Single-photon superradiant decay of cyclotron resonance in a *p*-type single-crystal semiconductor film with cubic structure

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We study a single-photon super-radiance under the conditions of cyclotron resonance in a perfect single-crystal *p*-type semiconductor film with cubic structure. We show that the rate of super-radiant emission scales with the film area, which allows one to specify the size of the film at which the probability of a single-photon super-radiance becomes much greater than the probabilities of other scattering channels. The power of super-radiant emission depends only on three fundamental constants: the electron charge q_e, the speed of light c, the electron mass m_e, and on the electric- to magnetic-field ratio.

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I. INTRODUCTION

The use of cyclotron resonance for the study of semiconductors began in the middle of the last century [1–5]. Since then, the cyclotron resonance has become a powerful tool for studying the structure of semiconductors, allowing investigation of their band structure, mechanisms of charge-carrier scattering, the influence of phonon-electron and phonon-hole interaction on their effective masses, and more (see review papers [6–8] and the references therein). In recent years, the development of submicron technologies paved the way for new methods of preparing low-dimensional semiconductor structures where quantum-size effects play a decisive role [9]. This makes possible the use of cyclotron resonance for the study of collective effects such as Dicke super-radiance, which has recently been observed experimentally in ultra-high-mobility two-dimensional (2D) electron gas in GaAs [10,11], and for electronic excitations in the InGaAs quantum well [12].

The effect of super-radiation, which has been well known for a long time (see review paper [13] and references therein), was discovered by Dicke [14], who showed that the system of N identical two level excited atoms undergoes a spontaneous coherent transition to the ground state. This is accompanied by the emission of N photons, the intensity of which scales as N^2, and the decay rate of which is $N\gamma$, where γ is the decay rate of an isolated atom. As was noticed in [14], super-radiant transition becomes possible if the system size L is much less than the photon wavelength λ ($L \ll \lambda$).

Another kind of super-radiance (so-called single-photon super-radiance) can occur when a single-photon Dicke state is formed: N identical two level atoms are in a symmetrical superposition of states with one excited atom and $N-1$ atoms in the ground state [15–21]. In this case, the decay rate of a single photon is also equal to $N\gamma$. As was shown in [16,20], a single-photon super-radiance can occur even if system size L is much greater than the photon wavelength λ. In this case, the photon decay rate also scales as N and the photon's emission results in a narrow radiation pattern.

Our paper is devoted to the study of a single-photon super-radiance under conditions of cyclotron resonance in a single-crystal semiconductor film with a cubic structure. It is assumed that the temperature is sufficiently low, so that there are no holes at the excited Landau level ($n = 1$), and the surface density of the holes at the lower Landau level ($n = 0$) is equal to $q_e B/2\pi\hbar$. Such a density of 2D holes results in the integer quantum Hall effect [22], where the Hall resistance of a semiconductor structure with 2D electron gas is quantized and depends only on fundamental constants—the electron charge and the Planck constant.

In general, a super-radiant transition in solids is difficult to observe, due to inherently fast decay channels for carriers. In semiconductors the main scattering channel for the electron and holes is the phonon channel. The time scale of the phonon relaxation of carriers in semiconductors is typically of the order of 10^{-13} s [23].

We show in the paper that, under the conditions of cyclotron resonance, the rate of the emission of one photon from a single-photon Dicke state is much greater than the probability of other hole scattering mechanisms, and hence, in this case, a single-photon super-radiance is the main relaxation mechanism. For example, for the static magnetic field $B = 10$ T [24] and film size $L > 0.2$ cm, the rate of a single-photon super-radiance in Ge film is more than 10^{14} s^{-1}. This value is an order of magnitude greater than the rate of hole scattering on phonons in a semiconductor (10^{13} s^{-1}) [23]. Therefore, under these conditions, the emission of phonons can be neglected, and the relaxation time is determined only by the mechanism of a single-photon super-radiance.

We also investigate the conduction and power of super-radiant emission of the two-dimensional hole gas and show that in this case the overall universal power generated in the film depends only on three fundamental constants q_e, c, m_e and on the ratio of intensities of the electric and magnetic fields.

The paper is organized as follows. In Sec. II we describe the cyclotron resonance spectrum of holes in a three-dimensional (3D) single crystal of Ge or Si in a strong homogeneous magnetic field and calculate the rate of spontaneous photon emission for a hole transition between Landau levels $n = 1$ and 0. In Sec. III we calculate the rate of a single-photon super-radiance and show that the system wave function is a symmetric superposition of single hole state products. In Sec. IV we calculate a surface current. The power of super-radiant emission and its radiation patterns are found in Sec. V.

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II. THE CYCLOTRON ENERGIES OF THE HOLES

We assume the film surface is oriented in the x-y plane, so that the z axis is directed along the [001] crystal axis. In order to study the cyclotron resonance it is necessary to know the energy spectrum of the holes. We take this spectrum as similar to that in a 3D single crystal. As is known, the wave function of the hole is a bispinor [3]. Accordingly, in 3D Si or Ge single crystals located in a strong homogeneous magnetic field B applied along the [001] axis, there are four energy levels for the first two Landau levels $n = 0,1$. In the framework of perturbation theory these energies were calculated in [25] up to the second order of magnitude under conditions $\hbar^2 k_z^2/2m_e \ll \hbar\omega_c;\quad \mu = 0.5(\gamma_3 - \gamma_2) < 1$, where $\omega_c = q_e B/m_e$ is the cyclotron frequency, to be

$$E_{\alpha,n} = E_{\alpha,n}^{(0)} + E_{\alpha,n}^{(1)} + E_{\alpha,n}^{(2)} \tag{1}$$

where the first subscript numbers the bispinor ($\alpha = 1,2$), and the second subscript numbers the Landau levels ($n = 0,1$).

For the subsequent study it is important that the energy spectrum of the holes in Ge and Si (1) is not equidistant relative to the quantum number n [3] and the energy $E_{\alpha,n}$ is independent on the quantum number k_x [25]. Expression (1) can be used for the calculation of hole energies in a film under the condition [25]

$$\frac{\pi^2\hbar^2}{2m_{\alpha,n}}\frac{(n')^2}{d^2} \ll \hbar\omega_c \tag{2}$$

where $m_{\alpha,n}$ is the effective mass of a hole in 3D single crystal [25], d is the film thickness, and n' is the number of de Broglie half waves across the film.

Condition (2) holds to a good accuracy for a magnetic field $B = 10$ T, film thickness $d = 2.0 \times 10^3$ Å, and $n' = 1$. It allows one to take zero approximation in Eq. (1), $E_{\alpha,n}^{(0)}$ for the calculation of the energy spectrum [25]:

$$E_{1,0}^{(0)} = \tfrac{1}{2}\hbar\omega_c(\gamma_2 - \gamma_1 + k), \tag{3a}$$

$$E_{1,1}^{(0)} = \tfrac{1}{2}\hbar\omega_c[3(\gamma_2 - \gamma_1) + k], \tag{3b}$$

$$E_{2,0}^{(0)} = -\tfrac{1}{2}\hbar\omega_c(\gamma_2 + \gamma_1 - 3k), \tag{4a}$$

$$E_{2,1}^{(0)} = -\tfrac{3}{2}\hbar\omega_c(\gamma_2 + \gamma_1 - k) \tag{4b}$$

where $\gamma_1,\gamma_2,\gamma_3$ are the Lattinger parameters. In Ge $\gamma_1 = 13.2, \gamma_2 = 4.4, \gamma_3 = 5.4$ [3], $k = -3.41$ [26]; in Si $\gamma_1 = 4.22, \gamma_2 = 0.5, \gamma_3 = 1.38$ [26].

The eigenvectors for energies (3a), (3b), (4a), and (4b) take on the form

$$\langle\psi_{\alpha,n}^{(0)}| = (0,(2-\alpha)u_n^*,0,(\alpha-1)u_n^*) \tag{5}$$

where u_n is the spatial part of the wave function:

$$u_n(k_x,x,y) = C_n\sqrt{\frac{1}{L_x d}}e^{ik_x x}e^{-\frac{\xi^2}{2}}H_n(\xi) \tag{6}$$

where $\xi = \sqrt{\frac{m_e\omega_c}{\hbar}}(y - R_e^2 k_x)$, $C_n = \frac{1}{\sqrt{2^n n!\sqrt{\pi}R_e}}$, $R_e = \sqrt{\frac{\hbar}{q_e B}}$ is a cyclotron radius, $H_n(\xi)$ are the Hermite polynomials, L_x is the film length along the x axis, and $k_x = \frac{2\pi}{L_x}n_x$, $n_x = 0, \pm 1, \pm 2, \ldots$.

The resonance transition is possible only between different Landau levels n which belong to the same bispinor. From expressions (3a), (3b), (4a), and (4b) we obtain the frequencies of the corresponding transitions:

$$\hbar\omega_\alpha = E_{\alpha,0}^{(0)} - E_{\alpha,1}^{(0)} = \hbar\omega_c C_\alpha, \quad \alpha = 1,2 \tag{7}$$

where $C_\alpha = (\gamma_1 + (-1)^\alpha\gamma_2)$.

In general, our approach is valid when $R_e \gg a_0$, where a_0 is the lattice constant (for Ge $a_0 = 5.6$Å). From $R_e = a_0$ we estimate the maximal value of the magnetic field to be $B_0 = 2.1 \times 10^3$ T. Therefore, our scheme for the calculation of the holes' spectrum is justified for $B \ll B_0$. On the other hand, the expressions (3a), (3b), (4a), and (4b) provide a good approximation if $|E_{\alpha,n}^{(0)}/\Delta| < 1$, where Δ is the spin-orbit splitting (for Ge $\Delta = 0.29$ eV). The calculations for Ge show that for magnetic fields $B = (1 \div 10)$ T the ratio $|E_{\alpha,n}^{(0)}/\Delta|$ does not exceed 0.12.

A. The rate of spontaneous photon emission under hole transition between $n = 1$ and 0 Landau levels

In the dipole approximation, the rate Γ_α for the hole transition between states $|\psi_{\alpha,1}^{(0)}\rangle$ and $|\psi_{\alpha,0}^{(0)}\rangle$ with the emission of a photon can be obtained from the conventional expression

$$\Gamma_\alpha = \frac{\omega_\alpha^3}{3\pi\varepsilon_0\hbar c^3}\left|\langle\psi_{\alpha,1}^{(0)}|q_e\hat{y}|\psi_{\alpha,0}^{(0)}\rangle\right|^2 \tag{8}$$

where $\langle\psi_{\alpha,1}^{(0)}|q_e\hat{y}|\psi_{\alpha,0}^{(0)}\rangle = q_e R_e\frac{1}{\sqrt{2}}\delta_{k_x,k'_x}$, and ε_0 is the electric constant.

Finally for Γ_α we obtain

$$\Gamma_\alpha = \frac{C_\alpha^3}{6\pi\varepsilon_0(2\pi)^3}\frac{q_e^2}{R_e\hbar}\left(\frac{\lambda_C}{R_e}\right)^3 \tag{9}$$

where $\lambda_C = 2\pi\hbar/m_e c$ is the electron Compton wavelength.

The lifetime of the state $|\psi_{\alpha,1}^{(0)}\rangle$ is given by the quantity $\tau_\alpha = 1/\Gamma_\alpha$. For the magnetic field strength $B = 10$ T we obtain from Eq. (9) the corresponding lifetimes $\tau_1 = 7.6 \times 10^{-5}$ s, $\tau_2 = 9.5 \times 10^{-6}$ s. These values are much greater than the lifetime of state $|\psi_{\alpha,1}^{(0)}\rangle$ against a phonon emission which is of the order of 10^{-13} s in semiconductors [23]. It would seem that under these conditions the photon decay channel is impossible. However, we will show in the next sections that due to the mechanism of a single-photon super-radiance the decay channel of the state $|\psi_{\alpha,1}^{(0)}\rangle$ against the photon emission becomes the dominating process.

III. SINGLE-PHOTON SUPER-RADIANCE

In order to estimate the rate of single-photon super-radiance we use the method of the non-Hermitian effective Hamiltonian [27], which has been applied to the study of microwave scattering on a chain of two level atoms [28]. We consider a one-dimensional chain of N noninteracting holes aligned along the y axis with the incident photon directed along the z axis. As a basis set of state vectors we take the states where one hole is in the excited state $|e\rangle$ and the other $N-1$ holes are in the ground state $|g\rangle$. Therefore, we have N vectors $|n\rangle = |g_1,g_2,\ldots g_{n-1},e_n,g_{n+1},\ldots g_{N-1},g_N\rangle$. The spontaneous emission of the excited hole results in a continuum of states

$|k\rangle = |g_1, g_2, \ldots\ldots g_{N-1}, g_N, k\rangle$, where all holes are in the ground state and there is one photon in the system. This process can be described by the non-Hermitian Hamiltonian

$$H = H_0 - iW \tag{10}$$

where H_0 is the Hamiltonian of holes, and operator W describes the interaction of the holes with the photon field.

The matrix elements of Eq. (10) in the $|n\rangle$ representation are

$$\langle m|H|n\rangle = \hbar\omega_\alpha \delta_{m,n} - i\langle m|W|n\rangle; (1 \leqslant m,n \leqslant N). \tag{11}$$

If the distance between holes along the direction of the photon scattering (z axis) is much less than the photon wavelength, the matrix element on the right-hand side of Eq. (11) takes the form [28]

$$\langle m|W|n\rangle = \hbar\sqrt{\Gamma_\alpha^{(m)}\Gamma_\alpha^{(n)}} \tag{12}$$

where $\Gamma_\alpha^{(n)}$ is the rate of spontaneous photon emission from a state where the nth hole is excited.

Due to the planar geometry of the film, the z coordinates of all holes in the chain are the same—they are in an identical arrangement relative to a wavefront. Therefore, we assume that the rate of spontaneous emission of holes is the same: $\langle m|W|n\rangle = \hbar\Gamma_\alpha$. Thus, we get a non-Hermitian $N \times N$ matrix, where the main diagonal elements are $\hbar\omega_\alpha - i\hbar\Gamma_\alpha$, and all off-diagonal elements are equal to $-i\hbar\Gamma_\alpha$:

$$\frac{1}{\hbar}\langle m|H|n\rangle = \begin{pmatrix} \omega_\alpha - i\Gamma_\alpha & -i\Gamma_\alpha & -i\Gamma_\alpha & \ldots & -i\Gamma_\alpha \\ -i\Gamma_\alpha & \omega_\alpha - i\Gamma_\alpha & \ldots & \ldots & -i\Gamma_\alpha \\ -i\Gamma_\alpha & -i\Gamma_\alpha & \omega_\alpha - i\Gamma_\alpha & \ldots & -i\Gamma_\alpha \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -i\Gamma_\alpha & -i\Gamma_\alpha & \ldots & \ldots & \omega_\alpha - i\Gamma_\alpha \end{pmatrix}. \tag{13}$$

The incident photon, when absorbed by the film, can excite any hole. As we do not know which of the N holes is excited, the wave function of the holes should be expressed as a superposition of the state vectors $|n\rangle$:

$$\Psi = \sum_{n=1}^{N} c_n|n\rangle. \tag{14}$$

It is not difficult to show that the solution of the Schrödinger equation $H\Psi = E\Psi$, with H and Ψ from Eqs. (13) and (14) respectively, has the following properties.

(1) There is a single state with energy $E_S = \hbar\omega_\alpha - i\hbar N\Gamma_\alpha$ the wave function of which is a symmetric coherent superposition of the state vectors $|n\rangle$, where all quantities c_n are the same:

$$|\Psi_S\rangle = \frac{1}{\sqrt{N}}\sum_{n=1}^{N}|n\rangle. \tag{15}$$

(2) There are $N-1$ degenerate states with energy $E = \hbar\omega_\alpha$, where all coefficients c_n in Eq. (14) satisfy the condition $\sum_{n=1}^{N} c_n = 0$. These states are dark, nondecaying states since their widths are equal to zero.

The collective state $|\Psi_S\rangle$, Eq. (15), which we call a single-photon Dicke state, can be formed by a single photon, which propagates normal to the film surface and interacts in phase with every hole in the plane of the film [16].

Therefore, under this condition, state (15) decays with a rate $N\Gamma_\alpha$, so that the rate of the spontaneous emission of a single hole Γ_α, Eq. (9), should be substituted with the quantity $\bar{\Gamma}_\alpha$:

$$\bar{\Gamma}_\alpha = N\Gamma_\alpha = \frac{2\pi}{3}\frac{q_e^2}{\varepsilon_0\hbar\lambda_\alpha}\left(\frac{L}{\lambda_\alpha}\right)^2, \tag{16}$$

which is the linewidth of a single-photon super-radiant emission. In Eq. (16) the quantity λ_α ($\alpha = 1,2$) is the wavelength of the emitted photon,

$$\lambda_\alpha = \frac{2\pi c m_e}{C_\alpha q_e B}, \tag{17}$$

and N is the number of holes which take part in the formation of the single-photon Dicke state (15):

$$N = \frac{q_e B}{2\pi\hbar}L^2 = \frac{1}{2\pi}\left(\frac{L}{R_e}\right)^2 \tag{18}$$

where $L = L_x = L_y$.

From the considerations given above we obtain the following estimations. For a magnetic field $B = 10$ T we estimate transition frequencies (7)—$\omega_1 = 1.6 \times 10^{13}$rad/s, $\omega_2 = 3.1 \times 10^{13}$rad/s—with corresponding wavelengths $\lambda_1 = 0.012$ cm and $\lambda_2 = 0.006$ cm. In the range $0.2 \leqslant L \leqslant 0.4$ cm the expression (18) gives $9.7 \times 10^9 \leqslant N \leqslant 3.9 \times 10^{10}$. Then, from expression (16), it follows that the rate of spontaneous hole emission from a Ge film is more than 10^{14} s^{-1}. Since the rate of the phonon scattering in semiconductors is of the order of 10^{13} s^{-1} (see, for example, [23]), we may neglect all scattering mechanisms except for a single-photon superradiance, which becomes, under these conditions, the main relaxation mechanism of excited holes.

IV. THE SURFACE CURRENT

First we define the ground state $|G\rangle$ of the ensemble of the holes: $|G\rangle = |g_1, g_2, \ldots, g_n, \ldots, g_{N-1}, g_N\rangle$ with the energy ε_G. Next we take the external time-dependent electric field, which is directed normal to the time-independent homogeneous strong magnetic field:

$$\hat{V}(y,t) = \begin{cases} 0, t < 0 \\ \hat{V}\cos(\omega t), t > 0 \end{cases} \tag{19}$$

where $\hat{V} = -yq_eE_y$. It is not difficult to show that this driving field gives rise to transitions only between the states $|G\rangle$ and $|\Psi_S\rangle$. The matrix element of the dipole operator between these states is $\langle\Psi_S|q_e\hat{y}|G\rangle = q_eR_e\sqrt{N/2}$, while the transition amplitudes between $|G\rangle$ and dark states are zero.

As the energy spectrum of holes in Ge and Si is not equidistant relative to the quantum number n [3], the evolution of the hole state vector $|\Psi(t)\rangle$, which accounts for the near resonant transitions at $\hbar\omega_\alpha \gg k_BT$, between states $|G\rangle$ and $|\Psi_S\rangle$ is as follows:

$$|\Psi(t)\rangle = |G\rangle a(t)e^{-i\frac{\varepsilon_G}{\hbar}t} + |\Psi_S\rangle b(t)e^{-i(\frac{\varepsilon_G}{\hbar}+\omega_\alpha - i\bar{\Gamma}_\alpha)t} \tag{20}$$

where the amplitudes $a(t)$ and $b(t)$ satisfy initial conditions $a(0) = 1$, $b(0) = 0$.

These amplitudes can be found near resonance $\omega \approx \omega_\alpha$, in the frame of conventional time-dependent perturbation theory: $a(t) = 1$,

$$b(t) = E_y\frac{q_eR_e\sqrt{N}}{2\sqrt{2}\hbar}\left(\frac{e^{i(\omega_\alpha - i\bar{\Gamma}_\alpha-\omega)t} - 1}{(\omega_\alpha - i\bar{\Gamma}_\alpha - \omega)}\right). \tag{21}$$

At resonance, $\omega = \omega_\alpha$, the condition $|b(t)| \ll 1$ sets an upper bound on the amplitude of the external electric field E_y: $(E_y \ll \frac{2\sqrt{2}\hbar\bar{\Gamma}_\alpha}{q_eR_e\sqrt{N}} \equiv E_\alpha^{\max})$.

From Eq. (20) we calculate a time-dependent steady-state part of the hole dipole moment $\lim_{t\to\infty}\langle\Psi(t)|q_e\hat{y}|\Psi(t)\rangle \equiv \langle q_ey(t)\rangle$, which causes the transitions between states $|G\rangle$ and $|\Psi_S\rangle$:

$$\langle q_ey(t)\rangle = E_y\frac{N(q_eR_e)^2}{2\hbar}\left(\frac{(\omega_\alpha-\omega)\cos\omega t + \bar{\Gamma}_\alpha\sin\omega t}{(\omega_\alpha-\omega)^2+\bar{\Gamma}_\alpha^2}\right). \tag{22}$$

From Eq. (22) we estimate the rate of change of the average dipole moment of the holes at resonant frequency $\omega = \omega_\alpha$:

$$\langle q_e\dot{y}(t)\rangle_r = E_y\frac{N(q_eR_e)^2\omega_\alpha}{2\hbar\bar{\Gamma}_\alpha}\cos(\omega_\alpha t). \tag{23}$$

From Eq. (23) we introduce the average velocity of a hole $\langle v\rangle$: $\langle q_e\dot{y}(t)\rangle = Nq_e\langle v\rangle$ and define a surface current density:

$$J_y(\omega = \omega_\alpha,t) = \frac{Nq_e\langle v\rangle}{L^2}. \tag{24}$$

And finally, from Eq. (24) we can estimate the current in the film and the conductivity of an ideal 2D system. Synchronous steady-state motion of the holes allows us to find the conductivity of a 2D system when the number of holes, which take part in the formation of the single-photon Dicke state (15), is equal to N [Eq. (18)].

V. THE ANGULAR DISTRIBUTION OF SUPER-RADIANT EMISSION

The total power, which is supplied to a film, gives rise to the transitions between the states $|G\rangle$ and $|\Psi_S\rangle$:

$$P_\alpha = \tfrac{1}{2}\sigma_\alpha E_y^2L^2. \tag{25}$$

We assume there are no dissipative losses, so that all this power is radiated into a free space.

In Eq. (25) a quantity σ_α is the conductivity at the frequency $\omega = \omega_\alpha$, which is obtained from Eqs. (23) and (24):

$$\sigma_\alpha = \frac{1}{4\pi}\frac{q_e^2\omega_\alpha}{\hbar N\Gamma_\alpha} = \frac{3}{4\pi}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left(\frac{\lambda_\alpha}{L}\right)^2. \tag{26}$$

Hence, we may express P_α in the following form:

$$P_\alpha = \frac{3}{8\pi}\sqrt{\frac{\varepsilon_0}{\mu_0}}\lambda_\alpha^2E_y^2 \tag{27}$$

where, as we noted before, the amplitude of the electric field satisfies the condition $E_y \ll E_\alpha^{\max}$.

In Ge with $B = 10$ T and $L = 0.2$ cm, the upper limit of the electric-field intensity $E_2^{\max} = 2.4\times10^3$ V/m, and from $E_y = 0.2E_2^{\max}$ we obtain the emission power $P_2 = 2.6\times10^{-7}$ J/s.

It is seen from Eq. (27) that the quantity $C_\alpha^2P_\alpha$, which we call the universal emission power, depends neither on the film dimension L nor on the material properties. It depends only on the fundamental constants q_e,c,m_e and on the electrical to magnetic field ratio E_y/B.

From an experimental point of view, it is important to know the angular distribution of a radiation field. An exact form of radiation pattern is given by the real part of the time-averaged power density $\langle\mathbf{S}\rangle = \frac{1}{2\mu_0}[\mathbf{E}\times\mathbf{B}^*]$, where $\mathbf{E}$ and $\mathbf{B}$ refer to the peak amplitudes of the oscillating quantities, $\mathbf{E}(t) = \mathbf{E}e^{i\omega t},\mathbf{B}(t) = \mathbf{B}e^{i\omega t}$. In what follows, we calculate the radiation pattern of spontaneous emission in a far-field region $(r \gg \lambda, r\lambda \geqslant L^2)$, where in a single electromagnetic plane wave a vector $\mathbf{E}$ is normal to a vector $\mathbf{B}$, and $E = cB$. Hence, in this region the time-averaged vector power density $\langle\mathbf{S}\rangle$ is simply a real number: $\langle\mathbf{S}\rangle = \frac{c}{2\mu_0}|\mathbf{B}|^2$.

The magnetic field in a far-field region is given by the expression [see the expression (A7) in the Appendix]

$$\mathbf{B}(\mathbf{r}) = -i\frac{\mu_0}{4\pi}[\mathbf{k}\times\mathbf{J}(\mathbf{k})]\frac{e^{ikr}}{r} \tag{28}$$

where r is a distance from a source of the field, $\mathbf{k}$ is the wave vector ($k = \omega/c$), which is directed along $\mathbf{r}$ in a far-field region, and $\mathbf{J}(\mathbf{k})$ is a spectral component of a source current $\mathbf{J}(\mathbf{r})$:

$$\mathbf{J}(\mathbf{k}) = \int_V\mathbf{J}(\mathbf{r})e^{-i(\mathbf{k}\cdot\mathbf{r})}d\mathbf{r}. \tag{29}$$

Therefore, for $\langle S\rangle$ we obtain

$$\langle S\rangle = \frac{1}{2r^2}\sqrt{\frac{\mu_0}{\varepsilon_0}}[\mathbf{k}\times\mathbf{J}(\mathbf{k})]^2. \tag{30}$$

In our case, the current in a square $L\times L$ film can be written as

$$\mathbf{J}(\mathbf{r}) = \begin{cases}0 & |x|,|y| > \frac{L}{2}\\ \mathbf{e}_yJ_y\delta(z) & |x|,|y| \leqslant \frac{L}{2}\end{cases} \tag{31}$$

where J_y is given in Eq. (24). In Eq. (31) the origin of coordinates is taken in the geometrical center of a film where the z axis is normal to the film plane. From Eq. (29) we find

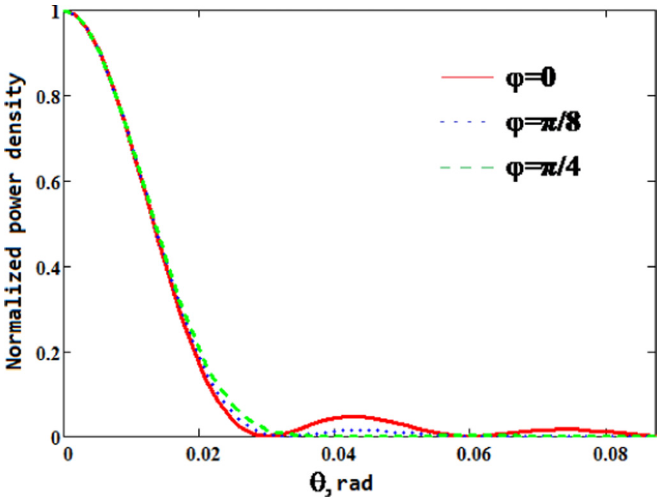

FIG. 1. Normalized radiated power density $f(\theta,\varphi)$ vs θ for fixed φ. $\lambda_2 = 0.006$ cm, $L = 0.2$ cm.

the spectral component $\mathbf{J}(\mathbf{k})$:

$$\mathbf{J}(\mathbf{k}) = \mathbf{e}_y J_y L^2 \frac{\sin\left(\frac{k_x L}{2}\right)}{\frac{k_x L}{2}} \frac{\sin\left(\frac{k_y L}{2}\right)}{\frac{k_y L}{2}} \tag{32}$$

where

$$\mathbf{k} = \mathbf{e}_x k_\alpha \sin(\theta)\cos(\varphi) + \mathbf{e}_y k_\alpha \sin(\theta)\sin(\varphi) + \mathbf{e}_z k_\alpha \cos(\theta), \tag{33}$$

$0 \leqslant \theta \leqslant \pi, 0 \leqslant \varphi \leqslant 2\pi$, $k_\alpha = 2\pi/\lambda_\alpha$, and $\mathbf{e}_x,\mathbf{e}_y,\mathbf{e}_z$ are unit vectors in the direction of the x axis, y axis, and z axis, respectively.

A substitution of Eq. (32) in Eq. (30) yields the radiated power density:

$$\langle S(\mathrm{r})\rangle = \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{J_y{}^2 L^4}{4\lambda^2 r^2} f(\theta,\varphi) \quad [\mathrm{W/m^2}] \tag{34}$$

where

$$f(\theta,\varphi) = \left(\cos^2\theta + \sin^2\theta\cos^2\varphi\right)\left(\frac{\sin\left(\frac{k_x L}{2}\right)}{\frac{k_x L}{2}} \frac{\sin\left(\frac{k_y L}{2}\right)}{\frac{k_y L}{2}}\right)^2 \tag{35}$$

is the normalized power density which defines the angular distribution of a super-radiant emission. Spherical angles θ and φ in Eqs. (33) and (35) coincide with those of vector $\mathbf{r}$ since in a far-field region vector $\mathbf{k}$ is directed along $\mathbf{r}$. We note that, except for the first factor in the right-hand side of Eq. (35), the expression for $f(\theta,\varphi)$ is similar to that of Fraunhofer diffraction on a square aperture.

In order to visualize the angle dependence of emission power density we draw the function $f(\theta,\varphi)$, Eq. (35), in three different coordinates. The plots are performed for $\lambda_2 = 0.006$ cm, $L = 0.2$ cm, so the far-field region corresponds to $r \geqslant L^2/\lambda_2 \approx 6.6$ cm. In Fig. 1 we show the dependence of normalized power density $f(\theta,\varphi)$ on θ for several fixed polar angles φ. A 3D plot of the normalized radiation density emitted in the upper half space is shown in Fig. 2. It is evident from these plots that for our parameters most of the power is radiated within a narrow region near a z axis, which corresponds to the

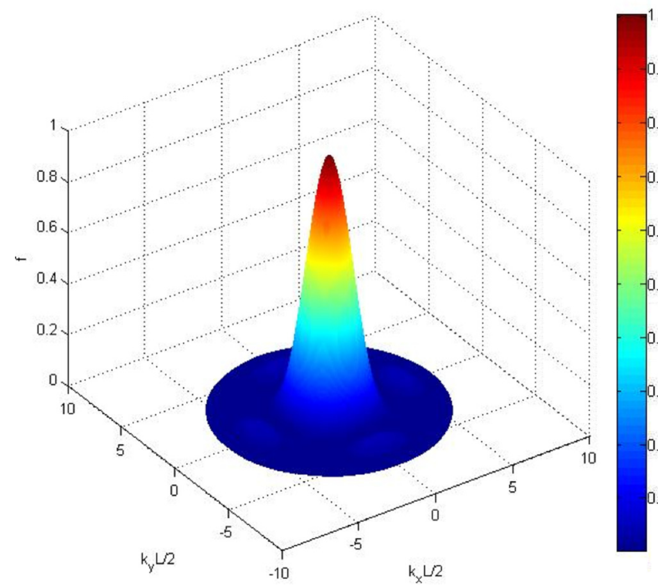

FIG. 2. 3D surface pattern of normalized radiation power density.

solid angle $\delta\Omega \approx \pi(\lambda_2/L)^2 = 2.82 \times 10^{-3}$ sr. The main and minor lobes can be seen in polar patterns of radiation power density as shown in Fig. 3.

VI. DISCUSSION

In the paper we study a single-photon super-radiance under the conditions of cyclotron resonance in a perfect single-crystal

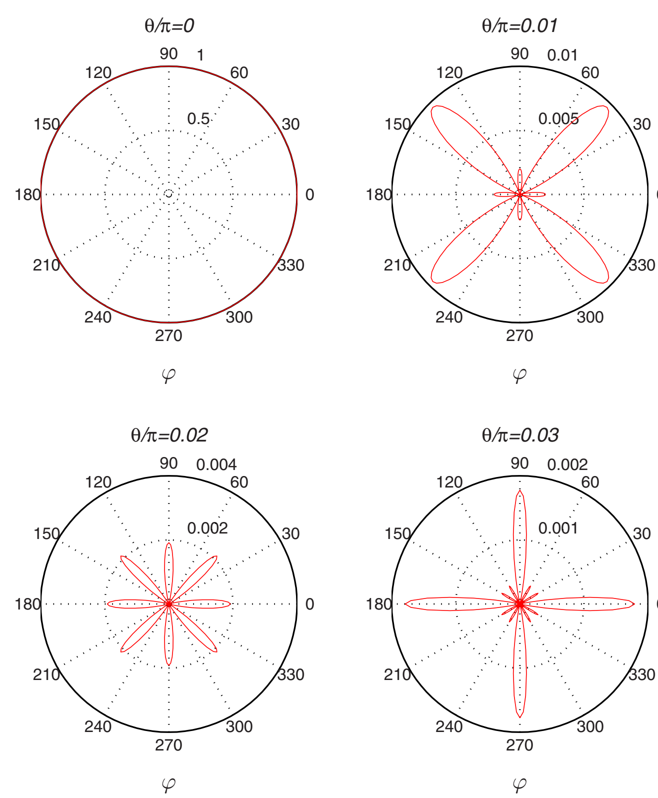

FIG. 3. Polar patterns of normalized radiation power density.

p-type semiconductor film with a cubic structure. We assume the film is at a sufficiently low temperature, so that we are able to take the initial hole density at the Landau level $n = 0$ to be $q_e B/2\pi\hbar$, with no holes at the excited Landau level $n = 1$.

We show that the rate of super-radiant emission, which results from the transition between the collective states $|G\rangle$ and $|\Psi_S\rangle$, scales as the film area, which allows one to specify the size of the film, at which the probability of a single-photon super-radiance becomes much greater than the probabilities of other scattering channels. For Ge in a static magnetic field of the order of 10 T and film dimension $L > 0.2$ cm, the rate of a single-photon super-radiance due to a hole transition is more than 10^{14} s^{-1}. This value is an order of magnitude higher than the rate of the phonon emission by a hole. Therefore, we may neglect all scattering mechanisms except for a single-photon super-radiance, which becomes, under these conditions, the main relaxation mechanism of excited holes.

We show that the universal power of super-radiant emission depends only on the fundamental constants q_e, c, m_e and on the electric to magnetic field ratio E_y/B.

We calculate the angular distribution of super-radiant emission and show that for our parameters most of the power is radiated within a narrow region near a z axis, which corresponds to the solid angle $\delta\Omega \approx \pi(\lambda_2/L)^2 = 2.82 \times 10^{-3}$ sr.

In conclusion we would like to mention several issues which may be important in the experimental realization of this effect.

A necessary condition for the formation of the single-photon Dicke state $|\Psi_S\rangle$, Eq. (15), is the existence of a single driving photon, which propagates normal to the film surface [16]. In principle, it could be arranged if the film under study is embedded in a resonant cavity the fundamental frequency of which is close to the transition frequency between Landau levels $n = 1$ and 0.

We showed in the paper that in order to obtain a large decay rate $N\Gamma_\alpha$, which overcomes other scattering channels, the film size L should be much greater than the photon wavelength λ. For large samples it leads to a reduction of the decay rate by a factor $(\lambda/L)^2$ [18], due to characteristic phase factors $e^{i\vec{k}\vec{r}_j}$, where $\vec{k}$ is the wave vector of the incident photon, and $\vec{r}_j$ is the hole position in the crystal volume. For a thin crystal film, which we consider here, the majority of the emitters are located near the film surface. In the case of the incident photon propagating normal to the film surface, all surface emitters experience nearly the same phase shift, so that in our case we may neglect the geometrical reduction of the decay rate.

As was shown above, for the formation of the quasistationary state $|\Psi_S\rangle$, with a large decay rate $N\Gamma_\alpha$, the transition frequencies ω_α and decay rates Γ_α for all emitters should be the same. This means that two-level systems (6), $u_0(k_x,x,y), u_1(k_x,x,y)$, spaced by different k_x along the y axis, need to be identical. To ensure this condition the film under study should be as ideal as possible. There cannot be local defects in the film located close to the maxima of the wave functions $u_0(k_x,x,y), u_1(k_x,x,y)$, the positions of which are determined by the magnitude of k_x.

We believe that the results obtained in our paper will help to open a new window for developing novel light sources based on super-radiance emission.

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APPENDIX: THE CALCULATION OF MAGNETIC FIELD IN A FAR-FIELD REGION

The magnetic field generated by a source current density $\mathbf{J}(\mathbf{r}')$ in an arbitrary point $\mathbf{r}$ of space can be found from Maxwell's equations in the following form [29]:

$$\mathbf{B}(\mathbf{r}) = -\mu_0 \int_V [\nabla_r G(\mathbf{r}-\mathbf{r}') \times \mathbf{J}(\mathbf{r}')] d\mathbf{r}' \tag{A1}$$

where the integration in Eq. (A1) is over the distribution of a source current density $\mathbf{J}(\mathbf{r}')$. The quantity $G(\mathbf{r}-\mathbf{r}')$ is the free-space Green's function of the scalar Helmholtz equation:

$$G(\mathbf{r}-\mathbf{r}') = \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} \tag{A2}$$

where k is the plane-wave wave vector, $k = \omega/c = 2\pi/\lambda$.

Below we use a spectral representation of Green's function (A2):

$$G(\mathbf{r}-\mathbf{r}') = \int \frac{d\mathbf{k}'}{(2\pi)^3} \frac{e^{i\mathbf{k}'\cdot(\mathbf{r}-\mathbf{r}')}}{k'^2 - k^2 - i\varepsilon} \tag{A3}$$

where a small imaginary quantity ε in the denominator of Eq. (A3) ensures the outgoing scattering wave solution of the Helmholtz equation.

Substitution of Eq. (A3) into Eq. (A1) yields the result

$$\mathbf{B}(\mathbf{r}) = -i\frac{\mu_0}{8\pi^3} \int_V \int_{\mathbf{k}'} [\mathbf{k}' \times \mathbf{J}(\mathbf{r}')] \frac{e^{i\mathbf{k}'\cdot(\mathbf{r}-\mathbf{r}')}}{\mathrm{k}'^2 - k^2 - i\varepsilon} d\mathbf{k}' d\mathbf{r}'. \tag{A4}$$

If we define a spectral current density

$$\mathbf{J}(\mathbf{k}') = \int_V \mathbf{J}(\mathbf{r}') e^{-i(\mathbf{k}'\cdot\mathbf{r}')} d\mathbf{r}', \tag{A5}$$

the expression (A4) can be rewritten as follows:

$$\mathbf{B}(\mathbf{r}) = -i\frac{\mu_0}{8\pi^3} \int_{\mathbf{k}'} [\mathbf{k}' \times \mathbf{J}(\mathbf{k}')] \frac{e^{i\mathbf{k}'\cdot\mathbf{r}}}{\mathrm{k}'^2 - k^2 - i\varepsilon} d\mathbf{k}'. \tag{A6}$$

When deriving Eq. (A6), the only implicit assumption we made was the existence of the spectral current density (A5). It can be rigorously proved that for any physical distribution of the current density $\mathbf{J}(\mathbf{r})$ in a restricted volume the spectral density $\mathbf{J}(\mathbf{k})$ always exists. In this case, $\mathbf{J}(\mathbf{k})$ is the integer function with a bounded spectrum.

In a far-field region the expression (A6) can be substantially simplified. In this region the electromagnetic waves are essentially plain waves with the only wave vector $\mathbf{k}_r$, which is directed along the vector $\mathbf{r}$: $\mathbf{k}_r = \frac{2\pi}{\lambda}\frac{\mathbf{r}}{r}$. Therefore, we may take the quantity $\mathbf{k} \times \mathbf{J}(\mathbf{k})$ at this point out of the integral in Eq. (A6) to obtain

$$\mathbf{B}(\mathbf{r}) = -i\mu_0 [\mathbf{k} \times \mathbf{J}(\mathbf{k})]|_{\mathbf{k}=\mathbf{k}_r} \int_{\mathbf{k}} \frac{d\mathbf{k}'}{(2\pi)^3} \frac{e^{i\mathbf{k}'\cdot\mathbf{r}}}{\mathrm{k}'^2 - k^2 - i\varepsilon}$$
$$= -i\frac{\mu_0}{4\pi} [\mathbf{k} \times \mathbf{J}(\mathbf{k})]|_{\mathbf{k}=\mathbf{k}_r} \frac{e^{ik_r r}}{r} \tag{A7}$$

where

$$\mathbf{k}_r = \mathbf{e}_x k \sin(\theta)\cos(\varphi) + \mathbf{e}_y k \sin(\theta)\sin(\varphi) + \mathbf{e}_z k \cos(\theta), \tag{A8}$$

$\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$ are unit vectors of the Cartesian coordinate system, and $k = 2\pi/\lambda$.

Spherical angles θ and φ in Eq. (A8) coincide with those of vector $\mathbf{r}$ since in a far-field region vector $\mathbf{k}$ is directed along $\mathbf{r}$.

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