



Self-focusing skyrmion racetracks in ferrimagnets

Se Kwon Kim,¹ Kyung-Jin Lee,^{2,3} and Yaroslav Tserkovnyak¹

¹*Department of Physics and Astronomy, University of California, Los Angeles, California 90095, USA*

²*Department of Materials Science and Engineering, Korea University, Seoul 02841, South Korea*

³*KU-KIST Graduate School of Converging Science and Technology, Korea University, Seoul 02841, South Korea*

(Received 10 February 2017; published 14 April 2017)

We theoretically study the dynamics of ferrimagnetic skyrmions in inhomogeneous metallic films close to the angular momentum compensation point. In particular, it is shown that the line of the vanishing angular momentum can be utilized as a self-focusing racetrack for skyrmions. To that end, we begin by deriving the equations of motion for the dynamics of collinear ferrimagnets in the presence of a charge current. The obtained equations of motion reduce to those of ferromagnets and antiferromagnets at two special limits. In the collective coordinate approach, a skyrmion behaves as a massive charged particle moving in a viscous medium subjected to a magnetic field. Analogous to the snake orbits of electrons in a nonuniform magnetic field, we show that a ferrimagnet with nonuniform angular momentum density can exhibit the snake trajectories of skyrmions, which can be utilized as racetracks for skyrmions.

DOI: [10.1103/PhysRevB.95.140404](https://doi.org/10.1103/PhysRevB.95.140404)

Introduction. A free particle with a magnetic moment precesses at a frequency proportional to the applied magnetic field and its gyromagnetic ratio, which is the ratio of its magnetic moment to its angular momentum. When a magnet is composed of equivalent atoms, its net magnetization and net angular momentum density are collinear with the proportionality given by the gyromagnetic ratio of constituent atoms. The magnetic and spin variables then represent the same degrees of freedom, and thus are interchangeable in describing the magnetization dynamics. One-sublattice ferromagnets and two-sublattice antiferromagnets are examples of such magnets.

When a magnet consists of inequivalent atoms, however, its magnetization and spin density can constitute independent degrees of freedom [1]. One such class of magnets is rare-earth transition-metal (RE-TM) ferrimagnetic alloys [2], in which the moments of TM elements and RE elements tend to be antiparallel due to superexchange. Because of the different gyromagnetic ratios between RE and TM elements, one can reach the angular momentum compensation point and the magnetization compensation point by varying the relative concentrations of the two species or changing the temperature. These compensation points (CPs) are absent in ferromagnets and antiferromagnets, which have been mainstream materials in spintronics [3], and thereby have been bringing a novel phenomenon to the field such as the ultrafast optical magnetization reversal [2]. In particular, the angular momentum CP may provide a tunable crossover between the ferromagnetic (away from the CP) [4] and antiferromagnetic (at the CP) [2,5] regimes of collective dynamics, with a promise for unusual behavior in the vicinity of the CP. Here, we are exploring this question in regard to the topological spin-texture dynamics.

Topological solitons in magnets [6] have been serving as active units in spintronics. For example, a domain wall, which is a topological soliton in quasi-one-dimensional magnets with easy-axis anisotropy, can function as a memory unit, as demonstrated in magnetic domain-wall racetrack memory [7]. Two-dimensional magnets with certain spin-orbit coupling can also stabilize another particlelike topological soliton, which is referred to as a skyrmion. Skyrmions have been gaining attention in spintronics as information carriers, an alternative to

domain walls, because of fundamental interest as well as their practical advantages such as a low depinning electric current [8]. Several RE-TM thin films such as GdFeCo and CoTb have been reported to possess perpendicular magnetic anisotropy and a bulk Dzyaloshinskii-Moriya interaction [9,10], and thus are expected to be able to host skyrmions under appropriate conditions.

In this Rapid Communication, we study the dynamics of skyrmions in metallic collinear ferrimagnets, in the vicinity of the angular momentum CP in RE-TM alloys. To that end, we first derive the equations of motion for the dynamics of general collinear magnets in the presence of an electric current. The resultant equations of motion reduce to those of ferromagnets and antiferromagnets at two limiting cases. The dynamics of a skyrmion is then derived within the collective coordinate approach [11]. Generally, it behaves as a massive charged particle in a magnetic field moving in a viscous medium. When there is a line in the sample across which the net angular momentum density reverses its direction, the emergent magnetic field acting on skyrmions also changes its sign across it. Motivated by the existence of a narrow channel in two-dimensional electron gas localized on the line across which the perpendicular magnetic field changes its direction [12], we show that, under suitable conditions, the line of the vanishing angular momentum in RE-TM alloys can serve as a self-focusing racetrack for skyrmions [13] as a result of the combined effects of the effective Lorentz force and the viscous force. We envision that ferrimagnets with a tunable spin density can serve as a natural platform to engineer an inhomogeneous emergent magnetic field for skyrmions, which would provide us a useful knob to control them.

Main results. The system of interest to us is a two-dimensional collinear ferrimagnet. Although the angular momentum can be rooted in either the spin or the orbital degrees of freedom, we will use the term, spin, as a synonym for angular momentum throughout for the sake of brevity. For temperatures much below than the magnetic ordering temperature, $T \ll T_c$, the low-energy dynamics of the collinear ferrimagnet can be described by the dynamics of a single three-dimensional unit vector \mathbf{n} , which determines the collinear structure of

the magnet [4]. Our first main result, which will be derived later within the Lagrangian formalism taken by Andreev and Marchenko [4] for the magnetic dynamics in conjunction with the phenomenological treatment of the charge-induced torques [14], is the equations of motion for the dynamics of \mathbf{n} in the presence of a charge current density \mathbf{J} and an external field \mathbf{h} to the linear order in the out-of-equilibrium deviations $\dot{\mathbf{n}}$, \mathbf{J} , and \mathbf{h} ,

$$\begin{aligned} s\dot{\mathbf{n}} + s_\alpha \mathbf{n} \times \dot{\mathbf{n}} + \rho \mathbf{n} \times \ddot{\mathbf{n}} \\ = \mathbf{n} \times \mathbf{f}_n + \xi (\mathbf{J} \cdot \nabla) \mathbf{n} + \zeta \mathbf{n} \times (\mathbf{J} \cdot \nabla) \mathbf{n}, \end{aligned} \quad (1)$$

where s is the net spin density along the direction of \mathbf{n} , s_α and ρ parametrize the dissipation power density $P = s_\alpha \dot{\mathbf{n}}^2$ and the inertia associated with the dynamics of \mathbf{n} , respectively, and $\mathbf{f}_n \equiv -\delta U / \delta \mathbf{n}$ is the effective field conjugate to \mathbf{n} with $U[\mathbf{n}]$ the potential energy [15]. Here, ξ and ζ are the phenomenological parameters for the reactive and dissipative torques due to the current, respectively.

When the inertia vanishes, $\rho = 0$, the obtained equations of motion are reduced to the Landau-Lifshitz-Gilbert equation for ferromagnets augmented by the spin-transfer torques [16,17], in which s_α/s and ξ can be identified as the Gilbert damping constant and the spin polarization of conducting electrons, respectively. When the net spin density vanishes, $s = 0$, it corresponds to the equations of motion for antiferromagnets [14]. The equations of motion for the dynamics of a two-sublattice ferrimagnet in the absence of an electric current and dissipation, $s_\alpha = 0$ and $\mathbf{J} = \mathbf{0}$, have been obtained by Ivanov and Sukstanskii [18].

The low-energy dynamics of rigid magnetic solitons in two-dimensional collinear magnets can be derived from Eq. (1) within the collective coordinate approach [11], where the dynamics of the order parameter is encoded in the time evolution of the soliton position, $\mathbf{n}(\mathbf{r}, t) = \mathbf{n}_0[\mathbf{r} - \mathbf{R}(t)]$. The resultant equations of motion for the position of a circularly symmetric soliton, which are obtained by integrating Eq. (1) multiplied by $\mathbf{n}_0 \times \partial_{\mathbf{R}} \mathbf{n}_0$ over the space, are our second main result,

$$M\ddot{\mathbf{R}} = Q\dot{\mathbf{R}} \times \mathbf{B} - D\dot{\mathbf{R}} + \mathbf{F}_U + \mathbf{F}_J, \quad (2)$$

where $M \equiv \rho \int dx dy (\partial_x \mathbf{n}_0)^2$ is the soliton mass [19], $D \equiv s_\alpha \int dx dy (\partial_x \mathbf{n}_0)^2$ is the viscous coefficient, $\mathbf{F}_U \equiv -dU/d\mathbf{R}$ is the internal force, and $(F_J)_i \equiv \int dx dy [\xi \mathbf{n}_0 \cdot (\mathbf{J} \cdot \nabla) \mathbf{n}_0 \times \partial_i \mathbf{n}_0 - \zeta \partial_i \mathbf{n}_0 \cdot (\mathbf{J} \cdot \nabla) \mathbf{n}_0]$ is the force due to the charge current. The first term on the right-hand side is the effective Lorentz force on the soliton, which is proportional to its topological charge

$$Q = \frac{1}{4\pi} \int dx dy \mathbf{n}_0 \cdot (\partial_x \mathbf{n}_0 \times \partial_y \mathbf{n}_0), \quad (3)$$

which measures how many times the unit vector $\mathbf{n}_0(\mathbf{r})$ wraps the unit sphere as \mathbf{r} spatially varies [20], and the fictitious magnetic field

$$\mathbf{B} \equiv B\hat{\mathbf{z}} = -4\pi s\hat{\mathbf{z}}. \quad (4)$$

According to the equations of motion, a skyrmion in chiral ferrimagnets, which is characterized by its topological charge $Q = \pm 1$, behaves as a massive charged particle in a magnetic field moving in a viscous medium. The fictitious magnetic field is proportional to the net spin density s along the direction of

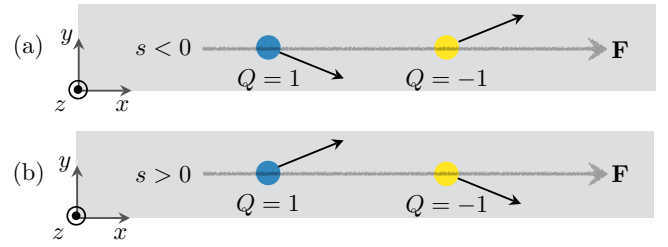


FIG. 1. Schematic illustrations of a steady-state skyrmion motion [Eq. (5)] in the presence of a current-induced force $\mathbf{F} = F\hat{\mathbf{x}}$. Four possible types are classified by its skyrmion charge Q and the sign of the net spin density s . See the main text for discussions.

the order parameter \mathbf{n} , which leads us to consider collinear magnets with tunable s to look for a possibly interesting dynamics of a skyrmion. The RE-TM ferrimagnetic alloys [2] are such materials. For example, $\text{Co}_{1-x}\text{Tb}_x$ has been shown to exhibit the vanishing angular momentum $s \approx 0$ at $x \approx 17\%$ at room temperature [10] by varying the chemical composition. As another example, the angular momentum compensation temperature of $\text{Gd}_{22\%}\text{Fe}_{75\%}\text{Co}_{3\%}$ has been reported as $T \approx 220$ K [21].

A skyrmion can be driven by an electric current, as can be seen in Eq. (2). In the presence of the corresponding current-induced force $\mathbf{F}_J \equiv F\hat{\mathbf{x}}$, the direction of which is defined as the x axis, the steady state of a skyrmion is given by

$$\dot{\mathbf{R}} \rightarrow \mathbf{V} = \frac{F}{B^2 + D^2} (D\hat{\mathbf{x}} - QB\hat{\mathbf{y}}). \quad (5)$$

See Fig. 1 for illustrations of a steady-state skyrmion motion for $F > 0$. The skyrmion with a topological charge $Q = 1$ moves down for $s < 0$ and up for $s > 0$, while moving to the right regardless of the sign of s . If the ferrimagnet is prepared in such a way that $s < 0$ for $y > 0$ and $s > 0$ for $y < 0$, the skyrmion with $Q = 1$ will move along the horizontal line $y = 0$ after a certain relaxation time because it is constantly pushed back to the line via the effective Lorentz force. Note that the skyrmion experiences no Lorentz force on the angular momentum compensation line, and thus will move as an antiferromagnetic skyrmion strictly along the line at a potentially higher speed compared to a ferromagnetic skyrmion [22]. A similar phenomenon has been predicted for a magnetic vortex moving around the interface between two ferromagnetic materials having opposite signs of the difference between the Gilbert damping constant α and the nonadiabatic spin-transfer torque coefficient η [23].

To corroborate the qualitative prediction, we numerically solve the equations of motion [Eq. (2)] in their dimensionless form,

$$I \frac{d^2 \tilde{\mathbf{R}}}{d\tilde{t}^2} + \frac{4\pi s Q}{s_\alpha} \frac{d\tilde{\mathbf{R}}}{d\tilde{t}} \times \hat{\mathbf{z}} + I \frac{d\tilde{\mathbf{R}}}{d\tilde{t}} = \tilde{F} \hat{\mathbf{x}}, \quad (6)$$

in which time, length, and energy are measured in units of the relaxation time $\tau \equiv \rho/s_\alpha$, the characteristic length scale for the skyrmion size l [24], and $\epsilon \equiv s_\alpha^2 l^2 / \rho$, respectively, where $I = \int dx dy (\partial_x \mathbf{n}_0)^2$ is a dimensionless number determined by the skyrmion structure. The symbols with the tilde will denote the dimensionless quantities throughout. Figure 2(a) shows the two trajectories of skyrmions of charge $Q = 1$

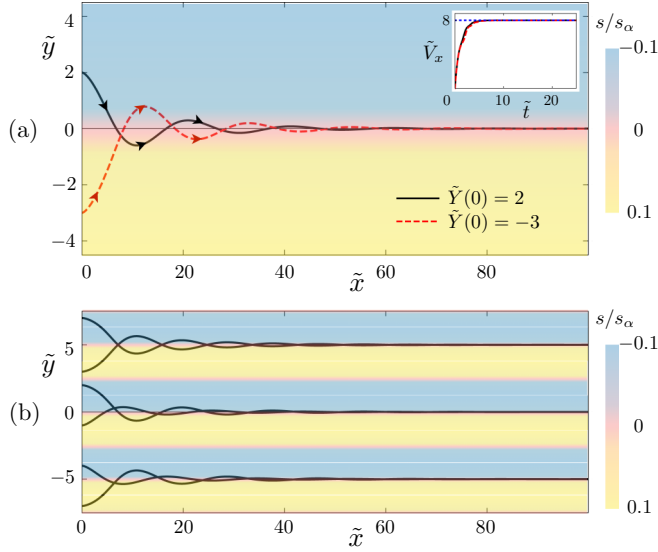


FIG. 2. Trajectories of skyrmions with the topological charge $Q = 1$ in the presence of a current-induced force $\mathbf{F} = F\hat{\mathbf{x}}$, which are obtained by numerically solving the dimensionless equations of motion for the dynamics of skyrmions in Eq. (6). (a) Two trajectories for the monotonic net angular momentum density s . The inset shows the convergence of the skyrmion velocities. (b) Multiple trajectories for the periodic net angular momentum density s . See the main text for detailed discussions.

departing from $(\tilde{X}, \tilde{Y}) = (0, 2)$ and $(\tilde{X}, \tilde{Y}) = (0, -3)$ with a zero initial velocity under the following configurations: $I = \pi/2$, $\tilde{F} = 4\pi$, and $s/s_\alpha = -0.1 \tanh(\tilde{y})$. We refer to the paths as skyrmion snake trajectories due to their shapes, analogous to the electronic snake orbits in an inhomogeneous magnetic field [12]. The inset shows that the skyrmion speed converges as $\tilde{V}_x \rightarrow \tilde{F}/I$ after a sufficiently long time, $\tilde{t} \gg 1$. Figure 2(b) depicts the multiple trajectories of skyrmions when the net spin density is spatially periodic, $s/s_\alpha = -0.1 \sin(2\pi \tilde{y}/5)$. Skyrmions are attracted to angular momentum compensation lines and their velocities converge to a finite value. This leads us to state our third main result: Self-focusing narrow guides for skyrmions can be realized in certain ferrimagnets such as the RE-TM alloys along the lines of the angular momentum compensation points, which can be useful in using skyrmions for information processing by, e.g., providing multiple parallel skyrmion racetracks in one sample [25].

The dynamics of collinear magnets. The derivation of the equations of motion for the dynamics of collinear magnets in [Eq. (1)] is given below, which follows the phenomenological approach taken for antiferromagnets by Andreev and Marchenko [4]. Within the exchange approximation that the Lagrangian is assumed invariant under the global spin rotations, we can write the Lagrangian density for the dynamics of the directional order parameter \mathbf{n} in the absence of an external field as

$$\mathcal{L} = -s\mathbf{a}[\mathbf{n}] \cdot \dot{\mathbf{n}} + \frac{\rho \dot{\mathbf{n}}^2}{2} - \mathcal{U}[\mathbf{n}], \quad (7)$$

to the quadratic order in the time derivative, where $\mathbf{a}[\mathbf{n}]$ is the vector potential for the magnetic monopole, $\nabla_{\mathbf{n}} \times \mathbf{a} = \mathbf{n}$ [26].

The first term accounts for the spin Berry phase associated with the net spin density along \mathbf{n} ; the second term accounts for the inertia for the dynamics of \mathbf{n} , which can arise due to, e.g., the relative canting of sublattice spins [15].

Next, the effects of an external field can be taken into account as follows. The conserved Noether charge associated with the symmetry of the Lagrangian under global spin rotations is the net spin density, and it is given by $\mathbf{s} = s\mathbf{n} + \rho\mathbf{n} \times \dot{\mathbf{n}}$. The magnetization in the presence of an external field \mathbf{H} can be then written as $\mathbf{M} = g_l s\mathbf{n} + g_t \rho\mathbf{n} \times \dot{\mathbf{n}} + \chi\mathbf{H}$, where g_l and g_t are the gyromagnetic ratios for the longitudinal and transverse components of the spin density with respect to the direction \mathbf{n} , respectively, and χ is the magnetic susceptibility tensor. The relation $\mathbf{M} = \partial\mathcal{L}/\partial\mathbf{H}$ [4] requires the susceptibility to be $\chi_{ij} = \rho g_t^2 (1 - n_i n_j)$, with which the Lagrangian is extended to

$$\mathcal{L} = -s\mathbf{a}[\mathbf{n}] \cdot \dot{\mathbf{n}} + \frac{\rho(\dot{\mathbf{n}} - g_t \mathbf{n} \times \mathbf{H})^2}{2} - \mathcal{U}[\mathbf{n}], \quad (8)$$

where $\mathcal{U}[\mathbf{n}]$ includes the Zeeman term, $-g_l s\mathbf{n} \cdot \mathbf{H}$. Finally, the dissipation can be accounted for by the Rayleigh dissipation function $\mathcal{R} = s_\alpha \dot{\mathbf{n}}^2/2$, which is half of the dissipation rate of the energy density $\mathcal{P} = 2\mathcal{R}$. The equations of motion obtained from the Lagrangian and the Rayleigh dissipation function are given by Eq. (1) without the current-induced torques.

Current-induced torques. To derive the torque terms due to an electric current, it is convenient to begin by phenomenologically constructing the expression for the charge current density \mathbf{J}^{pump} induced by the magnetic dynamics, and subsequently to invoke the Onsager reciprocity to obtain the torque terms as done for antiferromagnets in Ref. [14]. To the lowest order of the space-time gradients and to the first order in the deviations from the equilibrium, we can write two pumping terms that satisfy the appropriate spatial and spin-rotational symmetries, $\dot{\mathbf{n}} \cdot \partial_i \mathbf{n}$ and $\mathbf{n} \cdot (\dot{\mathbf{n}} \times \partial_i \mathbf{n})$. The resultant expression for the induced current density is given by

$$\mathbf{J}_i^{\text{pump}}/\sigma = \zeta \dot{\mathbf{n}} \cdot \partial_i \mathbf{n} + \xi \mathbf{n} \cdot (\partial_i \mathbf{n} \times \dot{\mathbf{n}}), \quad (9)$$

where σ is the conductivity.

To invoke the Onsager reciprocity that is formulated in the linear order in the time derivative of the dynamic variables, we turn to the Hamiltonian formalism instead of the Lagrangian formalism. We shall restrict ourselves here to the case of a vanishing external field for simplicity, but it can be easily generalized to the case of a finite external field. The canonical conjugate momenta of \mathbf{n} is given by $\mathbf{p} \equiv \partial\mathcal{L}/\partial\dot{\mathbf{n}} = \rho(\dot{\mathbf{n}} - g_t \mathbf{n} \times \mathbf{h}) - s\mathbf{a}$. The Hamiltonian density is then given by

$$\mathcal{H}[\mathbf{n}, \mathbf{p}] = \mathbf{p} \cdot \dot{\mathbf{n}} - \mathcal{L} = \frac{(\mathbf{p} + s\mathbf{a})^2}{2\rho} + \mathcal{U}, \quad (10)$$

which resembles the Hamiltonian for a charged particle subjected to an external magnetic field [27]. The Hamilton equations are given by

$$\dot{\mathbf{n}} = \frac{\partial\mathcal{H}}{\partial\mathbf{p}} \equiv -\mathbf{h}_p, \quad (11)$$

$$\dot{\mathbf{p}} = -\frac{\partial\mathcal{H}}{\partial\mathbf{n}} - \frac{\partial\mathcal{R}}{\partial\dot{\mathbf{n}}} \equiv \mathbf{h}_n - s_\alpha \dot{\mathbf{n}} = \mathbf{h}_n + s_\alpha \mathbf{h}_p, \quad (12)$$

where \mathbf{h}_p and \mathbf{h}_n are conjugate fields to \mathbf{p} and \mathbf{n} , respectively. In terms of the conjugate fields, the pumped charge current is given by $\mathbf{J}^{\text{pump}} = -\zeta \partial_i \mathbf{n} \cdot \mathbf{h}_p - \xi (\mathbf{n} \times \partial_i \mathbf{n}) \cdot \mathbf{h}_p$. By using the Onsager reciprocity and Ohm's law for the current $\mathbf{J} = \sigma \mathbf{E}$, we can obtain the torque terms in Eq. (1).

Discussion. Let us discuss approximations that have been used in this Rapid Communication. First, we have developed the theory for the dynamics of collinear magnets within the exchange approximation [4], in which the total energy is invariant under the simultaneous rotation of the constituent spins. The relativistic interactions including the magnetic anisotropy, which weakly break the exchange symmetry of the magnet, are added phenomenologically to the potential energy. Second, when studying the dynamics of skyrmions in inhomogeneous ferrimagnetic films, we have considered the nonuniform spin density s , while neglecting possible spatial variations of the other parameters such as inertia ρ or damping s_α . As long as skyrmions are attracted to the line of vanishing angular momentum due to the combined effects of the effective Lorentz force, the viscous force, and the current-induced force, smooth variations of various parameters away from it should not significantly affect the dynamics.

Ferrimagnetic RE-TM alloys not only have an angular momentum CP, which we have focused on in this Rapid Communication, but also a magnetic moment CP. Motivated by the attraction of skyrmions toward the angular momentum compensation lines that we have discussed, it would be worth looking for an interesting phenomenon that can occur on the magnetic moment compensation line. For example, since the magnetic moment governs the magnetostatic energy, there may be unusual magnetostatic spin-wave modes [28] localized at the line. In addition, we have considered the dynamics of a soliton in two-dimensional ferrimagnets driven by an electric current. In general, the dynamics of a soliton can be induced by other stimuli such as an external magnetic field [29] and a spin-wave excitation [30–32], which may exhibit peculiar features of ferrimagnets that are absent in ferromagnets and antiferromagnets.

Acknowledgments. We thank Kab-Jin Kim and Teruo Ono for enlightening discussions. This work was supported by the Army Research Office under Contract No. W911NF-14-1-0016 (S.K.K. and Y.T.) and by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIP) (NRF-2015M3D1A1070465) (K.-J.L.).

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