Fermi-edge singularity and related interaction induced phenomena in multilevel quantum dots

A. S. Goremykina and E. V. Sukhorukov
Département de Physique Théorique, Université de Genève, CH-1211 Genève 4, Switzerland
(Received 11 November 2016; published 12 April 2017)

We study the manifestation of the nonperturbative effects of interaction in sequential tunneling between a quasi-one-dimensional system of chiral quantum Hall edge channels and a multilevel quantum dot (QD). We use the formal scattering theory approach to the bosonization technique to present an alternative derivation of the Fermi-edge singularity effect and demonstrate the origin of its universality. This approach allows us to address, within the same framework, plasmon-assisted sequential tunneling to relatively large dots and investigate the interaction-induced level broadening. The results are generalized by taking into account the dispersion in the spectrum of plasmons in the QD. We then discuss their modification in the presence of neutral modes, which can be realized either in a QD with two chiral strongly interacting edge channels or in a three-dimensional QD in the Coulomb blockade regime. In the former case a universal behavior of the tunneling rate is discovered.

DOI: 10.1103/PhysRevB.95.155419

I. INTRODUCTION

The Fermi-edge singularity (FES), originally discovered [1] in the x-ray absorption spectra of metals, describes a divergence in the transition rate at low energies, which has a power-law dependence. There are two contributions to it, described by Mahan [2] and Anderson [3]. On one hand, the interaction between an electron and a hole in the final states increases the rate; on the other hand, the orthogonality catastrophe leads to its suppression. These results were confirmed [4] and the exact solution for the strong interaction case was provided. Since then this phenomenon has been extensively studied both experimentally [5,6] and theoretically [7–11] in various configurations.

Notably, the exact solution [4] demonstrates the universality of FES exponents and the nonperturbative character of this effect. Such bright manifestations of interactions also occur in one-dimensional (1D) Fermi systems, known as the Luttinger liquids [12]. Among experimentally accessible configurations the quantum Hall (QH) effect systems deserve special attention. In this regime, the edge states of the two-dimensional electron gases can be viewed as chiral 1D channels, whose direction of propagation is defined by the sign of the magnetic field. A convenient method to describe these states is to use the bosonization technique [13], which allows one to address the interactions nonperturbatively. Interestingly, this approach enables us to find the FES power law in a QH system, endowing it with a clear physical meaning. Namely, its manifestation was studied in tunneling to a single-level quantum dot (QD) [14], surrounded by a number of edge channels (see Fig. 1). In the low-energy limit, the tunneling rate dependence on the bias between the QD and the mth channel, where the electron tunnels from, acquires a form

$$\Gamma \propto \Delta\mu^{\alpha}, \quad \alpha = 2q_m + \sum q_n^2, \tag{1}$$

where $q_n < 0$ denotes the charge induced in the nth channel. In this paper we would like to complete this picture, by considering the tunneling rate at biases that reveal the structure of the energy levels of the QD.

To describe a QH system we apply the bosonization approach that provides means of depicting its physics in terms of the scattering bosons [15]. Being widely used [16–19] for its clarity and relative simplicity, this formalism remains relatively new. This novelty enables us to look at the FES from another point of view by studying it in the tunneling to the dot in the QH regime—an object extensively explored [20]. The benefit of such an approach is that in the low-energy limit the effect is universal and the theory can also be applied to a 3D QD. Therefore, we develop a formalism of the scattering theory of bosons in Sec. II and demonstrate its potential in the application to the FES phenomenon. On the other hand, we explain that the theory has a clear physical meaning. Namely, the scattering states, constituting the basis for the boson fields, can be expressed in terms of certain positive charges in the low-energy limit. Surprisingly, these turn out to be the charges (with a negative sign) induced in the channels around the QD when it is charged. Hence, the scattering problem of bosons becomes deeply connected to the electrostatic one, which explains the universality of the FES phenomenon. Then, in Sec. III, we go beyond the low-energy limit for the QD in QH regime, so that the scattering of the incoming wave in the channel leads to an excitation of collective modes in the QD.

The nature of these excitations is fully governed by two parameters: the coupling constant $\sigma = \sum_n q_n^2$ and the dispersion of plasmons in the dot. We first concentrate on the no-dispersion case. Then, if there is no interaction, i.e., $\sigma = 0$, the tunneling rate behaves as a set of steps as a function of the bias. The steps correspond to the free-fermion energy levels in the QD, implying that the bosonic and the fermionic pictures describe the same entity. If the interaction is “turned on,” the steps become smeared, due to the finite width that the energy levels acquire. The width is proportional to the coupling σ, but it also grows quadratically with the number of the energy level. Thus, even if the interaction is small, it leads to a nonperturbative effect, which we are able to describe analytically due to the correspondence between free-fermion levels and boson resonances in this geometry and the chirality of the boson fields. When there is interaction in the QD, the spectrum of plasmons acquires in general a weak dispersion. We then demonstrate splitting of the fermion levels (starting from the third one). It originates from the shift between different single- and multiplasmon processes corresponding to the excitation of a particular fermion level. The effect is

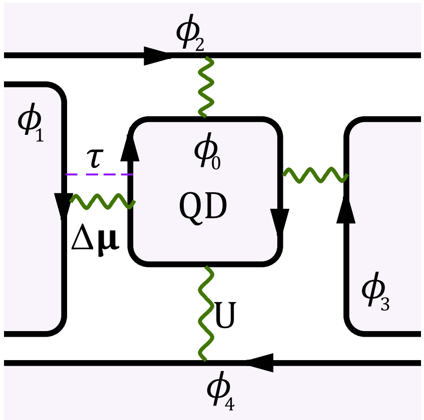

FIG. 1. A scheme of one of the possible system setups. A quantum dot in the QH regime, described by the bosonic field ϕ_0, interacts with $N = 4$ QH edge channels at the filling factor $\nu = 2$. The dashed line corresponds to tunneling, while the wavy lines depict Coulomb interactions. The rate of tunneling from one of the channels is studied as a function of the bias $\Delta\mu$ between the channel and the dot. Note that tunneling between copropagating QH edge channels is suppressed [21,22]; hence only the counterpropagating ones are considered. However, besides that, there is no formal difference between these two cases.

mostly pronounced for the third level, on which we dwell in detail.

The following discussion returns to QDs with a linear spectrum for plasmons. In Sec. IV, we study the tunneling rate to a QD with two chiral edge channels with strong long-range interaction. These can be decoupled into charged and neutral modes. By a proper choice of the bias only the lowest energy level of the charged mode can be excited. However, the heights of the steps stop being equal and acquire a universal structure which is a consequence of the strong interaction between the channels, leading to the charge fractionalization.

The previous case of a QD in the QH regime immediately suggests that somewhat similar effects might be seen in the rate of tunneling to a 3D QD in the situation considered in Sec. V. Indeed, this time there is again a separation between the charged and neutral modes, where by the latter we imply the energy levels of dimensional quantization. Using the formalism from Sec. II, we analyze this situation and show the result to resemble the one of Sec. IV. The difference is, however, in the fact that there is a direct coupling to the neutral modes so that the heights and the positions of the steps are arbitrary.

II. FERMI-EDGE SINGULARITY FROM SCATTERING THEORY

Let us consider a QD of the characteristic size L in the quantum Hall regime with the filling factor $\nu = 1$ interacting with N edge channels of a QH system at an integer filling factor, which we depict in Fig. 1. Such a complex can be realized in a 2D electron gas, where using electrostatic gates and forming quantum point contacts one can create potential barriers, allowing some of the edge channels to pass while others reflect [23]. To account for strong effects of interaction we first use the bosonization technique [13], which we recall briefly below. Next we describe the essence of the scattering theory for bosons, which is then applied to demonstrate the manifestation of the FES. The Coulomb interaction might be screened in a complicated way; therefore we do not assume any particular form of density-density interactions. We only require that the size L of the QD be much smaller than the wavelength of the density fluctuations in the edge channels (plasmons), thus taking into account the low-energy character of the FES. Note that we work in units in which $\hbar = c = e = 1$. To bosonize the electrons in the QD, we consider its edge as a one-dimensional channel with the glued ends, i.e., forming a ring. We express the electron operators in the channels, $\psi_n(x)$, and in the ring, $\psi_0(x)$, with the help of the boson fields $\phi_n(x)$ and $\phi_0(x)$:

$$\psi_n(x) \propto e^{i\phi_n(x)}, \quad n = 0,\dots,N, \tag{2}$$

where the field $\phi_n(x)$ is related to the charge density operator $\rho_n(x) = \frac{1}{2\pi}\partial_x\phi_n(x)$. The commutator of bosonic fields $[\phi_n(x),\phi_m(y)] = i\pi\delta_{nm}\,\mathrm{sgn}(x-y)$ together with the above definition guarantees the fermion commutation relations and the electron charge $[\psi_n(x),\rho_m(y)] = \psi_n(x)\delta(x-y)\delta_{mn}$.

Next, we write down the Hamiltonian of the interacting fermions in terms of the new bosonic fields:

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{int} + \mathcal{H}_t, \tag{3}$$

where the free Hamiltonian is given by

$$\mathcal{H}_0 = \frac{1}{4\pi}\sum_{n=0,\dots,N} v_n \int dx\{\partial_x\phi_n(x)\}^2. \tag{4}$$

We include the interaction part

$$\mathcal{H}_{int} = \frac{1}{8\pi^2}\sum_{nn'}\iint dxdy U_{nn'}(x,y)\partial_x\phi_n(x)\partial_y\phi_{n'}(y) \tag{5}$$

with arbitrary electrostatic potentials $U_{nn'}$. Finally, the last term describes tunneling between the mth channel and the QD at some point x_0:

$$\mathcal{H}_t = \tau(A + A^\dagger), \quad A = e^{i\{\phi_0(x_0)-\phi_m(x_0)\}}. \tag{6}$$

However, the exact position x_0 is of no interest because of the long-wavelength limit, allowing us to consider the bosonic field in a certain interaction region being independent of the coordinate.

Next, let us calculate the tunneling rate from one of the channels, say the mth one, to the QD when a bias $\Delta\mu$ is applied between them [24]. Notably, the change of the electro-chemical potential at the channel shifts the dot level due to the electrostatic interaction. The value $\Delta\mu$ takes this effect into account and will be written down explicitly later. Addressing the tunneling term as a perturbation, one can express the tunneling rate as the integral

$$\Gamma = |\tau|^2\int_{-\infty}^{\infty} dt\langle A(0)A^\dagger(t)\rangle. \tag{7}$$

Note that such an approach enables us to use the fact that the remaining part of the Hamiltonian, $H_0 + H_{int}$, has a quadratic form in the bosonic fields. Then the tunneling rate can be expressed in terms of their two-point correlators. The bosonic fields can be found from the equations of motion $\partial_t\phi_n(x) = i[\mathcal{H}_0 + \mathcal{H}_{int},\phi_n(x)], n = 0,\ldots,N$, that reveal

$$\partial_t\phi_n(t) + v_n\partial_x\phi_n(x) + \frac{1}{2\pi}\sum_{n'=0,\ldots,N}\int dy U_{nn'}(x,y)\partial_y\phi_{n'}(y) = 0. \quad (8)$$

The solution may be presented in the form

$$\phi_n(x,t) = \varphi_n(x,t) + \delta\phi_n(x,t), \quad (9)$$

where the zero mode $\varphi_n(x,t)$ and the fluctuating part $\delta\phi_n(x,t)$ read

$$\varphi_n(x,t) = -\mu_n t + \varphi_n^{(0)}(x), \quad (10)$$

$$\delta\phi_n(x,t) = \int_0^\infty \frac{d\omega}{\sqrt{\omega}}\sum_{n'=1}^{N}[\Phi_{n'n\omega}(x)e^{-i\omega t}a_{n'}(\omega) + \text{H.c.}]. \quad (11)$$

Basically, the zero modes solve the system (8) in the zero-frequency limit and satisfy the following equations:

$$\mu_n = v_n\partial_x\varphi_n^{(0)}(x) + \frac{1}{2\pi}\sum_{n'}\int dy\partial_y\varphi_{n'}^{(0)}(y)U_{nn'}(x,y). \quad (12)$$

Thus, zero modes describe stationary charge densities and corresponding phase shifts, while the deviations are taken into account by the fluctuating part. We expressed the latter in the second-quantized form in the basis of the scattering states $\Phi_{n'n\omega}(x)$ with the creation and annihilation operators $a_{n'}^\dagger(\omega)$, $a_{n'}(\omega)$ satisfying the usual bosonic commutation relations. The scattering states diagonalize the Hamiltonian $H_0 + H_{int}$ and satisfy specific boundary conditions. Namely, the scattering state $\Phi_{n'n\omega}(x)$ is described by an incoming plane wave in the channel n' which then scatters into all the other channels $n = 1,\ldots,N$. Thus, the scattering state presents the set of $N+1$ functions, enumerated by the second index, while the first index enumerates the scattering states.

To find correlation functions entering the expression (7) for the tunneling rate, we exploit the low-energy limit and perform the perturbation expansion of the scattering states in vicinity of the QD in frequency:

$$\Phi_{nl\omega}(x) = \Phi_{nl}^{(0)}(x) + i\omega\Phi_{nl}^{(1)}(x). \quad (13)$$

This means that we look at the asymptotic behavior of the scattering states in the region $x \sim L$, for which our approximation $\omega L/v \ll 1$ is valid. On the other hand, such an expansion may be understood as a way to describe how strong the scattering is. Specifically, the parameter $\omega L/v$ being small implies that almost the whole incident wave gets transmitted. Substituting now the expression (13) into (11) and into (8), we arrive at the system

$$0 = v_n\partial_x\Phi_{nl}^{(0)}(x) + \frac{1}{2\pi}\sum_{n'}\int dy\partial_y\Phi_{nn'}^{(0)}(y)U_{ln'}(x,y), \quad (14)$$

$$\Phi_{nl}^{(0)}(x) = v_n\partial_x\Phi_{nl}^{(1)}(x) + \frac{1}{2\pi}\sum_{n'}\int dy\partial_y\Phi_{nn'}^{(1)}(y)U_{ln'}(x,y). \quad (15)$$

Obviously, $\Phi_{nl}^{(0)}(x) = \text{constant}$ are the solutions of (14). Particularly, for our scattering problem

$$\Phi_{nl}^{(0)}(x) = \delta_{nl}, \quad n,l = 1,\ldots,N, \quad (16)$$

$$\Phi_{n0}^{(0)}(x) = \varepsilon_0, \quad (17)$$

where ε_0 needs to be defined.

To clarify the physical meaning of $\Phi_{nm}^{(0)}$, we note that substituting (16) and (17) into (15) brings us to the same kind of electrostatic equations that identify the zero modes (10). So we may treat the problem of finding the coefficients $\Phi_{n0}^{(0)}(x)$ as an electrostatic one and formally write its solution

$$q_n = \sum_{nn'} C_{nn'}\mu_{n'}, \quad (18)$$

where $q_n = \frac{1}{2\pi}\int dx\partial_x\varphi_n^{(0)}(x)$, $n = 0,1,\ldots,N$, are the charges in the channels and at the dot. The particular form of the interaction is of no interest and is generally described by a capacitance matrix $C_{nn'}$. Taking into account (16) and (17), and assuming that all channels are grounded, except for the nth one, leads to

$$q_n = C_{nn} + C_{n0}\varepsilon_0. \quad (19)$$

As there is no "charge" at the dot $q_0 = 0$, it is easy to define its "potential" [25]

$$\varepsilon_0 = -C_{n0}/C_{00}. \quad (20)$$

On the other hand, if one poses a question on how the QD with a unit charge gets screened by grounded channels, from $q_n = C_{n0}\mu_0$ and $q_0 = 1$ one gets

$$q_n = C_{n0}/C_{00}. \quad (21)$$

We then conclude that the "potential" ε_0, induced in the dot in the particular situation where the plane wave is incident in the channel n, is the same up to a sign as the charge induced in this channel in the setup when the dot is charged:

$$\varepsilon_0 = -q_n, \quad q_n < 0. \quad (22)$$

Thus, in the low-energy limit the scattering and the electrostatic problems are simply connected, which reflects the universality of the FES phenomenon. Finally, as mentioned above, the interaction between the biased channel m and the QD raises the dot's energy level by $\mu_0 = -\mu_m C_{m0}/C_{00}$ [compare to Eq. (20)], so we denote the difference in their potentials as $\Delta\mu = \mu_m - \mu_0$.

We now move to the calculation of the tunneling rate (7):

$$\Gamma \propto \int_{-\infty}^{\infty} dt\,\exp\{-i\Delta\mu t + \mathcal{K}_1(t) + \mathcal{K}_2(t)\}, \quad (23)$$

where we introduced the autocorrelator $\mathcal{K}_1(t)$ and the cross correlator $\mathcal{K}_2(t)$ of the bosonic fields:

$$\mathcal{K}_1(t) = -\sum_{n=0,1}\langle[\delta\phi_n(t) - \delta\phi_n(0)]\delta\phi_n(t)\rangle, \quad (24)$$

$$\mathcal{K}_2(t) = \langle\delta\phi_n(0)[\delta\phi_0(0) - \delta\phi_0(t)]\rangle - \langle[\delta\phi_0(0) - \delta\phi_0(t)]\delta\phi_n(t)\rangle. \quad (25)$$

To find them we use the spectral decomposition (11) of the fields in the channels and in the dot. Thereby, we arrive at the following result for the correlators:

$$\mathcal{K}_1(t) = -\left(1+\sum_{n=1}^{N} q_j^2\right)\int_0^\infty \frac{d\omega}{\omega}(1-e^{i\omega t}),$$
$$\mathcal{K}_2(t) = -2q_m\int_0^\infty \frac{d\omega}{\omega}(1-e^{i\omega t}).$$

Introducing a cutoff δ^{-1} at high energies, we calculate the above integral $\int_0^\infty \frac{d\omega}{\omega}(1-e^{i\omega t})e^{-\delta\omega} = \ln(\frac{\delta-it}{\delta})$. Finally, the formula (23) leads to the following outcome for the tunneling rate from the mth channel to the dot:

$$\Gamma \propto \int_{-\infty}^{\infty} dt \frac{e^{i\Delta\mu t}}{(\delta+it)^{1+\alpha}} = \frac{2\pi\theta(\Delta\mu)}{\Gamma(1+\alpha)}\Delta\mu^\alpha, \tag{26}$$
$$\alpha = 2q_m + \sum_{n=1}^{N} q_n^2, \quad q_m < 0. \tag{27}$$

Hence, we reproduce the FES power law for a quantum dot in the QH regime in the long-wavelength regime. However, this also implies that the same is true for any QD, since the FES has an electrostatic nature. Indeed, in terms of the scattering theory language we realize that it is the excitation of the lowest energy mode of the bosonic field $\delta\phi_0$ [i.e., of the scattering states $\Phi_{0n\omega}(x,\omega\to 0) = -q_n$] and of a zero mode that leads to appearance of the FES. In other words, the low-energy excitation of a charged mode defines the effect. We will eventually return to this discussion in Sec. V.

Thus, we have also demonstrated the potential of the used approach which will allow us to move to a significantly more complicated system, where we already do not confine ourselves by the low-energy limit.

To complete the formal description of the formalism, we make some comments on the role of the interaction between the copropagating channels before they get split close to the dot. The tunneling rate is determined by the local correlators and, in fact, it can be shown that the upstream interaction does not influence the local correlators in the vicinity of the QD. Namely, rewriting the boson fields in terms of the scattering states coming from the interacting region, one might see that the local correlators are the same as in the case with no interaction in the region upstream from the QD at all.

III. APPLICATION TO THE COLLECTIVE MODE ASSISTED TUNNELING

It has been pointed out in the previous section that the FES physics is universal in the low-energy limit. This is reflected in the particular power-law form of the tunneling rate as a function of the bias where the exponent is only defined by the charges induced in the channels around the dot. In this and following sections, we would like to study tunneling to a QD in a QH regime beyond the low-energy limit, meaning that the higher energy excitations of a charged mode will be considered. We will also answer the question of whether the FES effect "survives" in the tunneling to higher energy levels. We continue to work within the same framework and setup, but now assume that the characteristic size L of the dot can be larger than the wavelength of excitations λ, which in turn is much larger than the characteristic size W of the interaction region, implying relatively large QDs: $L \gg W$. This allows us to consider tunneling to excited states in a QD, nevertheless expecting the same level of universality as in the FES effect.

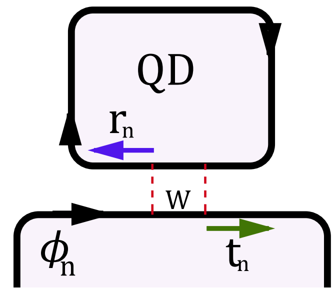

FIG. 2. Sketch of the scattering process between the QD and the nth channel. The arrows with the corresponding scattering coefficients r_n and t_n schematically represent the direction of a boson mode propagation.

In this case, the local interaction of the QD with each channel can be described by introducing the scattering coefficients r_n and t_n for the reflection and the transmission of the plasmons, respectively. We then derive the scattering states $\Phi_{m0\omega}$ at the dot explicitly. The procedure is as follows. Recall that the bosonic fields in the channels in the proximity of the dot are approximated by (11), (13), and (16) for $W \ll \lambda$, so the field in the mth channel has the following form:

$$\delta\phi_m(x,t) = \int_0^\infty \frac{d\omega}{\sqrt{\omega}}[e^{-i\omega t}a_m(\omega) + \text{H.c.}]$$
$$\equiv \int_0^\infty \frac{d\omega}{2\pi}[e^{-i\omega t}\delta\phi_m(\omega) + \text{H.c.}].$$

The next step would be to express the field $\delta\phi_0(\omega)$ at the dot in the interaction region with the mth channel in terms of all the fields $\delta\phi_n(\omega)$, $n = 1,\ldots,N$. There are two processes that define the value of $\delta\phi_0(\omega)$. First, it is the reflection of the fields in N channels to the dot and, second, the successive transmission of the field $\delta\phi_0(\omega)$ through all the interaction regions.

Note that the reflected field from the nth channel acquires a certain phase corresponding to the distance L_{mn} between the contacts of the QD with this channel and the mth one, i.e., the channel from which the tunneling occurs. Making use of the scattering coefficients r_n,t_n (see Fig. 2), we arrive at the following expression for the field $\delta\phi_0(\omega)$:

$$\delta\phi_0 = \sum_{n=1}^{N} r_n\delta\phi_n e^{i\frac{\omega}{v}L_{mn}} + \prod_{n=1}^{N} t_n e^{i\frac{\omega}{v}L}\delta\phi_0. \tag{28}$$

We use this result to rewrite the field $\phi_0(\omega)$ in terms of the scattering states $\Phi_{0m\omega}$:

$$\delta\phi_0(\omega) = \sum_{n=1}^{N}\Phi_{0n\omega}\delta\phi_n(\omega), \tag{29}$$
$$\Phi_{0n\omega} = \frac{r_n}{1-\prod_{n=1}^{N} t_n e^{i\frac{\omega}{v}L}}. \tag{30}$$

Note that we have ignored the phases $\exp(i\omega L_{mn}/v)$ as they cancel each other, which can be seen from the following expressions for the auto- and cross correlators:

$$\mathcal{K}_1 = -\int_0^\infty \frac{d\omega}{\omega}(1-e^{i\omega t})\left(1+\sum_{n=1}^N |\Phi_{0n\omega}|^2\right), \quad (31)$$

$$\mathcal{K}_2 = 2\int_0^\infty \frac{d\omega}{\omega}(1-e^{i\omega t})\mathrm{Re}(\Phi_{0m\omega}). \quad (32)$$

To analyze the form of the scattering states (30) in the low-energy limit, $\omega W/v \ll 1$, it is enough to expand the scattering coefficients up to the second order in frequency. Then using the unitarity of the scattering matrix we arrive at

$$r_n = i\omega\tilde{r}_n - \omega^2\tilde{r}_n\tilde{t}_n, \quad (33)$$

$$t_n = 1 + i\omega\tilde{t}_n - \omega^2\frac{\tilde{r}_n^2+\tilde{t}_n^2}{2}. \quad (34)$$

To understand the physical meaning of the coefficients $\tilde{r}_n$ and $\tilde{t}_n$, we note that on one hand $\Phi_{0m\omega}(\omega=0) = -q_m$ from (17) and (22), on the other hand $\Phi_{0m\omega}(\omega=0) = \tilde{r}_m/(L/v+\sum_n \tilde{t}_n)$. Finally, recalling (21) it becomes evident that $\tilde{r}_m$ and $L/v+\sum_n\tilde{t}_n$ are just the capacitances (up to a constant multiplier) between the channels C_{m0} and the self-capacitance C_{00}, respectively. We simply denote $\tau_C = L/v+\sum_n\tilde{t}_n$, since this is just the travel time of the plasmon in the dot. With this in mind, and using Eqs. (33) and (34), we rewrite the expression for the scattering states in the form

$$\Phi_{0n\omega} = \frac{i\omega\tau_C q_n}{1-\left(1-\frac{\sigma}{2}\omega^2\tau_C^2\right)e^{i\omega\tau_C}}, \quad (35)$$

where we introduced the dimensionless coupling constant

$$\sigma = \sum_{n=1}^N q_n^2 \quad (36)$$

characterizing the strength of the interaction.

To find the autocorrelator (31), we start with analyzing $|\Phi_{0n\omega}|^2$. Assuming that coupling is weak, $\sigma \ll 1$ (either due to the large number of channels, $N \gg 1$, or because of partial screening by a gate, $\sum_n q_n \ll 1$), the main contribution to the integral in (31) comes from the singularity at $\omega = 0$ and from the set of plasmon resonances in (35) at frequencies $\omega_l = \Delta\omega l, l = 1,2,\ldots$, where $\Delta\omega = 2\pi/\tau_C$. Evaluating these contributions separately, we can write

$$\mathcal{K}_1 = -(1+\sigma)\ln\left(\frac{\delta - it}{\delta}\right) - J, \quad (37)$$

$$J = \sum_{l=1}^\infty \frac{e^{-\varepsilon l}}{l}[1-\exp(il\Delta\omega t - \pi l^2\sigma\Delta\omega|t|)], \quad (38)$$

where ε is the high-energy cutoff parameter. Considering first the free-fermionic case of $\sigma = 0$, the sum in (38) can be evaluated explicitly, and we obtain

$$J_{free} = \ln(1-e^{i\Delta\omega t-\varepsilon}), \quad (39)$$

where we dropped the unimportant constant contribution. Therefore, the tunneling rate (23) is described by a set of steps as a function of a bias:

$$\Gamma_{free} \propto \int_{-\infty}^\infty dt\frac{e^{i\Delta\mu t}}{(\delta+it)}\frac{1}{1-e^{-i\Delta\omega t-\varepsilon}} \propto \sum_{n=0}^\infty \theta(\Delta\mu - n\Delta\omega), \quad (40)$$

where we set $\varepsilon = 0$ in the end of calculations. This result is in perfect agreement with the free-fermionic picture, as the steps correspond to the quantized energy levels of the QD. This happens because there is no interaction in the QD itself, as it is being screened. So the electronic and bosonic description are just the two alternative ways to look at the same system.

Returning now to the interaction case, instead of calculating the sum over l in (38), we formally represent $\exp(-J)$ as a Taylor series and change the sign of the integration variable t for convenience:

$$\Gamma(\Delta\mu) \propto \int_{-\infty}^\infty dt\frac{e^{i\Delta\mu t+\mathcal{K}_2^{(1)}}}{(\delta+it)^{1+\alpha}} \times \sum_{m=0}^\infty \frac{1}{m!}\left(\sum_{l=1}^\infty \frac{e^{-\varepsilon l}}{l}\exp(-il\Delta\omega t - l^2\sigma\pi\Delta\omega|t|)\right)^m. \quad (41)$$

Here, we also separate the Mahan term

$$\mathcal{K}_2^{(0)} = -2q_m\int_0^\infty\frac{d\omega}{\omega}(1-e^{-i\omega t}) = -2q_m\ln\left(\frac{\delta+it}{\delta}\right) \quad (42)$$

from the cross correlator $\mathcal{K}_2(-t) = \mathcal{K}_2^{(0)} + \mathcal{K}_2^{(1)}$ to complete the FES exponent α in the denominator. We show in the Appendix that the term $\mathcal{K}_2^{(1)}$ is negligible, as it can be considered perturbatively in the coupling σ.

After opening the brackets in Eq. (41), each term in the triple sum is of the form constant $\times \exp(-in\Delta\omega t - p\sigma\Delta\omega|t|)$, n and p being integers, and thus contributes to the nth step in the function $\Gamma(\Delta\mu)$. Recalling that $\sigma \ll 1$, and that we are interested in low energies, we can simply present $\Gamma(\Delta\mu)$ as a sum over steps [26]

$$\Gamma(\Delta\mu) = \frac{2\pi}{\Gamma(1+\alpha)}(\Delta\mu)^\alpha + \sum_{n=1}^{n_0}\Gamma_n, \quad \Delta\mu > 0, \quad (43)$$

and cut the sum at arbitrary $n_0 > \Delta\mu/\Delta\omega$. Here the first term is the FES contribution. Proceeding in this way, we evaluate the integral (41) term by term and present the result in terms of the functions

$$G(\gamma,\Delta\mu_n) = \frac{2\pi\,\mathrm{Im}\{(\gamma\Delta\omega + i\Delta\mu_n)^\alpha e^{i\pi\alpha/2}\}}{\sin(\pi\alpha)\Gamma(1+\alpha)}, \quad (44)$$

where we define $\Delta\mu_n \equiv \Delta\mu - n\Delta\omega$. Thus, e.g., the first three steps can be presented as

$$\begin{aligned}\Gamma_1 &= G(\pi\sigma,\Delta\mu_1),\\ \Gamma_2 &= \tfrac{1}{2}G(2\pi\sigma,\Delta\mu_2) + \tfrac{1}{2}G(4\pi\sigma,\Delta\mu_2),\\ \Gamma_3 &= \tfrac{1}{3}G(9\pi\sigma,\Delta\mu_3) + \tfrac{1}{2}G(5\pi\sigma,\Delta\mu_3) + \tfrac{1}{6}G(3\pi\sigma,\Delta\mu_3).\end{aligned} \quad (45)$$

The result of such evaluation is shown in Fig. 3.

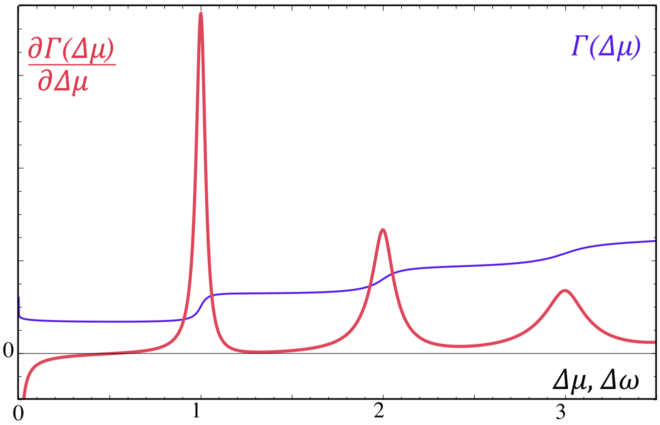

FIG. 3. The tunneling rate $\Gamma(\Delta\mu)$ and its derivative $\partial\Gamma(\Delta\mu)/\partial\Delta\mu$ is shown for the FES exponent $\alpha = -0.04$ and the coupling constant $\sigma = 0.01$. It is evaluated approximately according to (41) by keeping the finite number of terms. The values are presented in arbitrary units, as we study them up to constant multipliers.

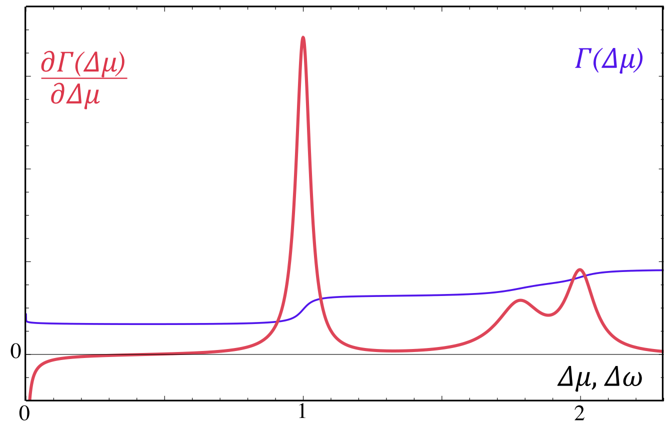

FIG. 4. Splitting of the second electron level in two levels corresponding to one- and two-plasmon processes in the case of weakly dispersive plasmons is shown. The tunneling rate $\Gamma(\Delta\mu)$ and its derivative $\partial\Gamma/\partial(\Delta\mu)$ are calculated using Eqs. (44)–(46) for $\alpha = -0.02$, $\sigma = 0.01$, and $\epsilon = 0.22$.

Quite remarkably, the results (43)–(45) show that single-electron levels in the QD can be viewed as plasmon resonances. Indeed, although each term in Eqs. (45) for Γ_n represents single- or multiplasmon process, and their broadening is caused by Coulomb interaction, in the free-fermionic limit $\alpha,\sigma \to 0$ they add to the single-electron excitation step: $\Gamma_n = \theta(\Delta\mu - n\Delta\omega)$. This is a consequence of the fact that we assume the linear spectrum of the plasmon in the QD, which is known to leave electrons effectively free. This leads us to the next idea to relax this limitation by considering a weak dispersion in the spectrum of plasmons. The immediate consequence of this is that one should expect splitting of the steps in Γ starting from $n = 2$, where it is also most pronounced.

Indeed, considering the situation where the Coulomb interaction in the QD is not screened by a gate, it is natural to assume the spectrum of plasmons to be weakly concave [27], so that the frequency of the second plasmon resonance acquires a small negative shift. To describe the first two peaks as well as the ground state analytically, we make use of an expression (43), introducing a small shift: $2\Delta\omega \to 2\Delta\omega - \epsilon$. Since two contributions to Γ_2 in Eq. (45) represent single- and two-plasmon processes, this splits the second step into two steps:

$$\Gamma_2 = \tfrac{1}{2}G(2\pi\sigma,\Delta\mu_2) + \tfrac{1}{2}G[(2-\epsilon)^2\pi\sigma,\Delta\mu_2+\epsilon], \qquad (46)$$

while Γ_1 and the FES contributions remain unchanged. Here, the lower energy step corresponds to the emission of one plasmon with the energy $2\Delta\omega - \epsilon$, while the higher energy step refers to the emission of two plasmons of the energy $\Delta\omega$. The width of the former one is larger and is defined by $(2-\epsilon)^2\pi\sigma\Delta\omega$ as compared to $2\pi\sigma\Delta\omega$ for the higher peak. The results of the calculation are shown in Fig. 4. Higher peaks, not shown in this figure, will show additional splittings corresponding to the number of emitted plasmons. If the coupling constant is small enough, $\sigma < \epsilon/\Delta\omega$, the effect should be clearly seen in experiment.

IV. TUNNELING TO A QD WITH NEUTRAL MODES

One of the conclusions that one can draw from the results of the previous section is that in the case of the linear plasmon spectrum the effects of interaction are controlled roughly by a single parameter, coupling constant σ. One one hand, this leads to the universality of the effect of level broadening, which is a nonperturbative interaction effect reflecting a subtle electron-plasmon correspondence. On the other hand, smearing of the steps in $\Gamma(\Delta\mu)$ results also in the suppression of the FES effect in the excited states, so that it is observable only in the transition to the ground state. Below, we demonstrate that including neutral modes in the QD helps to circumvent this limitation.

Let us consider the QD with two chiral channels with strong long-range interaction [28]. Without loss of generality of our main conclusion below, we can concentrate on the case of only one external channel. We denote the bosonic fields as $\phi_1(x)$ for the channel in the lead and $\phi_2(x)$, $\phi_3(x)$ for the outer and inner channel at the QD, respectively. Again, we study the tunneling rate to the QD according to (7), specifically, to its external channel. The interaction between the two channels at the QD can be "turned off" by a standard rotation of the basis:

$$\phi_2 = \frac{1}{\sqrt{2}}(\tilde{\phi}_2 + \tilde{\phi}_3), \qquad (47)$$

where $\tilde{\phi}_2(x)$ is the fast charge mode, and $\tilde{\phi}_3(x)$ is the slow dipole (neutral) mode. Hence, the tunneling operator in (6) will include both charge and neutral fields.

In the case of a strong interaction that we are dealing with, the neutral modes are much slower than the charge ones. This allows to consider biases $\Delta\mu$ smaller than the level spacing of the charge mode, so that only the neutral mode is excited. Formally, this can be expressed similarly to Eq. (30):

$$\delta\tilde{\phi}_2(\omega) = \frac{r_1}{1-t_1}\delta\phi_1(\omega), \quad \delta\tilde{\phi}_3(\omega) = \frac{r_2}{1-t_2 e^{ikL}}\delta\phi_1(\omega), \qquad (48)$$

where $r_{1,2}$ and $t_{1,2}$ are the scattering coefficients. They can be expanded as in Eqs. (33) and (34), which guarantees the unitarity of the scattering matrix. Using the analog of Eqs. (17) and (22), one can write $\delta\tilde{\phi}_2 = -\sqrt{2}q\delta\phi_1$ [29], where the charge $q < 0$ is induced in the channel outside the dot.

Repeating now the steps that lead to Eqs. (37) and (38), we observe that the charge mode contributes to the low-energy part of the correlator in (37) with the coupling constant $\sigma_c = q^2$, while the neutral mode mostly contributes to the term (38) with its own coupling σ_n to the field φ_1, which is, by all means, significantly smaller than for the charge mode. That is to say, there is almost no interaction between them. Therefore, in first approximation, one can set $\sigma_n = 0$, and the sum in (38) can be evaluated explicitly (as in the case of free electrons) with the result

$$\Gamma(\Delta\mu) \propto \int_{-\infty}^{\infty} dt \frac{e^{i\Delta\mu t}}{(\delta + it)^{1+\alpha}} \frac{1}{(1 - e^{i\Delta\omega t-\varepsilon})^{1/2}}$$
$$\propto \sum_{n=0}^{\infty} c_n \theta(\Delta\mu - n\Delta\omega)(\Delta\mu - n\Delta\omega)^{\alpha}, \qquad (49)$$

where the FES exponent $\alpha = 2q + q^2$, according to its universality, and $c_n = (-1)^n C^n_{-1/2}$.

The obtained result is strikingly different from the case of a strong interaction with a charged mode alone. Now the $\Gamma(\Delta\mu)$ curve gains a structure of steps that correspond to the excitation of the neutral mode. But in addition, each step carries a power-law singularity at its edge; i.e., the FES replicates itself as can be seen in Fig. 5. This happens because the neutral excitations are almost completely separated from the charge and there is nothing to smear the effect besides the temperature. So the energy is mostly spent on exciting the dipole mode, while a small excess part of it gives rise to long-range electron-hole excitations, which are responsible for the FES phenomenon.

We would like to mention also that in contrast to the free-electron case, the amplitude of the steps is weighted with the universal numbers c_n, which are independent of the details of the interaction. For example, the first four numbers are equal to 1, 1/2, 3/8, and 5/16. These numbers originate from the expansion of the function $(1 - e^{i\Delta\omega t-\varepsilon})^{-1/2}$ in (49) as a Taylor series, while the square root can be viewed as a manifestation of the charge fractionalization [30] caused by the strong interchannel interaction in the QD.

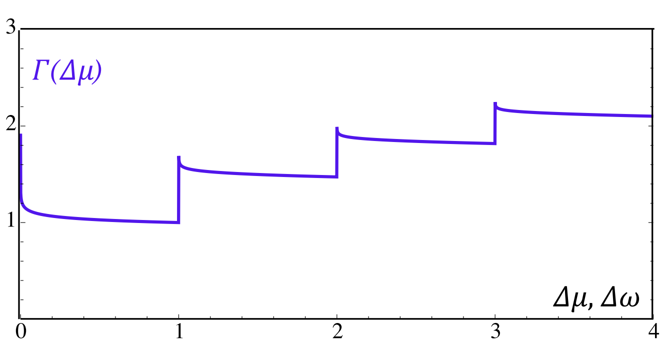

FIG. 5. The tunneling rate $\Gamma(\Delta\mu)$ (in arbitrary units) calculated according to Eq. (49) with the coupling to the charge mode $\alpha = -0.04$. It represents the case of tunneling from a QH channel to a QD with two chiral channels strongly interacting via a long-range potential. In this case the Hamiltonian can be diagonalized in the bias of a charge and a neutral mode. The bias can be chosen to be smaller than the energy of the first excited state for the charge mode. Then, the steps are explained by the excitation of the resonances of the neutral mode, while the interaction with the charge mode guarantees the appearance of the FES at each edge of the steps.

V. TUNNELING TO A 3D QD IN THE COULOMB BLOCKADE REGIME

In the previous section we have considered the situation in which the charge mode in the QD in the QH regime is separated by a Coulomb gap from the neutral mode, which is only weakly coupled to the charge mode outside the dot. However, because of the reduced dimensionality of the system and as a result of strong interactions, an electron is equally coupled to both modes, which has certain important consequences, as discussed above. Here we consider another quite common situation, where the QD is formed by a metallic or a semiconductor granula of small size, so that the charged mode is again separated by the Coulomb gap which results in the Coulomb blockade effect. However, this time the QD is a 3D system with the consequence that an electron has direct coupling to neutral modes. In order to analyze this situation, below we use the formalism introduced in Sec. II to derive the analog of the well-known $P(E)$ theory [31].

Having said that, we now consider the same QH system [32] with a 3D QD, described by the following Hamiltonian:

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{int} + \mathcal{H}_d + \mathcal{H}_t,$$

with $\mathcal{H}_0$ corresponding to the free Hamiltonian (4) of the QH channels. To model the 3D character of the dot and the Coulomb blockade, we compactify one of the bosonic fields $\phi_n(x)$ in (5) and take the zero limit for its length, $L \to 0$, which brings us to the following form of the interaction Hamiltonian:

$$\mathcal{H}_{int} = \frac{1}{8\pi^2} \sum_{nn'} \iint dx dy U_{nn'}(x,y)\partial_x\phi_n(x)\partial_y\phi_{n'}(y)$$
$$+ q \sum_n \int \frac{dx}{2\pi} U_n(x)\partial_x\phi_n(x) \qquad (50)$$

and the QD's energy

$$\mathcal{H}_d = \sum_k \varepsilon_k d_k^\dagger d_k + \frac{q^2}{2C}, \qquad (51)$$

where q is a charge at the QD and the second term corresponds to the charging energy. So we naturally arrive from the bosonic formalism at the fermionic description of the QD in this case. Finally, the tunneling Hamiltonian has the form

$$\mathcal{H}_t = A + A^\dagger, \quad A = \sum_k \tau_k d_k^\dagger e^{i(\phi_0 - \phi_m)}, \qquad (52)$$

so that the tunneling occurs from the mth channel to the kth level of the QD.

The field $\phi_0(x,t)$ represents the charged mode at the dot. However, as we consider the Coulomb blockade regime, i.e., the limit $L \to 0$, only a zero mode (10) as well the lowest energy mode in $\delta\phi_0(\omega)$ can be excited. The latter corresponds to the excitation of the "zero" scattering states $\Phi^{(0)}_{0n}$ defined in (17) and (22). Therefore, to calculate the tunneling rate we, basically, need to repeat the calculations in Sec. II. Thus, the part of the tunneling Hamiltonian (52) responsible for the charge mode reveals the FES contribution, while the neutral mode correlators lead to the appearance of the steps:

$$\Gamma(\Delta\mu) \propto \sum_k |\tau_k|^2 \theta(\Delta\mu - \varepsilon_k)(\Delta\mu - \varepsilon_k)^\alpha. \tag{53}$$

As in the case of a QD at the filling factor $\nu = 2$, considered in the previous section, the FES effect is replicated at each step corresponding to the excitation of a neutral mode. However, the important difference is that now neither the level spacing nor the amplitudes of the steps are regular and universal functions.

VI. CONCLUSION

Interaction of the QD with the QH edge channels results in various curious phenomena, which manifest themselves in the form of the tunneling rate to the dot. Working in the framework of boson scattering theory we managed to describe them considering a QD in the QH regime as a compactified boson field. We first demonstrated a particular convenience of this approach in describing a well-known FES phenomenon at low energies. Its universality can now be understood as a consequence of the connection between the scattering problem in the low-energy limit and the electrostatic problem of screening. Namely, the charges induced in the channels around the charged dot due to the interaction define the scattering states.

This method also allowed us to go to higher energies and consider the excitation of the collective modes in the QD, which is fully controlled by the interaction with the external channels and in the QD itself. If the interaction inside the dot is screened, so that the spectrum of plasmons is linear, there is a full correspondence between the free-fermion levels and the plasmon resonances. Thus, in the case of no interaction between the channels and the QD, the tunneling rate curve versus the bias is simply described by the set of steps, whereas in the presence of interaction, the steps become smeared. It is important to note that though we consider a relatively small coupling, the effect is nonperturbative. However, it was possible to describe the analytical solution due to the fermion-boson correspondence as well as the chirality of bosons. Next, we elaborated on the case of a weak dispersion in the spectrum of plasmons in the QD which leads to the splitting of fermion levels. This rather complicated behavior can be again easily explained in terms of plasmons.

Finally, a QD with two chiral strongly interacting edge channels also reveals interesting physics. Describing the system in terms of well-separated charge and neutral modes we showed that the tunneling rate acquires a universal form at low enough energies. Remarkably, a different setup with a 3D QD in the Coulomb blockade regime exhibits a similar behavior and can be treated within our general approach. Nevertheless, unlike the previous case, a direct coupling to the neutral mode leads to the nonuniversal structure of the result. All the discussed phenomena can be explored experimentally, which we strongly recommend [33].

ACKNOWLEDGMENTS

We thank Ivan Levkivskyi for useful discussions. This work was supported by the Swiss National Science Foundation.

APPENDIX A: CROSS CORRELATOR FOR THE MULTILEVEL QD

Let us justify why $\mathcal{K}_2^{(1)}$ can be considered as a perturbative correction in Eq. (41). We start with applying the expression for the scattering states (35) to (32) to obtain the cross correlator

$$\mathcal{K}_2 = -2q_m \int_0^\infty dx (1 - e^{ixt/\tau_C}) \frac{\sin x}{|\Delta|^2}, \tag{A1}$$

$$\Delta = 1 - [1 - (i\sigma/2)x^2]e^{ix}, \tag{A2}$$

where we introduced the dimensionless variable $x = \omega\tau_C$. The integral (A1) has a logarithmic divergence for small x cut by e^{-ixt/τ_C}. We explicitly singled out this divergence in Eq. (41) to find the tunneling rate. This is the Mahan contribution that defines the correct power-law exponent of the FES. The rest of the cross correlator can be written as

$$\mathcal{K}_2^{(1)} = -2q_m \int_0^\infty dx (1 - e^{-ix\frac{t}{C}}) \left(\frac{\sin x}{|\Delta|^2} - \frac{1}{x} \right). \tag{A3}$$

This part is proportional to $q_m \ll 1$ and here we demonstrate that the integral itself is of the order 1. First and foremost, we make sure that there are no more divergences. We eliminated the divergence for small x, but there is an ultraviolet cutoff at large x. However, this contribution is also canceled by the same cutoff in the FES [see Eq. (42)]. We also note that the term containing the time exponent vanishes for large values of x due to strong oscillations. Next, we have to check that by isolating $\mathcal{K}_2^{(0)}$, the Mahan term, we did not lose any additional contributions coming from the small x. Indeed, expanding around zero $\frac{\sin x}{|\Delta|^2} - \frac{1}{x} \sim \frac{1}{x - x^3\sigma/2} - \frac{1}{x} \sim x\sigma/2$, one can see that the deviation is small in σ. Finally, concerning the constant part of the correction, it can be left aside as we are describing the tunneling rate up to a multiplier. Consequently, the term $\mathcal{K}_2^{(1)}$ in the cross correlator represents a perturbative correction in coupling in the next order and thus can be neglected.

[1] K. Ohtaka and Y. Tanabe, Rev. Mod. Phys. **62**, 929 (1990).
[2] G. D. Mahan, Phys. Rev. **163**, 612 (1967).
[3] P. W. Anderson, Phys. Rev. Lett. **18**, 1049 (1967).
[4] P. Nozières and C. T. De Dominicis, Phys. Rev. **178**, 1097 (1969).
[5] A. K. Geim, P. C. Main, N. La Scala, L. Eaves, T. J. Foster, P. H. Beton, J. W. Sakai, F. W. Sheard, M. Henini, G. Hill, and M. A. Pate, Phys. Rev. Lett. **72**, 2061 (1994); D. H. Cobden and B. A. Muzykantskii, *ibid.* **75**, 4274 (1995); H. Frahm, C. von Zobeltitz, N. Maire, and R. J. Haug, Phys. Rev. B **74**, 035329 (2006); N. Maire, F. Hohls, T. Lüdtke, K. Pierz, and R. J. Haug, *ibid.* **75**, 233304 (2007).
[6] N. Ubbelohde, K. Roszak, F. Hohls, N. Maire, R. Haug, and T. Novotný, Sci. Rep. **2**, 374 (2012).
[7] K. A. Matveev and A. I. Larkin, Phys. Rev. B **46**, 15337 (1992).

[8] C. L. Kane, K. A. Matveev, and L. I. Glazman, Phys. Rev. B **49**, 2253 (1994); T. Ogawa, A. Furusaki, and N. Nagaosa, Phys. Rev. Lett. **68**, 3638 (1992); N. V. Prokof'ev, Phys. Rev. B **49**, 2148 (1994); Y. Oreg and A. M. Finkel'stein, *ibid.* **53**, 10928 (1996); A. Komnik, R. Egger, and A. O. Gogolin, *ibid.* **56**, 1153 (1997).

[9] P. Hawrylak, Phys. Rev. B **44**, 11236 (1991).

[10] B. Muzykantskii, N. d'Ambrumenil, and B. Braunecker, Phys. Rev. Lett. **91**, 266602 (2003).

[11] D. A. Abanin and L. S. Levitov, Phys. Rev. Lett. **93**, 126802 (2004).

[12] E. Fradkin, *Field Theories of Condensed Matter Physics* (Cambridge University Press, Cambridge, 2013).

[13] T. Giamarchi, *Quantum Physics in One Dimension* (Oxford University Press, Oxford, 2003).

[14] I. Chernii, I. P. Levkivskyi, and E. V. Sukhorukov, Phys. Rev. B **90**, 245123 (2014).

[15] E. V. Sukhorukov, Physica E (Amsterdam, Neth.) **77**, 191 (2016).

[16] E. V. Sukhorukov and V. V. Cheianov, Phys. Rev. Lett. **99**, 156801 (2007).

[17] Y. Oreg and A. M. Finkel'stein, Phys. Rev. Lett. **74**, 3668 (1995).

[18] A. O. Slobodeniuk, I. P. Levkivskyi, and E. V. Sukhorukov, Phys. Rev. B **88**, 165307 (2013).

[19] D. Ferraro, B. Roussel, C. Cabart, E. Thibierge, G. Fève, C. Grenier, and P. Degiovanni, Phys. Rev. Lett. **113**, 166403 (2014).

[20] M. Stone, H. W. Wyld, and R. L. Schult, Phys. Rev. B **45**, 14156 (1992); S. Jung, G. M. Rutter, N. N. Klimov, D. B. Newell, I. Calizo, A. R. Hight-Walker, N. B. Zhitenev, and J. A. Stroscio, Nat. Phys. **7**, 245 (2011); M. R. Geller and D. Loss, Phys. Rev. B **62**, R16298 (2000); J. M. Kinaret, Y. Meir, N. S. Wingreen, P. Lee, and X.-G. Wen, *ibid.* **45**, 9489 (1992).

[21] N. Ofek, A. Bid, M. Heiblum, A. Stern, V. Umansky, and D. Mahalu, Proc. Natl. Acad. Sci. U.S.A. **107**, 5276 (2010).

[22] E. Bocquillon, V. Freulon, J.-M. Berroir, P. Degiovanni, B. Plaçais, A. Cavanna, Y. Jin, and G. Fève, Nat. Commun. **4**, 1839 (2013).

[23] E. Weisz, H. K. Choi, M. Heiblum, Y. Gefen, V. Umansky, and D. Mahalu, Phys. Rev. Lett. **109**, 250401 (2012).

[24] The rate of tunneling in the opposite direction follows from the electron-hole symmetry.

[25] It is worth noting that the potentials μ_n in the electrostatic problem (18) are of 1/time dimensionality, while the charges are dimensionless. However, in the scattering problem (14) and (15) it is the potentials that are dimensionless and the charges get the time dimension. On the other hand, we are comparing the dimensionless potential ε_0 and the charge q_n, so that the fact that they are equal is self-consistent.

[26] Formally, the time integral in Eq. (41) does not vanish for $\Delta\mu < 0$, because the integrand is not an analytical function. Moreover, the sum inside the integral seems to be divergent. This problem is related to our approximate way of evaluating the contributions of the plasmon resonances in Eq. (31), which correctly describes the steps in $\Gamma(\Delta\mu)$ but fails far away from the steps. One can check that when expanding $\exp(-J)$ the phase space limitation in multiple integrals over ω solves this problem, and that correction to our result is small in σ.

[27] I. P. Levkivskyi and E. V. Sukhorukov, Phys. Rev. B **85**, 075309 (2012).

[28] I. P. Levkivskyi and E. V. Sukhorukov, Phys. Rev. B **78**, 045322 (2008).

[29] The additional factor of $\sqrt{2}$ in the case of two edge channels appears as a result of renormalization of the potential ε_0 in Eq. (18) corresponding to one edge channel in the QD. However, it cancels with $\sqrt{2}$ from the unitary transform (47). It leads, in the end, to the proper form of the FES exponent, as it should, due to the universality of this phenomenon.

[30] H. Inoue, A. Grivnin, N. Ofek, I. Neder, M. Heiblum, V. Umansky, and D. Mahalu, Phys. Rev. Lett. **112**, 166801 (2014).

[31] G.-L. Ingold and Y. Nazarov, in *Single Charge Tunneling*, edited by H. Grabert and M. H. Devoret (Plenum Press, New York, 1992).

[32] Although we stay within our framework and consider the QH system, it is not necessary for the effect, which would still be present if the tunneling was studied from an arbitrary metal. This statement relies on the fact of the universality of the FES [4]. Indeed, while the energy is mostly spent on the excitation of the energy levels of the dot, the rest goes on the electron-hole excitations that lead to the replication of the FES.

[33] A possible experimental technique could be introducing two unequal barriers, making the transport through the dot highly asymmetric [6]. In this case the tunneling rate is defined by the less-conducting barrier, which we are interested in.