Chiral Maxwell demon in a quantum Hall system with a localized impurity

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We investigate the role of chirality on the performance of a Maxwell demon implemented in a quantum Hall bar with a localized impurity. Within a stochastic thermodynamics description, we investigate the ability of such a demon to drive a current against a bias. We show that the ability of the demon to perform is directly related to its ability to extract information from the system. The key features of the proposed Maxwell demon are the topological properties of the quantum Hall system. The asymmetry of the electronic interactions felt at the localized state when the magnetic field is reversed joined to the fact that we consider energy-dependent (and asymmetric) tunneling barriers that connect such state with the Hall edge modes allow the demon to properly work.

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I. INTRODUCTION

Since Maxwell envisioned in a *gedankenexperiment* the possibility of an entity, an intelligent agent (a demon to Lord Kelvin) capable of separating warm and cold particles of a gas without performing work apparently violating the second law of thermodynamics, the idea attracted plenty of theoretical attention [1]. The apparent paradox was addressed on a basis in which information and entropy must be related [2]. Information is then a physical magnitude and it fulfills physical laws [3]. Erasing information implies energy dissipation [4–6] that compensates the entropy reduction suffered by a system in the presence of the demon and ensures the validity of the second thermodynamic law. Such information-to-energy conversion is regarded as the solution of the Maxwell demon paradox [3]. Nowadays, Maxwell's demon is regarded as a feedback control mechanism to convert information into energy. Even though Maxwell's idea was enunciated as a *gedankenexperiment*, present technologies have made possible to build it at small scales, for instance by using Brownian particles [7,8], single electrons [9], and lasers pulses [10]. However, demons for quantum systems [11] are hard to experimentally design and work and thus show a scarcer experimental activity due to technical difficulty of implementing a truly quantum demon [7,12] despite the possibility of improved performance [13]. It is worth mentioning that the great progress in the development of stochastic thermodynamics has resulted in theoretical proposals for stochastic Maxwell demons [14–20] and finally in an experimental implementation of an autonomous Maxwell demon using coupled quantum dots [21].

A key ingredient for the performance of stochastic Maxwell demons is the breakdown of detailed balance conditions [16] as direct consequence of a feedback mechanism. The breakdown of such relations is usually achieved through an asymmetry in the system, e.g., in the tunneling barriers of quantum dot systems [16,17]. Breaking the local detailed balance (LDB) condition creates an imbalance between forward and backward processes from which the demon can profit.

An extra degree of freedom for the demon to work is found in topological systems. Here, asymmetries, either kinetic (from tunneling events) or electrostatic, appear naturally in quantum Hall platforms due to the chirality of the edge states [22,23]. The out-of-equilibrium LDB is broken in quantum Hall (QH) systems with localized impurities due to a nonsymmetric electrostatic response of the system when the magnetic field is reversed [24]. Precisely, we benefit from this feature to devise a Maxwell demon feedback scheme in a QH device with an impurity. We show how the chirality of the edge channels favors the operation of a demon that pushes an electrical current against a bias. In the demon protocol, the drag of the electrical current opposite to the bias direction occurs by means of information-to-energy conversion as we demonstrate. The protocol requires two conditions to be satisfied: (i) electrostatic interactions must be asymmetric when the magnetic field or edge motion is reversed and (ii) tunneling events between the edge modes and the localized level must be energy dependent. Under these circumstances, the demon is able to convert information into work to drag an electrical current that moves contrary to the applied bias. Below, we describe the theoretical framework for the chiral demon protocol.

II. THEORETICAL APPROACH

As mentioned, our device is a topological setup consisting of a single quasilocalized state of energy ϵ_d and it can only be singly occupied since we restrict ourselves to the Coulomb-blockade regime with a sufficient strong Coulomb energy. Such level is tunnel coupled to chiral edge states of a QH system at filling factor $\nu = 1$ (see Fig. 1). The applied magnetic field defines the direction of transport (represented by

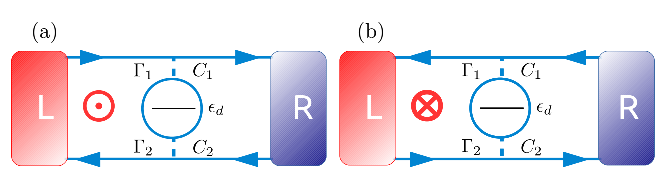

FIG. 1. The device composed by two electronic reservoirs (left and right), the QH bar with edge states represented with lines with arrows (representing direction of transport), and the localized impurity of energy level ϵ_d with tunnel couplings $\Gamma_{1(2)}$ and capacitive couplings $C_{1(2)}$. In (a) (for $B > 0$) the upper (lower) edge state travels from $L(R)$ to $R(L)$, instead in (b) (for $B < 0$) the upper (lower) edge state travels from $R(L)$ to $L(R)$.

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arrows) in each of the chiral modes. Tunneling events between chiral conducting states, $i = 1,2$, and the quasilocalized state are modeled by the tunneling rates $\Gamma_i^{0(1)}$ that depend on energy. Thus, tunneling rates are different depending on the dot charge state, either empty (0) or filled (1). Electrons in the quasilocalized state are capacitively coupled to those in the edge channels via the capacitances C_i [22,23,25]. Due to the chirality of the edge states, the coupling of the quasi-bound state to the left and right reservoirs changes by reversing the magnetic field that exchanges the direction of motion of such modes (see Fig. 1). In this manner, for $B > 0$ [Fig. 1(a)] the level is coupled to the left (L) reservoir through the upper capacitance (C_1) and to the right (R) reservoir through the bottom capacitance (C_2). Under these considerations, the quasilocalized level has a chemical potential given by $\mu_d^+ = \epsilon_d + \frac{e}{C_1+C_2}(C_1V_L + C_2V_R)$, being $V_{L(R)}$ the voltage in reservoir $L(R)$. Changing the field direction ($B < 0$) [Fig. 1(b)] leads to a reverse motion for the conducting modes, with a chemical potential $\mu_d^- = \epsilon_d + \frac{e}{C_1+C_2}(C_2V_L + C_1V_R)$. Note that the difference between both chemical potentials for $B < 0$ and $B > 0$ becomes

$$\mu_d^- - \mu_d^+ = e\eta(V_R - V_L), \tag{1}$$

where $\eta = \frac{C_1-C_2}{C_1+C_2}$. Such difference is null unless the setup is under nonequilibrium conditions and the capacitances are not symmetric, i.e., $\eta \neq 0$. Then, electrostatic interactions are not symmetric when B is reversed [25].

To describe transport through the interacting level, we employ the master-equation framework taking advantage of the fact that in the Coulomb-blockade regime transport happens sequentially. We permit two charge states, namely $|0\rangle$ and $|1\rangle$, whose occupation probabilities are governed by the master equations

$$\dot{p}_0 = -\left(W_{10}^L + W_{10}^R\right)p_0 + \left(W_{01}^L + W_{01}^R\right)p_1,$$
$$\dot{p}_1 = -\left(W_{01}^L + W_{01}^R\right)p_1 + \left(W_{10}^L + W_{10}^R\right)p_0. \tag{2}$$

The transition rates between those two states are given by W_{mn}^α where m is the final state of the dot, n is the initial state, and α is the reservoir to/from which the electron comes. Due to the demon protocol, described below and which needs to be taken into account in the master equation, the transition rates that fill the dot (W_{10}^α) are always given for a situation in which $B > 0$ and those that empty the dot (W_{01}^α) occur for $B < 0$. This is the basis for our Maxwell demon feedback scheme to work properly since both transition rates are not related to each other by a LDB condition [24] as we show below. These previous transition rates, in the sequential tunneling regime, are readily obtained from Fermi's golden rule and read as

$$W_{10}^{L(R)} = \Gamma_{1(2)}^0 f(\mu_d^+ - \mu_{L(R)}), \tag{3}$$

$$W_{01}^{L(R)} = \Gamma_{1(2)}^1[1 - f(\mu_d^- - \mu_{L(R)})], \tag{4}$$

being $f(\mu_d^{+/-} - \mu_{L(R)}) = 1/(1 + \exp\beta[\mu_d^{+/-} - \mu_{L(R)}])$ the Fermi distribution function of the reservoir $L(R)$ with electrochemical potential $\mu_{L(R)}$ and $\beta = 1/k_BT$, where T is the common temperature of all reservoirs. As we remarked before, these rates depend on the orientation of the magnetic field through their dependence on the quasi-bound-state chemical potential $\mu_d^{+/-}$. Due to the chirality of the system, the inversion

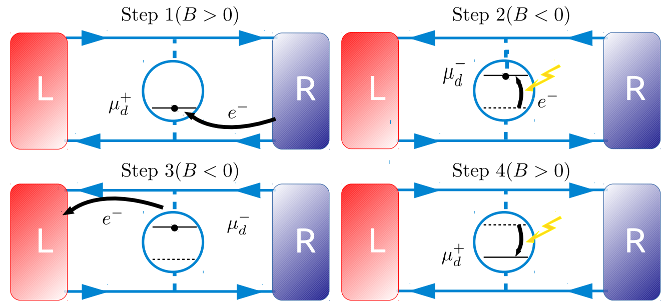

FIG. 2. Sketch of the performed action by the demon to push electrons against a bias. The demon's action is represented by yellow lightning (steps 2 and 4).

of the magnetic field changes the quasi-bound-dot energy at which transport is more favored and breaks the LDB as $\frac{W_{10}^L}{W_{01}^L} \sim \frac{\Gamma_1^0}{\Gamma_1^1} e^{-\beta(\epsilon_d - \frac{e\Delta V}{2})}\{1 - [1 - 2f(\epsilon_d - \frac{e\Delta V}{2})]\beta\eta\frac{e\Delta V}{2}\}$, being $\Delta V = V_L - V_R$ the applied bias. We also observe that an energy-dependent model for the tunneling rates $\Gamma_1^0 \neq \Gamma_1^1$ breaks the LDB even if capacitances are equal. However, we show here that the demon works properly and drags electricity against the voltage bias, solely when both requirements are met (i) asymmetric capacitances and (ii) energy-dependent tunneling rates with asymmetric barriers. We take the tunneling rate energy dependence to be within the WKB approximation. Here, the energy dependence of the tunneling rates is exponential, as shown experimentally [26]

$$\Gamma_\alpha^{0(1)} = \Gamma_\alpha e^{k_\alpha(\mu_d^{+/-} - E_\alpha)}, \tag{5}$$

where $\alpha = 1,2$ denotes the barrier, k_α models the energy dependence, and E_α is the top energy of the barrier. We profit from the fact that in this approximation barriers are asymmetric whenever $k_1 \neq k_2$.

III. DEMON PROTOCOL

As explained before, LDB is broken, either because of the energy dependence of the barriers or due to the distinct orientation of the magnetic field. Under these circumstances, we devise a working process for the demon (see Fig. 2) and investigate if it would be able to drive a current against a voltage bias ($V_L > V_R$), i.e., to drive a current from right to left. The process is as follows:

Step 1. Starting with $B > 0$ and an empty dot, the process is triggered when an electron enters the dot from the right contact (probability $\propto\Gamma_2^0$). Due to the orientation of the magnetic field, the energy of the electron in the dot is μ_d^+.

Step 2. The demon detects that the dot is singly occupied and accordingly it changes the direction of the magnetic field to $B < 0$, this raises the level energy for the localized state up to μ_d^-.

Step 3. Since now the energy of the electron inside the localized state has augmented it becomes easier to tunnel out through the upper barrier (probability $\propto\Gamma_1^1$) and as a consequence the dot is emptied. This leads to transport of one electron from the right to the left reservoir even though $V_L > V_R$.

Step 4. To finish, the demon detects the change in the localized state occupation and restores the initial magnetic field to $B > 0$.

Note that the demon pushes electrons against a bias; this operation is optimal as long as the localized state level energy is increased as much as possible, therefore helping charges climb against the applied bias. The energy difference after reversing the magnetic field is encountered in Eq. (1). Then, we need to take $\eta < 0$, i.e., $C_2 > C_1$, given that $V_L > V_R$. Therefore, the feedback scheme is more effective whenever η is close to -1 or at the very nonlinear regime (large $V_L - V_R$).

Although tunneling processes in Steps 1 and 3 are against the bias and thus less likely, this inconvenience can be overcome by cleverly engineering the tunneling rates [see below and Eq. (7)]. To implement theoretically the demon in our transport description, all these steps need to be incorporated in the master-equation description [Eq. (2)]. Then, the feedback scheme is taken into account by taking the transition rates at the correct magnetic field orientations.

IV. RESULTS

In order to characterize transport and to see whether the demon is able to drive a current against the applied bias or not, we calculate the currents using the probabilities from Eq. (2): $I_L = e(-W_{10}^L p_0 + W_{01}^L p_1)$. By this definition, currents are positive when particles go into a reservoir and negative otherwise, and fulfill the particle conservation relation $I_L + I_R = 0$. Then, solving Eq. (2) in the steady-state regime, the probabilities are easily obtained, yielding

$$I_L = \frac{e\left(W_{01}^L W_{10}^R - W_{10}^L W_{01}^R\right)}{W_{01}^R + W_{01}^L + W_{10}^R + W_{10}^L}. \tag{6}$$

We see from this expression that the first term accounts for particles entering the dot from the right and exiting through the left (positive contribution) and the second term accounts for particles entering the dot from the left and leaving through the right (negative contribution). In order to improve performance of the Maxwell demon, we want the first term to be bigger than the second one since we want a current flowing from right to left.

From the expressions of the transition rates and the current [Eqs. (3) and (6), respectively], we see that to favor the first term of the current we need to enhance $\Gamma_1^1\Gamma_2^0$ with respect to $\Gamma_1^0\Gamma_2^1$. From the expressions of the tunneling rates, we obtain their ratio:

$$\frac{\Gamma_1^1\Gamma_2^0}{\Gamma_1^0\Gamma_2^1} = e^{(k_1-k_2)(\mu_d^- - \mu_d^+)}. \tag{7}$$

It is observed then that we are able to favor the numerator by taking $k_1 > k_2$, given that $\mu_d^- - \mu_d^+ \geqslant 0$.

In Fig. 3 we represent the charge current in reservoir L. Current is negative as long as it flows with the bias according to our sign criterion. In Fig. 3(a), we observe that symmetric barriers lead to a current that always follows the bias independently of feedback strength η. However, in Fig. 3(b) when the tunneling barriers are different and for a sufficiently high-η value the current starts to flow opposite to the bias, becoming positive. This notable fact is thanks to the action of the demon. Importantly, the action of the demon, i.e., its ability

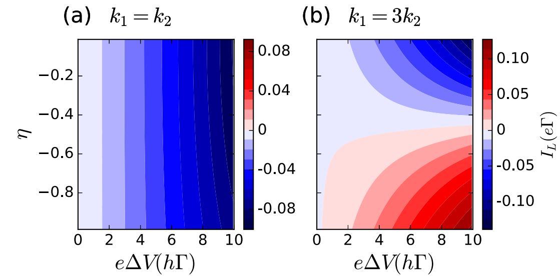

FIG. 3. Current to (positive) or from (negative) the left reservoir I_L for different values of η as a function of $\Delta V = V_L - V_R$. The current is negative when it flows in the bias voltage direction. The parameters have been chosen so that $k_1 = k_2$ in (a) and $k_1 = 3k_2$ in (b). We have taken $\Gamma_1 = \Gamma_2 = \Gamma$, $k_1 = 0.1/h\Gamma$, and $k_B T = 10h\Gamma$.

to change the energy level inside the dot $\mu_d^- - \mu_d^+ \propto \eta\Delta V$, then by increasing the applied voltage the current flowing against the bias enhances as well. The bias which in principle is an obstacle that needs to be overcome becomes rather a help because the action of the demon augments with it.

To fully characterize the demon and since it is not ideal, the energy flow from the system to the demon and the demon entropy information flow are evaluated. The energy current is related to the energy change that the demon causes on the electrochemical potential of the localized impurity. We calculate this current by measuring the number of electrons going in and out of the dot times their respective energies, yielding

$$J_D = (\mu_d^+ - \mu_d^-)\frac{\left(W_{10}^L + W_{10}^R\right)\left(W_{01}^L + W_{01}^R\right)}{W_{10}^L + W_{10}^R + W_{01}^L + W_{01}^R}. \tag{8}$$

Hence, we see that this energy current is proportional to the energy difference of the electrochemical potentials caused by the demon times the activity current which measures how many particles go in and out of the impurity. Our findings for the energy current carried by the demon are shown in Fig. 4. The energy injected by the demon increases with η and with the applied voltage, as expected. Comparing the results for symmetric barriers [Fig. 4(a)] and asymmetric ones [Fig. 4(b)] we show that the current energy does not change significantly. This indicates that the energy injected by the demon although present and necessary is not the key factor in pushing the

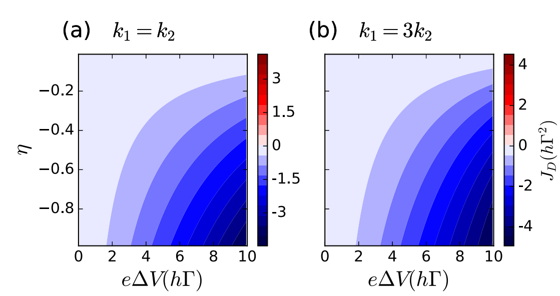

FIG. 4. Energy current (J_D) injected by the demon for different values of η. The parameters have been chosen so that $k_1 = k_2$ in (a) and $k_1 = 3k_2$ in (b). The rest of the parameters are the same as in Fig. 3.

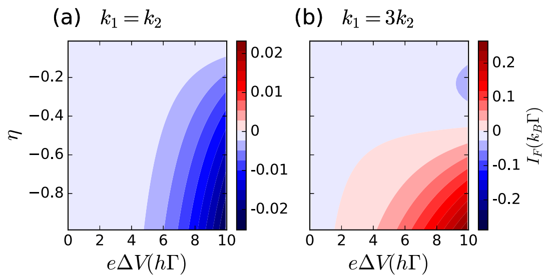

FIG. 5. Information current (I_F) of the demon for different values of η. The parameters have been chosen so that $k_1 = k_2$ in (a) and $k_1 = 3k_2$ in (b). The rest of the parameters are the same as in Fig. 3.

current against the bias. Let us now move on to the information entropy.

The final step in the characterization of the demon is the key quantity of an ideal Maxwell demon, i.e., the information that it can extract from the system in order to perform [16]. We start from the system information [14] given by Shannon's entropy $S = -k_B \sum_m p_m \ln p_m$, where p_m are the occupation probabilities given by Eq. (2). Then, the entropy balance $\dot{S} = \dot{S}_e + \dot{S}_i$ can be written as an entropy production $\dot{S}_i$ and an entropy flow $\dot{S}_e$ that satisfy $\dot{S}_i = -\dot{S}_e$.

The entropy flow in the system can be separated in the standard form of the entropy flow given by the exchanged heat in each reservoir (ν) over the temperature and an extra term accounting for the entropy flow to the demon $\dot{S}_D$:

$$\dot{S}_e = \sum_{\nu=L,R} \frac{J_{(\nu)}}{T} + \dot{S}_D. \tag{9}$$

The term $\dot{S}_D$ consists of two parts: the entropy flow caused by the energy flux J_D and an information flow I_F that powers the demon. This second term labeled I_F is the information flow extracted by the demon in its application of the feedback protocol:

$$I_F = \dot{S}_D - \frac{J_D}{T}. \tag{10}$$

This information current is the main characteristic of the Maxwell demon since it measures the information that the demon extracts from the system in order to be able to apply the necessary feedback scheme. Especially in the case of an ideal Maxwell demon ($J_D = 0$), it is the only measurable quantity associated to the Maxwell demon [16].

Figure 5 represents the information current for the demon. In Fig. 5(a) we observe that the information current is always negative meaning that information is flowing from the demon to the system. This corroborates that the demon is acting wrongly, being unable to extract information from the system. However, when barriers depend differently on energy [see Fig. 5(b)] the demon starts to be able to extract information from the system. Noticeably, we observe that the cases in which the demon drags an electrical current against the bias system coincide with the cases in which the demon is able to extract information from the system (compare Figs. 3 and 5). This is the definitive indicator that the extraction of information is the key to our demon's operation.

V. CONCLUSIONS

We have studied the effect of a nonideal Maxwell demon feedback on transport through a localized state in a quantum Hall system. We proposed a working principle for a Maxwell demon based on the chirality of the edge states. We showed that it is applicable under the following conditions: (i) asymmetry of the capacitive interactions as the magnetic field is reversed, under nonequilibrium conditions, and (ii) energy dependence of the tunneling probabilities through the barriers. We show with a precise feedback protocol how a demon is able to push electrical current against the applied bias voltage by extracting information from the system. We demonstrate that this is effectively the case: the demon works satisfactorily whenever it is able to extract information from the system. Our scheme opens an important avenue for the design of Maxwell demons that benefit from the topological properties of quantum matter in interacting systems since any means of switching the chirality would be well described by our theoretical description and would serve the same purpose as the reversal of the magnetic field.

It is to be noted that although the reversal of such high magnetic fields is hard to implement in the desired time scale of few nanoseconds with the experimental techniques available nowadays, this reversal is only meant as a reversal of the chirality of the edge states of the quantum Hall system. This reversal could be achieved, for example, using a fixed magnetic field of a few Tesla in one direction and then triggering a pulsed high magnetic field on the opposite direction to achieve the reversal of the magnetic field. These pulsed magnetic fields are triggered over typical time scales of microseconds [27] which is a reasonable operation time for the demon. Another possibility would be to move the sample between two well-localized fixed magnetic fields configured in opposite directions, achieving an effective reversal of the magnetic field. In both of these processes, the QD state could be preserved by lowering its energy and restoring it afterwards.

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