# Delocalized carriers in the $t$-$J$ model with strong charge disorder

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We show that electron-magnon interaction delocalizes the particle in a system with strong charge disorder. The analysis is based on results obtained for a single hole in the one-dimensional $t$-$J$ model. Unless there exists a mechanism that localizes spin excitations, the charge carrier remains delocalized even for a very strong charge disorder and shows subdiffusive motion up to the longest accessible times. Moreover, upon inspection of the propagation times between neighboring sites as well as a careful finite-size scaling, we conjecture that the anomalous subdiffusive transport may be transient and should eventually evolve into a normal diffusive motion.

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## I. INTRODUCTION

The many-body localization (MBL) represents a promising concept of macroscopic devices which do not thermalize [1–18] and may store quantum information [14,19]. Most of the inherent properties of MBL systems have been investigated using the generic one-dimensional (1D) disordered models of interacting spinless fermions [20–32]. Emerging characteristic features of MBL systems are the existence of localized many-body states in the whole energy spectrum that lead to vanishing of dc transport at any temperature [33–40], Poisson-like level statistics [41], and the logarithmic growth of entanglement entropy [5,7,42–46]. Numerical calculations of dynamical conductivity [35,38,39] and other dynamic properties based on the renormalization-group approach [7,36,47,48] indicate that in the vicinity of the transition to the MBL state the optical conductivity shows a characteristic linear $\omega$ dependence. In the presence of strong disorder but still below the MBL transition, several studies predict subdiffusive transport [35,36,49,50].

The presence of MBL has been rigorously shown so far only for the transverse-field Ising model [51], whereas indisputable numerical evidence is available mostly for interacting spinless fermions or equivalent spin Hamiltonians. However, in real systems, the particles are coupled to other degrees of freedom, and this coupling may be important not only for solids but also for cold-atom experiments. In particular, recent experiments [4,52,53] address the problem of MBL in the spin-1/2 Hubbard model, where charge carriers are coupled to spin excitation. On the other hand, well-established results [54,55] indicate that phonons destroy the Anderson localization; hence, they should destroy the MBL phase as well. Nevertheless, in contrast to phonons, the energy spectrum of many other excitations in the tight-binding models (e.g., the spin excitations) is bounded from above. It remains unclear whether strict MBL survives in the presence of the latter excitations. Solving this problem is important for answering the fundamental question of whether MBL exists also in more realistic models, including the Hubbard model [56,57]. The preliminary numerical results suggest that charge carriers may indeed be localized despite the presence of delocalized spin excitations [57]. Nonetheless, studies for larger systems and longer times are needed in order to eliminate the transient or finite-size (FS) effects.

We study the dynamics of a single charge carrier coupled to spin excitations which propagates in a system with a charge disorder. Our studies are carried out for the 1D $t$-$J$ model, which should be considered as a limiting case of the Hubbard model for large on-site repulsion. The common understanding of MBL is that it originates from a (single-particle) Anderson insulator [58–61] which persists *despite* the presence of carrier-carrier interaction [54,62]. The choice of a single particle (hole) in the $t$-$J$ model eliminates the latter interaction; hence, it should act in favor of localization. Note, however, that nontrivial many-body physics emerges from interaction between the spin excitations. The dimension of the Hilbert space in the present studies is of the same order as in the commonly studied model of spinless fermions; hence, the numerical results are obtained for rather large systems and long times far beyond the limitations of the Hubbard model.

We demonstrate that localization of charge carriers is possible only for localized spin excitations, whereas their dynamics is subdiffusive even for very strong disorder (or diffusive for weak disorder) when spins are delocalized. The latter result resembles the dynamics of interacting spinless fermions for strong disorder but still below the MBL transition [35,36,49,50]. Here, we demonstrate that the subdiffusive behavior originates from an extremely broad distribution of propagation times for transitions between the neighboring lattice sites. However, this distribution may also suggest that the subdiffusive behavior is a transient yet long-lasting phenomenon. The transition to the normal diffusive regime takes place at extremely long times and cannot be observed directly from numerical data.

## II. MODEL AND NUMERICAL METHODS

We study a single hole (a charge carrier) in the 1D $t$-$J$ model on $L$ sites with periodic boundary conditions

$$H = \sum_{i,\sigma}[-t_h c^\dagger_{i+1\sigma} c_{i\sigma} + \mathrm{H.c.} + \varepsilon_i n_{i\sigma}] + \sum_i J_i \vec{S}_{i+1}\vec{S}_i, \quad (1)$$

where $c^\dagger_{i\sigma}$ creates an electron with spin $\sigma$ at site $i$, $\vec{S}_i$ is the spin operator, and $n_{i\sigma} = c^\dagger_{i\sigma}c_{i\sigma}$. The states with doubly occupied sites are excluded ($n_{i\uparrow}n_{i\downarrow} = 0$), and for simplicity, the hopping integral is taken as the energy unit ($t_h = 1$). The on-site potentials $\varepsilon_i$ are random numbers that are uniformly distributed in the interval $[-W,W]$. In the special case $J_i = 0$, the spins are frozen (at least in the 1D case), and the hole dynamics should be the same as in the 1D Anderson insulators. For the same reason, the hole is localized also in the disordered $t$-$J_z$ model, where spin excitations are frozen.

Considering the Hamiltonian (1) as a large-$U$ limit of the Hubbard model, one finds that each $J_i$ depends on $\varepsilon_i$ and $\varepsilon_{i+1}$. In order to discuss also a more general case of localized spin excitations, $J_i$ and $\epsilon_i$ will be set independently of each other.

The transport properties will be discussed mainly from the numerical results for the time propagation of pure states $|\psi_t\rangle$. We take $|\psi_0\rangle = |\sigma_1 \cdots \sigma_{j-1} 0_j \sigma_{j+1} \cdots \sigma_L\rangle$ as an initial state, where the position of the hole $j$ and the spin configuration, $\sigma_i = \uparrow, \downarrow$, are chosen randomly. The latter choice means that the system is at infinite temperature. The essential information about the charge dynamics will be obtained from the hole density $\rho_i(t) = \langle\psi_t|1 - n_{i\uparrow} - n_{i\downarrow}|\psi_t\rangle$. Note that $\rho_i(t) \geqslant 0$ because states with doubly occupied sites are excluded and $\sum_i \rho_i(t) = 1$ because the system contains only a single hole. Then, one can define also the mean-square deviation of the hole distribution [63],

$$\sigma^2(t) = \sum_i i^2 \rho_i(t) - \left[\sum_i i \rho_i(t)\right]^2. \qquad (2)$$

Throughout the paper, the averaging over the charge disorder will be marked by a subscript d. We typically take $10^3$ realizations of the disorder.

In order to gain a deeper understanding of the anomalous charge dynamics, numerical data for the $t$-$J$ model will be compared with results for a classical particle which randomly walks on the same 1D lattice. As a toy model, we employ the continuous-time random walk in which a particle waits for a time $\tau$ on each site $i$ before jumping to the neighboring site $i-1$ or $i+1$. This model is well understood for various distributions of the waiting times $f(\tau)$ [64]. In particular, if the average waiting time is finite, $\bar{\tau} = \int d\tau\, f(\tau)\tau < \infty$, the model shows at long times normal diffusion, $\sigma^2(t) \propto t$. However, for a broad distribution of waiting times $f(\tau) \sim 1/\tau^{\mu+1}$, with $0 < \mu < 1$, $\bar{\tau}$ becomes infinite, and one obtains a subdiffusive transport with $\sigma^2(t) \propto t^\mu$ (see [64,65]).

## III. RESULTS

First, we apply the Lanczos propagation method [66] and study a system with weak charge disorder and homogeneous $J_i$, where one expects normal diffusion, i.e., $\sigma_d^2(t) \propto t$. Such linear behavior is indeed visible in Fig. 1(a) but only for short times. In order to explain the subsequent breakdown of this linear trend we have studied the toy model for exactly the same lattice and $p(\tau) \propto \theta(\tau_0 - \tau)$. The average waiting time $\bar{\tau} = \tau_0/2$ has been tuned to fit the linear regime in the $t$-$J$ model. Figures 1(a) and 1(b) show clear similarity between $\sigma^2(t)$ in both models. However, in the toy model any departure from the normal diffusion must originate from the FS effects. Due to these effects, the numerical results are of physical relevance only for small values of the mean-square deviation $\sigma_d^2 < \sigma_{\max}^2 \sim 10$.

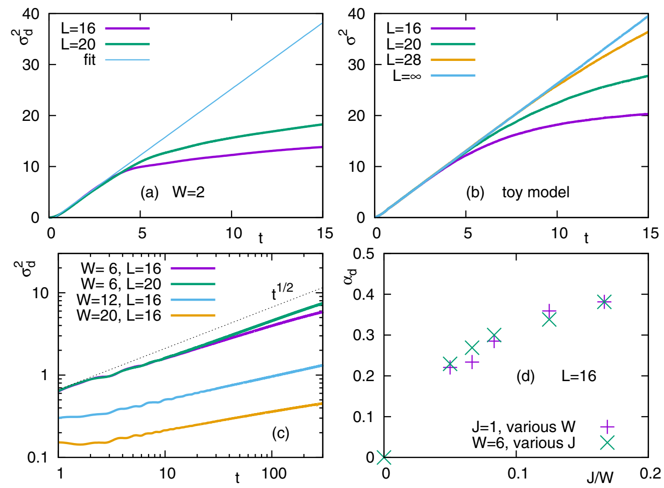


FIG. 1. (a) and (c) The mean-square deviation of the spatial distribution of holes vs time obtained for $J_i = 1$. Note logarithmic scale in (c). (b) Results for the toy model which reproduces the short-time linear regime in (a). (d) $\alpha_d$ obtained from fitting $\sigma_d^2(t) \propto t^{\alpha_d}$ for $L = 16$ and various $J$ and $W$.

In Figs. 1(c) and 1(d) we present results for the central problem of this work, i.e., for the hole dynamics in the $t$-$J$ model with strong charge disorder. The power-law dependence $\sigma_d^2(t) \propto t^{\alpha_d}$ is evident over at least two decades of time, and the transport is clearly subdiffusive. The exponent $\alpha_d \simeq 1/2$ for $W = 6$ and decreases further for stronger charge disorder. Within the studied time window $\sigma_d^2 \ll \sigma_{\max}^2$. Therefore, we do not expect any essential influence of the FS effects. The latter hypothesis is confirmed by numerical results on a larger system ($L = 20$) where only a slightly larger value of the mean-square deviation is obtained ($W = 6$), also shown in Fig. 1(c). Next, we have repeated the same calculations for various (homogeneous) exchange interactions $J_i = J$ and various disorder strengths. For each case we have obtained $\alpha_d$, and these exponents are shown in Fig. 1(d) as a function of $J/W$. Nearly overlapping points on this plot suggest that $\alpha_d = \alpha_d(J/W)$. Then, the charge localization ($\alpha_d = 0$) should occur only for $W \to \infty$ or for $J \to 0$. Localization in the former case is rather obvious, whereas the latter one is just the Anderson insulator. Otherwise, the transport is subdiffusive or diffusive.

The essential question is whether the model in Eq. (1) may show charge localization under some particular conditions. In the following we demonstrate that such localization is indeed possible, provided that spin excitations are also localized, i.e., when carriers are coupled to a reservoir which can absorb only limited energy [67]. In order to localize the latter degrees of freedom we put $J_i = 0$ for every second or every fourth site $i$; otherwise, we keep $J_i = 1$. Results are shown in Fig. 2(a) together with the data for the subdiffusive case ($J_i = J = 0.5$) and the Anderson insulator ($J_i = J = 0$). Within the time window that is accessible to our numerics, we do not observe a complete saturation of $\sigma_d^2(t)$ except for the Anderson insulator. However, since the increase is visibly slower than logarithmic, we conclude that that hole is indeed localized. An extremely slow charge dynamics within the localized regime is not very

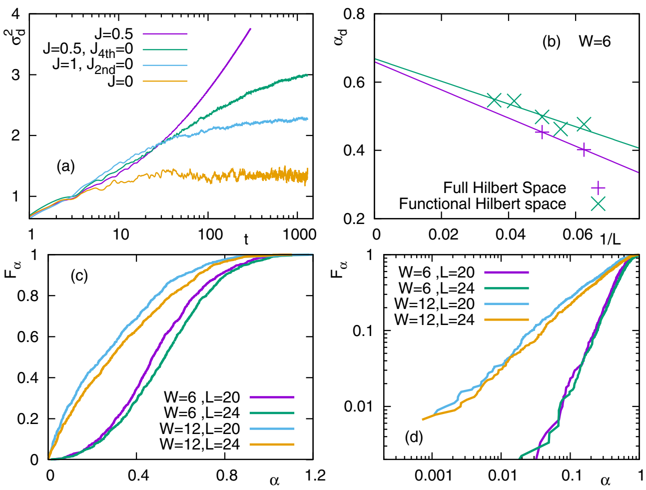

FIG. 2. (a) $\sigma_d^2$ for $L = 16$, $W = 6$ and various spatial distributions of $J_i$. The latter quantity takes the value denoted by $J$ except for every second or every fourth site $i$, where $J_i = J_{2\mathrm{nd}} = 0$ or $J_i = J_{4\mathrm{th}} = 0$, respectively. (b) The $1/L$ scaling of the exponent $\alpha_d$ for homogeneous $J_i = 1$. (c) and (d) Cumulative distribution functions, Eq. (4), for homogeneous $J_i = 1$. Results in (a) are obtained from diagonalization of the Hamiltonian in the full Hilbert space, while in (c) and (d) we used the functional space.

surprising. It has previously been reported also for the MBL phase in a system of interacting spinless fermions [68]. Note that the spatially averaged values of the exchange interaction $\langle J_i \rangle_i$ are the same for the subdiffusive system ($J_i = J = 0.5$) and for the localized case ($J_i = 0$ and 1 for odd and even $i$, respectively). Therefore, the average energy or the average coupling strength of the magnetic excitations is not essential for the charge localization. The finite localization length of magnetic excitations seems to be the key factor that enables localization of charge carriers.

The question arises whether the spin excitations have been artificially delocalized by introducing homogeneous exchange interaction $J$. When deriving the $t$-$J$ Hamiltonian from the disordered Hubbard model, one obtains also inhomogeneous $J$. For the neighboring sites $i$ and $j$ one obtains [69]

$$J_{ij} = \frac{4t_h^2 U}{U^2 - (\varepsilon_i - \varepsilon_j)^2}, \tag{3}$$

where $U$ is the Coulomb interaction in the Hubbard model. It is clear that $J_{ij}$ in the disordered system is not smaller than $J = 4t_h^2/U$ obtained for the homogeneous case. Since the disorder in the Hubbard model always enlarges the exchange interaction, such disorder alone should not localize the spin excitations. As follows from Eq. (3), elimination of the doubly occupied states is possible for $U > 2W$. In the case of strongly disordered systems (e.g., $W \sim 10$), the values of $J_{ij}$ obtained directly from Eq. (3) are too small to be studied numerically. It is the reason why we do not apply the latter equation in the present work.

From now on, we study the details of the subdiffusive transport in a system with homogeneous exchange interaction and, for simplicity, we set $J_i = 1$. Figure 2(b) shows the exponents $\alpha_d$ for various $L$. While the FS effects are not essential, they are not negligible either. Therefore, it is important to employ a method which allows to study even larger systems. In the case of a single carrier instead of diagonalizing the Hamiltonian in the full Hilbert space we use the limited functional Hilbert space [70]. Such an approach has successfully been applied to studies on the real-time dynamics of $t$-$J$ and Holstein models [71–74], and it is briefly explained also in Appendix A. In this approach one accounts for all spin excitations in the closest vicinity of the hole but only for selected more distant excitations. In contrast to the previous method, $L$ does not represent the geometric size of the lattice but the maximal distance between the hole and the spin excitation. However, in both approaches one is interested in the limit $L \to \infty$, and the corresponding $1/L$ scaling of $\alpha_d$ is shown in Fig. 2(b). Both methods obviously give the same extrapolated value of the exponent $\alpha_d$. However, diagonalization in the functional Hilbert space shows much weaker FS effects than the other approach.

Next, we check whether possible isolated cases with a localized hole have been overlooked when discussing results averaged over the charge disorder. To exclude the latter possibility, we have fitted $\sigma^2(t) \propto t^\alpha$ independently for each realization of the disorder, thus generating the distribution of the exponents $f(\alpha)$. The calculations have been carried out for times $t \leqslant 10^3$. In Figs. 2(c) and 2(d) we show the cumulative distribution function,

$$F_\alpha = \int_0^\alpha d\alpha' f(\alpha'), \tag{4}$$

which vanishes for small $\alpha$ according to the power law $\alpha^\mu$. As shown in Fig. 2(d), $\mu$ depends on the disorder strength but seems to be free from the FS effects. Therefore, we conclude that $F_{\alpha\to 0} = 0$ also in the thermodynamic limit. In contrast, $F_{\alpha\to 0} = F_0 > 0$ would indicate localization. For delocalized spin excitations in the $t$-$J$ model, the charge dynamics may be very slow, but the hole is never localized, at least not in the studied time window $(0,10^3)$.

It has recently been argued that the SU(2) symmetry precludes conventional MBL [75–77]. In order to show that the latter mechanism is not responsible for delocalization of holes in the present system, we have considered anisotropic spin-spin interaction. Results in Appendix B show that breaking the SU(2) symmetry does not lead to the charge localization even for $W = 20$.

Finally, we show that the hole dynamics in the $t$-$J$ model with strong charge disorder may be qualitatively understood by studying the classical toy model. The properties of the latter model are determined by the distribution of waiting times; hence, one should first specify which quantity obtained for the $t$-$J$ model bears the closest resemblance to the classical waiting time. Since the toy model describes the sequence of hoppings between the neighboring sites, in the $t$-$J$ model we define $\tau$ as the shortest time for which the mean-square deviation (2) equals the lattice constant, $\sigma^2(\tau) = 1$. Such a $\tau$ is well defined for each realization of the charge disorder, and one obtains the distribution of the waiting times $f(\tau)$ in the quantum model. Since we are particularly interested in the large-$\tau$ properties of $f(\tau)$, we study the integrated distribution

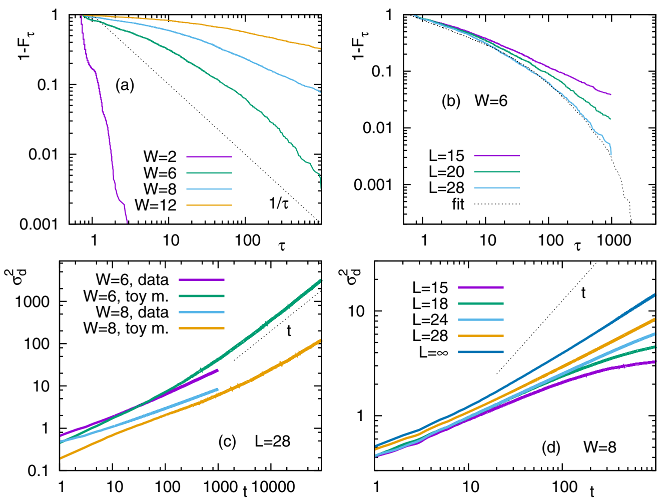


FIG. 3. (a) Integrated distribution of waiting times, Eq. (5), for $L = 28$ and various disorder strengths. The dashed line marks the border between diffusive and subdiffusive regimes in the toy model. (b) The same but for $W = 6$ and various $L$. Here, the dashed line shows the best fit. (c) $\alpha_d^2$ for the $t$-$J$ model with $L = 28$ (labeled "data") and the toy model. The latter results were obtained for the distribution of waiting times shown in (b) as the best fit. (d) $\sigma_d^2$ for various $L$ together with the linearly extrapolated value $1/L \to 0$. Results were obtained from diagonalization of the $t$-$J$ Hamiltonian in the limited functional Hilbert space.

function

$$\int_{\tau}^{\infty} d\tau' f(\tau') = 1 - F_\tau, \qquad (5)$$

where $F_\tau$ is the cumulative distribution. For the algebraically decaying $f(\tau) \sim 1/\tau^{\mu+1}$ one gets $1 - F_\tau \propto 1/\tau^{\mu}$, where $\mu = 1$ is the threshold value for the subdiffusive long-time behavior of the toy model.

Figure 3(a) shows the integrated distribution of the waiting times obtained in the $t$-$J$ model for the largest accessible systems and various strengths of the charge disorder. This distribution closely follows predictions of the toy model. In a system showing normal diffusion ($W = 2$) the distribution is very narrow, $1 - F_\tau$ decays much faster than $1/\tau$, and the average waiting time is quite short $\bar{\tau} \sim 1$. In strongly disordered subdiffusive systems $1 - F_\tau$ decays slower than $1/\tau$, and $\bar{\tau}$ should be very large, if not infinite. Therefore, our results strongly suggest that the subdiffusive transport originates from a very broad distribution of the waiting times. A broad distribution of the propagation times between the neighboring lattices sites has its origin in the strength of the charge disorder, which is by far the largest energy scale in the Hamiltonian. However, large disorder is frequently used in studies of systems showing MBL. Therefore, the present explanation of the subdiffusive transport may apply also to other strongly disordered systems with many-body interactions [35,36,49].

The integrated distributions of waiting times shown in Figs. 3(a) and 3(b) suggest that in the thermodynamics limit and for sufficiently large $\tau$ the decay of $1 - F_\tau$ may eventually become faster than $1/\tau$. Then, the average waiting time will be huge but finite, and the subdiffusive transport should be a long-lasting yet transient phenomenon. The time scale for the onset of the normal diffusion is far beyond the reach of any direct numerical studies of interacting quantum systems. However, such a long-time regime can still easily be studied in the toy model. In order to check this scenario, we have fitted numerically obtained results for the waiting times of the $t$-$J$ model, as shown in Fig. 3(b), and used this fit in the toy model. The resulting mean-square deviation of the particle distribution is shown in Fig. 3(c), confirming the onset of normal diffusion at $t \gtrsim 10^4$ for $W = 6$ and $t \gg 10^5$ for $W = 8$.

The conjecture with respect to the transiency of the subdiffusive transport may also be supported by the analysis of numerical results for the $t$-$J$ model without invoking the toy model. As shown in Fig. 3(b), the width of the distributions of the waiting times decreases when $L$ increases. Therefore, one may expect that properly carried out FS scaling may reveal at least a clear tendency for the transition to normal diffusion. In Fig. 3(d) we show results for $W = 8$ and various $L$ together with $\sigma_d^2(t)$ obtained from a linear in $1/L$ extrapolation to $1/L \to 0$. These extrapolated results together with the specific size dependences found for the exponents $\alpha_d$ [Fig. 2(b)], the cumulative distribution function $F_\alpha$ [Figs. 2(c) and 2(d)], and the distribution of the waiting times [Fig. 3(b)] consistently suggest that in the thermodynamic limit the transport may be normal-diffusive in the long-time regime. However, the related diffusion constant $D_{\rm norm}$ is extremely small,

$$\sigma_d^2(t) = 2D_{\rm anom} t^{\alpha_d} + 2D_{\rm norm} t, \qquad \alpha_d < 1, \qquad (6)$$

and the subdiffusive dynamics prevails up to very long times $t_{\rm norm} \sim (D_{\rm anom}/D_{\rm norm})^{1/(1-\alpha_d)}$. Such a time scale is too large to study directly within the present numerical approach. Consequently, the onset of the normal diffusion for long times should be considered the simplest but still conjectural explanation of our results. In order to confirm this hypothesis, one should develop an approach that allows one to study the dynamics of larger quantum systems for much longer times.

## IV. CONCLUSIONS

We have studied the dynamics of a single hole (charge carrier) in the $t$-$J$ model with strong charge disorder. Our main result is that localization of the charge carriers should be accompanied by localization of the spin degrees of freedom; otherwise, the charge dynamics is subdiffusive up to the longest times accessible to numerical calculations. This holds true also for $t$-$J$-like Hamiltonians with broken SU(2) symmetry. However, based on the distribution of propagation times between the neighboring sites and after careful finite-size scaling of the mean-square deviation, we conjecture that the subdiffusive transport is transient and may eventually be replaced by a normal diffusion. According to the latter conjecture, the delocalized magnetic excitations in the thermodynamic limit become an infinite heat bath, which, similar to electron-phonon coupling [54,55], restores nonzero albeit very small conductivity. Such an expectation is also supported by Refs. [78,79]. While this conjecture requires further study, the exceptionally broad distribution of propagation times indicates that the utmost care should be taken when formulating the claims about the asymptotic dynamics based on numerical results obtained for times $\sim 10^3$ of the inverse hopping integrals.

We believe that our qualitative claims should also be valid for other concentrations of charge carriers since each carrier is coupled to an infinite set of magnetic excitations, provided that the latter excitations remain delocalized. Contrary to this, localized spin excitations may absorb only limited energy; hence, sufficiently strong disorder in the charge sector localizes the carriers. Our studies have been carried out only for a single hole; hence, we have neglected the influence of charge disorder on the spin dynamics. While the results in Ref. [57] suggest that the charge disorder does not localize the spin degrees of freedom, the spin dynamics may be much slower than in systems without the charge disorder. However, an essential open problem is which results reported here for the $t$-$J$ model also remain valid for the Hubbard model [56,57]. Both models are equivalent, provided that the Hubbard repulsion is stronger than all other energy scales, including the disorder strength. Therefore, in strongly disordered systems the equivalence of both models is restricted to very strong repulsions when the coupling between charge carriers and spin excitations is too weak to study with purely numerical methods. For strong disorder and moderate interactions both models may lead to qualitatively different results.

## ACKNOWLEDGMENTS

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## APPENDIX A: LIMITED FUNCTIONAL HILBERT SPACE FOR THE $t$-$J$ MODEL

Generators of the limited functional Hilbert space (LFHS) are derived from off-diagonal parts of the Hamiltonian (1) in the main text,

$$O_1 = \sum_{i,\sigma} c^\dagger_{i+1\sigma} c_{i\sigma} + \text{H.c.},$$
$$O_2 = \sum_i S^+_{i+1} S^-_i + S^-_{i+1} S^+_i. \quad \text{(A1)}$$

The generating algorithm starts from a hole at a given position, e.g., $i = 0$ in a Néel state of spins, $|\psi^{(0)}\rangle = c_{0\sigma}|\text{Neel}\rangle$. We then apply the generator of basis $L$ times to generate the whole FHS:

$$\{|\psi^{(l)}\rangle\} = (O_1 + \tilde{O}_2)^l |\psi_{(0)}\rangle \quad \text{(A2)}$$

for $l = 0, \dots, L$. The operator $\tilde{O}_2$ acts only on pairs of spins that due to hole motion deviate from the original Néel state. $L$ represents the largest distance that the hole travels from its original position. In the case of LFHS we impose open boundary conditions. Sizes of LFHS span from $N_{\text{st}} \sim 4000$ for $L = 16$ up to $5 \times 10^5$ for the largest $L = 28$ used in our calculations. While even the largest size of LFHS seems rather small when performing exact-diagonalization procedures, that is not the case when performing time propagation as well as sampling over more than 1000 samples. To achieve sufficient accuracy of time propagation, we have used time-step size $\Delta t = 0.02$ and performed up to $5 \times 10^5$ time steps. The advantage of LFHS over the exact-diagonalization approach is that it significantly reduces the Hilbert space by generating spin excitations in the proximity of the hole, which in turn allows for investigations of larger system sizes. After completing generation of LFHS, we time evolve the wave function using the Hamiltonian in Eq. (A1) while taking advantage of the standard Lanczos-based diagonalization technique. The finite-size scalings of various quantities with increasing $L$ are presented in Figs. 2(b), 3(b), and 3(d) in the main text. The method has been successful in computing static and dynamic properties [70] as well as nonequilibrium dynamics [71–73] of correlated electron systems.

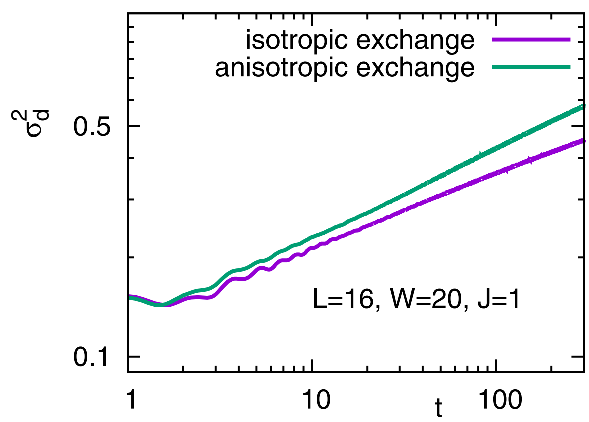


FIG. 4. Disorder-averaged mean-square deviations [see Eq. (2) in the main text] vs time for isotropic ($J^z = J = 1$) and anisotropic ($J^z = 2, J = 1$) exchange interactions.

## APPENDIX B: HOLE DYNAMICS IN THE ANISOTROPIC $t$-$J$ MODEL WITH STRONG CHARGE DISORDER

Our main result concerns the diffusive or subdiffusive charge transport which persists in the $t$-$J$ model despite the presence of very strong charge disorder, $W \sim 20$, unless the spin excitations are localized. It has recently been argued that the SU(2) symmetry precludes conventional many-body localization [75–77]. Hence, it is important to check whether the SU(2) invariance is responsible for the robust delocalized nature of charge carriers also in the $t$-$J$ model. In order to answer this question, we study a modified version of the $t$-$J$ model,

$$H = \sum_{i,\sigma} [-t_h c^\dagger_{i+1\sigma} c_{i\sigma} + \text{H.c.} + \varepsilon_i n_{i\sigma}] + \sum_i \left[J\left(S^x_{i+1} S^x_i + S^y_{i+1} S^y_i\right) + J^z S^z_{i+1} S^z_i\right]. \quad \text{(B1)}$$

When this version is compared to the Hamiltonian (1) in the main text, we have introduced anisotropic but site-independent exchange interaction such that the SU(2) invariance is broken for $J^z \neq J$. Otherwise, we use the same notation as in the main text.

In Fig. 4 we compare the disorder-averaged mean-square deviations $\sigma_d^2(t)$ obtained for isotropic ($J^z = J = 1$) and anisotropic ($J^z = 2, J = 1$) systems from the Lanczos propagation method [66]. Despite a very strong charge disorder, $W = 20$, breaking the SU(2) symmetry does not lead to localization of charge carriers. On the contrary, the subdiffusive transport $\sigma_d^2(t) \propto t^{\alpha_d}$ is clearly visible in both cases, and the exponent $\alpha_d$ is even slightly larger in a system with broken SU(2) symmetry. The latter result indicates that $\alpha_d$ is influenced by increasing $J^z$ rather than by breaking the SU(2) symmetry.


[1] A. Pal and D. A. Huse, Many-body localization phase transition, Phys. Rev. B **82**, 174411 (2010).

[2] M. Serbyn, Z. Papić, and D. A. Abanin, Local Conservation Laws and the Structure of the Many-Body Localized States, Phys. Rev. Lett. **111**, 127201 (2013).

[3] Y. Bar Lev and D. R. Reichman, Dynamics of many-body localization, Phys. Rev. B **89**, 220201 (2014).

[4] M. Schreiber, S. S. Hodgman, P. Bordia, H. P. Lüschen, M. H. Fischer, R. Vosk, E. Altman, U. Schneider, and I. Bloch, Observation of many-body localization of interacting fermions in a quasi-random optical lattice, Science **349**, 842 (2015).

[5] M. Serbyn, Z. Papić, and D. A. Abanin, Criterion for Many-Body Localization-Delocalization Phase Transition, Phys. Rev. X **5**, 041047 (2015).

[6] V. Khemani, R. Nandkishore, and S. L. Sondhi, Nonlocal adiabatic response of a localized system to local manipulations, Nat. Phys. **11**, 560 (2015).

[7] D. J. Luitz, N. Laflorencie, and F. Alet, Extended slow dynamical regime close to the many-body localization transition, Phys. Rev. B **93**, 060201(R) (2016).

[8] I. V. Gornyi, A. D. Mirlin, and D. G. Polyakov, Interacting Electrons in Disordered Wires: Anderson Localization and Low-$T$ Transport, Phys. Rev. Lett. **95**, 206603 (2005).

[9] E. Altman and R. Vosk, Universal dynamics and renormalization in many-body-localized systems, Annu. Rev. Condens. Matter Phys. **6**, 383 (2015).

[10] A. De Luca and A. Scardicchio, Ergodicity breaking in a model showing many-body localization, Europhys. Lett. **101**, 37003 (2013).

[11] C. Gramsch and M. Rigol, Quenches in a quasidisordered integrable lattice system: Dynamics and statistical description of observables after relaxation, Phys. Rev. A **86**, 053615 (2012).

[12] A. De Luca, B. L. Altshuler, V. E. Kravtsov, and A. Scardicchio, Anderson Localization on the Bethe Lattice: Nonergodicity of Extended States, Phys. Rev. Lett. **113**, 046806 (2014).

[13] D. A. Huse, R. Nandkishore, V. Oganesyan, A. Pal, and S. L. Sondhi, Localization-protected quantum order, Phys. Rev. B **88**, 014206 (2013).

[14] R. Nandkishore and D. A. Huse, Many-body localization and thermalization in quantum statistical mechanics, Annu. Rev. Condens. Matter Phys. **6**, 15 (2015).

[15] L. Rademaker and M. Ortuño, Explicit Local Integrals of Motion for the Many-Body Localized State, Phys. Rev. Lett. **116**, 010404 (2016).

[16] A. Chandran, I. H. Kim, G. Vidal, and D. A. Abanin, Constructing local integrals of motion in the many-body localized phase, Phys. Rev. B **91**, 085425 (2015).

[17] V. Ros, M. Müller, and A. Scardicchio, Integrals of motion in the many-body localized phase, Nucl. Phys. B **891**, 420 (2015).

[18] J. Eisert, M. Friesdorf, and C. Gogolin, Quantum many-body systems out of equilibrium, Nat. Phys. **11**, 124 (2015).

[19] M. Serbyn, M. Knap, S. Gopalakrishnan, Z. Papić, N. Y. Yao, C. R. Laumann, D. A. Abanin, M. D. Lukin, and E. A. Demler, Interferometric Probes of Many-Body Localization, Phys. Rev. Lett. **113**, 147204 (2014).

[20] R. Modak and S. Mukerjee, Many-Body Localization in the Presence of a Single-Particle Mobility Edge, Phys. Rev. Lett. **115**, 230401 (2015).

[21] C. Monthus and T. Garel, Many-body localization transition in a lattice model of interacting fermions: Statistics of renormalized hoppings in configuration space, Phys. Rev. B **81**, 134202 (2010).

[22] D. J. Luitz, N. Laflorencie, and F. Alet, Many-body localization edge in the random-field Heisenberg chain, Phys. Rev. B **91**, 081103(R) (2015).

[23] F. Andraschko, T. Enss, and J. Sirker, Purification and Many-Body Localization in Cold Atomic Gases, Phys. Rev. Lett. **113**, 217201 (2014).

[24] C. R. Laumann, A. Pal, and A. Scardicchio, Many-Body Mobility Edge in a Mean-Field Quantum Spin Glass, Phys. Rev. Lett. **113**, 200405 (2014).

[25] D. A. Huse, R. Nandkishore, and V. Oganesyan, Phenomenology of fully many-body-localized systems, Phys. Rev. B **90**, 174202 (2014).

[26] P. Ponte, Z. Papić, F. Huveneers, and D. A. Abanin, Many-Body Localization in Periodically Driven Systems, Phys. Rev. Lett. **114**, 140401 (2015).

[27] A. Lazarides, A. Das, and R. Moessner, Fate of Many-Body Localization Under Periodic Driving, Phys. Rev. Lett. **115**, 030402 (2015).

[28] R. Vasseur, S. A. Parameswaran, and J. E. Moore, Quantum revivals and many-body localization, Phys. Rev. B **91**, 140202(R) (2015).

[29] M. Serbyn, Z. Papić, and D. A. Abanin, Quantum quenches in the many-body localized phase, Phys. Rev. B **90**, 174302 (2014).

[30] D. Pekker, G. Refael, E. Altman, E. Demler, and V. Oganesyan, Hilbert-Glass Transition: New Universality of Temperature-Tuned Many-Body Dynamical Quantum Criticality, Phys. Rev. X **4**, 011052 (2014).

[31] E. J. Torres-Herrera and L. F. Santos, Dynamics at the many-body localization transition, Phys. Rev. B **92**, 014208 (2015).

[32] M. Távora, E. J. Torres-Herrera, and L. F. Santos, Inevitable power-law behavior of isolated many-body quantum systems and how it anticipates thermalization, Phys. Rev. A **94**, 041603(R) (2016).

[33] T. C. Berkelbach and D. R. Reichman, Conductivity of disordered quantum lattice models at infinite temperature: Many-body localization, Phys. Rev. B **81**, 224429 (2010).

[34] O. S. Barišić and P. Prelovšek, Conductivity in a disordered one-dimensional system of interacting fermions, Phys. Rev. B **82**, 161106(R) (2010).

[35] K. Agarwal, S. Gopalakrishnan, M. Knap, M. Müller, and E. Demler, Anomalous Diffusion and Griffiths Effects Near the Many-Body Localization Transition, Phys. Rev. Lett. **114**, 160401 (2015).
[36] S. Gopalakrishnan, M. Müller, V. Khemani, M. Knap, E. Demler, and D. A. Huse, Low-frequency conductivity in many-body localized systems, Phys. Rev. B **92**, 104202 (2015).
[37] Y. Bar Lev, G. Cohen, and D. R. Reichman, Absence of Diffusion in an Interacting System of Spinless Fermions on a One-Dimensional Disordered Lattice, Phys. Rev. Lett. **114**, 100601 (2015).
[38] R. Steinigeweg, J. Herbrych, F. Pollmann, and W. Brenig, Typicality approach to the optical conductivity in thermal and many-body localized phases, Phys. Rev. B **94**, 180401(R) (2016).
[39] O. S. Barišić, J. Kokalj, I. Balog, and P. Prelovšek, Dynamical conductivity and its fluctuations along the crossover to many-body localization, Phys. Rev. B **94**, 045126 (2016).
[40] M. Kozarzewski, P. Prelovšek, and M. Mierzejewski, Distinctive response of many-body localized systems to a strong electric field, Phys. Rev. B **93**, 235151 (2016).
[41] V. Oganesyan and D. A. Huse, Localization of interacting fermions at high temperature, Phys. Rev. B **75**, 155111 (2007).
[42] M. Žnidarič, T. Prosen, and P. Prelovšek, Many-body localization in the Heisenberg $XXZ$ magnet in a random field, Phys. Rev. B **77**, 064426 (2008).
[43] J. H. Bardarson, F. Pollmann, and J. E. Moore, Unbounded Growth of Entanglement in Models of Many-Body Localization, Phys. Rev. Lett. **109**, 017202 (2012).
[44] J. A. Kjäll, J. H. Bardarson, and F. Pollmann, Many-Body Localization in a Disordered Quantum Ising Chain, Phys. Rev. Lett. **113**, 107204 (2014).
[45] M. Serbyn, Z. Papić, and D. A. Abanin, Universal Slow Growth of Entanglement in Interacting Strongly Disordered Systems, Phys. Rev. Lett. **110**, 260601 (2013).
[46] S. Bera, H. Schomerus, F. Heidrich-Meisner, and J. H. Bardarson, Many-Body Localization Characterized from a One-Particle Perspective, Phys. Rev. Lett. **115**, 046603 (2015).
[47] R. Vosk, D. A. Huse, and E. Altman, Theory of the Many-Body Localization Transition in One-Dimensional Systems, Phys. Rev. X **5**, 031032 (2015).
[48] A. C. Potter, R. Vasseur, and S. A. Parameswaran, Universal Properties of Many-Body Delocalization Transitions, Phys. Rev. X **5**, 031033 (2015).
[49] M. Žnidarič, A. Scardicchio, and V. K. Varma, Diffusive and Subdiffusive Spin Transport in the Ergodic Phase of a Many-Body Localizable System, Phys. Rev. Lett. **117**, 040601 (2016).
[50] H. P. Lüschen, P. Bordia, S. Scherg, F. Alet, E. Altman, U. Schneider, and I. Bloch, Evidence for griffiths-type dynamics near the many-body localization transition in quasi-periodic systems, arXiv:1612.07173.
[51] J. Z. Imbrie, Diagonalization and Many-Body Localization for a Disordered Quantum Spin Chain, Phys. Rev. Lett. **117**, 027201 (2016).
[52] S. S. Kondov, W. R. McGehee, W. Xu, and B. DeMarco, Disorder-Induced Localization in a Strongly Correlated Atomic Hubbard Gas, Phys. Rev. Lett. **114**, 083002 (2015).
[53] P. Bordia, H. P. Lüschen, S. S. Hodgman, M. Schreiber, I. Bloch, and U. Schneider, Coupling Identical 1D Many-Body Localized Systems, Phys. Rev. Lett. **116**, 140401 (2016).
[54] D. M. Basko, I. L. Aleiner, and B. L. Altshuler, Metal-insulator transition in a weakly interacting many-electron system with localized single-particle states, Ann. Phys. (N.Y.) **321**, 1126 (2006).
[55] N. F. Mott, Conduction in glasses containing transition metal ions, J. Non-Cryst. Solids **1**, 1 (1968).
[56] R. Mondaini and M. Rigol, Many-body localization and thermalization in disordered Hubbard chains, Phys. Rev. A **92**, 041601(R) (2015).
[57] P. Prelovšek, O. S. Barišić, and M. Žnidarič, Absence of full many-body localization in the disordered Hubbard chain, Phys. Rev. B **94**, 241104(R) (2016).
[58] P. W. Anderson, Absence of diffusion in certain random lattices, Phys. Rev. **109**, 1492 (1958).
[59] N. F. Mott, Conduction in non-crystalline systems, Philos. Mag. **17**, 1259 (1968).
[60] B. Kramer and A. MacKinnon, Localization: Theory and experiment, Rep. Prog. Phys. **56**, 1469 (1993).
[61] F. Evers and A. D. Mirlin, Anderson transitions, Rev. Mod. Phys. **80**, 1355 (2008).
[62] L. Fleishman and P. W. Anderson, Interactions and the Anderson transition, Phys. Rev. B **21**, 2366 (1980).
[63] D. Schmidtke, R. Steinigeweg, J. Herbrych, and J. Gemmer, Interaction-induced weakening of localization in few-particle disordered Heisenberg chains, Phys. Rev. B **95**, 134201 (2017).
[64] J.-P. Bouchaud and A. Georges, Anomalous diffusion in disordered media: Statistical mechanisms, models and physical applications, Phys. Rep. **195**, 127 (1990).
[65] E. W. Montroll and H. Scher, Random walks on lattices. IV. Continuous-time walks and influence of absorbing boundaries, J. Stat. Phys. **9**, 101 (1973).
[66] T. J. Park and J. C. Light, Unitary quantum time evolution by iterative Lanczos reduction, J. Chem. Phys. **85**, 5870 (1986).
[67] K. Hyatt, J. R. Garrison, A. C. Potter, and B. Bauer, Many-body localization in the presence of a small bath, Phys. Rev. B **95**, 035132 (2017).
[68] M. Mierzejewski, J. Herbrych, and P. Prelovšek, Universal dynamics of density correlations at the transition to the many-body localized state, Phys. Rev. B **94**, 224207 (2016).
[69] M. M. Maśka, Ż. Śledź, K. Czajka, and M. Mierzejewski, Inhomogeneity-Induced Enhancement of the Pairing Interaction in Cuprate Superconductors, Phys. Rev. Lett. **99**, 147006 (2007).
[70] J. Bonča, S. Maekawa, and T. Tohyama, Numerical approach to the low-doping regime of the $t$-$J$ model, Phys. Rev. B **76**, 035121 (2007).
[71] S. Dal Conte, L. Vidmar, D. Golez, M. Mierzejewski, G. Soavi, S. Peli, F. Banfi, G. Ferrini, R. Comin, B. M. Ludbrook, L. Chauviere, N. D. Zhigadlo, H. Eisaki, M. Greven, S. Lupi, A. Damascelli, D. Brida, M. Capone, J. Bonca, G. Cerullo, and C. Giannetti, Snapshots of the retarded interaction of charge carriers with ultrafast fluctuations in cuprates, Nat. Phys. **11**, 421 (2015).
[72] D. Golež, J. Bonča, M. Mierzejewski, and L. Vidmar, Mechanism of ultrafast relaxation of a photo-carrier in antiferromagnetic spin background, Phys. Rev. B **89**, 165118 (2014).
[73] M. Mierzejewski, L. Vidmar, J. Bonča, and P. Prelovšek, Nonequilibrium Quantum Dynamics of a Charge Carrier Doped into a Mott Insulator, Phys. Rev. Lett. **106**, 196401 (2011).

[74] D. Golež, J. Bonča, L. Vidmar, and S. A. Trugman, Relaxation Dynamics of the Holstein Polaron, Phys. Rev. Lett. **109**, 236402 (2012).

[75] A. Chandran, V. Khemani, C. R. Laumann, and S. L. Sondhi, Many-body localization and symmetry-protected topological order, Phys. Rev. B **89**, 144201 (2014).

[76] A. C. Potter and R. Vasseur, Symmetry constraints on many-body localization, Phys. Rev. B **94**, 224206 (2016).

[77] I. V. Protopopov, W. W. Ho, and D. A. Abanin, The effect of $SU(2)$ symmetry on many-body localization and thermalization, arXiv:1612.01208.

[78] S. A. Parameswaran and S. Gopalakrishnan, Spin-catalyzed hopping conductivity in disordered strongly interacting quantum wires, Phys. Rev. B **95**, 024201 (2017).

[79] S. Gopalakrishnan, K. Ranjibul Islam, and M. Knap, Noise-induced subdiffusion in strongly localized quantum systems, arXiv:1609.04818.