RAPID COMMUNICATIONS

PHYSICAL REVIEW B **96**, 060301(R) (2017)

Weak quantum chaos

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(Received 1 February 2017; published 14 August 2017)

Out-of-time-ordered correlation functions (OTOCs) are presently being extensively debated as quantifiers of dynamical chaos in interacting quantum many-body systems. We argue that in quantum spin and fermionic systems, where all local operators are bounded, an OTOC of local observables is bounded as well and thus its exponential growth is merely transient. As a better measure of quantum chaos in such systems, we propose, and study, the density of the OTOC of extensive sums of local observables, which can exhibit indefinite growth in the thermodynamic limit. We demonstrate this for the kicked quantum Ising model by using large-scale numerical results and an analytic solution in the integrable regime. In a generic case, we observe the growth of the OTOC density to be linear in time. We prove that this density in general, locally interacting, nonintegrable quantum spin and fermionic dynamical systems exhibits growth that is at most polynomial in time—a phenomenon, which we term weak quantum chaos. In the special case of the model being integrable and the observables under consideration quadratic, the OTOC density saturates to a plateau.

DOI: 10.1103/PhysRevB.96.060301

Introduction. Quantum chaos was an active area of research in the 1980's and 1990's [1–3]. The main success of the field was a random matrix theory (RMT) classification of the universal properties of quantum systems whose classical counterparts are chaotic. The classical limits of such systems have positive Lyapunov exponents, which characterize exponential sensitivity to initial conditions—the so-called butterfly effect. However, since the (classical) definition of the Lyapunov exponent is based on the concept of phase-space trajectories, one cannot unambiguously translate it to the quantum realm.

Nevertheless, it has been argued that a weaker property of dynamical mixing—a decay of almost all connected temporal correlators—is sufficient to establish universal quantum chaotic behavior, such as random matrix statistics of energy spectra [4] or the universal exponential decay of Loschmidt echoes [5]. In the theory of dynamical systems, complex (mixing) dynamics that displays no exponential butterfly effect is referred to as weak chaos [6]. Examples of such dynamical systems include generic polygonal billiards in which nearby trajectories deviate only linearly with time, while correlation functions nevertheless exhibit mixing [7,8].

The study of dynamical mixing (now called scrambling) and Lyapunov chaos in quantum mechanics was recently revived by the high-energy physics community, initially in the context of the propagation of information in black hole backgrounds [9]. In 2014, Kitaev proposed to quantify chaos in quantum many-body systems [10] in terms of the out-of-time-ordered correlation function (OTOC),

$$C(x,t) = -\langle [w_x(t), v_0(0)]^2 \rangle_\beta, \tag{1}$$

where w_x, v_x are local observables and $\langle\cdot\rangle_\beta$ denotes the thermal expectation value at inverse temperature β. The concept is based on a work by Larkin and Ovchinnikov [11] from 1969, where OTOC was connected to the instability of semiclassical trajectories of electrons scattered by impurities in a superconductor. Consequently, extended quantum systems were defined as chaotic if the OTOC (1) of a pair of local observables w and v grows exponentially [11,12],

$$C(x,t) \propto e^{\lambda_L (t-|x|/v_B)}. \tag{2}$$

Motivated by the semiclassical picture, λ_L is referred to as the Lyapunov exponent and v_B the butterfly velocity.

A multitude of works examining the properties of quantum chaos has recently been written both from the high-energy perspective and from the condensed matter perspective [12–48].

In this Rapid Communication, we investigate systems with local interactions with an extensive number $N \to \infty$ of degrees of freedom, but with a finite local Hilbert space dimension D. In any model with a finite D (including all fermionic and spin lattice models), in which local operators u, v are bounded, the exponential growth in (2) can be bounded by operator norm inequalities (the triangular inequality $\|ab\| \leqslant \|a\|\|b\|$ and $\langle a\rangle_\beta \leqslant \|a\|$),

$$C(x,t) \leqslant 4\|v\|^2\|w\|^2. \tag{3}$$

Thus, the OTOC can only grow exponentially up to a finite (scrambling) time t^*, after which it remains bounded by a constant. This is consistent with the observations made in other works on OTOCs (of local observables) in fermionic systems where OTOCs were always observed to reach a plateau [29,31–35]. As already noted in Ref. [30], the only way for the exponential time evolution to persist to late times is if there is a small prefactor multiplying the exponential function in (2). Even in the Sachdev-Ye-Kitaev (SYK) model with long-range interactions, this prefactor is $1/N$, which becomes small as $N \to \infty$ [15]. Exponential growth (2) of the OTOC is therefore at best a transient effect in systems of interest to this work.

If interactions are local, $C(x,t)$ can be further bounded by the Lieb-Robinson theorem (LRT) [49] (see also Ref. [19]),

$$C(x,t) \leqslant 4\|v\|^2\|w\|^2 e^{-\mu \max\{0,|x|-v_{LR}t\}}. \tag{4}$$

In this case, for $t \ll t^* = |x|/v_{LR}$, the OTOC is even more suppressed. The interpretation of this effect is clear, namely, t^* is the time in which $C(x,t)$ enters the causal cone. Before

t^*, $C(x,t)$ is almost zero, while after t^* it is bounded by (3) and saturates at a plateau. The dynamics can only be nontrivial near the edge of the causal cone (or for $t \sim t^*$), where $C(x,t)$ can vary greatly. This is consistent with Refs. [29,31,46,47]. We note that chaos, as rigorously defined in classical dynamical systems [50,51], is not a transient effect but requires a $t \to \infty$ limit. Otherwise, phenomena such as the motion of an inverted pendulum close to a separatrix would be erroneously understood as "chaotic." Even in semiclassical theories, quantum chaos only emerges in the limit of the diverging Ehrenfest ($\propto -\ln\hbar$) time scale.

Another important fact is that momentum operators—the observables that Ref. [11] originally used to compute the Lyapunov exponent of the semiclassical trajectories—are unbounded. Therefore, if we wanted to preserve the semiclassical justification of the OTOC, which is necessary to be able to speak about quantum chaos, the quantum observables under consideration must have unbounded spectra.

These observations can be summarized in the intuitive statement that if chaos is to fully develop over a long time, the observables have to provide enough "space" for this to happen; they need to be unbounded. Indeed, this is the case with general observables in bosonic systems (usually studied in holography). However, this condition is not fulfilled by local observables in fermionic or spin systems, or more generally, in systems with a finite D. On the other hand, extensive observables in such theories do satisfy the unbounded spectrum criterium and therefore have the capacity to fully unveil the system's dynamical properties and quantum chaos. Motivated by this fact, we propose a different measure of quantum chaos: the density of the OTOC (DOTOC) of (nonlocal) extensive operators $V \equiv \sum_{x\in\Lambda} v_x$, $W \equiv \sum_{x\in\Lambda} w_x$, with w_x, v_x local. It is defined on a d-dimensional lattice Λ with N sites as the centralized second moment of the commutator

$$c^{(N)}(t) := -\frac{1}{N}\left(\langle[W(t),V(0)]^2\rangle_\beta - \langle[W(t),V(0)]\rangle_\beta^2\right). \quad (5)$$

The disconnected part, which is just the square of the standard dynamical susceptibility (i.e., the response function), has been subtracted to make the DOTOC well defined in the thermodynamic limit (TL) for any temperature. Because of the cyclicity of the trace, this term vanishes at $\beta = 0$ (this will occur in the model that we study below). Using the LRT and the clustering property of thermal states, which holds for any β in $d = 1$ [52] and for sufficiently high temperature in $d > 1$ [53], we rigorously prove in Sec. I of the Supplemental Material [54] that the DOTOC satisfies a uniform (in N) polynomial bound

$$c^{(N)}(t) \leqslant At^{3d}, \quad (6)$$

where A is an (N,t)-independent constant. The same bound thus holds in the TL, $c(t) := \lim_{N\to\infty} c^{(N)}(t)$.

Moreover, we report below the results of extensive numerical and analytical calculations, which demonstrate that possibly the simplest nontrivial locally interacting quantum chaotic spin system, the kicked Ising (KI) quantum spin chain [55,56], exhibits linear growth of the DOTOC of extensive magnetization observables, $c(t) \propto t$. An exception is the integrable KI model (equivalent to a free fermion model), for which we show analytically that its DOTOC of extensive quadratic observables (in fermionic variables) saturates, $c(t \to \infty) = \text{const}$. Since the KI model seems to be generic, we further conjecture that the bound (6) is not optimal and that typical one-dimensional, nonintegrable, and locally interacting models exhibit linear growth of DOTOCs.

As a consequence, theories under consideration here are not expected to exhibit any late-time butterfly effect but, as we know from results in the RMT, they can still be chaotic. In reference to classical mixing systems without the butterfly effect, we term the phenomenon of infinite polynomial growth of DOTOC's weak quantum chaos.

Kicked quantum Ising model. The Hamiltonian of the one-dimensional KI model consists of the Ising-interaction term $H_{\text{Ising}} = \sum_j J\sigma_j^x\sigma_{j+1}^x$ and the kick term $H_{\text{kick}} = \sum_j h(\sigma_j^z\cos\varphi + \sigma_j^x\sin\varphi)$,

$$H(t) = H_{\text{Ising}} + H_{\text{kick}}\sum_{n\in\mathbb{Z}}\delta(t-n), \quad (7)$$

where σ_j^α are local Pauli spin operators. The model has three parameters: the Ising coupling J, the magnitude of the external magnetic field h, and the inclination of the external magnetic field φ. KI is a periodic (in time) system with the Floquet propagator,

$$U = \mathcal{T}\{e^{-i\int_0^1 dt H(t)}\} = e^{-iJ\sum_j\sigma_j^x\sigma_{j+1}^x}e^{-ih\sum_j\left(\sigma_j^z\cos\varphi+\sigma_j^x\sin\varphi\right)}. \quad (8)$$

Because of the temporal periodicity, KI dynamics can be viewed as discrete in time, or as a quantum cellular automaton. The effect of a perturbation on a single lattice site propagates in a causal cone with speed 1. Namely, information can spread only by one site, left or right, within one period (kick of the magnetic field). Random matrix analysis [57,58] revealed that KI is chaotic.

The system has a further nice property of being integrable (quasifree) for a transverse magnetic field, $\varphi = 0$, and nonintegrable (and interacting) for $\varphi > 0$. Thus, φ serves as a handy parameter which allows us to study integrability breaking. See, e.g., Refs. [56,59] for a survey of the elementary dynamical properties of the KI model.

Here, we study the KI chain with N spins and evaluate the DOTOC (5) $c_\alpha^{(N)}(t)$ for a (nonlocal) extensive magnetization $W = V = M_\alpha = \sum_{j=1}^N \sigma_j^\alpha$, which can either be transverse ($\alpha = z$) or parallel ($\alpha = x$) to the direction of the Ising interaction. We take $\beta = 0$ as an infinite-temperature Gibbs ensemble is the only meaningful equilibrium state for periodically driven systems, which generically heat up to infinite temperature. We use three different approaches, two numerical methods for the general inclination ($0 \leqslant \varphi \leqslant \frac{\pi}{2}$) and an analytical solution for the transverse field case $\varphi = 0$. In the first, appropriate for small system sizes (up to $N \sim 12$), we used the exact numerical Floquet operator (8). The second method, used for intermediate system sizes (up to $N \sim 22$), was a Monte Carlo wave-function sampling based on typicality arguments (explained in Sec. II of Ref. [54]). As explained below, the analytical solution in the TL for the integrable (transverse) case and transverse magnetization M_z was found using fermionization.

Analytical solution. For the transverse field ($\varphi = 0$), KI is a quasifree model. If, furthermore, the (extensive) observable of interest is simple enough, the DOTOC allows for an analytic solution in terms of Jordan-Wigner transformation of Pauli spins into staggered Majorana fermion operators $w_{2j} = \sigma_j^x \prod_{k<j} \sigma_k^z$, $w_{2j+1} = \sigma_j^y \prod_{k<j} \sigma_k^z$ obeying the anti-commutation relations $\{w_i, w_j\} = 2\delta_{ij}$. The Floquet operator (8) then takes the following form,

$$U = e^{-J\sum_j w_{2j-1}w_{2j}} e^{-h\sum_j w_{2j}w_{2j+1}} = U_{\text{Ising}} U_{\text{kick}}, \quad (9)$$

with $U_{\text{Ising}} = \prod_j (\cos J - w_{2j-1}w_{2j} \sin J)$ and $U_{\text{kick}} = \prod_j (\cos h - w_{2j}w_{2j+1} \sin h)$. Now, the transverse magnetization can be expressed as a sum of quadratic Majorana operators,

$$M_z = -i \sum_{j\in\mathbb{Z}} w_{2j}w_{2j+1}, \quad (10)$$

which enables the analytic computation of the DOTOC of M_z [60].

Since the transverse field model is free [61], it is convenient to work in the Fourier transformed Majorana basis, $w(\theta) = \sum_j w_{2j} e^{i\theta j}$, $w'(\theta) = \sum_j w_{2j+1} e^{i\theta j}$, with shorthand notation $\underline{w}(\theta) = \binom{w(\theta)}{w'(\theta)}$. One can show (Sec. III. of Ref. [54]) that the Floquet propagator in the Heisenberg picture, $\mathcal{U}\underline{w}(\theta) := \begin{pmatrix} U^\dagger w(\theta) U \\ U^\dagger w'(\theta) U \end{pmatrix}$, takes the following form in a Fourier transformed Majorana basis,

$$\begin{aligned}\mathcal{U}(J,h,\theta) &= \mathcal{U}_{\text{kick}}(J,h,\theta)\mathcal{U}_{\text{Ising}}(J,h,\theta) \\ &= \begin{pmatrix} \cos(2h) & -\sin(2h) \\ \sin(2h) & \cos(2h) \end{pmatrix} \\ &\quad \times \begin{pmatrix} \cos(2J) & e^{i\theta}\sin(2J) \\ -e^{-i\theta}\sin(2J) & \cos(2J) \end{pmatrix}. \end{aligned} \quad (11)$$

This 2×2 unitary matrix valued symbol can be diagonalized as

$$\mathcal{U}(J,h,\theta) = V^\dagger(J,h,\theta)\begin{pmatrix} e^{i\kappa(J,h,\theta)} & \\ & e^{-i\kappa(J,h,\theta)} \end{pmatrix} V(J,h,\theta), \quad (12)$$

with the Floquet dispersion relation $\kappa(J,h,\theta) = \arccos[\cos(2J)\cos(2h) + \cos(\theta)\sin(2J)\sin(2h)]$. The matrix $V(J,h,\theta)$ is given explicitly in Sec. IV of Ref. [54].

Knowing that the KI Majorana fermions in the Fourier basis time evolve as $\underline{w}(\theta,t) = \mathcal{U}(\theta)^t \underline{w}(\theta,0)$ allows us to define the real space propagator as $K_{ab}^{kj}(t) := \langle w_{2k+a-1}\, w_{2j+b-1}(t)\rangle$, for $a,b \in \{1,2\}$. This equals the inverse Fourier transform of powers of $\mathcal{U}(\theta)$ (Sec. V of Ref. [54]),

$$K^{kj}(t) := K^{j-k}(t) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta e^{-i\theta(j-k)}\mathcal{U}^t(\theta). \quad (13)$$

Using the propagator (13), we can compute the infinite temperature OTOC of the transverse magnetization $c_z^{(N)}(t)$. First, we express the terms in (5) using (10), e.g., $\langle \sigma_i^z(t)\sigma_j^z\sigma_k^z(t)\sigma_l^z\rangle$ as an eight-fermion expectation value $\langle w_{2i}(t)w_{2i+1}(t)w_{2j}w_{2j+1}w_{2k}(t)w_{2k+1}(t)w_{2l}w_{2l+1}\rangle$. These are expressed as the product of four propagators (one for each time-dependent fermion) times an equal-time eight-fermion expectation value, with terms summed over four spatial and spin indices (see Sec. VI of Ref. [54] for details). Simple algebraic manipulations then lead to the final expression for the DOTOC in the TL,

$$\begin{aligned} c_z(t) = &\sum_{\substack{j,l_1,l_3\in\mathbb{Z} \\ s_0,s_j,p_1,p_3\in\{1,2\}}}^{j\neq 0} 4(-1)^{p_1+p_3} K_{S(p_1),1}^{R_1(p_1)}(t) K_{S(\tilde{p}_1),2}^{R_1(\tilde{p}_1)}(t) \\ &\times \Big[K_{S(p_3),1}^{R_3(p_3)}(t) K_{S(\tilde{p}_3),2}^{R_3(\tilde{p}_3)}(t) \\ &- (-1)^{s_j+s_0} K_{\tilde{S}(p_3),1}^{R_3(p_3)}(t) K_{\tilde{S}(\tilde{p}_3),2}^{R_3(\tilde{p}_3)}(t) \Big], \end{aligned} \quad (14)$$

where we used the following notation, $R_1 := (l_1 - j, l_1)$, $R_3 := (l_3 - j, l_3)$, $S := (s_j, s_0)$, together with the notation $v = [v(1), v(2)]$ for vector components and $\tilde{1} := 2$, $\tilde{2} := 1$. We can use formula (14) in two different ways. For intermediate times $t \sim 50$, we can perform the integral in (13) exactly and evaluate the sums in (14), which, because of the causal-cone spreading of information, now become finite sums (see Sec. VII A of Ref. [54] for details).

Furthermore, we can use the stationary phase approximation [in (12)–(14)] to compute the large-t asymptotics of the DOTOC. In this way, we prove that for large times, $c_z(t)$ is a constant (dependent only on J and h). In other words, the DOTOC of quadratic extensive observables in the integrable KI model saturates to a plateau. Details are explained in Sec. VII B of the Supplemental Material [54], which includes Refs. [49,52,53,59,62–67].

Results and discussion. In summary, we observe two distinct behaviours of the OTOC density for extensive observables in a one-dimensional KI model. For a generic situation, unless the model is integrable and the observable quadratic, the extensive DOTOC grows linearly with time. In fact, numerical results for finite system sizes saturate to a plateau at $t \sim N/2$, but this is simply due to a finite size effect—a consequence of the causal cone coming around the periodic boundary. This plateau grows with an increasing system size N and we expect that it disappears in the TL ($N \to \infty$). When the model is integrable (free) and the observable is simple (quadratic in fermion operators), the DOTOC saturates to a genuine plateau despite the fact that the spectrum of the observable is unbounded. The latter statement was proven in this work by finding an explicit analytic solution for $c_z(t)$ from which the expression for the height of the plateau could be found for any J,h. The results of the time dependence of the extensive DOTOC for different scenarios are presented and summarized in Fig. 1.

For the integrable case with $\varphi = 0$, the quasiparticle spectral gap closes on the line of $J = h$ in the parameter space and the system exhibits a Floquet analog of a quantum phase transition, i.e., $\kappa(J = h, \theta = 0) = -\kappa(J = h, \theta = 0) = 0$. It is interesting to ask whether the OTOC also reflects this transition in any way. What we find is that the plateau height ceases to be smooth for $J = h$. Beyond that, we also checked the slope of the DOTOC for longitudinal magnetization M_x and found a peak for J/h close to 1, where deviation could be attributed to finite size effects.

This work should be considered as a starting point for future investigations of quantum, weakly chaotic systems, which exhibit dynamical late-time mixing but do not display any exponential butterfly effect due to the locality of interactions and the finiteness of the local Hilbert space. In such systems,

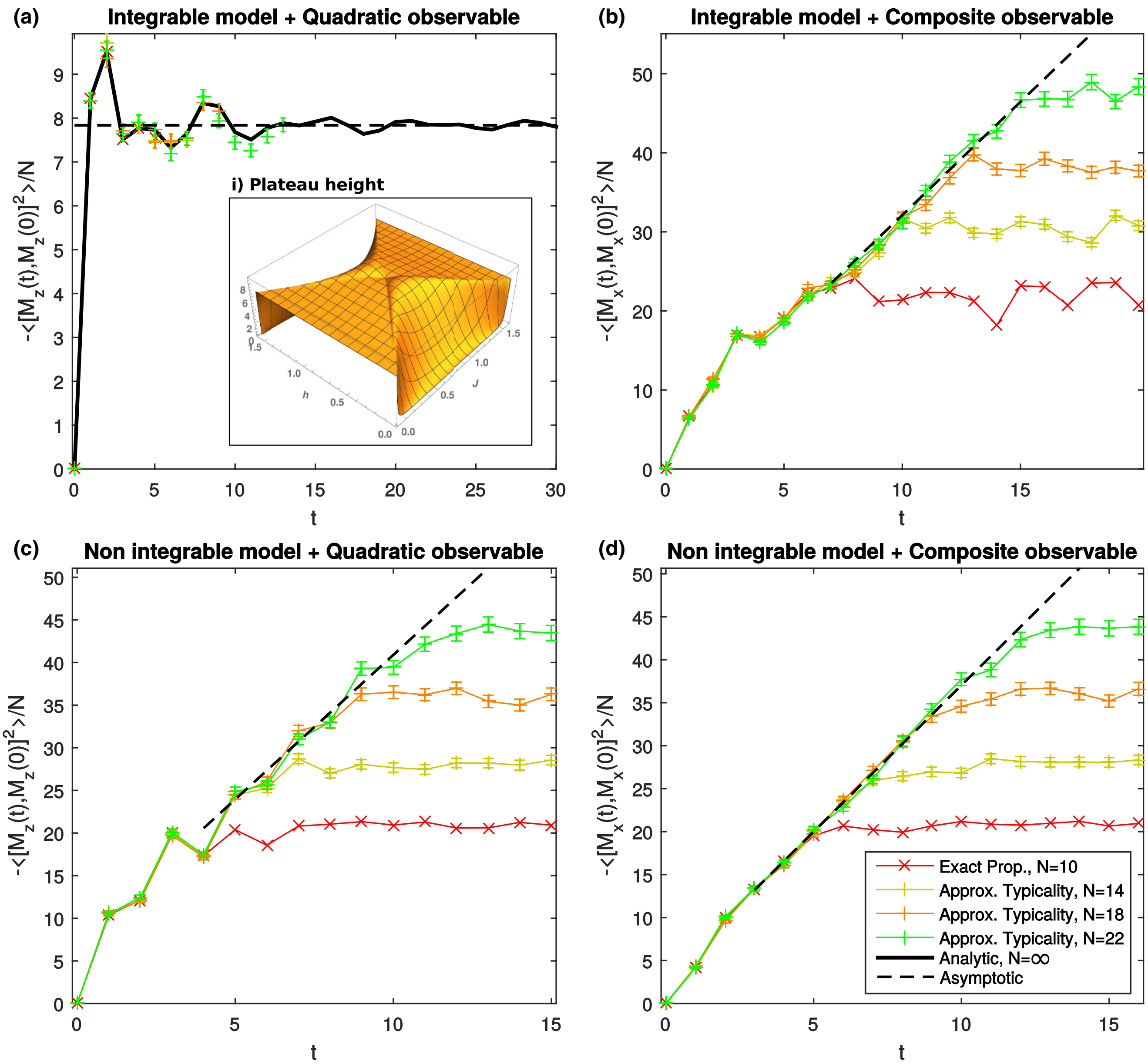

FIG. 1. Density of the OTOC of extensive observables for one-dimensional KI model (7) with periodic boundary conditions: In (a) and (b), the magnetic field is transversal ($\varphi = 0$) so the system is integrable (free), while in (c) and (d) the field is tilted ($\varphi = \frac{\pi}{4}$) so the model is nonintegrable. In (a) and (c), the observable is a sum of quadratic Majorana terms (10), while in (b) and (d) the observable is a sum of terms composed of infinite Majorana strings (composite). Here, $J = 0.7$, $h = 1.1$, but qualitatively similar behavior was found for other values of J,h. The numerically exact results for small system sizes are plotted with crosses. Results obtained with the numerical method based on typicality arguments (with a sample of 50×50 random vectors) are plotted with error bars. The analytical solution for the integrable case and quadratic observable is plotted with a bold black line. The asymptotic behavior in the limits $N \to \infty$ and $t \to \infty$ is plotted with a dashed line. In (a), the dashed line was obtained analytically. In other cases, it is a numerical extrapolation. Numerical results start to deviate around $t \sim N/2$ due to finite size effects. The inset (i) shows the dependence of the plateau height on J and h.

the standard OTOC rapidly plateaus and is therefore not a good measure of chaos. This observation led us to propose a different measure of chaos: the density of the OTOC of nonlocal extensive operators. We have proven (Sec. I of Ref. [54]) that such correlators always exhibit a polynomial bound and can thus be widely used to diagnose and classify quantum chaos. In the case of the nonintegrable KI model studied here, the growth is linear. Intuitively, it seems apparent that in locally interacting systems, information propagates slower than in an all-to-all interacting theory such as the SYK model. However, what is less apparent is that such systems can still be chaotic, a result established by RMT analysis [57].

Lastly, we note that in order to study chaos in strongly coupled, large-N theories (even in those that do exhibit the buttery effect), it would be interesting to extend holographic calculations to computations of OTOCs of nonlocal, smeared operators. For detailed future analyses, we will likely need to utilize the full machinery of holographic n-point function calculations [68–70] that will extend beyond studying gravitational shock waves [13,14].

Acknowledgments. We thank M. Medenjak for fruitful discussions and E. Ilievski for useful comments. The work has been supported by the ERC grant OMNES and Grants No. N1-0025 and No. P1-0044 of Slovenian Research Agency (ARRS). S.G. is supported in part by a VICI grant of the Netherlands Organization for Scientific Research (NWO) and by the Netherlands Organization for Scientific Research/Ministry of Science and Education (NWO/OCW).

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