. EXAMPLE WITH A CLOSED-FORM CUMULANT GENERATING FUNCTION

So far, we derived an efficiency bound for continuous quantum heat engines based on the fluctuation symmetry. We now proceed and describe a device where a closed-form expression for the cumulant generating function (CGF) $\mathcal{G}(\lambda) = \lim_{t\to\infty} \frac{1}{t} \ln \mathcal{Z}(\lambda)$ is achieved. Here, λ collectively refers to the three counting fields. From the CGF, all cumulants of the charge current, electronic energy current, and photonic current are available. The closed-form expression for the efficiency of the engine allows us to examine its actual performance under different conditions. Our model photoelectric heat engine is described by the Hamiltonian

$$\hat{H} = \hat{H}_s + \hat{H}_{\rm el} + \hat{H}_{\rm ph} + \hat{V}_{s-el} + \hat{V}_{s-ph}. \quad (17)$$

It comprises a single qubit $\hat{H}_s = \frac{\hbar\omega_0}{2}\hat{\sigma}_z$ of energy gap $\hbar\omega_0$. The photon bath is written in terms of bosonic creation $\hat{a}_k^\dagger$ and annihilation $\hat{a}_k$ operators, $\hat{H}_{\rm ph} = \sum_k \omega_k \hat{a}_k^\dagger \hat{a}_k$. The electronic circuit includes two sites (quantum dots) denoted by "d" and "a," each coupled to their respective metal leads, L and R. The corresponding Hamiltonian is

$$\hat{H}_{\rm el} = \epsilon_d \hat{c}_d^\dagger \hat{c}_d + \epsilon_a \hat{c}_a^\dagger \hat{c}_a + \sum_{\alpha,j} \epsilon_{\alpha,j} \hat{c}_{\alpha,j}^\dagger \hat{c}_{\alpha,j} + \sum_j v_{L,j} \hat{c}_{L,j}^\dagger \hat{c}_d + \sum_j v_{R,j} \hat{c}_{R,j}^\dagger \hat{c}_a + \text{H.c.} \quad (18)$$

Here, $\hat{c}$ ($\hat{c}^\dagger$) are fermionic annihilation (creation) operators. Energy is exchanged between the qubit and the reservoirs via the interaction terms

$$\hat{V}_{s\text{–}el} = g\hat{\sigma}_x(\hat{c}_d^\dagger \hat{c}_a + \hat{c}_a^\dagger \hat{c}_d), \quad \hat{V}_{s\text{–}ph} = \hat{\sigma}_x \sum_k g_k(\hat{a}_k^\dagger + \hat{a}_k). \quad (19)$$

In words, the excitation or relaxation of the qubit couples to the exchange of electrons between the two sites and the displacement of harmonic modes. The CGF is derived using a quantum master equation that is correct to second order in the electron-qubit and the photon-qubit couplings [26,27],

$$\mathcal{G}(\lambda) = -\tfrac{1}{2}(k_u + k_d) + \tfrac{1}{2}\sqrt{(k_u - k_d)^2 + 4\, k_u^\lambda k_d^\lambda}. \quad (20)$$

Here, $k_{d,u}^\lambda$ are the relaxation (d) and excitation (u) rate constants of the qubit, with transitions induced by the reservoirs, e.g.,

$$k_d^\lambda = \left[k_d^{el}\right]^\lambda + \left[k_d^{\rm ph}\right]^\lambda, \quad \left[k_d^{el}\right]^\lambda = \left[k_d^\lambda\right]^{L\to R} + \left[k_d^\lambda\right]^{R\to L}. \quad (21)$$

Specifically, $[k_d^\lambda]^{L\to R}$ describes a deexcitation process of the qubit, induced by an electron moving from the L to the R metal. It involves the release of energy at the right metal (where counting is performed),

$$\left[k_d^\lambda\right]^{L\to R} = \int \frac{d\epsilon}{2\pi}\big[f_L(\epsilon)(1 - f_R(\epsilon + \omega_0)) J_L(\epsilon) J_R(\epsilon + \omega_0) \times e^{-i(\lambda_c + (\epsilon+\omega_0)\lambda_e)}\big]. \quad (22)$$

Here, e.g., $J_L(\epsilon) = g\frac{\Gamma_L(\epsilon)}{(\epsilon-\epsilon_d)^2 + \Gamma_L(\epsilon)^2/4}$ is the spectral function of the L metal, determined by the dot-metal hybridization energy $\Gamma_\alpha(\epsilon) = 2\pi \sum_j |v_{\alpha,j}|^2 \delta(\epsilon - \epsilon_{\alpha,j})$. Transitions induced by the squeezed photon bath satisfy [26]

$$\left[k_d^\lambda\right]^{\rm ph} = \Gamma_{\rm ph}(\omega_0)\,[N(\omega_0) + 1]\, e^{-i\lambda_{\rm ph}\omega_0},$$
$$\left[k_u^\lambda\right]^{\rm ph} = \Gamma_{\rm ph}(\omega_0)\, N(\omega_0)\, e^{i\lambda_{\rm ph}\omega_0}. \quad (23)$$

Here, $\Gamma_{\rm ph}(\omega) = 2\pi \sum_k |g_k|^2 \delta(\omega - \omega_k)$, and $N(\omega_0)$ was defined below Eq. (7).

Under the transformations $\lambda_c \to -\lambda_c + i\beta_{\rm el}(\mu_R - \mu_L), \lambda_e \to -\lambda_e$, and $\lambda_{\rm ph} \to -\lambda_{\rm ph} - i(\beta_{\rm eff} - \beta_{\rm el})$, the rates modify to

$$\left[k_d^\lambda\right]^{el} \to \left[k_u^\lambda\right]^{el} e^{\beta_{\rm el}\omega_0},$$
$$\left[k_d^\lambda\right]^{\rm ph} \to \left[k_u^\lambda\right]^{\rm ph} e^{\beta_{\rm el}\omega_0}. \quad (24)$$

Similarly, the excitation rates translate as $[k_u^\lambda]^{\rm ph} \to [k_d^\lambda]^{\rm ph} e^{-\beta_{\rm el}\omega_0}$, $[k_u^\lambda]^{el} \to [k_d^\lambda]^{el} e^{-\beta_{\rm el}\omega_0}$. Note that the transformation of the phonon-bath-induced rates are performed with the effective temperature defined in Eq. (8). Given these rules for the rate constants, we confirm that the CGF (20) obeys the FR in Eq. (11). The electron charge current is given by

$$\langle I_c \rangle = \frac{\partial \mathcal{G}(\lambda)}{\partial(i\lambda_c)}\Big|_{\lambda=0} = \frac{k_d \frac{\partial(k_u^\lambda)}{\partial(i\lambda_c)} + k_u \frac{\partial(k_d^\lambda)}{\partial(i\lambda_c)}}{k_u + k_d}. \quad (25)$$

An analogous expression is written for $\langle I_{\rm ph} \rangle$. In Fig. 3, we display the averaged efficiency of the engine $\langle \eta \rangle = -\Delta\mu \langle I_c \rangle / \langle I_{\rm ph} \rangle$ for certain parameters, once we set $T_{\rm ph} > T_{\rm el}$

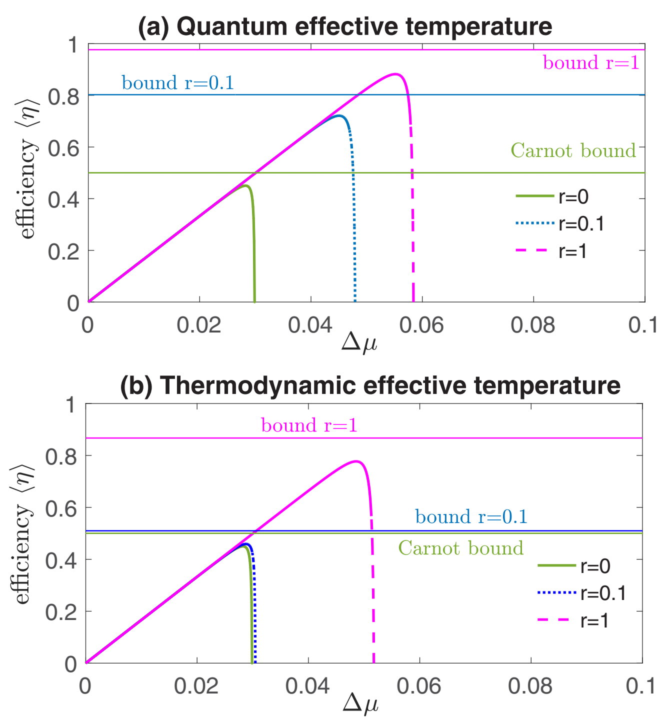

FIG. 3. Efficiency of a photoelectric heat engine. Comparison of results making use of the (a) quantum effective temperature (9) and (b) thermodynamical effective temperature (10). Parameters are $\epsilon_d = -0.03$, $\epsilon_a = 0.03$, $\omega_0 = 0.06$, $g = 0.1$, $\Gamma_{L,R} = 0.01$, $\Gamma_{\rm ph} = 0.1$, all in eV, $T_{\rm el} = 30$ K, and $T_{\rm ph} = 60$ K. The horizontal lines correspond to bounds, given by Eqs. (13) and (15).

and $\mu_R > \mu_L$. The device operates as a photoelectric engine when heat is absorbed from the photon bath and charge current is flowing against the potential bias. We operate it in the quantum regime, $\omega_0\beta_{\rm ph} \sim 10$, and reveal a significant enhancement of efficiency, largely exceeding the Carnot bound for small squeezing, $r = 0.1$.

IV. SUMMARY

We investigated the operation of heat engines coupled to a squeezed thermal bath. Based on the fluctuation symmetry, we derived a generalized quantum Carnot efficiency bound, as well as other thermodynamical linear-response operational bounds. We exemplified our approach with a quantum mechanical full-counting statistics description of a photoelectric device. In multilevel systems, it may be necessary to define multiple effective temperatures for a noncanonical bath, corresponding to different transitions in the system. The identification of an effective temperature here and in other studies [7,11] was achieved in the limit of weak coupling between the qubit and the environment. Quantum systems that are *strongly* coupled to equilibrium thermal reservoirs are expected to bring in new design rules for energy-conversion devices [28–34]. The description of heat engines that are strongly coupled to noncanonical reservoirs remains a challenge for future work.

ACKNOWLEDGMENTS

D.S. and B.K.A. acknowledge support from a Natural Sciences and Engineering Research Council of Canada Discovery Grant, the Canada Research Chair program, and the CQIQC at the University of Toronto. J.H.J. acknowledges support from the National Natural Science Foundation of China (Grant No. 11675116) and the Soochow University faculty start-up funding.

APPENDIX: EFFICIENCY BOUND FOR ABSORPTION REFRIGERATION WITH A NONCANONICAL PHOTON BATH

The fluctuation symmetry in Eq. (11) can be further generalized to the case with the left and right electron leads prepared at different temperatures. In this case, the fluctuation relation translates to

$$\begin{aligned}\mathcal{Z}(\lambda_c,\lambda_e,\lambda_{\rm ph}) = \mathcal{Z}[&-\lambda_c+i(\beta_R\mu_R-\beta_L\mu_L),-\lambda_e-i(\beta_R-\beta_L),\\ &-\lambda_{\rm ph}-i(\beta_{\rm eff}-\beta_L)].\end{aligned} \tag{A1}$$

This implies that

$$\langle e^{-(\beta_R\mu_R-\beta_L\mu_L)N+(\beta_R-\beta_L)E_e-(\beta_L-\beta_{\rm eff})Q_{\rm ph}}\rangle = 1, \tag{A2}$$

and, following Jensen's inequality, we receive

$$(\beta_R\mu_R-\beta_L\mu_L)\langle N\rangle-(\beta_R-\beta_L)\langle E_e\rangle+(\beta_L-\beta_{\rm eff})\langle Q_{\rm ph}\rangle \geqslant 0. \tag{A3}$$

In order to operate the device as a refrigerator, we assume that $\beta_{\rm eff} < \beta_L < \beta_R$, set $\mu = \mu_L = \mu_R$, and demand that $\langle Q_{\rm ph}\rangle > 0, \langle Q_e\rangle \equiv \langle E_e\rangle - \mu\langle N\rangle \geqslant 0$, where $\langle Q_e\rangle$ is the net heat absorbed from the cold (R) bath, with the remaining heat being dumped into the hot (L) bath. The refrigeration efficiency is defined as the ratio of the heat extracted from the cold bath, $\langle Q_e\rangle$, to the heat absorbed from the "work" environment, $\langle Q_{\rm ph}\rangle$, $\langle\eta_{\rm ref}\rangle \equiv \langle Q_e\rangle/\langle Q_{\rm ph}\rangle$. Following the inequality in Eq. (A3), we immediately receive the general bound

$$\langle\eta_{\rm ref}\rangle \leqslant \frac{\beta_L-\beta_{\rm eff}}{\beta_R-\beta_L}. \tag{A4}$$

This expression was obtained under more restrictive conditions in Ref. [25] by assuming the dynamics obeys a quantum master equation of Lindblad form. Since $\beta_{\rm eff} < \beta_{\rm ph}$, the refrigeration efficiency for the squeezed case equals or exceeds the classical Carnot value.

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