Supersymmetry in the fractional quantum Hall regime

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Supersymmetry (SUSY) is a symmetry transforming bosons to fermions and vice versa. Indications of its existence have been extensively sought after in high-energy experiments. However, signatures of SUSY have yet to be detected. In this paper we study a condensed matter realization of SUSY on the edge of a Read-Rezayi quantum Hall state, given by filling factors of the form $v = \frac{k}{k+2}$, where k is an integer. As we show explicitly, this strongly interacting state exhibits an $\mathcal{N} = 2$ SUSY. This allows us to use a topological invariant—the Witten index—defined specifically for supersymmetric theories, to count the difference between the number of bosonic and fermionic zero modes in a circular edge. In this system, we argue that the edge hosts k + 1 protected zero modes. We further discuss the stability of SUSY with respect to generic perturbations and find that much of the above results remain unchanged. In particular, these results directly apply to the well-established v = 1/3 Laughlin state, in which case SUSY is a robust property of the edge theory. These results unveil a hidden topological structure on the long-studied Read-Rezayi states.

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Since its discovery, the quantum Hall effect has led to a plethora of remarkable new physical phenomena. The integer quantum Hall (IQH) effect [1], for instance, is a paradigmatic example of noninteracting topological phases, characterized by bulk topological invariants and gapless edge modes. The strongly interacting fractional quantum Hall (FQH) states [2], on the other hand, present even more striking properties, such as the existence of fractionally charged anyonic bulk excitations.

A subset of the fractional states are non-Abelian ones, whose bulk excitations are non-Abelian anyons, and whose edges realize nontrivial interacting conformal field theories (CFTs). While the recent interest in non-Abelian phases is mostly driven by their exotic bulk excitations and the possibility of using them as resources in topological quantum computation, these states are also a natural playground for experimentally studying one-dimensional (1D) conformal field theories (CFTs).

In this paper we will study the edge CFTs of Read-Rezayi (RR) states at filling $\nu = \frac{k}{k+2}$. The simplest example is given by the v = 1/3 Laughlin state, corresponding to k = 1, which constitutes the most prominent FQH state [2]. RR states with k > 1, on the other hand, are widely believed to be energetically unfavorable compared to other competing states at the same filling factors in the lowest Landau level. In the first excited Landau level, however, numerical works indicate that the particle-hole conjugates of these states may be the ground states in the corresponding filling factors [3–19]. Indeed, the plateaus observed at $\nu =$ 5/2 and $\nu = 12/5$ are strong candidates for realizing the particle-hole conjugates of the k = 2 and k = 3 states. As we will show, supersymmetry (SUSY)-a symmetry transforming bosons to fermions and vice versa-emerges naturally in these states.

In general, SUSY is a space-time symmetry which constitutes the only possible extension of the Poincaré group consistent with the symmetries of the scattering matrix [20]. It has attracted attention given that it solves several open problems in high-energy physics and cosmology [21–26]. In particular, its existence implies that the strengths of the three fundamental forces of the standard model unify at the same energy scale [27]. Furthermore, if it is in fact a symmetry of nature, it will provide natural candidates for dark matter particles.

Despite its many features, the existence of this symmetry has not been confirmed in high energy experiments so far. This has recently sparked interest in realizing SUSY in condensed matter systems. In particular, signatures of space-time SUSY have been proposed at critical points in topological superconductors [28–30], topological insulators [31,32], and topological semimetals [33]. $\mathcal{N} = 1$ SUSY has been discovered in strongly interacting Majorana chains [34]. Recently [35], $\mathcal{N} = 2$ SUSY was shown to generically exist in translation invariant lattice systems with an odd number of Majorana degrees of freedom per unit cell.

In this paper, we demonstrate that the low-energy description of the edge of RR states gives rise to an $\mathcal{N} = 2$ SUSY, generated by two fermionic charges Q_1, Q_2 . Combining this with the existence of conformal symmetry, we find that the edge of our incompressible state realizes an $\mathcal{N} = 2$ superconformal theory in 1+1 spacetime dimensions.

Once we establish the emergence of SUSY in our quantum Hall system, we will turn to study its implications. To do so, we will introduce the so-called Witten index, which is a fundamental topological invariant measuring the number of bosonic zero modes minus the number of fermionic zero modes in a theory containing SUSY. While SUSY constrains this difference to be zero for any finite energy, at exactly zero energy the constraint is lifted. Being a topological invariant, this index is highly stable and is in particular completely independent of temperature.

We note that traditionally in the study of topological phases, topological indices result from properties of the bulk insulating phase and dictate the edge physics through the bulk-boundary correspondence. The topological invariant we study, on the other hand, is an explicit property of the gapless

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edge theory itself, revealed by supersymmetry. We discuss the possibility of understanding it from the point of view of the bulk topological quantum field theory and highlight its connection to the entanglement spectrum of a bipartition of the state in the bulk. Intriguingly, while topological invariants are generically defined for *noninteracting* symmetry protected topological phases, supersymmetry provides us with tools to define a topological invariant for the *strongly interacting* RR states.

Clearly, any perturbation preserving supersymmetry does not alter the above structure. As we will see, SUSY is a robust property of the $\nu = 1/3$ Laughlin state. However, in other experimentally relevant states, given by filling factors of the form $3 - \frac{k}{k+2}$, intermode interaction terms generally break SUSY. However, imprints of SUSY may still be observed. In particular, for weak SUSY breaking terms, the difference between the number of bosonic and fermionic states near zero energy is still given by the Witten index. Finally, we briefly discuss the possibility of measuring the robust zero modes in a small circular edge configuration.

I. THE SYSTEM

The system we study is made of a two-dimensional electron gas in the quantum Hall regime. We will be interested in studying fermionic RR states at filling $v = \frac{k}{k+2}$. However, for pedagogical reasons, we start from a different system made of two layers; the first contains bosons while the second contains fermions.

Both layers are in the quantum Hall regime, and we fix their densities such that the filling of the fermionic (bosonic) layer is v = 1 (v = k/2). While such a fermion-boson double layer system is beyond experimental reach, studying it will provide a clear demonstration of the emergence of a supersymmetric low-energy sector on the edge. Furthermore, as we will later argue, the edge theory of realistic fermionic RR states can be mapped to the supersymmetric theory on the edge of the fermion-boson double layer. This will prove the existence of SUSY on the edge of fermionic RR states.

Focusing first on the auxiliary fermion-boson double layer system, we explicitly write the edge theories of the two layers. The fermionic layer is made of the trivial v = 1 IQH state, whose edge contains a chiral free fermion field described by the Hamiltonian

$$H_{\psi} = -iv \int dx \psi^{\dagger} \partial \psi, \qquad (1)$$

with the usual anticommutation relations $\{\psi^{\dagger}(x), \psi(x')\} = \delta(x - x')$.

The bosonic layer, on the other hand, is more complicated. It is assumed to be in a bosonic RR state, whose edge realizes a strongly interacting SU(2)_k CFT with central charge $c = \frac{3k}{k+2}$. The corresponding Hamiltonian is given by

$$H_{\mathrm{SU}(2)_k} = \frac{v}{4\pi(k+2)} \int dx \,\mathcal{J} \cdot \mathcal{J},\tag{2}$$

where the currents $\mathcal{J}^z, \mathcal{J}^{\pm}$ satisfy an affine Kac-Moody algebra at level *k* [36]. Note that the Hamiltonians above and throughout the text are assumed to be normal ordered. A simple representation of these currents is given in terms of *k*

bosons φ_a , with $\mathcal{J}^m = \sum_{a=1}^k J_a^m$ where

$$J_a^{\pm} = e^{\pm i\sqrt{2}\varphi_a} \quad \text{and} \quad J_a^3 = \frac{1}{\sqrt{2}} \frac{\partial\varphi_a}{2\pi}, \tag{3}$$

the bosonic degrees of freedom satisfy the commutation relations

$$[\varphi_a(x),\varphi_b(x')] = i\pi \delta_{ab} \operatorname{sign}(x - x').$$
(4)

We emphasize that the above fields are chiral, as they appear on the edge of a two-dimensional quantum Hall system.

The SU(2)_k theory can be further decomposed into two mutually commuting sectors: a U(1) charge mode with central charge $c_{U(1)} = 1$ and a SU(2)_k/U(1) theory with $c_{\mathbb{Z}_k} = 2\frac{k-1}{k+2}$, describing \mathbb{Z}_k parafermions, i.e.,

$$H_{\mathrm{SU}(2)_k} = H^b_\rho + H_{\mathbb{Z}_k}.$$
(5)

This can be done by defining $\varphi_a = \frac{1}{\sqrt{k}}\varphi_{\rho}^b + \vec{d}_a \cdot \vec{\varphi}_{\sigma}$, where the k vectors \vec{d}_a (each of dimension k - 1) satisfy the relations

$$\sum_{a=1}^{k} \vec{d}_{a} = 0, \quad \sum_{a=1}^{k} d_{a}^{\alpha} d_{a}^{\beta} = \delta_{\alpha\beta}, \quad \vec{d}_{a} \cdot \vec{d}_{b} = \delta_{ab} - \frac{1}{k}.$$
 (6)

The Hamiltonian describing the charge sector of the bosonic RR state is given by

$$H^b_\rho = \frac{v}{4\pi} \int dx \left(\partial_x \varphi^b_\rho\right)^2. \tag{7}$$

The neutral parafermionic sector is an inherently strongly interacting CFT. In terms of the neutral bosons, we write the Hamiltonian

$$H_{\mathbb{Z}_k} = \frac{v}{2\pi(k+2)} \int dx \left[\left(\partial_x \vec{\varphi}_\sigma \right)^2 + \sum_{a \neq b} e^{i\sqrt{2}(\vec{d}_a - \vec{d}_b) \cdot \vec{\varphi}_\sigma} \right].$$
(8)

Indeed, the Hamiltonian $H_{\mathbb{Z}_k}$ describes a critical system, whose low-energy sector coincides with the parafermionic sector of our edge theory [37]. We emphasize that the above bosonic representation is not unique but simply a useful one [38]. In terms of the above, we can write the parafermion operator as $\Psi = \sum_{a} e^{\sqrt{2}i\vec{d}_a\cdot\vec{\varphi}_{\sigma}}$.

We can decompose the Hilbert space of the full fermionboson double layer into the following two mutually commuting sectors: The first sector describes the total charge degrees of freedom and is given by the Hamiltonian

$$H_{\rm U(1)} = \frac{k}{k+2} H_{\rho}^{b} + \frac{2}{k+2} H_{\psi} - H_{\rm int}, \qquad (9)$$

with

$$H_{\rm int} = \frac{v}{2(k+2)\pi} \sqrt{\frac{k}{2}} \int dx \psi^{\dagger} \psi \partial_x \varphi_{\rho}^b.$$
(10)

The remaining sector is governed by the Hamiltonian

$$H_{\text{SUSY}} = H_{\mathbb{Z}_k} + \frac{k}{k+2}H_{\psi} + \frac{2}{k+2}H_{\rho}^b + H_{\text{int}}.$$
 (11)

Note that $H_{\text{double}} = H_{\psi} + H_{\rho}^b + H_{\mathbb{Z}_k} = H_{\text{U}(1)} + H_{\text{SUSY}}$, with $[H_{\text{U}(1)}, H_{\text{SUSY}}] = 0.$

II. $\mathcal{N} = 2$ SUSY AND THE FERMIONIC READ-REZAYI EDGE THEORY

As we demonstrate in Appendix C, the Hamiltonian H_{SUSY} exhibits an $\mathcal{N} = 2$ SUSY. This means that there exist two independent fermionic currents G_1, G_2 satisfying the $\mathcal{N} = 2$ superconformal algebra (shown explicitly in Appendix C). The associated fermionic charges, generating the SUSY transformations (transforming bosons into fermions and viceversa), are given by

$$Q_1 = \frac{1}{\sqrt{k+2}} \int dx (\psi \mathcal{J}^+ + \psi^{\dagger} \mathcal{J}^-), \qquad (12)$$

$$Q_2 = \frac{i}{\sqrt{k+2}} \int dx (\psi^{\dagger} \mathcal{J}^- - \psi \mathcal{J}^+).$$
(13)

Note that $Q_1^{\dagger} = Q_1$ and $Q_2^{\dagger} = Q_2$. Being conserved charges, they further satisfy $[H_{\text{SUSY}}, Q_a] = 0$, as can be verified directly by using the (anti)commutation relations of the fields ψ and \mathcal{J} .

In the above analysis we focused on the auxiliary fermionboson system. Recall, however, that we are interested in a fermionic RR state at filling $v = \frac{k}{k+2}$ (without an additional unrealistic bosonic subsystem). The Hamiltonian describing such a fermionic RR system is similar to the one describing the bosonic RR state. In particular, the neutral sector, described by Eq. (8), remains unchanged. However, the charged degrees of freedom are now described by the Hamiltonian

$$H_{\rho}^{f} = \frac{v}{4\pi} \frac{k+2}{k} \int dx \left(\partial_{x} \varphi_{\rho}^{f}\right)^{2}, \qquad (14)$$

where φ_{ρ} is a boson field satisfying

$$\left[\varphi_{\rho}^{f}(x),\varphi_{\rho}^{f}(x')\right] = i\pi \frac{k}{k+2} \operatorname{sign}(x-x').$$
(15)

In terms of these, the electron operator is given by $\psi_{el}^{\dagger} = \Psi e^{i\frac{k}{k+2}\varphi_{p}^{f}}$. This operator has a scaling dimension of 3/2 and a unit of electric charge. It can therefore be thought of as a dressed version of the microscopic electron field, which commutes with the nontrivial bulk Hamiltonian. Remarkably, the full Hamiltonian describing the fermionic RR state

$$H_{RR} = H_{\rho}^{f} + H_{\mathbb{Z}_{k}} \tag{16}$$

can be mapped to the supersymmetric Hamiltonian H_{SUSY} studied in the fermion-boson auxiliary system.

To make a connection with the fermionic RR state, we start from the fermion-boson auxiliary system and bosonize the fermion operator by introducing an extra boson ϕ_1 , satisfying $\psi^{\dagger} = e^{i\phi_1}$ and $[\phi_1(x),\phi_1(y)] = i\pi \operatorname{sign}(x - y)$. In terms of the bosonized fermion the Hamiltonian H_{SUSY}

becomes

$$H_{\text{SUSY}} = \frac{v}{2(k+2)\pi} \int dx \left[\frac{k}{2} (\partial \phi_1)^2 - 2\sqrt{\frac{k}{2}} \partial \varphi_\rho^b \partial \phi_1 + \left(\partial \varphi_\rho^b \right)^2 + (\partial \vec{\varphi}_\sigma)^2 + \sum_{a \neq b} e^{i\sqrt{2}(\vec{d}_a - \vec{d}_b) \cdot \vec{\varphi}_\sigma} \right].$$
(17)

Introducing the new bosonic field

$$\varphi_{\rho}^{f} = \frac{k}{k+2} \left(\phi_{1} - \sqrt{\frac{2}{k}} \varphi_{\rho}^{b} \right), \tag{18}$$

the Hamiltonian H_{SUSY} reads

$$H_{\text{SUSY}} = \frac{v}{4\pi} \frac{k+2}{k} \int dx \left(\partial \varphi_{\rho}^{f}\right)^{2} + \frac{v}{2(k+2)\pi} \int dx \left((\partial \vec{\varphi}_{\sigma})^{2} + \sum_{a \neq b} e^{i\sqrt{2}(\vec{d}_{a} - \vec{d}_{b}) \cdot \vec{\varphi}_{\sigma}} \right)$$
$$= H_{\rho}^{f} + H_{\mathbb{Z}_{k}}, \tag{19}$$

coinciding with the Hamiltonian of the fermionic RR edge theory. It follows from the definition that $[\partial \varphi_{\rho}^{f}(x), \varphi_{\rho}^{f}(y)] = \frac{2i\pi k}{k+2}\delta(x-y)$ so we identify this field with the charge mode of the RR state.

We point out that the central charge of the auxiliary fermionboson double layer, given by $c_{\text{double}} = \frac{3k}{k+2} + 1$, is larger than the central charge of the fermionic RR state, $c_{RR} = \frac{3k}{k+2}$. Indeed, the auxiliary system consists of an additional total charge degrees of freedom, given by

$$\varphi_{\text{tot}} = \frac{1}{\sqrt{k+2}} \left(\phi_1 + \sqrt{\frac{k}{2}} \varphi_{\rho}^b \right). \tag{20}$$

However, this mode commutes with the Hamiltonian H_{SUSY} defined above. We therefore emphasize that the above mapping is between the fermionic RR system and the neutral sector of the fermion-boson double layer.

The above line of arguments shows that the well established fermionic RR state possesses $\mathcal{N} = 2$ supersymmetry. It is worth noting that the results reported above for the edge of a RR state are related to previous studies connecting parafermionic CFTs with $\mathcal{N} = 2$ superconformal theories [39–41].

III. SUSY AND ITS CONSEQUENCES

The presence of SUSY can be demonstrated explicitly by writing two Hermitian fermionic conserved currents, G_1 and G_2 , satisfying the superconformal algebra shown in Appendix C. In the fermionic RR edge CFT, these two currents take the form

$$G_{1} = \frac{1}{\sqrt{k+2}} (\psi_{el}^{\dagger} + \psi_{el}),$$

$$G_{2} = \frac{i}{\sqrt{k+2}} (\psi_{el}^{\dagger} - \psi_{el}).$$
(21)

Note, in particular, that the above results apply to the $\nu = 1/3$ Laughlin state (see Appendix C for the details of this simple case).

By integrating over space, we get two fermionic charges, Q_1 and Q_2 , satisfying [42]

$$Q_1^2 = Q_2^2 = H_{\rm SUSY},$$
 (22)

with $\{Q_1, Q_2\} = 0$, and $H_{SUSY} = H_{\rho}^f + H_{\mathbb{Z}_k}$, as shown explicitly in Appendix C.

We further define the complex supersymmetric current operator $\mathcal{G}^{\pm} = \frac{1}{\sqrt{2}}(G^1 \pm iG^2) = \sqrt{\frac{2}{k+2}}e^{\mp i\frac{k+2}{k}\varphi_p^f}\Psi^{\pm}$. Intriguingly, this fermionic current is given by the physical (annihilation/creation) electron operator defined above (up to a multiplicative constant), which indeed has conformal dimension 3/2. The fermionic charge, generating the SUSY transformations, is given by the spatial integral of \mathcal{G} .

The associated complex fermionic charge is given by $Q = \frac{Q_1+iQ_2}{\sqrt{2}} = \sqrt{\frac{2v}{k+2}} \int dx \psi_{el}$. By definition, the complex charge satisfies $Q^2 = (Q^{\dagger})^2 = 0$ and can be used to write the Hamiltonian in the convenient form

$$H_{\rm SUSY} = \frac{1}{2} \{ Q^{\dagger}, Q \}.$$
 (23)

fermions

Applying Q on any bosonic state, consisting solely of density excitations, generates a fermionic state with the same energy, as a consequence of SUSY.

This simple structure allows us to study the ground state of H_{SUSY} . Here we follow the arguments presented in Ref. [43] to define an appropriate topological invariant. From Eq. (23) we see that $[H_{SUSY}, Q] = 0$. This implies that for each bosonic state $|\xi\rangle$ with energy E > 0 [44], there is a fermionic state with the same energy, given by $|f\rangle \sim Q|\xi\rangle$. To be more precise, assuming a normalized state $|\xi\rangle$, the normalized fermionic partner of this state is given explicitly by $|f\rangle = \frac{1}{\sqrt{E}}Q|\xi\rangle$.

For the zero energy states, on the other hand, the relation between fermionic and bosonic states is broken, and one generally has a different number of fermionic and bosonic zero modes, as depicted schematically in Fig. 1. To characterize this difference, it is useful to introduce the so-called Witten index $W = \text{tr}(-1)^F$, where F = 0 for a bosonic state and F = 1 for a fermionic state. We note that a natural realization of W is

bosons

 E_4



FIG. 1. A generic spectrum of a supersymmetric theory. States at positive energies come in degenerate Fermi-Bose pairs, related by the action of the SUSY charge Q. At zero energy, no such constraint exists and the number of fermionic and bosonic zero modes can be different. The difference between the number of bosonic and fermionic states is called the Witten index W and constitutes a topological invariant.

given in terms of the angular momentum operator L^z , such that $(-1)^F = \exp(2\pi i L^z)$. It is easy to see that the W operator measures the difference between the number of bosonic and fermionic states at zero energy, i.e., $W = (N_B - N_F)_{E=0}$. For our system this difference has been calculated in Ref. [45] and is given by

$$W = (N_{\rm B} - N_{\rm F})_{E=0} = \begin{cases} 0 & k & \text{odd} \\ 1 & k & \text{even} \end{cases}$$
 (24)

As we argue below, this quantity is a topological invariant characterizing the RR edge theory [43,45].

In order for the number of zero modes to change, a state must change its energy from zero to some positive value, or vice versa. However, since positive energy states come in Bose-Fermi pairs, the change in the number of bosonic and fermionic zero modes must be identical, meaning W must remain fixed as long as SUSY is preserved. This prompts us to regard W as a topological invariant.

The relation (23) can be used to show that the zero modes of H_{SUSY} are given by the states that satisfy $Q|\xi\rangle = 0$ but cannot be written as $|\xi\rangle = Q|\chi\rangle$ for some state $|\chi\rangle \neq 0$ [43]. Clearly, since $Q^2 = 0$, any state of the form $|\rho\rangle \equiv Q|\chi\rangle$ is annihilated by Q. Positive energy eigenstates $|\xi\rangle$ of H_{SUSY} which are annihilated by Q can indeed be written as $|\xi\rangle = Q|\chi\rangle$, with $|\chi\rangle = \frac{1}{2E}Q^{\dagger}|\xi\rangle$.

The zero modes $|\rho\rangle$ of H_{SUSY} , on the other hand, are annihilated by Q (this follows from $\langle \rho | H_{\text{SUSY}} | \rho \rangle = \frac{1}{2} ||Q|\rho\rangle||^2 = 0$) but cannot be written as Q times some other state. This is so because if $|\rho\rangle = Q|\mu\rangle$, then the states $|\rho\rangle$ and $|\mu\rangle$ have the same energy E = 0 (as $[H_{\text{SUSY}}, Q] = 0$). However, any state of zero energy satisfies $Q|\mu\rangle = 0$, leading to a contradiction.

We refer to the space of solutions of $Q|\xi\rangle = 0$ as the Kernel of the operator Q, or Ker(Q). Additionally, the space of states which can be written as $|\xi\rangle = Q|\chi\rangle$ for some state $|\chi\rangle$ is referred to as the image of the operator Q, or Im(Q). Using these definitions, the number of zero modes is given by the dimension of the space spanned by states belonging to Ker(Q)but not to Im(Q), also referred to as the cohomology of the operator Q [43]:

$$N = (N_B + N_F)_{E=0} = \dim\left(\frac{\operatorname{Ker}Q}{\operatorname{Im}Q}\right).$$
 (25)

In our case, H_{SUSY} can be obtained by a coset decomposition of the Super-AKM algebra [41,46] of $SU(2)_k$, into its U(1) subalgebra, and the $\mathcal{N} = 2$ SUSY sector $SU(2)_k/U(1)$. The number of zero modes in this case is given by

$$N = k + 1, \tag{26}$$

and is also known as the dimension of the chiral ring of the SUSY system [45].

The above topological invariant W and the number of zero modes N are associated with the edge of the quantum Hall system but can be connected with properties of the bulk through the bulk-boundary correspondence. In particular, if we consider a bipartition in the quantum Hall system (Fig. 2), we will find that the entanglement Hamiltonian along the boundary of the bipartition is precisely given by the Hamiltonian of the low-energy chiral edge theory [47], which exhibits SUSY, and in particular displays W and N as characteristics of the spectrum. This prompts us



FIG. 2. Performing a cut in real space in the bulk of the quantum Hall state, here displayed on the surface of a cylinder, reveals the entanglement spectrum, given by the thermal density matrix of the chiral edge theory. This indicates that edge invariants W and N can be connected to characteristics of the bulk.

to expect that supersymmetry may arise directly from bulk properties.

IV. PHYSICAL PERTURBATIONS

Being a symmetry which does not occur commonly in condensed matter systems, it is natural to ask to which extent SUSY is robust. For $\nu = 1/3$, any perturbation within the low-energy chiral Luttinger liquid theory merely renormalizes the Fermi velocity. In this case, SUSY is indeed protected as long as coupling to other low-energy degrees of freedom can be neglected.

The particle-hole conjugate of the RR states in the excited Landau level, given by $v = 3 - \frac{k}{k+2}$ are prominent candidates for describing the plateaus observed at v = 5/2 and 12/5. In these cases, the edge is described by three co-propagating fermionic modes and one counterpropagating RR edge mode. Density-density interactions between the RR edge mode and the fermionic channels generally break SUSY. However, we note that the zero modes associated with the parafermionic sector remain unchanged, as the latter is charge neutral. Furthermore, if the SUSY breaking perturbations are weak, the overall shape of the spectrum should weakly deviate from the supersymmetric spectrum presented in Fig. 1.

V. DISCUSSION

In this paper, we have shown that quantum Hall states at filling $v = \frac{k}{k+2}$ constitutes a condensed matter realization of an $\mathcal{N} = 2$ supersymmetric conformal field theory. This allowed us to use the Witten topological index, defined specifically for supersymmetric theories, to count the difference between the bosonic and fermionic zero modes. We further discussed the stability of the above against perturbations expected to occur in a physical realization. Remarkably, SUSY was found to be a particularly robust property of the Laughlin state at filling v = 1/3.

General arguments dictate that the bipartite entanglement properties of the bulk of the system should also display the SUSY. Given this connection, we expect that SUSY invariants could be discovered directly from the bulk lowenergy topological field theory. The results presented here, and in particular the presence of SUSY on the edge, provide a strong indication that a supersymmetric bulk theory, such as the one presented in Ref. [48], is indeed adequate to study the low-energy physics of such quantum Hall states. One can in principle measure the predicted zero modes by creating a small circular edge, in which case the positive energy states acquire an energy $E \propto 1/L$, where L is the circumference of the edge. Thus, the zero modes become effectively isolated from the rest of the spectrum. If, in addition, coupling to the external edge is taken into account, the zero modes are generally expected to slightly deviate from zero energy and can therefore be distinguished. Such coupling terms decay exponentially as a function of the width of the system and are therefore much smaller than the intraedge separation of states. By measuring the energy spectrum in this case, one can in principle observe the k + 1 zero modes.

It is interesting to consider if any of the ideas discussed here can be generalized to other Abelian states. Given that the electron field in a v = 1/m theory has conformal dimension m/2, the trivial generalization of taking those fields to be the generators of supersymmetry clearly does not work. On the other hand, the mathematical structure of such Abelian states is similar, making it natural to expect the ideas presented here extend to those states as well.

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APPENDIX A: FROM THE HAMILTONIAN TO THE ENERGY-MOMENTUM TENSOR

In this section, we demonstrate how the Hamiltonian can be obtained from the corresponding energy-momentum tensor. As we will use the CFT formulation to demonstrate the existence of $\mathcal{N} = 2$ SUSY, the energy-momentum tensor will play a central role in our discussion.

If we impose periodic boundary conditions in the x direction (i.e., we take a circular edge), the coordinates τ and x define a cylinder. As is well known, one can then apply a conformal transformation that goes from the cylinder to a plane:

$$z = e^{2\pi(v\tau + ix)/L}$$

where L is the circumference of the edge. Time ordering on the physical cylinder corresponds to radial ordering on the transformed plane. While the resulting plane has no immediate physical significance, working on it greatly simplifies the analysis of conformal field theories.

Defining the stress-energy tensor T(z) on the plane, one can extract the Hamiltonian according to

$$H = \frac{2\pi v}{L} L_0 \quad \text{with} \quad L_0 = \frac{1}{2\pi i} \int_C dz z T(z),$$

where C is a circular contour going around the origin. The connection to the physical cylinder is seen by performing the

transformation from z to x (working in fixed time τ) and recalling that conformal fields transform according to

$$\phi(z) \to \phi(x) \left(\frac{\partial x}{\partial z}\right)^h.$$

For example, using the above, we will find that the supersymmetric Hamiltonian H_{SUSY} (11) obtained from the auxiliary fermion-boson system corresponds to the zero mode of the energy-momentum tensor (see also Sec. C)

$$T_{\text{SUSY}}(z) = \frac{1}{k+2} \left[-\frac{k}{2} (:\psi^{\dagger} \partial \psi : -: \partial \psi^{\dagger} \psi :) - 2\mathcal{J}^{3} : \psi^{\dagger} \psi : + \frac{1}{2} (:\mathcal{J}^{+}\mathcal{J}^{-} : +:\mathcal{J}^{-}\mathcal{J}^{+} :) \right] (z), \quad (A1)$$

where $\mathcal{J}^3, \mathcal{J}^{\pm}$ are the SU(2)_k currents defined in the next section. As we will show in the next sections, this theory exhibits an $\mathcal{N} = 2$ superconformal algebra.

APPENDIX B: OPERATOR PRODUCT EXPANSION (OPE)

In this section we list the OPEs necessary to derive the $\mathcal{N} = 2$ SUSY algebra. Using the OPEs of a free complex fermion and boson fields as building blocks, other relations can be obtained. In particular, we explicitly show how the OPEs of the SU(2) Affine Kac-Moody (AKM) algebra can be represented in terms of bosonic vertex operators.

1. Complex Fermion

The OPE of a complex fermion is given by

$$\psi^{\dagger}(z)\psi(w) = \frac{1}{z-w} + :\psi^{\dagger}\psi(w):$$
$$+ (z-w):\partial\psi^{\dagger}\psi(w): + \cdots, \quad (B1)$$

where the ellipsis represent higher powers of (z - w). The normal ordering of the field \mathcal{O} is represented as usual by the symbol : \mathcal{O} : and corresponds to the substraction of all the singular terms in the limit where the arguments coincide. For future reference, we also include here the OPE of Majorana fields, obtained from the complex fermion field by $\psi_1 = (\psi^{\dagger} + \psi)/\sqrt{2}$ and $\psi_2 = (\psi^{\dagger} - \psi)/\sqrt{2}i$. The OPE of a pair of Majorana fields takes the form

$$\psi_a(z)\psi_b(w) \sim \frac{\delta_{ab}}{z-w} + :\psi_a\psi_b(w): +(z-w):\partial\psi_a\psi_b(w):$$
(B2)

here the symbol \sim means that both sides are equal up to higher (positive) powers of (z - w).

An additional useful relation is

$$-: \psi_a \partial \psi_a(z): \psi_a(w) \sim \frac{\psi_a(w)}{(z-w)^2} + \frac{2\partial \psi_a(w)}{z-w} + \cdots$$
(B3)

2. Free Boson

The OPE between bosonic fields ϕ_a is given by $\partial \phi_a(z) \partial \phi_b(w) \sim -\delta_{ab}/(z-w)^2 + \cdots$. This corresponds to the familiar commutation relation of bosonic fields in the Luttinger liquid theory, expressed in the language of CFT [36].

Using this OPE and the Baker-Hausdorff-Campbell relation, the OPE between two vertex operators reads

$$e^{i\alpha\phi_a(z)}e^{i\beta\phi_b(\omega)} = \frac{1}{(z-w)^{-\alpha\beta}}e^{(i(\alpha\phi_a(z)+\beta\phi_a(w)))}\delta_{ab}.$$
 (B4)

Using the Wick theorem and the OPE between bosonic fields, it is also straightforward to show that

$$\partial \phi_a(z) e^{i\alpha\phi_b(w)} \sim -i\alpha \frac{e^{i\alpha\phi_a(w)}}{z-w} \delta_{ab} + \cdots$$
 (B5)

and

$$- \left(\partial\phi_a(z)\right)^2 e^{i\alpha\phi_b(w)} \sim \left(\alpha^2 \frac{e^{i\alpha\phi_a(w)}}{(z-w)^2} + \frac{\partial(e^{i\alpha\phi_a(w)})}{z-w}\right) \delta_{ab} + \cdots$$
(B6)

3. $SU(2)_k$ currents

The previous relations allow us to construct a representation of the SU(2) Affine Kac-Moody (AKM) algebra at level one. Defining $J_a^{\pm} = e^{i \pm \sqrt{2}\phi_a}$ and $J_a^3 = \frac{i}{\sqrt{2}}\partial\phi_a$ and using the relations between vertex operators of bosonic fields discussed above, we find

$$J_a^3(z)J_b^3(w) \sim \frac{1/2\delta_{ab}}{(z-w)^2} + \cdots,$$
 (B7)

$$J_{a}^{+}(z)J_{b}^{-}(w) \sim \frac{\delta_{ab}}{(z-w)^{2}} + \frac{2J_{a}^{3}(w)\delta_{ab}}{z-w} + \cdots, \quad (B8)$$

$$J_{a}^{3}(z)J_{b}^{\pm}(w) \sim \frac{\pm \delta_{ab}J_{a}^{\pm}(w)}{z-w} + \cdots$$
 (B9)

Defining $J^1 = (J^+ + J^-)/2$ and $J^2 = (J^+ - J^-)/2i$, the previous relations can be brought into the compact form

$$J_a^m(z)J_b^n(w) \sim \left(\frac{1/2}{(z-w)^2}\delta^{mn} + \frac{i\varepsilon^{mnl}J_a^l(w)}{z-w}\right)\delta_{ab} + \cdots,$$
(B10)

where the sum over repeated indices is assumed. The SU(2) indices (m,n,l) run from 1 to 3 and ε^{mnl} is the Levi-Civita antisymmetric tensor, which parametrizes the structure constants of SU(2).

Adding *k* different mutually commuting currents, we obtain a representation of the SU(2) AKM algebra at level *k*. To be specific, defining $\mathcal{J}^m = \sum_{a=1}^k J_a^m$, we find

$$\mathcal{J}^{m}(z)\mathcal{J}^{n}(w) \sim \frac{k/2}{(z-w)^{2}}\delta^{mn} + \frac{i\varepsilon^{mnl}\mathcal{J}^{l}(w)}{z-w} + :\mathcal{J}^{m}\mathcal{J}^{n}(w): + \frac{i}{2}\varepsilon^{mnl}\partial\mathcal{J}^{l}(w) + \cdots$$
(B11)

where we have included the first nonsingular term. In this representation of the SU(2)_k currents, the normal ordered product appearing above is $: \mathcal{J}^+\mathcal{J}^-(w) := \sum_{a \neq b} J_a^+ J_b^- - \sum_a (\partial \phi_a)^2$, and similarly for the other combinations.

For future use, it is useful to work in a basis of charge and neutral degrees of freedom. This can be done by defining $\phi_a = \frac{1}{\sqrt{k}} \varphi_{\rho} + \vec{d}_a \cdot \vec{\varphi}_{\sigma}$, with the \vec{d} vectors defined in Eq. (6) of the main text. In terms of these, the currents take the form

$$\mathcal{J}^{\pm} = \sum_{a=1}^{k} e^{\pm i(\sqrt{\frac{2}{k}}\varphi_{\rho} + \sqrt{2}\vec{d}_{a}\cdot\vec{\varphi}_{\sigma})} \quad \text{and} \quad \mathcal{J}^{3} = i\sqrt{\frac{k}{2}}\partial\varphi_{\rho}.$$
(B12)

When computing OPEs of a product of fields, it is generally important to keep terms that, although are nonsingular within a single OPE, multiplied by singular terms coming from other fields could still give a nontrivial (i.e., singular) contribution. An example of this occurs in the computation of the following OPE

$$- (: \psi^{\dagger}\psi : \mathcal{J}^{3}(z))([\psi\mathcal{J}^{+} + \psi^{\dagger}\mathcal{J}^{-}](w))$$

$$\sim \frac{\psi(z)\mathcal{J}^{+}(w) + \psi^{\dagger}(z)\mathcal{J}^{-}(w)}{(z-w)^{2}} + \frac{\psi(z)}{2}\partial\mathcal{J}^{+}(w)$$

$$+ \frac{\psi^{\dagger}(z)}{2}\partial\mathcal{J}^{-}(w) + \cdots$$
(B13)

$$\sim \frac{[\psi \mathcal{J}^+ + \psi^{\dagger} \mathcal{J}^-](w)}{(z-w)^2} + \frac{[(\partial \psi)\mathcal{J}^+ + (\partial \psi^{\dagger})\mathcal{J}^-](w)}{z-w} + \frac{1}{2} \frac{[\psi \partial \mathcal{J}^+ + \psi^{\dagger} \partial \mathcal{J}^-](w)}{z-w} + \cdots,$$
(B14)

where the top equation is obtained by using (B1) and (B11). The bottom result is obtained expanding the fields at z in Taylor series, i.e., $\psi(z) = \psi(w) + (z - w)\partial\psi(w) + \cdots$.

APPENDIX C: $\mathcal{N} = 2$ SUPERCONFORMAL ALGEBRA

As we discussed in the main text, to explicitly show the supersymmetric structure of the RR theory, we first study an auxiliary fermion-boson double layer, whose edge consists of a chiral fermion coupled to an $SU(2)_k$ CFT (which corresponds to a bosonic Read-Rezayi state). Using the OPEs introduced previously, we show that the neutral sector of this fermionic + bosonic theory satisfies an $\mathcal{N} = 2$ superconformal algebra. Finally, by bosonizing the fermion field, we show that the neutral part can be mapped to the fermionic RR edge CFT.

First, we identify the U(1) current associated with the total charge in the boson-fermion double layer:

$$J_{U(1)}(z) = \frac{1}{\sqrt{k+2}} (\mathcal{J}(z) + :\psi^{\dagger}\psi:(z)).$$
(C1)

The energy-momentum tensor associated with the total charge degrees of freedom, $T_{U(1)}$, is given by the OPE of $J_{U(1)}$ with itself. Clearly, the remaining part, T_{SUSY} , of the total energy-momentum tensor describes neutral degrees of freedom. It is defined such that

$$T_{fb} = T_{\psi} + T_{\rho}^{b} + T_{\mathbb{Z}_{k}} = T_{U(1)} + T_{SUSY}.$$
 (C2)

We therefore find that the energy-momentum tensor corresponding to the neutral sector of the auxiliary theory is given PHYSICAL REVIEW B 95, 205144 (2017)

by

$$T_{\text{SUSY}}(z) = \frac{1}{k+2} \bigg[-\frac{k}{2} (:\psi^{\dagger} \partial \psi : -: \partial \psi^{\dagger} \psi :) - 2\mathcal{J}^{3} :\psi^{\dagger} \psi :$$
$$+ \frac{1}{2} (:\mathcal{J}^{+}\mathcal{J}^{-} : +:\mathcal{J}^{-}\mathcal{J}^{+} :) \bigg] (z).$$
(C3)

We will find below that the OPE between $T_{U(1)}$ and T_{SUSY} is nonsingular.

Using the OPEs outlined in the previous section, it is possible to show that the energy-momentum tensor T_{SUSY} satisfies the full $\mathcal{N} = 2$ superconformal algebra [41], which reads

$$T(z)T(w) \sim \frac{c/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w},$$
 (C4)

$$T(z)G^{\alpha}(w) \sim \frac{\frac{3}{2}G^{\alpha}(w)}{(z-w)^2} + \frac{\partial G^{\alpha}(w)}{z-w},$$
(C5)

$$T(z)J(w) \sim \frac{J(w)}{(z-w)^2} + \frac{\partial J(w)}{z-w},$$
(C6)

$$J(z)G^{\alpha}(w) \sim i\epsilon^{\alpha\beta}\frac{G^{\beta}(w)}{z-w},$$
(C7)

$$J(z)J(w) \sim \frac{c/3}{(z-w)^2},$$
 (C8)

together with the OPE between the fermionic currents

$$G^{\alpha}(z)G^{\beta}(w) \sim \left[\frac{\frac{2}{3}c}{(z-w)^{3}} + \frac{2T(w)}{z-w}\right]\delta^{\alpha\beta} + i\left[\frac{2J(w)}{(z-w)^{2}} + \frac{\partial J(w)}{z-w}\right]\epsilon^{\alpha\beta}, \quad (C9)$$

where

$$G^{1}(z) = \frac{1}{\sqrt{k+2}}(\psi(z)\mathcal{J}^{+}(z) + \psi^{\dagger}(z)\mathcal{J}^{-}(z)),$$

$$G^{2}(z) = \frac{i}{\sqrt{k+2}}(\psi^{\dagger}(z)\mathcal{J}^{-}(z) - \psi(z)\mathcal{J}^{+}(z))$$

are the two fermionic currents and $J(z) = \frac{2}{k+2} [\mathcal{J}^3(z) - \frac{k}{2} \psi^{\dagger} \psi]$ is a U(1) current. The central charge here is c = 3k/(k+2). The tensor $\epsilon^{\alpha\beta}$ ($\alpha, \beta = 1, 2$) is antisymmetric with $\epsilon^{12} = 1$. In terms of the complex fermionic currents $\mathcal{G}^{\pm} = \frac{1}{\sqrt{2}} (G^1 \pm i G^2)$, the relation (C9) reads

$$\mathcal{G}^{-}(z)\mathcal{G}^{+}(w) \sim \frac{\frac{2}{3}c}{(z-w)^{3}} + \frac{2T(w)}{z-w} - \frac{2J(w)}{(z-w)^{2}} - \frac{\partial J(w)}{z-w},$$
(C10)

and $\mathcal{G}^+(z)\mathcal{G}^+(w) \sim \mathcal{G}^-(z)\mathcal{G}^-(w) \sim 0$. To illustrate the results above, we compute explicitly an example of the OPE between two fermionic currents (here we use the shorthand

notation $A(z) \equiv A_z$

$$G_{z}^{1}G_{w}^{1} \sim \frac{1}{k+2} [\psi_{z}\psi_{w}^{\dagger}\mathcal{J}_{z}^{+}\mathcal{J}_{w}^{-} + \psi_{z}^{\dagger}\psi_{w}\mathcal{J}_{z}^{-}\mathcal{J}_{w}^{+}] \sim \frac{1}{k+2} \left(\frac{1}{z-w} + :(\psi\psi^{\dagger})_{w}: +(z-w):(\partial\psi\psi^{\dagger})_{w}:\right) \\ \times \left(\frac{k}{(z-w)^{2}} + \frac{2\mathcal{J}_{w}^{3}}{z-w} + :(\mathcal{J}^{+}\mathcal{J}^{-})_{w}: +i\partial\mathcal{J}_{w}^{3}\right)$$
(C11)

$$+\frac{1}{k+2}\left(\frac{1}{z-w}+:\psi^{\dagger}\psi_{w}:+(z-w):\partial\psi^{\dagger}\psi_{w}:\right)\left(\frac{k}{(z-w)^{2}}-\frac{2\mathcal{J}_{w}^{3}}{z-w}+:\mathcal{J}^{-}\mathcal{J}_{w}^{+}:-i\partial\mathcal{J}_{w}^{3}\right).$$
 (C12)

Here we have used the OPE of $\psi^{\dagger}(z)\psi(w)$ given in (B1) and the one analogous for $\psi(z)\psi^{\dagger}(w)$. The OPE between two SU(2)_k currents is given by (B11). Rearranging and keeping the singular terms, we find

$$G^{1}(z)G^{1}(w) \sim \frac{\frac{2k}{k+2}}{(z-w)^{3}} + \frac{\frac{2}{k+2}}{z-w} \bigg[-\frac{k}{2} (:\psi^{\dagger}\partial\psi: -:\partial\psi^{\dagger}\psi:) - 2\mathcal{J}^{3}:\psi^{\dagger}\psi: +\frac{1}{2} (:\mathcal{J}^{+}\mathcal{J}^{-}:+:\mathcal{J}^{-}\mathcal{J}^{+}:) \bigg] (w),$$

where we recognize the first term on the right hand side of (C6) with $T = T_{SUSY}$ (C3). All the other relations (C4)–(C8) follow in a similar way.

1. An explicit analysis of the Laughlin state k = 1, v = 1/3

The simplest and most prominent example of a FQH edge theory endowed with $\mathcal{N} = 2$ SUSY is the $\nu = 1/3$ Laughlin state. This filling fraction is realized in the RR series by taking k = 1. In this case, the parafermion sector vanishes and the theory becomes Abelian. To illustrate the general results described above, we now explicitly demonstrate how they arise this simple case. In particular, we show that the SUSY algebra appears naturally in this case by making use of the vertex OPE.

The electron operator is given by $\psi_{el}^{\dagger} = e^{3i\varphi_{\rho}}$ and the fermionic current operator is given by $\mathcal{G}^{+} = \sqrt{\frac{2}{3}}\psi_{el}$. The OPE of two supercurrent operators is then

$$\mathcal{G}^{-}(z)\mathcal{G}^{+}(w) = \frac{2}{3}\psi_{\rm el}^{\dagger}(z)\psi_{\rm el}(w) = \frac{2/3}{(z-w)^3}e^{3i(\varphi_{\rho}(z)-\varphi_{\rho}(w))},$$

where in the last equality we have used that $\varphi_{\rho}(z)\varphi_{\rho}(w) \sim 1/3\ln(z-w)$. Expanding the difference in the exponent in Taylor series around w, we have

$$\mathcal{G}^{-}(z)\mathcal{G}^{+}(w) = \frac{2/3}{(z-w)^3} \exp\left(3i\sum_{n\geq 1} \frac{(z-w)^n}{n!} \partial^n \varphi_\rho\right)$$
$$= \frac{2/3}{(z-w)^3} \left(1 + 3i(z-w)\partial\varphi_\rho\right)$$
$$- \frac{9}{2}(z-w)^2(\partial\varphi_\rho)^2 + \frac{3i}{2}(z-w)^2\partial^2\varphi_\rho + \cdots\right)$$
(C13)

$$= \frac{2/3}{(z-w)^3} + \frac{2(i\partial\varphi_{\rho})}{(z-w)^2} + \frac{2(-\frac{3}{2}(\partial\varphi_{\rho})^2)}{(z-w)} + \frac{\partial(i\partial\varphi_{\rho})}{(z-w)} + \cdots$$
(C14)

which corresponds to the expression (C10) for c = 1, $T(z) = -\frac{3}{2}(\partial \varphi_{\rho})^2$, and $J(z) = -i \partial \varphi_{\rho}$. The different OPEs that give rise to the full superconformal algebra (C4)–(C8) can be obtained in a similar way.

2. Mode expansion

Using the previous OPEs, it is possible to find the (anti)commutation relations between the different modes of the fields. These modes are given in the Laurent expansion of the fields, i.e.,

$$T(z) = \sum_{n \in \mathbb{Z}} \frac{L_n}{z^{n+2}}, \quad L_m = \oint_0 \frac{dz}{2\pi i} z^{m+1} T(z)$$

$$G(z) = \sum_{n \in \mathbb{Z}} \frac{G_n}{z^{n+\frac{3}{2}}}, \quad G_m = \oint_0 \frac{dz}{2\pi i} z^{m+\frac{1}{2}} G(z)$$

$$J(z) = \sum_{n \in \mathbb{Z}} \frac{J_n}{z^{n+1}}, \quad J_m = \oint_0 \frac{dz}{2\pi i} z^m J(z).$$
 (C15)

The (anti)commutation relations between the different modes are (in the Ramond sector)

$$[L_{n}, L_{m}] = (n - m)L_{n+m} + \frac{c}{12}n(n^{2} - 1)\delta_{n+m}$$

$$[L_{n}, G_{m}] = \left(\frac{n}{2} - m\right)G_{n+m}$$

$$[L_{n}, J_{m}] = -mJ_{n+m}, [J_{n}, G_{m}^{\alpha}] = i\epsilon^{\alpha\beta}G_{n+m}^{\beta},$$

$$[J_{n}, J_{m}] = \frac{c}{3}n\delta_{m+n},$$
 (C16)

together with the commutation of the fermionic modes

$$\{G_n^{\alpha}, G_m^{\beta}\} = \delta^{\alpha\beta} \left(\frac{c}{3} \left(n^2 - \frac{1}{4}\right) \delta_{n+m} + 2L_{n+m}\right) + i\epsilon^{\alpha\beta} (n-m)J_{n+m}.$$
(C17)

The combinations $\mathcal{G}_n^{\pm} = \frac{1}{\sqrt{2}} (G_n^1 \pm i G_n^2)$ correspond to the electron operator, which is also the fermionic current, with anticommutation relations

$$\{\mathcal{G}_{n}^{+},\mathcal{G}_{m}^{-}\} = \frac{c}{3} \left(n^{2} - \frac{1}{4} \right) \delta_{n+m} + 2L_{n+m} + (n-m)J_{n+m},$$
(C18)

and $\{\mathcal{G}_n^+, \mathcal{G}_m^+\} = \{\mathcal{G}_n^-, \mathcal{G}_m^-\} = 0$. The SUSY charge Q, defined in the main text, corresponds to $\sqrt{\frac{2\pi v}{L}}\mathcal{G}_0^+$. Plugging this in, we find that

$$\frac{1}{2}\{Q,Q^{\dagger}\} = H_{\text{SUSY}}.$$
(C19)

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