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Dissipation in mesoscale superfluids

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We investigate the maximum speed at which a driven superfluid can flow through a narrow constriction with a size on the order of the healing length. Considering dissipation via the thermal nucleation of quantized vortices, we calculate the critical velocity for superfluid ^{4}He and ultracold atomtronic circuits, identify fundamental length, and velocity scales, and are thus able to present results obtained in widely different temperature and density ranges in a universal framework. For ultranarrow channels, we predict a drastic reduction in the critical velocity as the energy barrier for flow reducing thermally activated phase slip fluctuations is suppressed.

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The flow of dissipationless atomic supercurrents in neutral superfluids is one of the most dramatic manifestations of macroscopic quantum coherence [1–3], with applications to matter wave interferometry [4–6]. Recently, there has been increased interest in dimensionally confined superfluids, due to progress in manufacturing nanoscale channels and fountain effect devices for studying the flow of superfluid helium [7–24] and the availability of trapped nonequilibrium atomic Bose-Einstein condensates [25–39]. Common to these experiments in vastly different density and interaction regimes is an observed increase in dissipation for highly confined systems.

In general, superflow is possible at speeds less than a superfluid critical velocity set by the Landau criterion $v_c \leqslant \min \varepsilon(p)/p$ below which there are no accessible excitations $\varepsilon(p)$ with momentum p [40]. Among the different types of excitations in superfluids, quantized vortices [1,41–44] give rise to the smallest v_c. For flow through a cylindrical channel, if the total kinetic energy is converted into vortex rings with the size a of the constriction, the Landau criterion predicts a critical velocity $v_{c,F} \sim (\kappa/a)\ln(a/\xi_0)$ [42], where $\kappa = h/m$ is the quantum of circulation for condensed bosons of mass m and ξ_0 is a characteristic length scale of the superfluid. This prediction (due to Feynman) has been born out by nearly a half-century worth of superfluid massflow observations with *temperature independent* critical velocities [2,45]. However, it must ultimately break down as the constriction radius approaches ξ_0. Moreover, any observed temperature dependence of v_c can only be described by the existence of an energy activation barrier for the creation of vortices.

In this Rapid Communication, we consider confined mesoscale superflow through quasi-one-dimensional (1D) constrictions with a characteristic size a approaching the temperature (T) dependent correlation (healing) length $\xi(T)$, and find a strong increase in dissipation when $a/\xi(T)$ approaches one. Going beyond previous studies [2,36], we (i) quantitatively predict the temperature, size, and drive dependence of the critical velocity without adjustable parameters, (ii) use a paradigmatic orifice geometry to model the enhancement of vortex creation in spatially inhomogeneous flow near a sharp boundary, which significantly lowers critical velocities, (iii) point out the universality between high-density ^{4}He [8,18] and low-density atomic condensates [25,27,28,36], by characterizing constrictions via the dimensionless length a/ξ_0 and measuring velocities in units of $v_0 = \kappa/(4\pi\xi_0)$, and (iv) describe the crossover to the purely 1D limit, a Luttinger liquid in the thermal regime. Predictions are expected to be logarithmically accurate in the critical regime while corrections of order unity may arise when extrapolating to lower T.

We begin by considering superflow between reservoirs with a chemical potential difference $\Delta\mu$ (pressure difference ΔP) connected by prototypical geometric constrictions with either a "channel" or "orifice" shape as seen in Fig. 1. Channel flow is spatially homogeneous with a constant superfluid velocity v_s that is representative of flow through long narrow cylindrical pores. Flow through an orifice can be studied by considering a hyperbolic surface of revolution connecting two bulk reservoirs where the potential flow problem can be solved analytically [46]. The solution is characterized by divergent flow near the boundaries as seen in Fig. 1(c), where the creation of line vortices (Fig. 2, left) is facilitated. Ring vortices in the center of the orifice (Fig. 2, right) are not strongly affected by the spatial dependence of flow near the boundaries.

The energy cost for creating a quantized vortex is due to (i) the kinetic energy of circular superflow around the core region, and (ii) the loss of condensation energy within the core [41]. For a vortex ring of radius R (length $\mathcal{L} = 2\pi R$) or a line vortex (length $\mathcal{L}$) in a constriction of radius a, the combination of these effects yields

$$E_{\text{tension}} = \frac{\mathcal{L}}{4\pi}\rho_s\kappa^2\left(\ln\frac{\ell}{\xi} + \alpha\right), \tag{1}$$

where $\ell = a$ for line vortices, $\ell = R$ for ring vortices, and the constant α depends of the vortex type and model of the core. Solving the Gross-Pitaevski (GP) equation, one finds that $\alpha = 0.385$ for line vortices [47] and $\alpha = \ln 8 - 2 + 0.385 \simeq 0.464$ for ring vortices [48]. Obtaining a more accurate model of the vortex core is possible via numerical simulations in low [49] and high density superfluids [50,51]. The results are consistent with the GP value of α and show only weak density dependence. The reduction of the kinetic energy of superflow due to interaction with the vortex is

$$E_{\text{flow}} = \kappa\rho_s \iint \boldsymbol{v}_s \cdot d\boldsymbol{S}, \tag{2}$$

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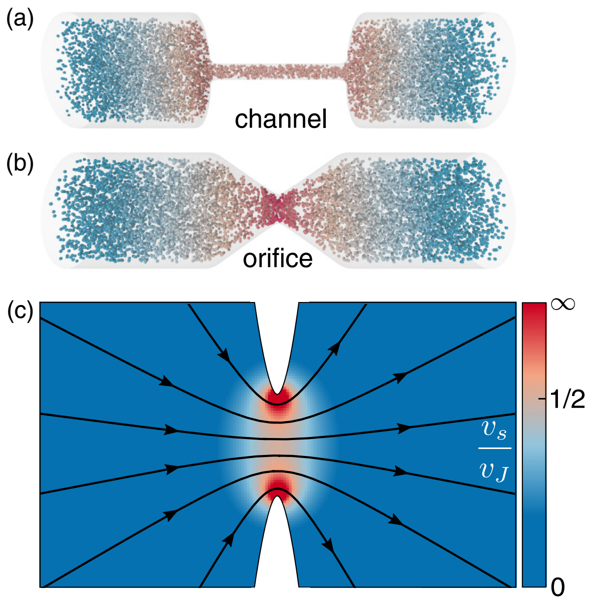

FIG. 1. (a) A current of superfluid atoms driven through a long channel with a homogeneous (radius independent) flow profile. (b) Flow through a narrow orifice formed from the surface of revolution of a hyperbola around the flow axis. (c) Velocity field $\boldsymbol{v}_s$ for the potential flow though an orifice in units of the average flow speed v_J. The flow direction is indicated by black lines, with the magnitude diverging as a power law at the orifice boundary [46].

where the integral is over the area bounded by the vortex ring, or between the vortex line and boundary. The total energy is $E = E_{\text{tension}} + E_{\text{flow}}$. To unify the description of driven quantum fluids, we employ the Josephson relation in 3D [52] $\kappa^2\rho_s(T)\xi(T) = 4\pi^2 k_B T_c$, where $\rho_s(T)$ is the superfluid mass density for a transition temperature T_c and $\xi(T) = \xi_0(1 - T/T_c)^{-\nu}$ and ν is the correlation length critical exponent. We numerically checked that the Josephson relation is valid to within 20% down to $T/T_c \approx 0.7$, for details see Ref. [46]. For flow through an orifice with speed v_J, we obtain the energy barrier for a line vortex located a distance x from its center:

$$\beta_c E_{\text{line}}(x) = 2\pi\frac{a}{\xi}\sqrt{1-\left(\frac{x}{a}\right)^2}\left[\ln\left(1-\frac{|x|}{a}\right)+\ln\frac{a}{\xi}+\alpha\right] - \frac{v_J}{v_0}\left(\frac{a}{\xi}\right)^2\frac{\pi^2\xi}{2\xi_0}\left(1-\frac{x}{a}\right) \tag{3}$$

and that for a centered ring vortex with radius R:

$$\beta_c E_{\text{ring}}(R) = 2\pi^2\frac{R}{\xi}\left(\ln\frac{R}{\xi}+\alpha\right) - \frac{v_J}{v_0}\left(\frac{a}{\xi}\right)^2\frac{\pi^2\xi}{\xi_0}\left[1-\sqrt{1-\left(\frac{R}{a}\right)^2}\right], \tag{4}$$

where $\beta_c = 1/(k_B T_c)$. From these expressions (and those for channel flow derived in the Supplemental Material [46]) we observe the emergence of natural length (ξ_0) and velocity $[v_0 = \kappa/(4\pi\xi_0)]$ scales that are essential for constructing a universal theory of dissipative superfluids.

The velocity of superflow at finite T is limited by the thermal nucleation of quantized vortices which traverse the flow lines leading to a change of phase of the superfluid order parameter by $\pm 2\pi$ (see, e.g., Ref. [53]). The decay of a persistent current is then governed by vortex energetics via

$$\Gamma = \Gamma_0[e^{-\beta E_{\max}(+v_J)} - e^{-\beta E_{\max}(-v_J)}], \tag{5}$$

where $h\Gamma = \Delta\mu = m\Delta P/\rho_s$ drives total mass flow $J = \rho_s \iint \boldsymbol{v}_s \cdot d\boldsymbol{S} \equiv \pi a^2 \rho_s v_J$ and $E_{\max} \equiv \max_{\mathcal{L}} E$ is the saddle point of the vortex excitation energy over the domain of the channel or orifice. The difference of rates in Eq. (5) corresponds to the contributions from vortices which reduce and increase the superflow, respectively. The attempt rate Γ_0 is related to the phase space available for vortex excitations and is geometry dependent:

$$\Gamma_0 = \frac{1}{\tau_{GL}}\frac{La}{\xi^2}\begin{cases}\frac{\pi a}{\xi} & \text{vortex ring} \\ 2\pi & \text{vortex line}\end{cases}, \tag{6}$$

with $\tau_{\text{GL}}^{-1} = 16k_B(T_c - T)/h$ the Ginzburg-Landau relaxation rate. The decay rate in Eq. (5) contains the main contribution from integration over zero modes, corresponding to a translation of the vortex with action S_v in both time and space. In addition, there are Jacobians due to the change of coordinates from the superfluid phase Φ to the radius R of the vortex ring or the location x of the vortex line, and a contribution from integration over the negative eigenvalue mode. As previously discussed [54,55], the Jacobian is $\sqrt{S_v/2\pi}$ for the zero modes, and the negative eigenvalue mode contributes a factor of similar magnitude. We have verified that the combination of all these factors is of order unity and thus neglect them. Other modifications to the pre-factor in Eq. (5) could result when considering the microscopic details of dynamics and vortex evolution inside the constriction [56–61] and would introduce quantitative logarithmic corrections to the nucleation theory.

Evaluation of the critical velocity from Eq. (5) proceeds as follows. For a given flow profile and vortex type, we maximize the vortex energy as a function of $\pm v_J$ over the spatial domain of possible configurations. This leads to critical vortex positions $x^*(\pm v_J) \in (-a,a)$ for line vortices

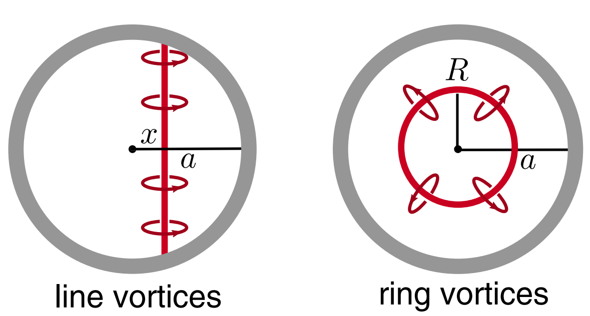

FIG. 2. Two vortex types that can be nucleated in a confined geometry. (Left) A line vortex located a distance x from the center of the orifice or channel. The vortex line begins and ends on the boundary, with the flow circulating around it. (Right) A vortex line can detach from the boundary and close on itself forming a vortex ring of radius R. Arrows indicate the circulation of quantized flow around the vortex core.

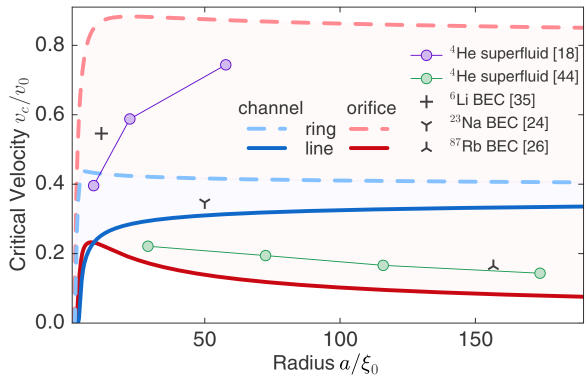

FIG. 3. Upper and lower bounds on the critical velocity are indicated by lines from the channel ($L = 10^3\xi_0$, blue) and orifice ($L = 10\xi_0$, red) flow profiles for both ring (dashed) and line vortices (solid) at $T = 0.7T_c$, $\hbar\Gamma = 0.1k_BT_c$, and $\nu \simeq 0.6717$. Symbols show experimental massflow results for superfluid helium [18,45] and Bose-Einstein condensates [25,27,36]. Details are provided in the Supplemental Material [46] where we discuss the relationship between the zero temperature healing length and ξ_0 in the weakly interacting Bose gas [62].

and critical radii $R^*(\pm v_J) \in (0,a)$ for ring vortices. Vortices with a length smaller than the critical one will tend to collapse, or anihilate at boundaries, while those larger can proliferate, leading to dissipation. For a fixed constriction radius a/ξ_0, temperature T/T_c and external drive potential Γ_0/Γ, Eq. (5) can be numerically solved self-consistently giving the critical velocity when $v_J = v_c$.

When $a/\xi_0 \gg 1$, the boundaries of the constriction no longer play an important role and only uniform channel flow is relevant. Due to the resulting large critical velocities, energy increasing fluctuations can be neglected and the saddle point energies can be found analytically. The resulting pair of transcendental equations

$$\frac{R^*}{\xi} = \frac{1}{\pi^2}\frac{T}{T_c}\frac{\ln\frac{\Gamma_0}{\Gamma}}{\ln\frac{R^*}{\xi} + \alpha - 1}, \tag{7}$$

$$\frac{v_c}{v_0} = \frac{\xi_0}{R^*}\left(\ln\frac{R^*}{\xi} + \alpha + 1\right) \tag{8}$$

can be solved numerically for R^* and v_c [46].

Our main results for the critical velocity in both the channel and orifice flow profiles are shown as lines in Fig. 3. When employing the scales v_0 and ξ_0, mass flow measurements in drastically different density, interaction, and temperature regimes are well bounded by the vortex nucleation theory, and experiments on confined superfluid ^{4}He and low-dimensional Bose-Einstein condensates can be directly compared. For both flow profiles, line vortices have lower activation energies than ring vortices, giving smaller velocities and a lower bound on v_c. An absolute upper bound is provided by ring vortices in the orifice flow profile. In the limit $a \gg \xi_0$, where the geometry approaches that of bulk flow, we recover the intrinsic superfluid velocity due to the nucleation of vortex rings analyzed by Langer and Fisher [43]. For ^{4}He, we have used a vortex core size determined from specific heat measurements [46,63], and have thus been able to correct a long-standing inconsistency of 27 orders of magnitude in the applied pressure difference employed in Ref. [43] to obtain agreement with experiments. For tight constrictions, both the lower and upper bound turn towards smaller velocities indicative of enhanced dissipation.

While details of additional experiments are discussed in the Supplemental Material [46], Fig. 3 includes data from two ^{4}He studies whose critical velocities display a clear temperature dependence—a signature of activated behavior. Harrison *et al.* [45] measured flow in networks of imperfect pores of varying radii, which should provide a lower bound on flow speeds, behavior consistent with our results. Recent measurements on single nanopores by Duc *et al.* [18] exhibit drastically different behavior: large critical velocities that *decrease* as the radius approaches the correlation length. While the microscopic details of the flow profiles are not known, v_c for the narrowest pore is bounded by the channel prediction, consistent with the reported aspect ratio of 10:1 and suggesting a crossover to strongly dissipative quasi-1D flow.

Figure 3 also includes results from analogous neutral atomtronic circuits using utracold bosonic and fermionic condensates [3]. Here, the "orifice" can be replaced with a quantum point contact between two resonantly coupled Fermi gases [37] or channel-like flow can be driven by the discharge of an bosonic atom capacitor [29] or by dragging an optical potential through a simply [27,36] or multiply connected [28,30,33] Bose-Einstein condensate. For the latter, our nucleation theory yields $v_c \approx 100\ \mu\mathrm{m\,s}^{-1}$ for the drag and $v_c \approx 1\ \mathrm{mm\,s}^{-1}$ for the toroidal flow in remarkable agreement with measurements and more microscopic theoretical analysis [33,35,64].

Intuition for the dissipation in narrow pores with radius $a \approx \xi$ can be gained by considering the unoptimized energy of line vortices with position $x = 0$ at the center of the channel. This approximation is qualitatively correct since narrow pores can be expected to be in the channel flow regime with line vortex activation energies comparable to temperature. The vortex energy is found from the sum of Eqs. (1) and (2) with $x = 0$:

$$\beta_c E_{\mathrm{line}}(x=0) = 2\pi\frac{a}{\xi}\left(\ln\frac{a}{\xi} + \alpha\right) - \frac{\pi^2\xi}{2\xi_0}\frac{v_s}{v_0}\left(\frac{a}{\xi}\right)^2, \tag{9}$$

with resulting critical velocity

$$\frac{v_c}{v_0} = \frac{2}{\pi^2}\frac{\xi\xi_0}{a^2}\frac{T}{T_c}\sinh^{-1}\left\{\frac{1}{64\pi}\frac{\xi^2}{aL}\left(1-\frac{T}{T_c}\right)^{-1} \times \exp\left[\frac{2\pi a}{\xi}\frac{T_c}{T}\left(\ln\frac{a}{\xi}+\alpha\right)\right]\right\}. \tag{10}$$

As $a/\xi \to 1^+$, the activation energy in the exponent of Eq. (10) decreases rapidly, and the small multiplier of the exponential is no longer compensated. As a consequence, the critical velocity drops by several orders of magnitude.

In the quasi-1D ($a \lesssim \xi$) limit, there are no transverse degrees of freedom, and the system can be described in analogy to fluctuating superconducting wires by computing the resistance due to thermally activated phase slips [65,66]. Translated into the language of 1D superfluidity, the phase slip

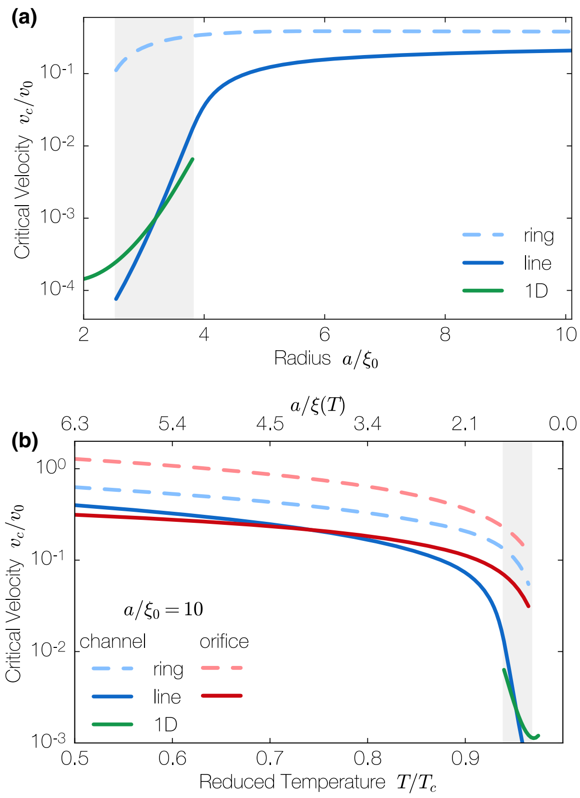

FIG. 4. (a) The critical velocity in a quasi-1D superfluid channel of length $L = 10^3\xi_0$ as a function of radius for $T/T_c = 3/4$, $h\Gamma = k_B T_c$, and $\xi(3T_c/4) \simeq 2.5\xi_0$. The solid and dashed blue lines extending over the full domain of the plot were computed via the vortex nucleation theory, while the green line is for 1D, Eq. (11). (b) The temperature dependence of the critical velocity for a channel ($L = 10^3\xi_0$) and orifice ($L = 10\xi_0$) with $a = 10\xi_0$. As $T \to T_c$, the correlation length ξ diverges, thus reducing the effective channel width $a/\xi(T)$ and lowering v_c. The shaded gray bars demarcates the radii where $1 \leqslant a/\xi(T) \leqslant 3/2$ and in this region the upper bound due to ring vortices is no longer expected to be relevant.

energy is (see Ref. [46])

$$\beta_c E_{1D} = \pi\left(\frac{a}{\xi}\right)^2\left(\frac{4}{3\sqrt{2}} - \pi\frac{\xi}{\xi_0}\frac{v_s}{v_0}\right). \tag{11}$$

For line vortices in the channel flow profile, the tension scales with $(a/\xi)^2$ in contrast to the linear dependence in Eq. (9). This difference is unimportant at the crossover $a/\xi \approx 1$ and the critical velocity from the nucleation of line vortices and the 1D theory should be in agreement as demonstrated in Fig. 4(a). The lower bound on the critical velocity due to line vortices drops three orders of magnitude, and crosses over to the 1D result for single mode channels. An analogous crossover from the linear to nonlinear Josephson junction regime has been observed in superflow through an array of orifices near T_c [8].

Figure 4(b) shows the T dependence of v_c for constrictions with $a/\xi_0 = 10$ and we observe a reduction as $T \to T_c$. Qualitatively, this is due to the fact that $\xi(T)$ increases as $T \to T_c$, thus reducing the ratio $a/\xi(T)$, which determines the effective constriction radius. Experiments on ^{4}He have reported an apparent temperature power-law scaling of v_c [18], which we have confirmed is the spurious result of an interplay between the thermal activation energy and vortex attempt rate. As $T \to 0$, vortices will no longer be thermally activated and dissipation will be dominated by the nucleation of quantum phase slips [67,68].

In summary, we considered two characteristic confined flow geometries and the thermal activation of representative low energy excitations, ring and line vortices, inside them. The resulting bounds they place on the critical velocity of neutral superflow through narrow constrictions agree with a large body of measurements on confined superfluid ^{4}He and low-dimensional ultracold gases. As the confinement radius approaches the healing length, we find an exponential suppression of the critical velocity of three orders of magnitude. The experimental observation of this dramatic reduction would be a clear signal of entering the strongly fluctuating mesoscale regime.

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