PHYSICAL REVIEW B **96**, 075127 (2017)

Duality and universal transport in mixed-dimension electrodynamics

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(Received 31 May 2017; published 14 August 2017)

We consider a theory of a two-component Dirac fermion localized on a (2+1)-dimensional brane coupled to a (3+1)-dimensional bulk. Using the fermionic particle-vortex duality, we show that the theory has a strong-weak duality that maps the coupling e to $\tilde{e} = (8\pi)/e$. We explore the theory at $e^2 = 8\pi$ where it is self-dual. The electrical conductivity of the theory is a constant independent of frequency. When the system is at finite density and magnetic field at filling factor $\nu = \frac{1}{2}$, the longitudinal and Hall conductivity satisfies a semicircle law, and the ratio of the longitudinal and Hall thermal electric coefficients is completely determined by the Hall angle. The thermal Hall conductivity is directly related to the thermal electric coefficients.

DOI: 10.1103/PhysRevB.96.075127

I. INTRODUCTION

Recent developments have revealed powerful dualities between seemingly different nonsupersymmetric quantum field theories in (2+1) dimensions. A special case, the bosonic particle-vortex duality, has been known for decades [1,2]. More recently, a new duality between a free fermion theory and a gauge theory called QED_3 was suggested in the context of the construction of a particle-hole symmetric version of the composite fermion theory of the half-filled Landau level [3]. Later, the duality was shown to be related to the electromagnetic duality in the bulk [4,5]. Most recently, the duality was shown to be a particular case in a web of dualities that follows from a relativistic version of flux attachment [6,7]. Many more examples of dualities have been recently discovered, including those of self-dual theories [8–11].

In this paper we consider a simplest quantum theory living in mixed (2+1) and (3+1) dimensions. The theory involves a single two-component Dirac fermion Ψ living in a (2+1)D coupled to a massless U(1) gauge field (photons) A_μ living in (3+1)D [12]:

$$S = \int d^3x\, i\bar{\Psi}\gamma^\mu(\partial_\mu - iA_\mu)\Psi - \frac{1}{4e^2}\int d^4x\, F_{\mu\nu}^2. \qquad (1)$$

This theory has been previously considered in various contexts [13–15]. Most recently, it was considered in Ref. [3] as an example of a relativistic theory exhibiting fractional quantum Hall effect. The theory is similar to the low-energy effective theory describing graphene—Dirac fermions in (2+1) dimensions interacting through a 3D Coulomb potential—with two notable differences: there is only one (instead of four) two-component Dirac fermion, and the photon propagates with the same velocity as the fermion (instead of 300 times faster as in graphene).

The bosonic version of the theory has been known for some time [16–18]. In analogy with the bosonic case, we find that the theory exhibits a strong-weak duality, which combines the electromagnetic duality in the bulk and the fermionic particle-vortex duality on the brane. The duality maps e to $\tilde{e} = 8\pi/e$, and the theory is self-dual at $e^2 = 8\pi$. Provided that the theory is conformal at this coupling, we find nontrivial consequences for the transport of the U(1) charge. In particular, we find that the electrical conductivity is equal to a universal value:

$$\sigma = \sigma_0 \equiv \frac{1}{4\pi}\left[\sigma_0 \equiv \frac{e^2}{2h}\right]. \qquad (2)$$

The expressions in the square brackets $[\cdots]$ on the right-hand side in this and later equations correspond to the standard normalization of current and gauge field, in which the electric charge e stays in the covariant derivative in Eq. (1). Remarkably, the electrical conductivity is independent of the ratio between the frequency and the temperature, ω/T, and hence has the same value in the ballistic ($\omega \gg T$) and hydrodynamic ($\omega \ll T$) regimes. This behavior has previously been noted in the strongly coupled large-N theory living on a stack of M2 branes [19].

Moreover, we find that when one turns on a charge density n and a magnetic field B satisfying the condition $n = B/(4\pi)$ (or filling factor $\nu = \frac{1}{2}$ in the quantum Hall terminology), there are nontrivial relationships between electrical and thermal transport coefficients. The longitudinal and Hall conductivities satisfy a semicircle law:

$$\sigma_{xx}^2 + \sigma_{xy}^2 = \sigma_0^2. \qquad (3)$$

The ratio of the longitudinal and Hall thermoelectric coefficients is directly related to the Hall angle $\theta_H = \arctan(\sigma_{xy}/\sigma_{xx})$. In addition, the thermal Hall conductivity is related directly to the thermoelectric coefficients.

The plan of this paper is as follows. We describe the model in Sec. II and derive its self-duality in Sec. III. In Sec. IV we extract the consequences of the self-duality. Section V contains concluding remarks.

II. MIXED-DIMENSION QED

We start to recall some feature of the model (1) which we will call $\mathrm{QED}_{4,3}$. The coupling constant e is dimensionless. Physically, e determines the force between two charges located infinitely far from the brane, and hence it does not run. The theory is scale invariant at small e, but the situation at large e is not clear. The large-N version of (1) is conformal for all values of e, including $e = \infty$ where the theory becomes N-flavor QED_3. It is expected that there exist a critical value N_c, below which QED_3 undergoes spontaneous chiral symmetry breaking. For $N < N_{\mathrm{crit}}$ then one expects $\mathrm{QED}_{4,3}$ to be conformal only for sufficiently small e, $e < e_{\mathrm{crit}}(N)$.

Analytic estimates for N_c based on a truncation of the Schwinger-Dyson equation are typically of order 6–9 [20,21], but a recent numerical simulation [22] suggests that scale invariance persists at $N = 2$, implying that $N_{\rm crit} < 2$. There has not been any numerical study of $N = 1$. The "strong version" of the conjectured duality between QED_3 and free Dirac fermion [4,5] would imply that $\mathrm{QED}_{4,3}$ is scale invariant even at $e^2 = \infty$.

Applying the fermionic particle-vortex duality to the (2+1)D part of (1), the model is mapped to

$$S = \int d^3x \left[i\bar{\psi}\gamma^\mu(\partial_\mu - ia_\mu)\psi - \frac{1}{4\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda \right] - \frac{1}{4e^2}\int d^4x\, F_{\mu\nu}^2, \tag{4}$$

where ψ is the "composite fermion," or the fermionic vortex. Since a_μ is now a field propagating in (2+1)D, care is needed to define the theory (4) on a compact manifold—to avoid parity anomaly one should either restrict the path integral over the a field configurations with even fluxes or introduce another gauge field (for discussions of this point see Refs. [6,7]). For the questions in which we are interested in this paper, this subtlety will not play an important role. At first sight (4) and (1) appear to be very different theories; however we will show that they are the same theory with different coupling constant.

III. DERIVATIONS OF SELF-DUALITY

A. A simple derivation

The most straightforward way to see the self-duality is to rewrite both theories in the form of theories with nonlocal current-current interactions. Integrating over A_μ in Eq. (1) we obtain a nonlocal action in $2 + 1$ dimensions,

$$S = \int d^3x\, i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - i\frac{e^2}{2}\int d^3x\, d^3x'\, j_{\Psi\mu}\frac{1}{\sqrt{\partial^2}}j^\mu_\Psi, \tag{5}$$

where $\frac{1}{\sqrt{\partial^2}}$ is the (3+1)D Feynman propagator subject to the constraint $z = z' = 0$. On the other hand, integrating over A_μ in the dual theory (4) leads to

$$S = \int d^3x i\bar{\psi}\gamma^\mu(\partial_\mu - ia_\mu)\psi - \frac{ie^2}{(8\pi)^2}\int d^3x d^3x' f_{\mu\nu}\frac{1}{\sqrt{\partial^2}}f^{\mu\nu}, \tag{6}$$

where $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, and now integrating over a_μ we get

$$S = \int d^3x i\bar{\psi}\gamma^\mu\partial_\mu\psi - i\frac{(8\pi)^2}{2e^2}\int d^3x d^3x' j_{\psi\mu}\frac{1}{\sqrt{\partial^2}}j^\mu_\psi, \tag{7}$$

which has the same form as Eq. (5), with the replacement $e \to 8\pi/e$.

B. Alternative derivation through bulk electromagnetic duality

There is another derivation of the self-duality which reveals the connection to electromagnetic duality in the bulk. We first note that across the brane, the density and current on the brane determines the jump of the perpendicular (to the brane) component of the electric field and the parallel components of the magnetic field,

$$\Delta E_z = e^2\rho, \qquad \Delta\mathbf{B}_\parallel = e^2\,\mathbf{j}\times\hat{\mathbf{z}}. \tag{8}$$

In contrast, B_z and the $\mathbf{E}_\parallel$ are continuous across the brane.

Without losing generality, we can impose an orbifold condition

$$A_\mu(z) = A_\mu(-z), \quad \alpha = t,x,y, \tag{9a}$$

$$A_z(z) = -A_z(-z). \tag{9b}$$

One can see that by decomposing the fields into symmetric and antisymmetric (under $z \to -z$) parts, $A_\mu = A^s_\mu + A^a_\mu$, the action then decomposes into

$$S[\psi, A^s_\mu, A^a_z] + S[A^a_\mu, A^s_z]. \tag{10}$$

The fields A^a_μ and A^s_z do not couple to the brane degrees of freedom and can be integrated away. With the orbifold condition (9), Eqs. (8) completely determine the boundary values of the perpendicular component of the electric field and the parallel components of the magnetic field,

$$E_z(z = \pm\epsilon) = \pm\frac{1}{2}e^2\rho, \tag{11a}$$

$$\mathbf{B}_\parallel(z = \pm\epsilon) = \pm\frac{1}{2}e^2\mathbf{j}\times\hat{\mathbf{z}}. \tag{11b}$$

In contrast $\mathbf{E}_\parallel$ and B_z are continuous at $z = 0$.

We now analyze the composite fermion theory (4). First let us write down the field equations. A_μ satisfy the Maxwell equation in the bulk and the boundary conditions (11), where the charge density and current are

$$\rho = -\frac{1}{4\pi}b, \tag{12a}$$

$$\mathbf{j} = -\frac{1}{4\pi}\mathbf{e}\times\hat{\mathbf{z}}. \tag{12b}$$

Varying S with respect to a_μ we also find, at $z = 0$,

$$\rho^{\rm CF} = \frac{1}{4\pi}B_z, \tag{13a}$$

$$\mathbf{j}^{\rm CF} = \frac{1}{4\pi}\mathbf{E}_\parallel\times\hat{\mathbf{z}}. \tag{13b}$$

Instead of dealing with A_μ, we now perform an operation electromagnetic duality in the bulk. We introduce a dual electromagnetic field $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$ related to $\mathbf{E}$ and $\mathbf{B}$ by

$$\mathbf{E} = -\,\mathrm{sgn}(z)\tilde{\mathbf{B}}, \tag{14a}$$

$$\mathbf{B} = \mathrm{sgn}(z)\tilde{\mathbf{E}}. \tag{14b}$$

Note that the transformation has a discontinuity at $z = 0$. In the bulk $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$ satisfy the same free Maxwell equations as $\mathbf{E}$ and $\mathbf{B}$. Note that the orbifold conditions (9) are preserved by the duality transformation (14).

With Eq. (12), Eqs. (11) become, after the EM duality,

$$\tilde{B}_z = -\frac{e^2}{2}\rho = \frac{e^2}{8\pi}b, \tag{15a}$$

$$\tilde{\mathbf{E}}_\parallel = \frac{e^2}{2}\mathbf{j}\times\hat{\mathbf{z}} = \frac{e^2}{8\pi}\mathbf{e}. \tag{15b}$$

That means we can now extend the gauge field a_μ to the whole (3+1)D space, taking for the value of the field in the bulk $a_\mu = \frac{8\pi}{e^2}\tilde{A}_\mu$. The bulk Lagrangian for a_μ is

$$S_{\text{bulk}}[a] = -\frac{1}{4e^2}\left(\frac{e^2}{8\pi}\right)^2 f_{\mu\nu}^2 = -\frac{1}{4\tilde{e}^2} f_{\mu\nu}^2, \tag{16}$$

where $\tilde{e} = 8\pi/e$. The electromagnetic duality operation (14) introduces jumps in components of $f_{\mu\nu}$,

$$e_z(z=\pm\epsilon) = \frac{8\pi}{e^2}\tilde{E}_z(z=\pm\epsilon) = \pm\frac{8\pi}{e^2}B_z, \tag{17a}$$

$$\mathbf{b}_\parallel(z=\pm\epsilon) = \frac{8\pi}{e^2}\tilde{\mathbf{B}}_\parallel(z=\pm\epsilon) = \mp\frac{8\pi}{e^2}\mathbf{E}_\parallel. \tag{17b}$$

By using Eq. (13), these equations can be written as

$$e_z(z=\pm\epsilon) = \pm 4\pi\frac{8\pi}{e^2}\rho_{\text{CF}} = \pm\frac{1}{2}\tilde{e}^2\rho_{\text{CF}}, \tag{18a}$$

$$\mathbf{b}_\parallel(z=\pm\epsilon) = \pm 4\pi\frac{8\pi}{e^2}\mathbf{j}_{\text{CF}}\times\hat{\mathbf{z}} = \pm\frac{1}{2}\tilde{e}^2\mathbf{j}_{\text{CF}}\times\hat{\mathbf{z}}, \tag{18b}$$

which have exactly the same form as Eq. (11).

Thus the action for the composite fermion can be written as

$$S = \int d^3x\, i\bar{\psi}\gamma^\mu(\partial_\mu - ia_\mu)\psi - \frac{1}{4\tilde{e}^2}\int d^4x\, f_{\mu\nu}^2. \tag{19}$$

When $e^2 = 8\pi$ this action coincides with the action for the original electron. This is the self-dual point.

C. Comparison with the model in half space

In the literature, one frequently considers a model where the gauge field propagates in one half space and the fermion is localized on the boundary of the half space. In this case (see, e.g., Ref. [7]) we need a bulk θ term with $\theta = \pi$ to properly define the partition function. The duality transformation considered above becomes a combination of S and T transformations. To see that, let us define the complex coupling constant

$$\tau = \frac{\theta}{2\pi} + i\frac{2\pi}{e^2} \tag{20}$$

and recall that the operations S and T act on the constant as

$$\mathsf{S}: \tau \to -1/\tau, \tag{21}$$

$$\mathsf{T}: \tau \to \tau + 1. \tag{22}$$

The composite operator $\mathsf{ST}^{-2}\mathsf{ST}^{-1}$ maps τ onto

$$\tau \to \tau' = \frac{\tau - 1}{2\tau - 1}. \tag{23}$$

In particular, starting from $\theta = \pi$, one also ends up with $\theta' = \pi$:

$$\tau = \frac{1}{2} + i\frac{2\pi}{e^2} \to \tau' = \frac{1}{2} + i\frac{e^2}{4\times 2\pi}. \tag{24}$$

The self-dual point is at $e^2 = 4\pi$. This is twice smaller than the value $e^2 = 8\pi$ we found for the model living in the whole space. The factor of 2 difference accounts for the fact that in our current model the electric field lines are restricted to one half space; hence the strength of the Coulomb interaction is twice larger than in the model living in the whole space with the same value of e^2.

IV. CONSEQUENCES OF SELF-DUALITY

A. Electrical conductivity at zero chemical potential and zero magnetic field

Now let us explore the consequences of duality for the conductivity. Let us introduce the conductivity tensor, which is denoted as σ_{ij} on the electron side and $\tilde{\sigma}_{ij}$ on the composite fermion side.

On the electron side Ohm's law reads

$$j^i = \sigma^{ij}E_j, \tag{25}$$

and on the composite fermion side

$$j^i_{\text{CF}} = \tilde{\sigma}^{ij}e_j. \tag{26}$$

Using the duality dictionary,

$$j^i = -\frac{1}{4\pi}\epsilon^{ij}e_j, \tag{27}$$

$$j^i_{\text{CF}} = \frac{1}{4\pi}\epsilon^{ij}E_j, \tag{28}$$

one can easily find

$$\sigma = -\frac{1}{(4\pi)^2}\epsilon\,\tilde{\sigma}^{-1}\epsilon, \quad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{29}$$

Let us assume the electron theory to be at zero chemical potential and zero magnetic field, but finite temperature. The conductivity tensor is diagonal: $\sigma_{ij} = \sigma\delta_{ij}$. Equation (29) now implies

$$\sigma(e)\tilde{\sigma}(\tilde{e}) = \frac{1}{(4\pi)^2}. \tag{30}$$

In particular, at the self-dual point $e^2 = \tilde{e} = 8\pi$,

$$\sigma = \sigma_0. \tag{31}$$

At zero temperature, the conductivity is just the coefficient appearing in the current-current correlation function:

$$\langle J_\alpha(p)J_\beta(q)\rangle = \frac{\sigma}{\sqrt{q^2}}(q^2 g_{\mu\nu} - q_\mu q_\nu)(2\pi)^3\delta^{(3)}(p+q). \tag{32}$$

For a free Dirac fermion, $e^2 = 0$, $\sigma = 1/16$ [23]. Our result thus indicates that the conductivity at $e^2 = 8\pi$ is by a factor of $4/\pi \approx 1.273$ times larger than at $e^2 = 0$. This can be compared with the weak-coupling result derived in Ref. [24],

$$\sigma = \frac{1}{16}\left(1 + C\frac{e^2}{4\pi} + O(e^4)\right), \quad C = \frac{92 - 9\pi^2}{18\pi}. \tag{33}$$

If one naively substitutes $e^2 = 8\pi$, one finds that the one-loop correction enhances the conductivity by a factor of 1.112. Obviously at such large coupling higher-loop effects cannot be neglected.

From Eq. (30) we also find the conductivity at infinite coupling,

$$\sigma(e^2 = \infty) = \frac{1}{\pi^2}\left[\frac{2}{\pi}\frac{e^2}{h}\right]. \tag{34}$$

Moreover, our result (31) is also applicable at finite temperature, where it implies that the conductivity is independent of frequency. In particular the conductivity has the same value in the hydrodynamic regime $\omega \ll T$ and in the ballistic regime $\omega \gg T$. A frequency-independent finite-temperature conductivity has been found in Ref. [19] for the theory living on a stack of N M2 branes in M theory in the limit of large N, which has been traced back to the electromagnetic duality in the holographic description.

The result continues to be true in the presence of duality symmetric disorder. Such disorder can be introduced, e.g., as a randomly fluctuating mass term of the fermion, or by placing random electric charges and magnetic monopoles in the bulk near the brane so that the statistical properties of this random ensemble of electric and magnetic charges is invariant under electromagnetic duality.

One can also consider transport at nonzero wave vectors, where it is characterized by the longitudinal and transverse conductivities. Again from Eq. (29) it follows that at $e^2 = 8\pi$,

$$\sigma_\perp(\omega,q)\sigma_\parallel(\omega,q) = \sigma_0^2. \tag{35}$$

This exact relationship has been found previously in the context of holography [19] and the bosonic self-dual theory [18]. When $q = 0$ the longitudinal and transverse conductivities are equal and one recovers Eq. (31).

B. Electric and thermal transport at filling factor $\nu = \frac{1}{2}$

1. Electrical conductivities

We now consider our system in a finite magnetic field and finite density, so that the filling factor is 1/2. At zero temperature and at weak coupling, the system forms an integer quantum Hall state with the zero-energy Landau level completely filled. Now the conductivity tensor has nonzero off-diagonal elements (the Hall conductivity). For simplicity we only consider transport at zero wave number when

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix}. \tag{36}$$

From the duality mapping between the density and the magnetic fields, it follows that at the self-dual point the dual theory is exactly the original theory, but with the filling factor $-\frac{1}{2}$. That means $\tilde{\sigma}_{xx} = \sigma_{xx}$, $\tilde{\sigma}_{xy} = -\sigma_{xy}$, or $\tilde{\sigma} = \sigma^T$, and Eq. (29) implies

$$\sigma_{xx}^2 + \sigma_{xy}^2 = \sigma_0^2. \tag{37}$$

At zero temperature and in the absence of disorder, we have an integer quantum Hall state with $\sigma_{xx} = 0$ and $\sigma_{xy} = \frac{1}{4\pi}$ and Eq. (37) is trivially satisfied. Turning on the temperature, a nonzero σ_{xx} is induced by scatterings of the charge fermions on photons in the bulk. In the limit when the temperature is very large (compared to the scale set by the magnetic field and the density), $\sigma_{xx} = \frac{1}{4\pi}$ as we have derived at zero field and zero chemical potential. As the temperature changes the conductivities vary between these two extreme, following a quarter circle in the (σ_{xx}, σ_{xy}) plane. This behavior is reminiscent of the semicircle law in quantum Hall transitions [25–27].

2. Thermoelectric coefficients

We now apply the duality technique to the thermoelectric transport. Introducing the thermoelectric coefficients α_{xx} and α_{xy},

$$j^i = \sigma^{ij}E_j + \alpha^{ij}\partial_j T, \tag{38}$$

$$j^i_{\rm CF} = \tilde{\sigma}^{ij}e_j + \tilde{\alpha}^{ij}\partial_j T. \tag{39}$$

By using again the duality mapping, we find

$$\alpha = \frac{1}{4\pi}\epsilon\tilde{\sigma}^{-1}\tilde{\alpha}. \tag{40}$$

Again, at the self-dual point $\tilde{\alpha} = \alpha^T$, and Eq. (40) determines the ratio α_{xy}/α_{xx} in terms of σ_{xy}/σ_{xx}. If we introduce the Hall angle $\theta_H = \arctan(\sigma_{xy}/\sigma_{xx})$, then

$$\frac{\alpha_{xy}}{\alpha_{xx}} = \tan\left(\frac{\pi}{4} + \frac{\theta_H}{2}\right). \tag{41}$$

This can be written in terms of components of the Seebeck tensor, defined through $E_i = S_{ij}\partial_j T$ when there is no current, $j_i = 0$,

$$\frac{S_{xy}}{S_{xx}} = \tan\left(\frac{\pi}{4} - \frac{\theta_H}{2}\right). \tag{42}$$

Note that S_{xx} is the usual Seebeck coefficient and S_{xy}/B is the Nernst coefficient.

Again one can discuss two limits. In the low-temperature quantum Hall regime, $\theta \to \pi/2$, the Hall thermoelectric coefficient α_{xy} dominates over the longitudinal coefficient α_{xx}. In the very high temperature regime, $T \gg n^{1/2}$, our result indicates that the ratio α_{xx}/α_{yy} tends to 1, in contrast to the electric conductivities where σ_{xx} dominates over σ_{xy}. This is not completely surprising: the thermoelectric coefficients break charge conjugation and hence are zero when $n = B = 0$, and one can show that α_{xx} is proportional to n and α_{xy} to B when n and B are small. At filling factor $\nu = 1/2$, n and B are proportional to each other; hence α_{xx} and α_{xy} are of the same order of magnitude.

3. Thermal Hall coefficient

Finally, we consider the transport of heat at filling factor $\nu = \frac{1}{2}$. The heat current is

$$q_i = -T\alpha_{ij}E_j - \bar{\kappa}_{ij}\partial_j T, \tag{43}$$

where $\bar{\kappa}_{ij}$ is the thermal conductivity tensor in the absence of electric field, which is related to the thermal conductivity tensor in the absence of electric current κ_{ij} by $\kappa = \bar{\kappa} - T\alpha\sigma^{-1}\alpha$. As the heat current is invariant under electromagnetic duality, it has to be given by the same expression in the dual description,

$$q_i = -T\tilde{\alpha}_{ij}e_j - \tilde{\bar{\kappa}}_{ij}\partial_j T. \tag{44}$$

Following the duality maps we obtain a connection relating the thermal conductivity tensors on the two sides the duality,

$$\bar{\kappa} = \tilde{\bar{\kappa}} - T\tilde{\alpha}\tilde{\sigma}^{-1}\tilde{\alpha} = \tilde{\kappa}. \tag{45}$$

Thus, the thermal conductivity at zero field on one side of the duality is equal to the thermal conductivity at zero current on the other side. At the self-dual point, $\tilde{\alpha} = \alpha^T$ and $\tilde{\bar{\kappa}} = \bar{\kappa}^T$. Equation (45) then establishes a direct relationship between the thermal Hall conductivity and the thermoelectric coefficients:

$$\bar{\kappa}_{xy} = -\kappa_{xy} = \frac{T}{2}\frac{\alpha_{xx}^2 + \alpha_{xy}^2}{\sigma_0} = \frac{T}{2}\left(S_{xx}^2 + S_{xy}^2\right)\sigma_0. \quad (46)$$

We also find that $\bar{\kappa}_{xx} = \kappa_{xx}$, but otherwise there is no constraint on this coefficient. Not surprisingly, the values of the kinetic coefficients in the N M2-brane theory [28] also respects an analogous constraint (in the dc regime). We note that relationships similar to Eqs. (40) and (45) have been found in Ref. [29] in the context of a holographic model with bulk electromagnetic duality.

V. CONCLUSION

We have shown that the simple model of a (2+1)D fermion coupled to a three-dimensional U(1) gauge field, $QED_{4,3}$, exhibits weak-strong duality, and is self-dual at a particular value of the coupling constant. From the self-duality we derive the value of the conductivity at zero density and magnetic field and show that it is independent of frequency. At finite magnetic field and filling factor $\nu = \frac{1}{2}$ we were able to derive a semicircle law satisfied by the longitudinal and Hall conductivities, relate the ratio of the diagonal and Hall thermoelectric coefficients with the Hall angle, and derive a relationship between the thermal Hall conductivity and the thermoelectric coefficients.

We note here that all results obtained above can be transferred to the bosonic model, at zero density and magnetic field and at filling factor $\nu = 1$. This may be interesting since the $\nu = 1$ bosonic quantum Hall state can be a Fermi liquid. Most formulas derived in the paper remain valid if one replaces the value of σ_0 by $\sigma_0 = 1/(2\pi)$.

It remains to be seen whether self-dual systems are accessible experimentally [30]. If they are, the predictions made in this paper may serve as tests of self-duality in such systems.

Note added. Recently we became aware of work by D. F. Mross, J. Alicea, and O. I. Motrunich [31] which partially overlaps with our paper. We thank them for sharing their unpublished work with us.

ACKNOWLEDGMENTS

The authors thank Clay Córdova, Olexei Motrunich, Cenke Xu, Luis Melgar, Subir Sachdev, and Nathan Seiberg for discussions. This work is supported, in part, by US DOE Grant No. DE-FG02-13ER41958 and a Simons Investigator Grant from the Simons Foundation. Additional support was provided by the Chicago MRSEC, which is funded by NSF through Grant No. DMR-1420709.

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