θ_0 thermal Josephson junction

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We predict the thermal counterpart of the anomalous Josephson effect in superconductor/ferromagnet/superconductor junctions with noncoplanar magnetic texture. The heat current through the junction is shown to have the phase-sensitive interference component proportional to $\cos(\theta - \theta_0)$, where θ is the Josephson phase difference and θ_0 is the texture-dependent phase shift. In the generic trilayer magnetic structure with the spin-filtering tunnel barrier θ_0 is determined by the spin chirality of magnetic configuration and can be considered as the direct manifestation of the energy transport with participation of spin-triplet Cooper pairs. In case of the ideal spin filter the phase shift is shown to be robust against spin relaxation caused by the spin-orbital scattering. Possible applications of the coupling between heat flow and magnetic precession are discussed. For the nonideal spin filters with practically relevant parameters we show that θ_0 is much larger than the phase shift of the equilibrium Josephson current.

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I. INTRODUCTION

During recent years large attention has been devoted to the emerging field of phase-coherent caloritronics in hybrid superconducting structures [1]. The mechanism of phase-sensitive heat transport is based on the thermal counterpart of the Josephson effect [2–6] which occurs in the system consisting of two superconductors S_1 and S_2 separated by a weak link and residing at temperatures T_1 and T_2, respectively. The nonzero temperature bias (for definiteness we assume that $T_1 > T_2$) generates a stationary heat flow from S_1 to S_2 given by the heat current-phase relation (HCPR)

$$\dot{Q}_{\rm tot}(T_1,T_2,\theta) = \dot{Q}_{qp} - \dot{Q}_{\rm int}\cos\theta, \tag{1}$$

where θ is the phase difference between superconducting electrodes. Here the first term is the usual quasiparticle heat current while the second one describes the contribution of energy transfer with participation of Cooper pairs. In accordance with Onsager symmetry the heat current is time-reversal invariant since the phase-coherent term in Eq. (1) does not change under the phase inversion $\dot{Q}_{\rm tot}(\theta) = \dot{Q}_{\rm tot}(-\theta)$.

Experimentally the interplay of heat transport and Josephson phase difference has been studied starting from the observations of thermoelectric effects in superconductor/normal metal/superconductor junctions [7–12]. Recently the existence of coherent thermal currents (1) has been confirmed in experiments using the Josephson heat interferometry with tunnel contacts [1,13,14]. Subsequently the number of possible applications has been suggested including heat interferometers [13–16], diodes [17], transistors [15,18,19], phase-tunable ferromagnetic Josephson valves [20,21], and the probes of topological Andreev bound states [22]. The direction $\dot{Q}_{\rm int}$ in Eq. (1) can be controlled in experiments providing the realization of $0-\pi$ thermal Josephson junction [23].

In the present paper we report on the possibility to obtain the generalized HCPR of the form

$$\dot{Q}_{\rm tot}(T_1,T_2,\theta) = \dot{Q}_{qp} - \dot{Q}_{\rm int}\cos(\theta-\theta_0), \tag{2}$$

which can have an arbitrary phase shift θ_0 in contrast to the Eq. (1) studied in all previous works [2–6]. This effect takes place in the systems with broken time-reversal and chiral symmetries such as the S/F/S junctions with noncoplanar magnetic textures or spin-orbital interaction. It can be considered as the thermal counterpart of the anomalous Josephson effect characterized by the generalized current-phase relation (CPR) [24–44]

$$I(\varphi) = I_c \sin(\theta - \varphi_0)\ . \tag{3}$$

Here I_c is the critical current and φ_0 is an arbitrary phase shift which however in the general case is different from that in the generalized HCPR $\theta_0 \neq \varphi_0$.

We demonstrate the phase-shifted HCPR (2) using a generic example of Josephson spin valve [20,21,45,46] that contains three noncoplanar magnetic vectors, see Fig. 1. It consists of two ferromagnetic layers (F) with exchange fields $\boldsymbol{h}_{1,2}$ interacting with the superconducting electrodes (S), separated by the spin-filter barrier with the magnetic polarization directed along $\boldsymbol{m}$. Recently the spin-filter effect in superconductor/ferromagnet structures has been demonstrated by using ferromagnetic insulators (FI) for example europium chalcogenides [47–52] or GdN tunneling barriers [53]. The role of outer $F_{1,2}$ contacts is to induce effective exchange fields in the superconducting electrodes. In case of metallic ferromagnets this can be achieved through the inverse of the proximity effect [54–57]. Alternatively $F_{1,2}$ can be ferromagnetic insulators and induce the effective exchange field in $S_{1,2}$ as a result of the spin-mixing scattering of conduction electrons [58].

II. MODEL

To calculate the currents across spin-filtering barriers we use generalized Kuprianov-Lukichev boundary conditions [59], that include spin-polarized tunnelling at the SF interfaces [58,60]. The matrix tunneling current from S_1 to S_2 is given by

$$\check{I}_{12} = [\check{\Gamma}\check{g}_1\check{\Gamma}^{\dagger},\check{g}_2], \tag{4}$$

where $\check{g}_k$ for $k=1,2$ are the matrix Green's functions (GF) in the superconducting electrodes S_k. The spin-polarized tunneling matrix has the form $\check{\Gamma} = t_+\hat{\sigma}_0\hat{\tau}_0 + t_-(\boldsymbol{m}\hat{\boldsymbol{\sigma}})\hat{\tau}_3$, where $\boldsymbol{m}$ is the direction of barrier magnetization,

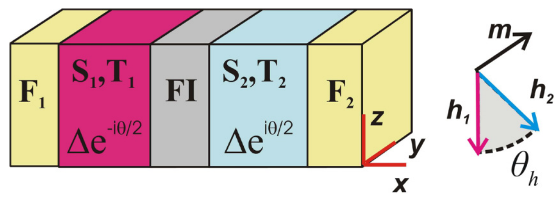

FIG. 1. The sketch of an FS-FI-SF system under the thermal bias with the superconducting electrodes $S_{1,2}$ residing at different temperatures $T_{1,2}$. The exchange fields $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ in ferromagnetic electrodes F_1 and F_2 form a noncoplanar system with the spin polarization $\boldsymbol{m}$ of the ferromagnetic barrier (FI).

$t_\pm = \sqrt{(1 \pm \sqrt{1-P^2})/2}$ and P is the spin-filter efficiency of the barrier that ranges from 0 (no polarization) to 1 (100% filtering efficiency).

The matrix GF is given by $\check{g} = \begin{pmatrix} g^R & g^K \\ 0 & g^A \end{pmatrix}$, where g^K is the Keldysh component and $g^{R(A)}$ is the retarded (advanced) GF determined by the equation [56]

$$[i\varepsilon\tau_3 - i(\boldsymbol{h}\cdot\boldsymbol{S})\tau_3 - \check{\Delta} + \check{\Sigma}_s, \check{g}] = 0. \quad (5)$$

Here ε is the energy, $\check{\Delta} = \Delta\tau_1 e^{i\tau_3\varphi}$ is the order parameter with the amplitude Δ and phase φ, $\boldsymbol{h}$ is the exchange field, and $\boldsymbol{S} = (\sigma_1,\sigma_2,\sigma_3)$, $\sigma_{1,2,3}$, and $\tau_{1,2,3}$ are the Pauli matrices in spin and Nambu spaces, respectively. We include the spin-orbital (SO) scattering process which lead to the spin relaxation described by [56,61] $\check{\Sigma}_s = (\boldsymbol{S}\cdot\check{g}\boldsymbol{S})/8\tau_{so}$, where τ_{so} is the SO scattering time. Due to the normalization condition $\check{g}^2 = 1$ the Keldysh component can be written as $g^K = (g^R - g^A)f_L$, where $f_L = f_L(\varepsilon)$ is the distribution function. We assume that it has an equilibrium form $f_L^{(1,2)} = \tanh(\varepsilon/2T_{1,2})$ characterized by the different temperatures $T_{1,2}$ in the electrodes $S_{1,2}$.

Proximity of the outer ferromagnetic layers shown in Fig. 1 induces Zeeman splitting of electronic states which acts as an effective exchange field in the superconducting electrodes. We assume that superconducting layers are thin enough to neglect the spatial variations of the spectral GFs (retarded and advances) so that up to leading order they retain their bulk values in the presence of a homogeneous exchange field

$$g^R = \tau_3[g_{03} + g_{33}(\boldsymbol{\sigma}\boldsymbol{h})] + \tau_1[g_{01} + g_{31}(\boldsymbol{\sigma}\boldsymbol{h})] \quad (6)$$

and $g^A = -\tau_3 g^{R\dagger}\tau_3$. The terms diagonal in Nambu space (τ_3) correspond to the normal correlations which determine the total density of states (DOS) $N_+ = \mathrm{Re} g_{03}$ and the DOS difference between the spin-up and spin-down subbands $N_- = \mathrm{Re} g_{33}$. The off-diagonal components (τ_1) describe spin-singlet g_{01} and spin-triplet g_{31} superconducting correlations which appear due to the exchange splitting [62].

The tunneling heat current $\dot{Q}$ across the Josephson junction (JJ) is given by the general expression

$$R_N\dot{Q} = \frac{1}{16e^2}\int_{\infty}^{\infty} d\varepsilon\varepsilon\, \mathrm{Tr}(\check{I}_{12}^K), \quad (7)$$

where R_N is the normal-state resistance of the tunneling barrier and e is the electron charge.

III. RESULTS

At first we calculate the heat current assuming that the temperature difference is small and expanding distribution functions in $S_{1,2}$ electrodes $f_L^{(1,2)} = f_0 + (T_{1,2} - T)\frac{\partial f_0}{\partial T}$ where $T = (T_1 + T_2)/2$ and $f_0 = \tanh(\varepsilon/2T)$. In accordance with Eq. (7) the total heat conductance $\kappa = \dot{Q}/(T_1 - T_2)$ can be written as the superposition of three terms

$$\kappa/\kappa_N = \kappa_{qp} - \kappa_c\cos\theta - \kappa_s\sin\theta, \quad (8)$$

where $\kappa_N = \pi^2 T/(3e^2R_N)$ is the normal state thermal conductance of the junction. We will refer to the different contributions in (8) as the quasiparticle κ_{qp}, the usual κ_c, and phase-shifting κ_s interference terms. Assuming $S_{1,2}$ superconductors to be identical we get expressions for components in Eq. (8):

$$\kappa_{qp} = \int_{-\infty}^{\infty} d\varepsilon F\{N_+^2 + [r(\boldsymbol{h}_{1\perp}\boldsymbol{h}_{2\perp}) + h_{1\parallel}h_{2\parallel}]N_-^2\}, \quad (9)$$

$$\kappa_c = \int_{-\infty}^{\infty} d\varepsilon F \quad (10)$$
$$\{r(\mathrm{Im} g_{01})^2 + [rh_{1\parallel}h_{2\parallel} + (\boldsymbol{h}_{1\perp}\boldsymbol{h}_{2\perp})](\mathrm{Im} g_{31})^2\},$$

$$\kappa_s = \chi P\int_{-\infty}^{\infty} d\varepsilon F(\mathrm{Im} g_{31})^2, \quad (11)$$

where $\chi = \boldsymbol{m}\cdot(\boldsymbol{h}_1\times\boldsymbol{h}_2)$ is spin chirality, $r = \sqrt{1-P^2}$, and $h_\parallel = (\boldsymbol{m}\boldsymbol{h})$ and $\boldsymbol{h}_\perp = \boldsymbol{h} - h_\parallel\boldsymbol{m}$ are the exchange field components parallel and perpendicular to the FI barrier polarization, $F(\varepsilon) = (6\varepsilon^2/\pi^2T^2)\partial f_0/\partial\varepsilon$.

The quasiparticle and the usual phase-sensitive contributions (9,10) have been analyzed for the coplanar magnetic configuration [21]. The term κ_s (11) is nonzero only in the noncoplanar case $\chi \neq 0$ and it produces the phase shift of HCPR in Eq. (2) given by $\theta_0 = \arctan(\kappa_s/\kappa_c)$. Comparing different parts of the conductance [(9), (10), (11)] one can see that κ_s is qualitatively different from the others since it stems exclusively from the triplet part of the condensate associated with the GF component g_{31}. In the general case of a nonideal spin filter $P \neq 1$ the usual interference part κ_c has contributions from both spin-singlet and spin-triplet Cooper pairs. Thus one can conclude that the nontrivial phase shift $\theta_0 \neq 0$ of the HCPR is a direct experimentally measurable evidence of the transport of spin-triplet Cooper pairs across the tunnel junction.

IV. DISCUSSION

Physically the phase shifts in HCPR (2) as well as in CPR (3) appear as a result of the additional phase picked up by the spin-triplet Cooper pairs when tunneling between two superconductors with noncollinear exchange fields through the spin-polarizing barrier. To understand this phenomenon on a qualitative level let us consider the magnetic configuration $\boldsymbol{h}_1 = h_1\boldsymbol{z}$, $\boldsymbol{h}_2 = h_2\boldsymbol{x}$, and $\boldsymbol{m} = \boldsymbol{y}$. The spin-triplet condensates in S_1 and S_2 are described by the wave functions $\Psi_{t1} \sim |\uparrow,\downarrow\rangle_z + |\downarrow,\uparrow\rangle_z$ and $\Psi_{t2} \sim e^{i\varphi}(|\uparrow,\downarrow\rangle_x + |\downarrow,\uparrow\rangle_x)$, respectively, where the spin quantization axes are set by the directions of exchange fields $\boldsymbol{h}_{1,2}$. Assuming the spin filter to be ideal $P = 1$ we find for the tunneling amplitude

$\langle\Psi_{2t}|\hat{P}_y|\Psi_{1t}\rangle \sim ie^{i\varphi}$, where $\hat{P}_y$ is the projection operator acting on each of the single-electron states $\hat{P}_y|\uparrow\rangle_z = \frac{1}{2}|\uparrow\rangle_y$ and $\hat{P}_y|\downarrow\rangle_z = \frac{-i}{2}|\uparrow\rangle_y$. Thus one can see that the spin-filtering provides an additional $\pi/2$ phase in the tunneling amplitude of spin-triplet Cooper pairs, which is the origin of the anomalous Josephson effect [44] and the phase-shifted HCPR studied here.

On a quantitative level let us analyze the particular configuration $\boldsymbol{h}_{1,2} \perp \boldsymbol{m}$ when the expressions (10), (11) yield the interference contributions which are proportional to each other $\kappa_{\rm int} = \int_{-\infty}^{\infty} d\varepsilon F(\varepsilon)({\rm Im} g_{31})^2$ so that $\kappa_c = -\cos\theta_h \kappa_{\rm int}$ and $\kappa_s = \sin\theta_h \kappa_{\rm int}$, where θ_h is the angle between $\boldsymbol{h}_{2\perp}$ and $\boldsymbol{h}_{1\perp}$ shown schematically in Fig. 1. Hence for the ideal spin filter $P = 1$ the phase shift of HCPR is determined by the geometry of magnetic configuration $\theta_0 = \theta_h$ although the overall amplitude of $\kappa_{\rm int}$ is strongly suppressed by the spin relaxation. To demonstrate this we find the order parameter and spectral functions (6) self-consistently taking onto account the presence of exchange field and SO scattering rate which can vary in wide limits corresponding to [63] $(\Delta_0\tau_{so})^{-1} \approx 0.2$ in Al and to [64] $(\Delta_0\tau_{so})^{-1} \approx 500$ in Nb, where Δ_0 is the bulk superconducting gap at $h = 0$, $\tau_{so} = \infty$, and $T \to 0$.

Let us consider the calculated dependencies of quasiparticle $\kappa_{qp}(T)$ and interference $\kappa_{\rm int}(T)$ parts shown in Figs. 2(a) and 2(c), respectively, for the fixed exchange field $h = 0.5\Delta_0$. First there is a nonmonotonic dependence $\kappa_{qp} = \kappa_{qp}(T)$ which increases above the normal state value as the temperature goes down below T_c. This behavior is explained by the DOS enhancement near the gap edge. At lower temperatures $T \ll T_c$ the quasiparticles are frozen out which results in the exponential drop of the heat conductance. Of interest is the evolution of the peak amplitude in the $\kappa_{qp}(T)$ dependence with increasing spin relaxation rate. Initially with increasing τ_{so}^{-1} from zero to small values the peak of $\kappa_{qp}(T)$ is suppressed while at the larger values of τ_{so}^{-1} it is restored. This tendency reveals the evolution of DOS $N_+(\varepsilon)$ and the anomalous function ${\rm Im} g_{01}(\varepsilon)$ with increasing τ_{so}^{-1} shown in Fig. 3. For $\tau_{so}^{-1} \ll T_{c0}$ the singularities of spectral functions are smeared. However at larger values of $\tau_{so}^{-1} > T_{c0}$ both N_+ and ${\rm Im} g_{01}$ again develop the peaks although without the spin-splitting features. At the same time the spin-triplet components g_{33} and g_{13} are strongly suppressed by the SO scattering.

The temperature dependence of interference thermal conductance $\kappa_{\rm int}(T)$ is also nonmonotonic due to the similar mechanism as discussed above. As shown in Fig. 2(c) the maximum of $\kappa_{\rm int}(T)$ is strongly suppressed by the SO scattering which tends to remove the spin-dependent components g_{31} of the GFs. However for the fully-polarizing ideal spin filter $P = 1$ according to the equitation (10) the suppression of $\kappa_{\rm int}$ does not affect the phase shift of HCPR which is fixed by the angle between exchange fields $\boldsymbol{h}_{1,2}$ as discussed above.

The situation is different for $P < 1$ when κ_c and κ_s are no longer proportional to each other. The qualitative difference between the usual κ_c and phase-shifting κ_s contributions in this case is determined by the transport of spin-singlet Cooper pairs which is only possible if both the spin projections can pass the spin filter. This contribution is described by the first term in the r.h.s of Eq. (10) which yields a nonzero contribution since $r \neq 0$. In this case the phase shift of HCPR is suppressed

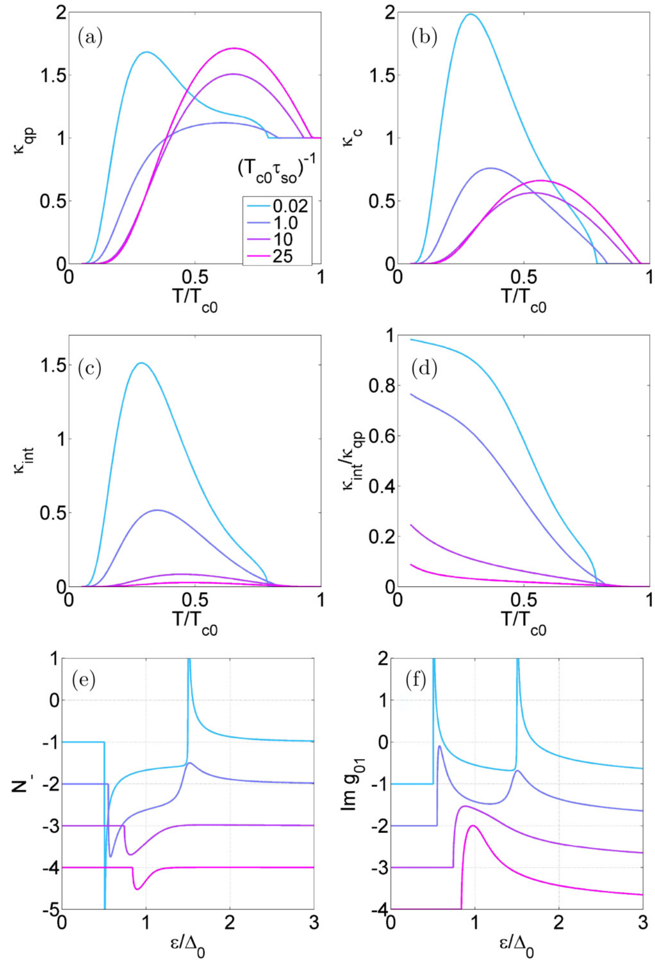

FIG. 2. Temperature dependencies of heat conductance contributions (a) κ_{qp}, (c) $\kappa_{\rm int}$, and their ratio (d) $\kappa_{\rm int}/\kappa_{qp}$ for the SFS structure shown in Fig. 1 with $\boldsymbol{h}_{1,2} \perp \boldsymbol{m}$ and the 100% spin filtering $P = 1$. (b) κ_c for the same structure but with $P = 0.8$ and $\theta_h = \pi/4$. Note that the quantities κ_{qp}, κ_c, and κ_s defined by Eqs. (9), (10), and (10) are dimensionless. (e) Total DOS $N_+(\varepsilon)$ and (f) spin-singlet anomalous function ${\rm Im} g_{01}(\varepsilon)$. The exchange field is $h = 0.5\Delta_0$ and the values of spin relaxation rate $(T_{c0}\tau_{so})^{-1}$ shown in (a) are the same for all panels.

by the SO scattering which leads to the decrease of spin-triplet correlations so that $\kappa_s \to 0$. At the same time the contribution of spin-singlet Cooper pairs survives keeping $\kappa_c \neq 0$ in the limit $\tau_{so} \to 0$ as shown in Fig. 2(b).

It is instructive to compare the phase-shifted HCPR (2) and CPR (3) calculated for the spin valve shown in Fig. 1 using the general matrix current (4). The usual $I_0 = I_c \cos\varphi_0$ and anomalous $I_{an} = I_c \sin\varphi_0$ Josephson currents through tunnel barrier are given by

$$\frac{R_N I_0}{\pi e T} = \sum_{\omega_n} \left[r\left(g_{01}^2 + h_{1\parallel} h_{2\parallel} g_{31}^2\right) + (\boldsymbol{h}_{1\perp}\boldsymbol{h}_{2\perp}) g_{31}^2 \right] \quad (12)$$

$$\frac{R_N I_{an}}{\pi e T} = \chi P \sum_{\omega_n} g_{31}^2, \quad (13)$$

where $\chi = \boldsymbol{m}\cdot(\boldsymbol{h}_1 \times \boldsymbol{h}_2)$ is the spin chirality, and g_{01} and g_{31} are the spin-singlet and spin-triplet components of the

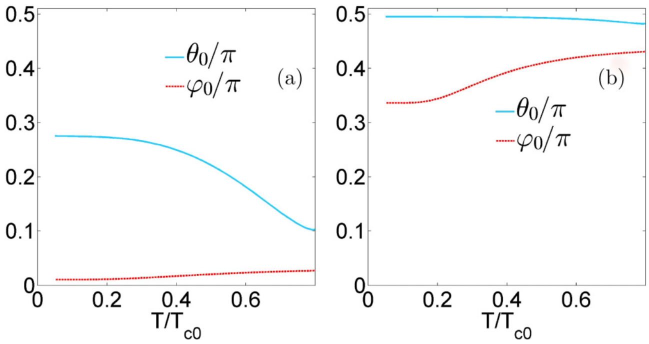

FIG. 3. The phase shifts of CPR $\varphi_0 = \varphi_0(T)$ and HCPR $\theta_0 = \theta_0(T)$ for (a) $P = 0.8$ ($r = 0.6$) and (b) $P = 0.9999$ ($r = 0.014$). Exchange splitting $h = 0.5\Delta_0$ and SO relaxation $(T_{c0}\tau_{so})^{-1} = 1$. The magnetic configuration is that $\boldsymbol{h}_{1,2} \perp \boldsymbol{m}$ and $\theta_h = \pi/2$.

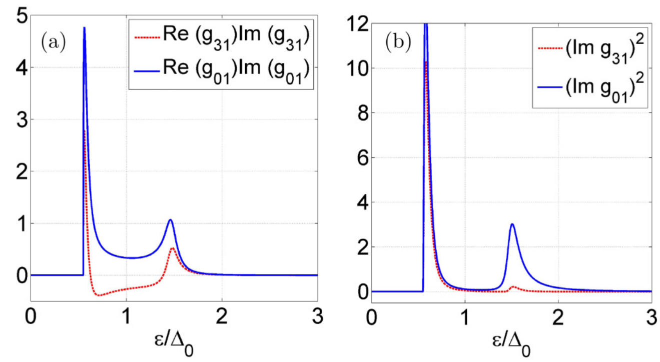

FIG. 4. Spectral amplitudes of the different components of (a) Josephson charge currents (12),(13) and (b) heat currents (10),(11). The blue solid lines and red dashed lines correspond to the contributions of spin-singlet and spin-triplet Cooper pairs, respectively. Exchange splitting $h = 0.5\Delta_0$, temperature $T = 0.1T_{c0}$, and SO relaxation $(T_{c0}\tau_{so})^{-1} = 1$.

Matsubara GF written in the form (6) analytically continued to the imaginary frequencies $\varepsilon \to i\omega_n$ with $\omega_n = (2n+1)\pi T$.

Expressions for the Josephson current (12),(13) are dual to that of the interference heat conductance (10),(11). The relation between Eqs. (12),(13) and (10),(11) can be understood by replacing the Matsubara summations with real-frequency integrals according to the rule $\pi T \sum_{\omega_n} g_{k0}^2 = \int_{-\infty}^{\infty} d\varepsilon f_0 \mathrm{Re} g_{k1} \mathrm{Im} g_{k1}$ for $k = 1,3$.

Similar to the phase-shifting term κ_s the anomalous current I_{an} is mediated by spin-triplet component g_{31}. Therefore φ_0-Josephson effect is the directly observable signature of the equilibrium spin-triplet charge current. As discussed above the θ_0- thermal Josephson effect is the signature of the nonequilibrium heat transport with participation of spin-triplet Cooper pairs.

For the ideal spin filter $P = 1$ Eqs. (10),(11) and (12),(13) yield temperature-independent phase shifts of CPR and HCPR $\varphi_0 = \theta_0 = \theta_h$, although in the general case θ_0 and φ_0 can be quite different, as shown in Figs. 3(a) and 3(b). For nonideal spin filters when the difference $1 - P$ has experimentally reasonable values we find that the thermal phase shift θ_0 is much larger than that of the CPR $\theta_0 \gg \varphi_0$, see Fig. 3(a). Thus the observability of anomalous HCPR in Josephson spin valves is less restrictive to the spin-filter properties than that of the φ_0-JJ.

The relative suppression of anomalous Josephson effect as compared to its thermal counterpart can be understood considering spectral densities of the singlet and triplet contributions to the heat (10),(11) and charge (12),(13) currents shown in Fig. 4. As one can see the triplet amplitude $\mathrm{Im} g_{31} \mathrm{Re} g_{31}$ is sign changing in the interval $\Delta_0 - h < \varepsilon < \Delta_0 + h$ while the singlet contribution is always positive $\mathrm{Re} g_{01} \mathrm{Im} g_{01} > 0$. Hence at low temperatures when the distribution function can be set $f_0 = 1$ in this energy interval, the integral contribution to usual Josephson current appears to be much larger than the anomalous one.

Quantitatively, the relative amplitude of usual and anomalous Josephson currents at small temperatures can be calculated analytically for small exchange fields $h \ll \Delta$ and negligible SO relaxation. In this case we can use expressions for the Matsubara Green's functions $g_{01} = \Delta/\sqrt{\omega^2 + \Delta^2}$ and $g_{31} = i d g_{01}/d\omega$. Then the summation at $T \to 0$ gives $T \sum_\omega g_{01} = \pi\Delta$ and $T \sum_\omega g_{31} = -(\pi/8)h/\Delta$. E.g., for $h = \Delta/2$, $\chi = 1$, and $P \approx 1$ that gives $I_{an}/I_0 \approx 0.03/r$. Hence this estimation explains why we have found significant values of φ_0 only for the almost ideal spin filter $P = 0.9999$ when the singlet-channel transparency coefficient in Eq. (12) is rather small $r = 0.014$.

At larger temperatures the spectral gap becomes suppressed so that the distribution function f_0 cannot be considered constant in Eqs. (12) and (13). Therefore the positive and negative parts of the triplet spectral amplitude in Fig. 4(a) acquire different weights. This leads to the enhancement of the triplet current and the resulting growth of phase shift φ_0 shown in Fig. 3.

The behavior of heat current is completely different from that of the charge current described above. It is determined by the amplitudes $(\mathrm{Im} g_{01})^2$ and $(\mathrm{Im} g_{31})^2$ which have quite similar peaks at the energies close to the gap edge, which provide the largest contribution at low temperature due to the weigh factor F in Eqs. (10) and (11). Thus at $T \ll T_c$ the usual κ_c and phase-shifting κ_s terms in the heat conductance have the same order of magnitude and θ_0 can reach large values as compared to φ_0. At elevated temperatures the HCPR is determined by larger energies where the spin-singlet [blue solid line in Fig. 4(b)] contribution is significantly larger than the spin-triplet one [dashed red line in Fig. 4(b)]. That results in the suppression of θ_0 at $T \to T_c$ which can be seen in Fig. 3(a).

The predicted effect of phase-shifted HCPR can be experimentally observed using the Josephson heat interferometer [1,13] consisting of the temperature-biased superconducting quantum interference device (SQUID) with the usual JJ in the one part and θ_0-JJ in the other part. In this case following the derivation in Ref. [1] one can show that the interference pattern of the heat current across the SQUID $\dot{Q}_{\mathrm{int}}$ contains a spontaneous shift as a function of the external magnetic flux Φ so that $\dot{Q}_{\mathrm{int}} = \dot{Q}_{\mathrm{int}}(\Phi - \Phi_e)$. For the ideal spin filter when $\varphi_0 = \theta_0 = \theta_h$ we get $\Phi_e = \theta_h \Phi_0/2\pi$.

The θ_0-shifted HCPR (3) provides an interesting possibility to couple the heat transport with magnetization dynamics. Oscillations of moments $\boldsymbol{h}_{1,2}$ and $\boldsymbol{m}$ driven by the Larmour precession around the effective field [28] in the generic thermomagnetic circuit (Fig. 1) produce the time-dependent spin chirality $\chi = \chi(t)$ and hence generate the nonstationary phase shifts $\varphi_0 = \varphi_0(t)$ and $\theta_0 = \theta_0(t)$. Thus according to Eqs. (2) and (3) one can generate alternating heat and charge currents at the Larmour frequency which can be controlled by external magnetic and the anisotropy fields. The other possible application is based on the effective conversion of spin currents inside the ferromagnet or ferromagnetic insulator into the electronic heat and charge currents across the attached Josephson junctions. Based on the discussed effect it is in principle possible to implement the superconducting JJ detector of magnetic precession associated with magnons in FI layer [65–67] or the skyrmion motion inside ferromagnets which can be used for the racetrack magnetic memory applications [68].

V. CONCLUSIONS

To summarize, we have found the thermal counterpart of the anomalous Josephson effect. Under the conditions of broken time-reversal and chiral symmetries the interference heat current acquires an arbitrary phase shift θ_0 which substantially generalizes the previously found forms of HCPR. For the generic example of the noncoplanar Josephson spin valve (Fig. 1) θ_0 is determined by the nonzero spin chirality $\chi = \boldsymbol{m} \cdot (\boldsymbol{h}_1 \times \boldsymbol{h}_2) \neq 0$. The phase shift is demonstrated to be the direct and experimentally measurable evidence of the heat transport with participation of spin-triplet Cooper pairs. We show that for experimentally realistic nonideal spin filters θ_0 is generically much larger than the phase shift φ_0 of the equilibrium Josephson current. Therefore the proposed effect is much less restrictive in terms of the spin-filtering properties than the φ_0-Josephson effect. In view of possible applications the proposed effect allows us to change the heat conductance of the system in a continuous way by rotating magnetic vectors. For this purpose it is preferable to use magnetic elements with different coercivity fields [50] or anisotropies.

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