Deformation-induced spin-orbit interaction in the Hubbard chain

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The effect of the electron-electron coupling of the Hubbard type on the one-dimensional electron system with the spin-orbit interaction (SOI), caused by the deformation of the elastic subsystem, is studied. As a rule, the electron-electron repulsion suppresses the deformation-induced SOI. In particular, at half-filling there is no lattice-induced SOI for the repulsive Hubbard chain. The attraction between electrons, instead, mostly enhances the deformation-induced SOI. The effect of the external magnetic field on such a SOI, caused by the deformation of the lattice, is considered.

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In recent years, systems where the spin-orbit interaction (SOI) plays a crucial role in low-dimensional electron systems, such as edge or surface states of topological insulators [1] or semiconductor nanowires [2], have attracted the attention of researchers. The SOI manifests itself as the effect of an external or internal electric field on a moving charged particle with spin. It is of special importance in spintronics [3] where the spin of electrons in electronic devices is manipulated and detected. In low dimensions, the semiconductor device structure may give rise to an internal electric field, and hence a SOI [4] of the Rashba or Dresselhaus type [5]. In one space dimension the effects of electron-electron interactions are very important because they can strongly change the properties of the system. For example, the Hubbard repulsion can cause a gap for charge excitations in the half-filled electron band (the Mott gap). Recently several quasi-one-dimensional correlated electron systems in which both the SOI and the Hubbard interaction play a crucial role, have been synthesized and studied, see, e.g., Ref. [6]. This is why it is important to investigate the effects of the SOI together with the interactions between electrons.

Among other low-dimensional electron systems, the ones where the SOI is connected with the properties of the lattice (e.g., the quantum spin Hall effect with the strain-induced SOI [7,8]) recently became a topic of intensive studies. For example, the SOI plays a major role in mechanically controlled spin transport [9]. The action of the shear components of the strain are equivalent to the action of the electric field [7,10], thus producing the SOI. The lattice-induced SOI can reveal itself in semiconductor systems [4,7], nanostructures [11], and ultracold atomic systems [12].

In the present paper the spontaneous onset of the SOI due to the coupling of one-dimensional electrons with the elastic subsystem is studied for the interacting electrons. The electron-electron interaction is considered in the on-site Hubbard form. Utilizing the exact Bethe ansatz solution we have shown that the Hubbard repulsion (attraction) between electrons can drastically change the deformation-induced SOI in the considered model. We have shown that the repulsion between electrons mostly reduces the lattice-induced SOI. For example, for the half-filled band the large repulsion between electrons destroys the lattice-induced SOI. Instead, the attraction can enhance the deformation-induced SOI. The influence of the external magnetic field directed along the SOI-distinguished direction has been studied.

To start with we study the behavior of electrons with the SOI on a one-dimensional lattice (for definiteness we consider the ring or the wire on the xy plane, hence only the z component of the spin enters the Hamiltonian),

$$\mathcal{H}_0 = -\sum_{j\sigma}[t'(\psi^\dagger_{j+1,\sigma}\psi_{j,\sigma} + \psi^\dagger_{j,\sigma}\psi_{j+1,\sigma}) + i\alpha\sigma(\psi^\dagger_{j+1,\sigma}\psi_{j,\sigma} - \psi^\dagger_{j,\sigma}\psi_{j+1,\sigma}) + \mu n_{j,\sigma}], \quad (1)$$

where $\psi^\dagger_{j,\sigma}$ creates an electron at site j with spin projection σ, t' is the hopping integral (related to the effective mass of electrons), α is the SOI parameter, caused by the deformation of the lattice, and μ is the chemical potential. The first two terms of the Hamiltonian can be combined into an effective complex hopping parameter [13],

$$\mathcal{H}_0 = -\sum_{j\sigma}[t\psi^\dagger_{j+1,\sigma}\psi_{j,\sigma}e^{i2\pi\sigma\varphi} + \text{H.c.}], \quad (2)$$

where $t' + i\alpha\sigma = t\exp(i2\pi\sigma\varphi)$ (we denote $t = \sqrt{t'^2 + \alpha^2/4}$) and the phase factor $\varphi = (1/\pi)\arctan(\alpha/2t')$ is caused by the SOI. A gauge transformation removes the phase factor from the Hamiltonian for the open chain or transfers it into a spin-dependent twisted boundary condition for the ring. Note that this gauge transformation can also be performed for the Hubbard chain with the SOI. The corresponding Hamiltonian reads $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\text{int}}$, where

$$\mathcal{H}_{\text{int}} = U\sum_j n_{j,\uparrow}n_{j,\downarrow} \quad (3)$$

represents the on-site interaction between electrons of strength U and $n_{j,\sigma} \equiv \psi^\dagger_{j,\sigma}\psi_{j,\sigma}$.

Our goal is to find the equilibrium value of the deformation-induced parameter of the spin-orbit coupling. To find it, we consider the balance between the gain of the energy of the electron subsystem modeled by the Hubbard chain due to the deformation-induced SOI $E = Le$, and the loss of the energy of the elastic subsystem due to that deformation, taken in the simplest approximation, caused by the nonzero deformation $LA\delta^2/2$ where the parameter A is related to the off-diagonal

(shear) components of the elastic modulus. Suppose for the deformation-induced SOI we have $\alpha = f\delta$, where f is the renormalization constant, related to the electron-lattice coupling (the gradient of the strain). Then in the equilibrium we have $A\delta + \partial e/\partial\delta = 0$, which is the condition for the equilibrium value of $\delta = \delta_s$. The deformation-induced SOI in the equilibrium state is $\alpha_s = f\delta_s$. Obviously for leading order in $1/L$ we have $(\partial e/\partial\delta) = (\partial e/\partial t)(f^2\delta/4t)$. Then there can exist the trivial solution $\delta_s = 0$, which describes the absence of the deformation-induced SOI, and nonzero solution(s) (which show the possibility of the spontaneous onset of the SOI due to the electron-lattice coupling), of the equation,

$$\frac{4At}{f^2} + \frac{\partial e}{\partial t} = 0. \tag{4}$$

The solutions depend on the values of A, f, on the hopping integral t', the Hubbard constant U, on the values of the chemical potential μ, the magnetic-field H (or the number of electrons and their magnetic moments), and the temperature. Summarizing, the deformation of the elastic subsystem can cause the SOI in the one-dimensional correlated electron system. To find the energy of the Hubbard chain we utilize the exact solution, see, e.g., Ref. [14].

The Hamiltonian $\mathcal{H}$ is diagonalized by Bethe's ansatz in terms of two sets of parameters $\{k_j\}$ and $\{\Lambda_\beta\}$, called rapidities, which satisfy the following Bethe ansatz equations [15,16] (the Bethe ansatz for the Hubbard model with twisted boundary conditions was introduced in Refs. [17,18]; the twist in the spin sector was first introduced in Ref. [17]):

$$\begin{aligned}\exp(ik_jL + i\pi\varphi) &= \prod_{\beta=1}^{M}\frac{\sin(k_j) - \Lambda_\beta - iu}{\sin(k_j) - \Lambda_\beta + iu},\\ &\prod_{j=1}^{N}\frac{\Lambda_\beta - \sin(k_j) - iu}{\Lambda_\beta - \sin(k_j) + iu}\\ &= -e^{i2\pi\varphi}\prod_{\gamma=1}^{M}\frac{\Lambda_\beta - \Lambda_\gamma - i2u}{\Lambda_\beta - \Lambda_\gamma + i2u}.\end{aligned} \tag{5}$$

Here $u \equiv U/4t$, L is the number of sites in the chain, N is the number of electrons, and M is the number of spin-down electrons. From now on we consider the most important sector with $0 \leqslant N \leqslant L$ and $M \leqslant N/2$. The generalization for other sectors can be performed straightforwardly. The energy of the eigenstate can be expressed as the function of rapidities as

$$E = -2t\sum_{j=1}^{N}\cos(k_j). \tag{6}$$

Note that the Bethe ansatz solution can be obtained only if the external magnetic field is aligned with the axis of the SOI [15]. For other directions of the magnetic field, the SOI together with the magnetic field destroy the exact integrability and cause a gap for low-energy states, which then yields an exponential dependence of the correlation functions. Equation (5) is written for the ring geometry. For the open chain it has similar structures with standing waves instead of plane waves for the ring, however the SOI-caused phase factor is trivially removed by a gauge transformation. Also, it is known that for the periodic boundary conditions the phase factors reveal themselves in the finite-size corrections (on the order of $1/L$) [14–16]. We do not consider those corrections in this paper.

First we concern ourselves with the ground state. Let us first concentrate on the ground-state behavior of the repulsive $U > 0$ Hubbard chain with the deformation-induced SOI. For leading order in $1/L$ (in the thermodynamic limit $L, N, M \to \infty$ with the ratios $n = N/L$ and $m = M/L$ fixed) the ground state of the repulsive Hubbard chain is described by the set of integral equations for the densities of rapidities or for the dressed energies of low-energy states, related to those rapidities [14]. In general for the values of $0 \leqslant n \leqslant 1$ and $m \leqslant n/2$ the ground state is organized by the filling of two Fermi seas for unbound electron states with real rapidities $k_{j=1}^{N}$ (such unbound electrons carry charges $-e$ and spins 1/2) and spinons with the rapidities $\lambda_{\beta=1}^{M}$ (spinons carry zero charges and spin 1/2). Let us denote by $\rho(k)$ $[\varepsilon(k)]$ the density of rapidities (the dressed energies) for unbound electron states and by $\sigma(\lambda)$ $[\phi(\lambda)]$ the density of rapidities (the dressed energies) for spinons. Then the ground-state behavior is described by the solution of the second-order Fredholm integral equations for densities [19],

$$\begin{aligned}\rho(k) &= \frac{1}{2\pi} + \cos(k)\int_{-Q}^{Q} d\lambda\, a_1[\sin(k) - \lambda]\sigma(\lambda),\\ \sigma(\lambda) &= \int_{-B}^{B} dk\, a_1[\sin(k) - \lambda]\rho(k)\\ &\quad - \int_{-Q}^{Q} d\lambda' a_2(\lambda' - \lambda)\sigma(\lambda'),\end{aligned} \tag{7}$$

where $a_j(x) = ju/\{\pi[x^2 + (ju)^2]\}$. The limits of integration B and Q are fixed via the conditions,

$$\begin{aligned}\int_{-B}^{B} dk\,\rho(k) &= n,\\ \int_{-Q}^{Q} d\lambda\,\sigma(\lambda) &= m \equiv \frac{n}{2} - M^z,\end{aligned} \tag{8}$$

where M^z is the magnetic moment per site. The dressed energies are determined from the following set of integral equations:

$$\begin{aligned}\epsilon(k) &= -2t\,\cos(k) - \mu - (H/2)\\ &\quad - \int_{-Q}^{Q} d\lambda\, a_1[\sin(k) - \lambda]\phi(\lambda),\\ \psi(\Lambda) &= H + \int_{-B}^{B} dk\,\cos(k) a_1[\sin(k) - \lambda]\epsilon(k)\\ &\quad - \int_{-Q}^{Q} d\lambda' a_2(\lambda' - \lambda)\phi(\lambda').\end{aligned} \tag{9}$$

The limits of integration are related to the values of the chemical potential and the magnetic field, μ and H via $\epsilon(\pm B) = 0$ and $\phi(\pm Q) = 0$. It is often useful to determine Fermi velocities for unbound electron states and spinons. They can be written as $v_c = \epsilon'(B)/2\pi\rho(B)$ and $v_s = \phi'(Q)/2\pi\sigma(Q)$ where primes denote the derivatives with respect to k and λ, respectively. The number of electrons in the framework of the grand canonical

ensemble can be controlled by the chemical potential μ. On the other hand, for the canonical ensemble, states are determined by the number (density) of particles n. The ground-state energy per site can be written as

$$e_0 = -\int_{-B}^{B} dk\, \rho(k)\left(\mu + \frac{H}{2} + 2t\, \cos(k)\right) + H\int_{-Q}^{Q} d\lambda\, \sigma(\lambda) = \frac{1}{2\pi}\int_{-B}^{B} dk\, \epsilon(k). \quad (10)$$

We are in position now to look for the value of the spontaneous onset of the deformation-induced SOI in the repulsive Hubbard chain.

For $U = 0$ (the noninteracting case) the Bethe ansatz equations describe periodic boundary conditions for free electrons. Formally, from the viewpoint of the written above integral equations, it corresponds to the case of $\lim_{u\to 0} a_j(x) \to \delta(x)$ with trivial solutions. Notice, however, that in the weak-coupling limit $u \ll 1$ for $\sin(B) < Q$, the solutions are discontinuous. Hence, in the noninteracting case one needs to solve integral equations first and only then take the limit $U \to 0$. It yields $\rho(k) = \pi^{-1}$ and $\epsilon(k) = -2\mu - 2t\, \cos\, k$ for $0 \leqslant |k| \leqslant \arcsin(Q)$, $\rho(k) = (2\pi)^{-1}$ and $\epsilon(k) = -\mu - (H/2) - 2t\, \cos(k)$ for $|k| > \arcsin(Q)$, $\sigma(\lambda) = [2\pi\, \cos(\arcsin\, \lambda)]^{-1}$ and $\phi(k) = -\mu - (H/2) - 2t\, \cos(k)$ for $0 \leqslant |\lambda| \leqslant \sin\, B$, and $\sigma(Q) = 0$ and $\phi(\lambda) = H$ otherwise. It follows that $n = \pi^{-1}B + m$ and $m = \pi^{-1}\arcsin\, \lambda_0$, i.e., $B = k_{F,\uparrow}$ and $\arcsin\, Q = k_{F,\downarrow}$. The ground-state energy per site is well known $e_0 = -(2t/\pi)X$, where

$$X = \sum_{\pm}\{\sin[\pi(n \pm 2M^z)] - \pi(n \pm 2M^z)\cos[\pi(n \pm 2M^z)]\}. \quad (11)$$

It is easy to obtain the nonzero equilibrium value for the SOI, caused by the deformation in the noninteracting electron chain,

$$\alpha_s = \sqrt{\frac{f^4X^2}{\pi^2A^2} - 4(t')^2}. \quad (12)$$

The formula obviously makes sense for the non-negative expression under the square root $|X| > 2\pi At'/f^2$. It is easy to check that this configuration has lower energy than the energy of the configuration with $\alpha_s = 0$. For the empty band $n = M^z = 0$ there is no deformation-induced SOI. In the grand canonical ensemble analytic results can be presented too, e.g., for zero magnetic-field $H = 0$. In that case the nonzero equilibrium value of the SOI is

$$\alpha_s = \sqrt{\frac{y}{2\pi^2A^2} - 4(t')^2}. \quad (13)$$

where $y = 4(f^4 \pm \sqrt{f^4 - 4\pi^2A^2\mu^2})$. Also, such an expression is valid only for $\mu \leqslant |f^2/2\pi A|$.

Let us turn to the case that is more important to us $U \neq 0$. In several limiting cases we can obtain analytic results.

Consider the situation of zero magnetic-field $H = 0$ in which $M^z = 0$. This case is related to $Q = \infty$. Then the ground state of the Hubbard chain with repulsion is described by the following integral equations:

$$\rho(k) = \frac{1}{2\pi} + \frac{\cos(k)}{2\pi}\int_{-B}^{B} dk' K[\sin(k) - \sin(k')]\rho(k'),$$
$$\epsilon(k) = -\mu - 2t\, \cos(k) + \frac{1}{2\pi}\int_{-B}^{B} dk' \cos(k')K[\sin(k) - \sin(k')]\epsilon(k'), \quad (14)$$

where the kernel is given by

$$K(x) = \int_0^{\infty} d\omega\, \cos(\omega x)\frac{2}{1 + \exp(-2u\omega)}. \quad (15)$$

At the half-band filling $n = 1$ we have $B = \pi$, and since the number of electrons is fixed, the chemical potential can be chosen to be zero $\mu = 0$. The charge excitations (unbound electron states) are gapped. The contribution from spinons is equivalent to the one of the spin-1/2 isotropic Heisenberg chain with the antiferromagnetic interaction. The behavior of spinons in this case can be described via the solution of the following integral equations:

$$\sigma(\lambda) = \int_{-\pi}^{\pi} dk\, a_1[\sin(k) - \lambda] - \frac{1}{2\pi}\int_{-Q}^{Q} d\lambda\, a_2(\lambda - \lambda')\sigma(\lambda'),$$
$$\phi(\lambda) = H - t\int_{-\pi}^{\pi} dk\, \cos^2(k)a_1[\sin(k) - \lambda] - \int_{-Q}^{Q} d\lambda' a_2(\lambda - \lambda')\phi(\lambda'). \quad (16)$$

The density and the dressed energy of the unbound electron states can be written as

$$\rho(k) = \frac{1}{2\pi} + \cos(k)\int_{-Q}^{Q} d\lambda\, a_1[\sin(k) - \lambda]\sigma(\lambda),$$
$$\epsilon(k) = -\frac{H}{2} - 2t\, \cos(k) + \int_{-Q}^{Q} d\lambda\, a_1[\sin(k) - \lambda]\phi(\lambda). \quad (17)$$

The density of the ground-state energy for the half-filled band is

$$e_{0h} = -\frac{U}{2} - 4t\int_0^{\infty} d\omega \frac{J_0(\omega)J_1(\omega)}{\omega[1 + \exp(2u\omega)]}, \quad (18)$$

where $J_n(x)$ are the Bessel functions.

For the small Hubbard repulsion $u \ll 1$ in zero magnetic-field $H = 0$ one gets [20]

$$e_0 \approx -\frac{4t}{\pi} - \frac{U}{4} - \frac{7\zeta(3)U^2}{16\pi^3 t} + \cdots, \quad (19)$$

where $\zeta(3) \approx 1.20$ is the Riemann ζ function. It implies the equilibrium value of the deformation-induced SOI,

$$\alpha_s = \sqrt{\frac{f^4}{\pi^2A^2} - 4(t')^2}. \quad (20)$$

It means that the weak Hubbard coupling in the main approximation yields a similar value of the equilibrium lattice-induced

SOI as in the noninteracting case. Analyzing other terms we obtain

$$\alpha_s = \sqrt{\frac{f^4 y_1^2}{\pi^2 A^2} - 4(t')^2}, \tag{21}$$

where $y_1 \approx 1 - 7\zeta(3)A^2U^2/64f^4$. We can conclude that the small Hubbard repulsion suppresses the deformation-induced SOI.

On the other hand, for the large Hubbard repulsion $u \gg 1$ the ground state is described by the following integral equations:

$$\begin{aligned}\sigma(\lambda) &= n a_1(\lambda)_{u=1} - \int_{-Q/u}^{Q/u} d\lambda' a_2(\lambda-\lambda')_{u=1}\sigma(\lambda'),\\ \phi(\lambda) &= H - \pi H_s a_1(\lambda)_{u=1}\\ &\quad - \int_{-Q/u}^{Q/u} d\lambda' a_2(\lambda-\lambda')_{u=1}\phi(\lambda'),\end{aligned} \tag{22}$$

where for the large u we have $H_s \approx (t/\pi u)[2\pi n - \sin(2\pi n)]$. The density and the dressed energy of unbound electron states can be written as ($k \leqslant B$)

$$\begin{aligned}\rho(k) &= \frac{1}{2\pi} + \frac{\cos(k)}{u}\int_{-Q/u}^{Q/u} d\lambda\, a_1(\lambda)_{u=1}\sigma(\lambda),\\ \epsilon(k) &= -2t[\cos(k) - \cos(B)].\end{aligned} \tag{23}$$

The ground-state energy per site in this case is equal to

$$e_0 = -\frac{2t}{\pi}[\sin(B) - 2B\,\cos(B)], \tag{24}$$

where B is determined from the equation,

$$n = \frac{B}{\pi} + \frac{2\,\sin(B)}{u}\int_{-Q/u}^{Q/u} a_1(\lambda)_{u=1}\sigma(\lambda). \tag{25}$$

Finally we get for the ground-state energy at the half-filling for $u \gg 1$ [21],

$$e_0 \approx -\frac{U}{2} - \frac{4t^2\ln 2}{U} - \frac{[6\zeta(3)t^4]}{U^3} + \cdots. \tag{26}$$

For leading order in the u^{-1} approximation, it yields $\alpha_s = 0$, i.e., at the half-filling in the zero magnetic field the large Hubbard repulsion destroys the lattice-induced SOI in the chain.

Now let us turn to the situation with almost half-band filling $n \sim 1$. In this case the ground-state energy can be written as

$$e_0 = e_{0h} - tC_1(1-n) - \frac{tC_2(1-n)^3}{3} + \cdots, \tag{27}$$

where

$$C_1 = 2 - 2u - 4\int_0^\infty d\omega \frac{J_1(\omega)}{\omega[1+\exp(2u\omega)]}, \tag{28}$$

and

$$\begin{aligned}C_2 = \frac{1}{4\pi^2}&\left(1 - \int_0^\infty d\omega \frac{J_0(\omega)}{[1+\exp(2u\omega)]}\right)\\ &\times\left(1 - 2\int_0^\infty d\omega \frac{\omega J_1(\omega)}{[1+\exp(2u\omega)]}\right).\end{aligned} \tag{29}$$

Considering the leading in the $(1-n)$ approximation we can see that the deviation from the half-filling of the band enhances the equilibrium deformation of the lattice and, thus, the lattice-induced SOI in the repulsive Hubbard chain, comparing to the half-filled band. In the framework of the grand canonical ensemble it is useful to connect the density of electrons with the number of electrons, yielding $1 - n \approx \sqrt{C_1 t - \mu}/\sqrt{C_2 t}$.

In the opposite case of the small filling of the band $n \ll 1$, the ground-state energy of the repulsive Hubbard chain is

$$e_0 = -\left(2t + \frac{U}{2}\right)n + \frac{\pi t n^3}{3} + \cdots. \tag{30}$$

For the leading order in n it implies the following value for the lattice-induced SOI:

$$\alpha_s = \sqrt{\frac{f^4 n^2}{A^2} - 4(t')^2}. \tag{31}$$

For the grand canonical ensemble one can use the connection between the band filling and the chemical potential $n \approx (\pi t)^{-1}\sqrt{\mu - 2t - (U/2)}$.

Consider now the case of the nonzero magnetic field. For $H > H_s$ where

$$H_s = \frac{8U}{\pi}\int_{-\pi n}^{\pi n} dk\,\cos(k)\frac{\cos(k) - \cos(\pi n)}{\sin^2(k) + u^2}, \tag{32}$$

the system is in the spin-polarized state with $\rho(k) = (2\pi)^{-1}$ and $B = \pi n$ (the Fermi sea for spinons is empty, i.e., $Q = 0$). The ground-state energy per site is $e_0 = -(2t/\pi)[\sin(\pi n) - \pi n\,\cos(\pi n)]$. We get for the equilibrium deformation-induced SOI,

$$\alpha_s = \sqrt{\frac{f^4[\sin(\pi n) - \pi n\,\cos(\pi n)]^2}{\pi^2 A^2} - 4(t')^2}. \tag{33}$$

For small band filling we obtain $\alpha_s = 0$, i.e., the strong magnetic field destroys the lattice-induced SOI in the repulsive Hubbard chain for this case. For small magnetic fields using the Wiener-Hopf method one gets $Q = (2u/\pi)\ln(H/aH_s)$, where $a = \pi^{3/2}/\sqrt{(2e)}$. On the other hand, for $H > H_s$ we have $Q = 0$ and $B = \pi n$, hence, getting the same situation as considered above. Near and below the spin-saturation point we get $Q = u[(H_s - H)/H_s]^{1/2}$. Hence, for $0 \leqslant H \leqslant H_s$, the lattice-induced SOI is decaying to zero at $H \to H_s$ with the growth of the field value H for small band fillings.

Now let us turn to the consideration of the on-site attraction between electrons. For the attraction $U < 0$ the ground state is organized by the filling of Fermi seas with $N - 2M$ unbound electron states with real k_j rapidities and M Cooper-like pairs with complex conjugated rapidities $\sin(k_\beta) = \Lambda_\beta \pm iu$. In the thermodynamic limit the ground state is given by the solution of the second-order Fredholm integral equations for the density of unbound electron states $\rho(k)$ and density of pairs $\sigma'(\Lambda)$,

$$\begin{aligned}\rho(k) &= \frac{1}{2\pi} - \cos(k)\int_{-Q}^{Q} d\Lambda a_1[\sin(k) - \Lambda]\sigma'(\Lambda),\\ \sigma'(\Lambda) &= \frac{1}{\pi}\mathrm{Re}[1 - (\Lambda - i|u|)^2]^{-1/2}\end{aligned}$$

$$- \int_{-Q}^{Q} d\Lambda' a_2(\Lambda' - \Lambda)\sigma'(\Lambda') - \int_{-B}^{B} dk\, a_1[\sin(k) - \Lambda]\rho(k). \quad (34)$$

Here we have to use the change $u \to |u|$ in the definition of $a_j(x)$. The limits of integration, which play the role of Fermi points for the Fermi seas B and Q, are determined from the following conditions:

$$n - 2m = \int_{-B}^{B} dk\, \rho(k), \quad m = \int_{-Q}^{Q} d\Lambda\, \sigma'(\Lambda). \quad (35)$$

We also can introduce the ground-state dressed energies for unbound electron states $\epsilon(k)$ and pairs $\psi(\Lambda)$, which are determined from the following set of integral equations:

$$\begin{aligned}\epsilon(k) &= -2t\, \cos(k) - \mu - H/2 - \int_{-Q}^{Q} d\Lambda\, a_1[\sin(k) - \Lambda]\psi(\Lambda),\\ \psi(\Lambda) &= -4t\, \mathrm{Re}[1 - (\Lambda - iu)^2]^{1/2} - 2\mu - \int_{-Q}^{Q} d\Lambda' a_2(\Lambda' - \Lambda)\psi(\Lambda') - \int_{-B}^{B} dk\, \cos(k) a_1[\sin(k) - \Lambda]\epsilon(k). \end{aligned} \quad (36)$$

The ground-state energy per site can be obtained as

$$e_0 = -2t \int_{-B}^{B} dk\, \cos(k)\rho(k) - 4t \int_{-Q}^{Q} d\Lambda\, \mathrm{Re}[1 - (\Lambda - iu)^2]^{1/2}\sigma'(\Lambda)\,. \quad (37)$$

For external magnetic-field $H < H_c$ only paired states are populated and have a Fermi sea (i.e., $B = 0$) where

$$H_c = -4t - 2\mu - 2\int_{-Q}^{Q} d\Lambda\, a_1(\Lambda)\psi(\Lambda). \quad (38)$$

There is also a saturation field H_s so that for $H > H_s$ the ground state consists of only spin-polarized unpaired particles and the system has a magnetization of $M^z = n/2$.

For the most important case of $H < H_c$ we have $B = 0$ and $M^z = 0$. The ground-state energy per site of the Hubbard chain with attraction between the electrons for the small band filling $n \ll 1$ is equal to

$$e_0 = -4tn\sqrt{1 + u^2}. \quad (39)$$

Then the equilibrium deformation-induced SOI is

$$\alpha_s = 2\sqrt{\frac{f^4 n^2(1 + u^2)}{A^2} - (t')^2}. \quad (40)$$

Clearly, the attractive Hubbard interaction in this case enhances the lattice-induced SOI. In the grand canonical ensemble we can use the connection between the chemical potential and the filling n,

$$\frac{\sqrt{(1 + u^2)}\mu}{2t} = -1 - \frac{\pi^2 n^2(1 - 2u^2)}{8(1 + u^2)}. \quad (41)$$

For $H > H_s$ we have $Q = 0$ and $m = 0$. All electrons are unbound with the density $\rho = (2\pi)^{-1}$. Then the ground-state energy is $e_0 = -2t\, \sin(\pi n)/\pi$, and the equilibrium lattice-induced SOI is

$$\alpha_s = 2\sqrt{\frac{4f^4 \sin^2(\pi n)}{\pi^2 A^2} - (t')^2}. \quad (42)$$

Notice that, for the Hubbard chain with attraction between electrons, the spin saturation does not destroy the deformation-induced SOI as for the repulsive case. For the grand canonical ensemble we can use the connection between the band filling and the chemical potential $n = 1 - \pi^{-1} \arccos[(2\mu + H)/2t]$.

For nonzero temperatures we have to minimize the free energy instead of the ground-state one for $T = 0$. It is not difficult to show using the Bethe ansatz solution [14] that, at high temperatures of $T \gg t$, only the trivial equilibrium configuration with $\delta_s = 0$ exists. At low temperatures the free energy per site can be approximated by $f_{\mathrm{low}} \approx e_0 - \gamma T^2/2 + \cdots$, where γ is the Sommerfeld coefficient, which can be calculated in the case of two Fermi seas as $\gamma = (\pi/3)[v_c^{-1} + v_s^{-1}]$ for the repulsive Hubbard chain and $\gamma = (\pi/3)[v_c^{-1} + v_p^{-1}]$, where v_p is the Fermi velocity of pairs $v_p = \psi'(Q)/2\pi\sigma'(Q)$. If one of the Fermi seas is absent (for some ranges of the values of the band fillings/chemical potential or the magnetization/magnetic field), the contribution to the free energy from the related eigenstates becomes exponentially small. [At the critical points of quantum phase transitions the contribution is proportional to $T^{-1/2}$.] In a zero magnetic field at half-filling for the repulsive Hubbard chain and for the attractive Hubbard chain for $H < H_c$ we have $v_c = 0$ and $v_s = v_p = 2tI_1(2\pi/|u|)/I_0(2\pi/|u|)$. Minimization of the free energy yields the condition for the equilibrium deformation and, hence, for the deformation-induced SOI at low temperatures. We can see that at low temperatures a nonlinear (e.g., the cubic) equation for the equilibrium deformation-induced SOI can appear. The analysis shows that the phase transition to the low-temperature phase with the spontaneous SOI, induced by the deformation of the lattice, can take place in the attractive Hubbard chain at $H < H_c$ and for the repulsive Hubbard chain for the non-half-filled case.

Our results are generic for one-dimensional fermion systems with the deformation-induced SOI. In particular, in the continuum limit (where one has the gas of fermions with the δ-function repulsion/attraction instead of the Hubbard chain) our results remain essentially the same (for the gapless phases because the phase with the Mott gap is absent in the continuum limit).

To summarize, we have studied the spontaneous onset of the spin-orbit interaction due to the deformation of the lattice in the correlated electron chain. We have shown that the on-site Hubbard repulsion between electrons mostly suppresses the deformation-induced SOI in the ground state. On the other hand, the attraction between electrons mostly enhances the lattice-induced SOI. The external magnetic field in the repulsive Hubbard chain also suppresses the lattice-caused SOI, whereas it does not destroy the deformation-induced SOI even in the spin-saturated phase for the attraction

between electrons. The analysis of thermodynamics implies the presence of a phase transition between the nonzero value of the deformation-induced SOI at low temperatures and the zero SOI at high temperatures. The predicted effect of the electron-electron interaction on the deformation-induced SOI can be very important, e.g., in the application of quantum nanowires with mechanically controlled spin transport in spintronics.

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