

Coulomb blockade in fractional topological superconductors

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We study charge transport through a floating mesoscopic superconductor coupled to counterpropagating fractional quantum Hall edges at filling fraction $\nu = 2/3$. We consider a superconducting island with finite charging energy and investigate its effect on transport through the device. We calculate conductance through such a system as a function of temperature and gate voltage applied to the superconducting island. We show that transport is strongly affected by the presence of parafermionic zero modes, leading at zero temperature to a zero-bias conductance quantized in units of $\nu e^2/h$ independent of the applied gate voltage.

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Introduction. Topological superconductors, characterized by the presence of localized Majorana zero-energy modes (MZMs), have recently generated significant excitement in the condensed matter and quantum information communities [1–5]. Much of this excitement is due to the prediction that MZMs obey non-Abelian braiding statistics [6–8], and as such have potential applications in topological quantum computation [5]. Theory predicts that MZMs may be realized in semiconductor-superconductor heterostructures [9–12], and there is mounting experimental evidence for their existence in semiconductor nanowires [13–22]. More recently, a number of proposals [23–29] were put forward describing how to realize a scalable platform for topological quantum computation using mesoscopic superconducting islands hosting two or more MZMs. The interplay between charging energy in mesoscale islands and topological degrees of freedom is an outstanding open problem.

In a normal-superconductor-normal (N-S-N) junction consisting of a gated s -wave superconducting island, the conductance through the device has $2e$ periodicity with the gate charge [30,31]. The transport is dominated by the coherent Cooper-pair transmission through the island. In contrast, an N-TSC-N junction has e periodicity due to the presence of MZMs [32–36] which enable coherent single-electron transmission between opposite ends of a nanowire (i.e., an electron propagates coherently over distances much larger than the superconducting correlation length). This effect is at the heart of some of the recent measurement-only quantum computation proposals with Majorana zero modes [28,29]. An interesting question is whether this coherent transmission phenomenon has some analog in *fractional* 1D topological superconductors (ftSCs).

One-dimensional (1D) ftSCs are characterized by the modes at their end points that may accommodate a discrete fraction of an electron charge e^* at no energy cost. These modes, known as parafermionic zero modes, are a generalization of the more well-known Majorana zero modes, which can accommodate only electrons at no cost. According to a classification theorem [37] parafermionic zero modes are forbidden in a generic purely one-dimensional system.

However, 1D ftSCs may exist in *effectively* 1D systems that emerge at the boundary of a 2D region that already admits fractionalized excitations, such as a 2D electron gas in a fractional quantum Hall (fQH) state. There have been several proposals for realizing these fractional topological superconductors in solid-state systems [38–41]. Recently, Clarke *et al.* [42] argued that ftSCs may lead to an interesting and unique set of circuit elements when the proximitizing superconductor is grounded (i.e., has no charging energy). In this paper we consider a device (shown in Fig. 1) with a floating ftSC and investigate the effect of charging energy on transport in such a system. We find that the transport properties of an ftSC in the presence of charging energy are drastically different from that in Majorana islands [20,32,34–36,43]. Floating metallic islands coupled to QH edges have been already realized experimentally [44,45]. Therefore, we believe that our proposal is within the experimental reach, and is particularly suitable for graphene-based ftSC proposals.

Theoretical model. We consider the transport through a mesoscopic superconducting island connecting the counterpropagating edge modes bordering two regions of spin-unpolarized fQH state at a filling fraction $\nu = 2/(2n + 1)$ (see Fig. 1). We assume that edge states are strongly coupled to the superconductor in the region $x_1 < x < x_2$, and are completely decoupled outside. Each edge state can be described using the K -matrix formalism [46,47] with the corresponding Lagrangian $\mathcal{L}_0^{R/L} = \mathcal{L}_\rho^{R/L} + \mathcal{L}_\sigma^{R/L}$, where

$$\mathcal{L}_\rho^{R/L} = \frac{2n + 1}{8\pi} \partial_x \phi_\rho (\pm \partial_t \phi_\rho - V_\rho \partial_x \phi_\rho), \quad (1)$$

$$\mathcal{L}_\sigma^{R/L} = \frac{1}{8\pi} \partial_x \phi_\sigma (\pm (-1)^n \partial_t \phi_\sigma - V_\sigma \partial_x \phi_\sigma). \quad (2)$$

Here R/L denotes the right/left propagating edge modes. The fields ϕ_ρ and ϕ_σ correspond to charge and neutral modes, respectively. In particular, we note that the operator $e^{i\phi_\rho}$ creates a spinless quasiparticle with charge $e^* = \nu e$. Note that we assume here that the state is unpolarized and neglect spin- $SU(2)$ symmetry-breaking terms such as $V_{\rho\sigma} \partial_x \phi_\rho \partial_x \phi_\sigma$.

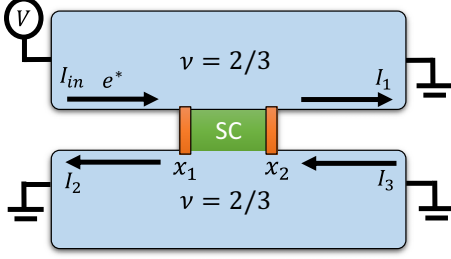


FIG. 1. Schematic diagram of the device layout. Two counter-propagating fQH edges with the filling fraction $\nu = 2/3$ are coupled to a floating mesoscopic s -wave superconductor with a charging energy E_C . In the appropriate parameter regime (see below), two parafermionic zero modes are localized at x_1 and x_2 . Here the distance $|x_2 - x_1|$ should be much larger than superconducting coherence length. The low-energy transport is dominated by the coherent charge e^* transmission through the superconducting island.

In terms of the chiral fields, electron operators at either side of the superconducting island can be written as

$$\psi_{\uparrow}^{R/L} = \frac{1}{\sqrt{2\pi a}} e^{i(\frac{1}{\nu}\phi_{\rho}^{R/L} - \frac{1}{2}\phi_{\sigma}^{R/L})}, \quad (3)$$

$$\psi_{\downarrow}^{R/L} = \frac{1}{\sqrt{2\pi a}} e^{i(\frac{1}{\nu}\phi_{\rho}^{R/L} + \frac{1}{2}\phi_{\sigma}^{R/L})}, \quad (4)$$

where a is a short-distance cutoff. We now introduce the nonchiral bosonic variables $\phi_{\rho,\sigma}^R = \varphi_{\rho,\sigma} + \theta_{\rho,\sigma}$ and $\phi_{\rho,\sigma}^L = \varphi_{\rho,\sigma} - \theta_{\rho,\sigma}$ where the charge and spin fields satisfy the following commutation relations:

$$[\theta_{\rho}(x), \varphi_{\rho}(x')] = -2\pi \nu i \Theta(x' - x), \quad (5)$$

$$[\theta_{\sigma}(x), \varphi_{\sigma}(x')] = 2\pi i (-1)^n \Theta(x' - x). \quad (6)$$

Here $\Theta(x' - x)$ is the Heaviside theta function. The total charge density now reads

$$\rho = \frac{\partial_x(\phi_{\rho}^R - \phi_{\rho}^L)}{2\pi} = \frac{\partial_x \theta_{\rho}}{\pi}, \quad (7)$$

and the current operator for the corresponding segment of the four-terminal device shown in Fig. 1 is given by [48]

$$I_{in} = \frac{e}{2\pi} \dot{\phi}_{\rho}^R(x_1), \quad I_1 = \frac{e}{2\pi} \dot{\phi}_{\rho}^R(x_2), \quad (8)$$

$$I_2 = \frac{e}{2\pi} \dot{\phi}_{\rho}^L(x_1), \quad I_3 = \frac{e}{2\pi} \dot{\phi}_{\rho}^L(x_2). \quad (9)$$

The injected current in the linear response regime is given by $\langle I_{in} \rangle = V e^2 / h$, whereas the injected current $\langle I_3 \rangle = 0$ since both contacts upstream of the bottom right edge are grounded. Therefore, we can define differential conductances for the two different drain electrodes $G_1 = d\langle I_1 \rangle / dV$ and $G_2 = d\langle I_2 \rangle / dV$ with the constraint $G_1 + G_2 = \nu e^2 / h$ due to current conservation. After including interaction terms across the superconducting island

$$\mathcal{L}_C^{RL} = -\frac{1}{4\pi \nu} V_{\rho}^C \partial_x \phi_{\rho}^R \partial_x \phi_{\rho}^L - \frac{1}{8\pi} V_{\sigma}^C \partial_x \phi_{\sigma}^R \partial_x \phi_{\sigma}^L, \quad (10)$$

one arrives at the effective action $S_{\rho} + S_{\sigma}$ with

$$S_{\rho} = \frac{1}{2\pi} \int dx d\tau \frac{K_{\rho}}{\nu} \left[\frac{(\partial_{\tau} \varphi_{\rho})^2}{\nu} + \nu (\partial_x \varphi_{\rho})^2 \right], \quad (11)$$

$$S_{\sigma} = \frac{1}{2\pi} \int dx d\tau \frac{K_{\sigma}}{2} \left[\frac{(\partial_{\tau} \varphi_{\sigma})^2}{\nu} + \nu (\partial_x \varphi_{\sigma})^2 \right], \quad (12)$$

where $K_{\rho,\sigma} = \sqrt{\frac{V_{\rho,\sigma} + V_{\rho,\sigma}^C}{V_{\rho,\sigma} - V_{\rho,\sigma}^C}}$. From now on we will assume a weak repulsive interaction between the charge modes and an attractive interaction between the neutral modes such that $K_{\rho} \gtrsim 1 > K_{\sigma}$.

Next we consider various perturbations induced by the superconducting trench (of width smaller than the SC coherence length). In the Supplemental Material [47], we analyze the single-particle and two-particle processes across the superconducting trench and calculate the scaling dimension of the corresponding operators. One can show that the neutral mode is gapped out in the singlet channel for $K_{\sigma} < 1$ and, as a result, θ_{σ} is pinned. Henceforth we will assume that the gap for the neutral modes is the largest energy scale in the problem which effectively makes the system spinless. In the charge sector, there are two important bulk perturbations involving one fermion from each edge: a spin-conserving backscattering process $\psi_{\uparrow}^{\dagger R} \psi_{\uparrow}^L + \psi_{\downarrow}^{\dagger R} \psi_{\downarrow}^L + \text{H.c.} \propto \cos(\frac{2}{\nu} \theta_{\rho})$ and a superconducting pairing term in the singlet channel $\psi_{\uparrow}^R \psi_{\downarrow}^L - \psi_{\downarrow}^R \psi_{\uparrow}^L + \text{H.c.} \propto \cos(\frac{2}{\nu} \varphi_{\rho})$.

For $\nu = 2/3$, both terms are relevant at $K_{\rho} \sim 1$ and flow to strong coupling. For other filling fractions ($\nu = \frac{2}{2n+1}$ with $n > 1$), only one of these terms may be made relevant at a time by adjusting K_{ρ} . However, given that θ_{ρ} and φ_{ρ} are dual variables, these terms compete with each other and cannot order simultaneously. Henceforth we focus on the limit when superconducting pairing is relevant ($K_{\rho} > \frac{1}{2\nu}$) and dominates over the backscattering term to open a pairing gap in the trench; see detailed discussion in Refs. [38–40]. As a result, the backscattering term is suppressed in the bulk but may be important at the boundaries of the superconducting region (x_1, x_2). Note that the system with a grounded superconductor was considered in Ref. [42] where it was shown that the parafermionic zero modes emerging at the end of the superconductor lead to a spectral flow of the boundary conditions and strongly modify transport properties of the system. In the present case, we consider a floating superconducting island with a finite charging energy $E_C \gg T$, with T being the temperature. Thus, in contrast with Ref. [42], uncorrelated Andreev processes at x_1 and x_2 are suppressed in our case.

Taking into account the above considerations, one can now write an effective low-energy model for the system. For concreteness, we consider the case $\nu = 2/3$, which may be the most experimentally relevant situation given that both the pairing term (leading to a topological phase) and the spin-conserving backscattering term (that reforms the $\nu = 2/3$ state from separated edges) are relevant when interactions are weak ($K_{\rho} \sim 1$). In the limit of weak backscattering at $x_{1/2}$, the corresponding Hamiltonian becomes

$$H = H_0 + H_B + H_P + H_C, \quad (13)$$

where H_0 describes the two decoupled edges and

$$H_B = -Dr_1 \cos(3\theta_\rho(x_1)) - Dr_2 \cos(3\theta_\rho(x_2)), \quad (14)$$

$$H_P = -\frac{\Delta}{2\pi a} \int_{x_1^+}^{x_2^-} dx \cos(3\varphi_\rho(x)), \quad (15)$$

$$H_C = E_C \left(\frac{\theta_\rho(x_2) - \theta_\rho(x_1)}{\pi} - \mathcal{N}_g \right)^2. \quad (16)$$

Here $r_{1,2} \ll 1$ are the reflection amplitudes at $x = x_{1,2}$, respectively, Δ is the induced SC gap, E_C is the charging energy determined by the geometric capacitance of the island, a is the short-distance cutoff, and $x_i^\pm \equiv x_i \pm 0^+$. The charge on the island, given by $[\theta_\rho(x_2) - \theta_\rho(x_1)]/\pi$, can be tuned with the dimensionless gate voltage $\mathcal{N}_g = C_g V_g$, where C_g and V_g are gate capacitance and voltage, respectively. We implicitly assume here that due to the presence of a metallic island and strong hybridization between edge states and states in the metal, normal-state level spacing in the domain $x \in (x_1, x_2)$ becomes negligibly small.

High-temperature limit. We first analyze the high-temperature limit $E_C \gg T \gg \Delta$ when the island is in the normal state. At energies below E_C , charge fluctuations will be suppressed, resulting in the constraint $\theta_\rho^- \equiv \theta_\rho(x_2) - \theta_\rho(x_1) = \pi \mathcal{N}_g$. In terms of the fluctuating field $\theta_\rho^+ \equiv \theta_\rho(x_2) + \theta_\rho(x_1)$, the boundary backscattering Hamiltonian H_B is given by

$$H_B^{(\text{eff})} = -Dr(\mathcal{N}_g) \cos\left(\frac{3}{2}\theta_\rho^+ - \beta(\mathcal{N}_g)\right), \quad (17)$$

where $\beta(\mathcal{N}_g)$ is some unimportant phase and $r(\mathcal{N}_g)$ reads

$$r(\mathcal{N}_g) = \text{sgn}\left(\cos\frac{3\pi\mathcal{N}_g}{2}\right) \sqrt{r_1^2 + r_2^2 + 2r_1 r_2 \cos(3\pi\mathcal{N}_g)}. \quad (18)$$

As a result of pinning of θ_ρ^- , the RG equation for $r(\mathcal{N}_g)$ in the case $D < E_C$ becomes $dr/dl = (1 - 3K_\rho/4)r$. For $K_\rho < 4/3$, the backscattering term is relevant, and flows to the strong coupling limit with θ_ρ^+ pinned. Using the condition $r(D_c) \sim 1$, we find the strong-coupling crossover scale D_c :

$$D_c \sim E_C r(\mathcal{N}_g)^{\frac{4}{4-3K_\rho}}. \quad (19)$$

In the intermediate regime $D_c \ll D \ll E_C$, the backscattering term remains small and can be taken into account perturbatively.

The differential tunneling conductance in different temperature regimes can be evaluated using the Kubo formula [49]

$$G_i = \frac{1}{2T} \int_{-\infty}^{\infty} dt \Pi_i\left(it + \frac{1}{2T}\right), \quad \Pi_i(\tau) = \langle I_i(\tau) I_i(0) \rangle. \quad (20)$$

Here τ is imaginary time and I_i is the corresponding expression for the current operator; see Eq. (8). The resulting conductance $G_1(T)$ for $\max\{D_c, \Delta\} \ll T \ll E_C$ is given by

$$\frac{G_1(T)}{G_0} = \nu \left(1 - c_1 r(\mathcal{N}_g)^2 \left(\frac{E_C}{T} \right)^{\frac{4-3K_\rho}{2}} \right), \quad (21)$$

where $G_0 = e^2/h$ and c_1 is an $O(1)$ numerical constant.

Let us now consider the case $\Delta \ll T \ll D_c$ [50], where backscattering becomes large and the system flows to strong coupling, thereby pinning the field θ_ρ^+ at the boundary. In order to calculate the conductance in this case, we first need to perform a duality transformation. The leading irrelevant operator, which shifts $\frac{3}{2}\theta_\rho^+$ by 2π , is given by

$$H_{\text{dual}} = -D\lambda(D) \cos(\delta\varphi_{\text{out}} - \delta\varphi_{\text{in}}), \quad (22)$$

where $\delta\varphi_{\text{out}} = \varphi_\rho(x_2^+) - \varphi_\rho(x_1^-)$ and $\delta\varphi_{\text{in}} = \varphi_\rho(x_2^-) - \varphi_\rho(x_1^+)$. Equation (22) describes a process of correlated tunneling of charge e^* at x_1 and x_2 preserving the total charge in the island. The scaling dimension of this operator is $4/3K_\rho$, in keeping with its role as the dual of the Hamiltonian $H_B^{(\text{eff})}$. The RG flow for λ reads $d\lambda/dl = (1 - 4/3K_\rho)\lambda$. Let us now consider transport at this fixed point. The pinning of boundary fields θ_ρ^\pm implies that

$$\dot{\phi}_\rho^R(x_1) = \dot{\phi}_\rho^L(x_1), \quad \dot{\phi}_\rho^R(x_2) = \dot{\phi}_\rho^L(x_2). \quad (23)$$

Thus there is strong backscattering at x_1 and x_2 resulting in $\langle I_1 \rangle \rightarrow 0$. Assuming $\Delta \ll T \ll D_c$, the conductance $G_1(T)$ can be calculated perturbatively in λ . Using the Kubo formula (20) and the current-conservation constraint at x_1 (i.e., $G_2 = \nu G_0 - G_1$), one finds that

$$\frac{G_1(T)}{G_0} \sim \lambda(D_c)^2 \left(\frac{T}{D_c} \right)^{\frac{8}{3K_\rho}-2} \sim r(\mathcal{N}_g)^{-\frac{8}{3K_\rho}} \left(\frac{T}{E_C} \right)^{\frac{8}{3K_\rho}-2}, \quad (24)$$

where we used $\lambda(D_c) \sim 1$. Thus transport through the island in this temperature regime is dominated by the inelastic processes and is suppressed at low temperatures.

Low-temperature limit. Let us now consider the low-temperature limit $T \ll \Delta$. We expect that transport properties will be significantly modified due to the presence of parafermionic zero modes [38–40,42]. In the limit $\Delta \ll D_c$, the effective Hamiltonian at the scale $D \sim D_c$ is given by Eq. (22) with $\lambda(D) = \lambda(D_c)(D/D_c)^{4/3K_\rho-1}$. Upon lowering the bandwidth to $D \sim \Delta$, the SC pairing H_P opens a gap in the spectrum and suppresses fluctuations of $\delta\varphi_{\text{in}}$. It is illuminating to rewrite the low-energy boundary Hamiltonian (22) in terms of the parafermionic zero modes. Using the right-moving representation [51], the effective Hamiltonian becomes

$$H_{\text{dual}} = -\frac{1}{2} D \lambda^*(D) e^{i\phi_\rho^R(x_2^+)} e^{-i\phi_\rho^R(x_1^-)} \alpha_2^{R\dagger} \alpha_1^R + \text{H.c.}, \quad (25)$$

where $\alpha_{1,2}^R$ are parafermionic operators localized at $x_{1/2}$. One should keep in mind that the system hosting two parafermionic zero modes ($N_m = 2$) does not have ground-state degeneracy since charge on the island is fixed by the charging energy. If, however, the number of zero modes $N_m > 2$, ground-state degeneracy will be restored and the process considered above provides a way of measuring which ground state the system is in. Hamiltonian (25) describes a coherent transfer of charge e^* quasiparticles through the superconducting island and is reminiscent of the single-electron coherent transmission in Majorana systems [32,36].

Let's now analyze transport properties at low temperature $T \ll \Delta$. One may notice that the scaling dimension of $\lambda(D)$ for $D < \Delta$ is halved to $2/3K_\rho$. Thus the boundary term (25) becomes relevant for $2/3 < K_\rho$, and $\lambda(D)$ grows under RG

and reaches a strong coupling limit at the new scale:

$$D_s \sim \Delta \cdot \lambda(\Delta)^{\frac{3K_\rho}{3K_\rho-2}}. \quad (26)$$

Using $\lambda(\Delta) \sim (\Delta/D_c)^{\frac{4}{3K_\rho-1}}$, the differential conductance can be calculated perturbatively in the limit $D_s \ll T \ll \Delta$ yielding

$$\frac{G_1(T)}{G_0} \sim r(\mathcal{N}_g)^{-\frac{8}{3K_\rho}} \left(\frac{\Delta}{E_C}\right)^{\frac{8}{3K_\rho}-2} \left(\frac{\Delta}{T}\right)^{2-\frac{4}{3K_\rho}}. \quad (27)$$

Notice that the above expression matches Eq. (24) at $T \sim \Delta$.

Finally, let's consider the low-temperature regime $T \ll D_s, \Delta$. At $D < D_s$, the boundary condition for the fields becomes $\varphi_\rho(x_2^+) - \varphi_\rho(x_1^-) = \text{const}$, which leads to the following conservation law for the chiral fields:

$$\dot{\phi}_\rho^R(x_2) + \dot{\phi}_\rho^L(x_2) - \dot{\phi}_\rho^R(x_1) - \dot{\phi}_\rho^L(x_1) = 0. \quad (28)$$

Using current conservation, one finds that $\langle I_1 \rangle = \langle I \rangle$ and $\langle I_2 \rangle = 0$. As a result, we conclude that zero-temperature conductance $G_1 = \nu e^2/h$ and is independent of \mathcal{N}_g , which is very different from the Majorana case [36]. Finite-temperature corrections to the conductance can be calculated by perturbing the above result with the leading irrelevant operator at the strong coupling fixed point $D \ll D_s$:

$$H_B^{\text{eff}} \sim -Dr(D) \cos\left(\frac{3\theta_\rho^+}{2}\right). \quad (29)$$

Given that φ_ρ is pinned in the domain $x_1 < x < x_2$, the RG flow for $r(D)$ becomes $dr/dl = (1 - 3K_\rho/2)r$. Thus, at the energy scale $D \ll D_s$, one finds that

$$\tilde{r}(D) = \tilde{r}(D_s) \left(\frac{D}{D_s}\right)^{\frac{3K_\rho}{2}-1}. \quad (30)$$

By perturbatively evaluating corrections to the conductance using the Kubo formula (20) (see Ref. [52] for details) one finds

$$\frac{G_1(T)}{G_0} = \nu \left(1 - c_2 r^4(\mathcal{N}_g) \left(\frac{E_C}{\Delta}\right)^{4-3K_\rho} \left(\frac{T}{\Delta}\right)^{3K_\rho-2}\right). \quad (31)$$

Here c_2 is an $O(1)$ numerical coefficient. This is a counterintuitive result. Despite the fact that the backscattering term D_c was initially large (i.e., $D_c \gg \Delta$), the low-energy transport properties are characterized by a universal value of the conductance. In other words, ground-state properties of the system are independent of \mathcal{N}_g (i.e., effective charging energy is renormalized to zero by quantum fluctuations). (See Fig. 2.)

Let's compare our results for the Coulomb blockade in the fractional TSC systems with the corresponding case in the Majorana counterparts [32,34–36,43]. In the Majorana systems the backscattering operator is marginal [43] and the zero-temperature conductance G_1 is dependent on \mathcal{N}_g : it reaches

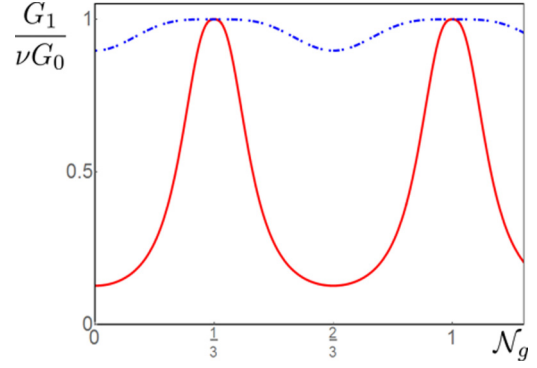


FIG. 2. Schematic plot of the differential conductance $G_1(\mathcal{N}_g)$ in the low-temperature $T \ll D_s, \Delta$ (dash-dot blue line) and high-temperature $\Delta \ll T \ll D_c, E_C$ (solid red line) limits. Here we assume symmetric contacts $r_1 = r_2$.

maximum of the order of e^2/h at the charge degeneracy points and gets significantly reduced in the Coulomb valleys. In stark contrast, we find *quantized* conductance G_1 in the fractional TSC systems. This drastic difference originates from the fact that backscattering operators for charge- e^* quasiparticles are not allowed between fractional QH edges separated by the trivial vacuum and backscattering is therefore dominated by fermionic processes having higher scaling dimension. As a result, quantum charge fluctuations are much stronger in fTSC systems than in Majorana systems.

Conclusion. Coulomb blockade of charge transport across a mesoscopic superconducting island manifests itself through the oscillations of the conductance with the gate voltage \mathcal{N}_g . In Majorana islands the periodicity of the oscillations corresponds to an increment of charge by e , whereas in fractional topological superconductors this periodicity is determined by the fractional quasiparticle charge e^* . In this paper we have developed a framework for studying the Coulomb blockade effect in QH-superconductor heterostructures. By considering the specific fractional topological superconductor proposal based on $\nu = 2/3$ QH state, we show that dependence of the differential conductance on gate voltage and temperature is quite nontrivial. At zero temperature the conductance approaches a quantized value of $\nu e^2/h$. The dependence on gate voltage appears only at finite temperature with the amplitude of gate-voltage oscillations increasing with temperature [see Eq. (31)]. The conductance decreases with increasing temperature until T reaches the superconducting gap scale Δ and then increases again to the quantized value for $\Delta \ll T \ll E_C$.

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