Tunneling of heat between metals

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We provide a theory of how heat tunnels between two parallel metal surfaces separated by an air gap. Two contributions are calculated: (1) electron-electron interactions and (2) photon fields from surface plasmons. Both contributions can transfer energy through low energy electron pairs. Our concern is with heat flow for small air gaps, on the order of nanometers. In that case the contribution from electron-electron interactions is most important. The contribution from photons is more important at larger separations.

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I. INTRODUCTION

We calculate the heat transfer between two parallel metal surfaces separated by an air or vacuum gap. In the Coulomb gauge, the Coulomb interactions are separate from the electromagnetic interactions. We consider both contributions to the heat transfer: (1) Coulomb interactions between charge fluctuations and (2) photonic interactions between current fluctuations. We find the Coulomb interactions are important at small air gaps, which is our main result.

This field has a long history. Hargreaves [1] first measured the tunneling of heat between two metals in 1969. In 1971 Polder and van Hove [2] provided a theory using only electromagnetic interactions. They and Hargreaves summarized the earlier, mostly Russian, work in this field. Polder and van Hove used the fluctuation dissipation theorem to evaluate the current-current correlation function. Many subsequent papers refined their theory [3–19]. Many of the recent electromagnetic theories have included the wave-vector dependence of the dielectric response, either through dielectric functions $\varepsilon(q,\omega)$ or by using skin-effect theories. There have been two excellent reviews [14,19]. Modern measurements between metals was done in [20–24]. Song *et al.*'s measurements [22] go down to 100 nm gap separation. In that gap size photonic heat transfer is certainly the important contribution. Two very recent measurements [23,24] go down to nanometer distances, where they observe a large heat transfer. Reference [23] reports "We find an extraordinary large heat flux which is more than five orders of magnitude larger than black body radiation and four orders of magnitude larger than the values predicted by conventional theory of fluctuational electrodynamics." Here we present the theory of heat transfer by charge fluctuations, which should provide the explanation of these measurements.

The theory of heat transfer is closely related to the theory of van der Walls interactions: they have the same Feynman diagram. For the van der Waals interaction between two neutral atoms, the potential goes as R^{-6} at short separations R, where Coulomb fluctuations dominate. At larger separations, the potential goes as R^{-7} where photon fluctuations dominate. The same difference applies to heat transfer: Coulomb fluctuations dominate at small values of separation, while photon fluctuations dominate at larger separations.

There have been measurements [25,26] and theories [27,28] of the tunneling of heat between a metal and an insulator. Prunnila and Melthaus [29] provided a theory of heat tunneling between two polar insulators, where optical phonons generate electric fields. Acoustic phonons in piezoelectric insulators also generate electric fields. There is interesting recent work on heat transfer between insulators [30–34]. This paper is concerned only with heat transfer between metals for small air gaps.

The tunneling of heat could occur, for example, during an STM measurement [26]. It may be important for thermionic devices, or metal-oxide-metal tunneling devices. Several groups [6,17,25] suggested that the energy transfer was due to the coupling of the surface polaritons on the two surfaces. These papers provided the impetus for the present calculation. We find that surface plasmons are an important mechanism, but not the only one. The other is electron-electron interactions.

Our calculation differs from previous calculations in many important respects. First, we include Coulomb interactions between density fluctuations, which we find to be a large effect at small gap separations. This contribution was not previously considered. The above citations are all concerned with electromagnetic heat transfer by photons. Secondly, we use quantum mechanics to evaluate the current-current correlation functions, and get a very different answer than previous workers who used the fluctuation-dissipation theorem. Our method of quantizing the electromagnetic fields uses the Brillouin formula for the energy density [35,36], which was not done before. We also used for the electronic current the expression which include spin rotations, which was also not done before. Thus our final formulas are totally different than found previously.

Figure 1 shows the Feynman diagram for the correlation energy between two metals separated by a gap. We will evaluate this expression below. Actually, electron-electron interactions need to be screened, so each metal has a string of bubble diagrams which provide screening. In this case, each bubble is a charge fluctuation, and the dashed line is the Coulomb interaction. This screening is describe by the surface dielectric function, which we derived previously [37]. This process is similar to the screened van der Waals interaction.

Figure 1 also describes another physical process. This is the interaction of two metals by electromagnetic fluctuations. In this case, the dashed lines are photons, while the bubble is a current fluctuation. Since the photons are confined between two metals, they are waveguide modes or surface plasmons. Both of these processes, charge fluctuations and current fluctuations, are present and must be calculated separately.

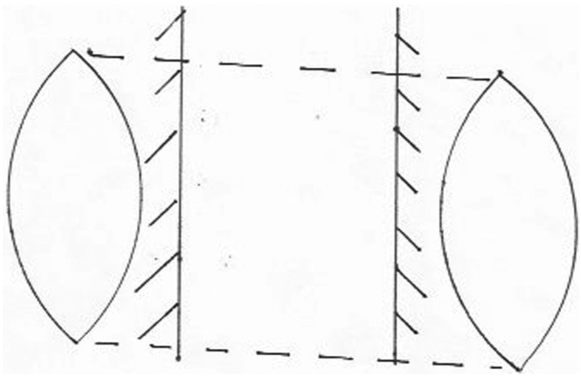

FIG. 1. Diagram for correlation energy and surface plasmon interactions.

The electromagnetic heat transfer is the topic of all prior theories.

II. ELECTRON-ELECTRON INTERACTIONS

One method of calculating interaction between metals is using electron-electron interactions. Here we derive (i) the ground state energy and (ii) the rate of energy transfer between metal surfaces. The center of the coordinate system is at the center of the gap of width $2d$. The electron wave function on the left and right are

$$\phi_L(\mathbf{r}) = \sqrt{\frac{2}{LA}} e^{i\mathbf{k}\cdot\rho} \sin[k_i(z+d)], \quad z < -d, \tag{1}$$

$$\phi_R(\mathbf{r}) = \sqrt{\frac{2}{LA}} e^{i\mathbf{p}\cdot\rho} \sin[p_i(z-d)], \quad z > d. \tag{2}$$

The Coulomb interaction between these states is

$$V_{LR} = \frac{e^2}{4\pi\varepsilon_0} \int \frac{d^3r_1 d^3r_2}{|\mathbf{r}_1 - \mathbf{r}_2|} \phi_L^*(\mathbf{k}_f, k_f; \mathbf{r}_1)\phi_L(\mathbf{k}_i, k_i; \mathbf{r}_1)\phi_R^*(\mathbf{p}_f, p_f; \mathbf{r}_2)\phi_R(\mathbf{p}_i, p_i; \mathbf{r}_2),$$

where $(\mathbf{k},\mathbf{p})$ are two dimensional wave vectors in the plane of the gap. Go to center of mass, then $d^2\rho_1 d^2\rho_2 = d^2\rho d^2R$. The integral over d^2R sets $\mathbf{k}_f + \mathbf{p}_f = \mathbf{k}_i + \mathbf{p}_i$. Define the momentum transfer as $\mathbf{q} = \mathbf{k}_f - \mathbf{k}_i = \mathbf{p}_i - \mathbf{p}_f$. Then we get

$$\int \frac{d^2\rho}{[\rho^2 + (\delta z)^2]^{1/2}} e^{i\mathbf{q}\cdot\rho} = \frac{2\pi}{q} e^{-q|\delta z|} = \frac{2\pi}{q} e^{-q(z_2 - z_1)}. \tag{3}$$

The integrals over $dz_1 dz_2$ give

$$2\int_{-\infty}^{-d} dz_1 e^{qz_1} \sin[k_i(z_1+d)] \sin[k_f(z_1+d)] = q\, e^{-qd} I(k_i,k_f), \tag{4}$$

$$I(k_i,k_f) = \left[\frac{1}{q^2 + k_-^2} - \frac{1}{q^2 + k_+^2}\right], \tag{5}$$

$$k_\pm = k_i \pm k_f, \tag{6}$$

$$2\int_d^\infty dz_2 e^{-qz_2} \sin[p_i(z_2-d)] \sin[p_f(z_2-d)] = q\, e^{-qd} I(p_i,p_f). \tag{7}$$

The Coulomb interaction is now

$$V_{LR} = \frac{2\pi e^2}{4\pi\varepsilon_0 AL^2} \sum_{\mathbf{q},k_i,k_f,p_i,p_f} q\, e^{-2qd} I(k_i,k_f) I(p_i,p_f) \rho_L(\mathbf{q},k_f,k_i)\rho_R(-\mathbf{q},p_f,p_i), \quad \rho_L(\mathbf{q},k_f,k_i) = \sum_{\mathbf{k},\sigma} C^\dagger_{\mathbf{k}+\mathbf{q},k_f,\sigma} C_{\mathbf{k},k_i,\sigma}. \tag{8}$$

Each metal has electron-electron interactions among their own electrons. From [37]

$$V_{LL} = V_{LL3} - V_{LL2}, \tag{9}$$

$$V_{LL2} = \frac{e^2}{4\varepsilon_0 AL^2} \sum_{\mathbf{q}} q \left[\sum_{k_f,k_i} I(k_f,k_i)\rho_L(\mathbf{q},k_f,k_i)\right]\left[\sum_{k'_f,k'_i} I(k'_f,k'_i)\rho_L(-\mathbf{q},k'_f,k'_i)\right],$$

$$V_{LL3} = \frac{e^2}{4\varepsilon_0 AL^2} \sum_{\mathbf{q},q_z,k_i,k'_i} \frac{1}{q^2+q_z^2} \tilde{\rho}_L(\mathbf{q},q_z,k_i)\tilde{\rho}_L(-\mathbf{q},q_z,k'_i), \tag{10}$$

$$\tilde{\rho}_L(\mathbf{q},q_z,k_i) = \sum_{\mathbf{k},\sigma} C^\dagger_{\mathbf{k}+\mathbf{q},k_i+q_z,\sigma} C_{\mathbf{k}+\mathbf{q},k_i,\sigma} + \mathrm{sgn}(k_i - q_z) C^\dagger_{\mathbf{k},|k_i-q_z|,\sigma} C_{\mathbf{k},k_i,\sigma}.$$

A similar form is found for V_{RR}.

Note that, in comparison to Ref. [37], we do not include the term for the back side of the metal slab. In Ref. [37] we had a canonical ensemble with a fixed number of electrons. When an external charge was on one side of the slab, the screening charge for the external source had to come from the back side of the slab, since the interior is charge neutral. That is the origin of the anomalous contribution from the back side of the slab. However, if the slab is grounded, then the screening charge comes from ground, and there is no contribution from the backside. This backside contribution is independent of the thickness of the slab. Another, different, contribution from the back side comes from the mutual interaction of the surface plasmons on the two surfaces. This latter contribution depends upon thickness. Here we assume the slabs are grounded and there is no contribution from the backside of the two slabs.

A. Correlation functions

Following Ref. [37], we define three kinds of correlation functions.

(i) The correlation $\langle\rho\rho\rangle$ gives

$$\Lambda_L(q,i\omega_n) = -\frac{e^2 q}{2\varepsilon_0 A L^2}\sum_{k_f,k_i} I(q,k_f,k_i)^2 \int_0^\beta d\tau\, e^{i\omega_n\tau}\langle T_\tau \rho_L(\mathbf{q},k_f,k_i:\tau)\rho_L(-\mathbf{q},k_i,k_f:0)\rangle$$

$$= -\frac{e^2 q}{2\varepsilon_0 A L^2}\sum_{\mathbf{k},k_f,k_i,\sigma} I(q,k_f,k_i)^2\frac{n_F(E_i)-n_F(E_f)}{E_i-E_f-i\omega_n}, \tag{11}$$

$$E_i = \frac{\hbar^2}{2m}\left(k^2+k_i^2\right),\quad E_f = \frac{\hbar^2}{2m}\left[(\mathbf{k}+\mathbf{q})^2+k_f^2\right]. \tag{12}$$

(ii) The correlation $\langle\tilde\rho\tilde\rho\rangle$ gives the 3D polarization function of the electron gas:

$$\chi_L(q,q_z,i\omega_n) = \frac{e^2}{\varepsilon_0\Omega}\frac{1}{q^2+q_z^2}\sum_{\mathbf{k},k_i\sigma}\frac{n_F(E_i)-n_F(E_f)}{E_i-E_f-i\omega_n}. \tag{13}$$

(iii) The correlation $\langle\rho\tilde\rho\rangle$ gives the correlation function called U:

$$U_L(q,q_z,i\omega_n) = \chi_L(q,q_z,i\omega_n) - U_{Lb}(q,q_z,i\omega_n), \tag{14}$$

$$U_{Lb}(q,q_z,i\omega_n) = \frac{e^2}{2\varepsilon_0\Omega}\sum_{\mathbf{k},k_i,\sigma}\left[\frac{n_F(E_i)-n_F(E_{f+})}{E_i-E_{f+}-i\omega_n}\frac{1}{q^2+(2k_i+q_z)^2}+\frac{n_F(E_i)-n_F(E_{f-})}{E_i-E_{f-}-i\omega_n}\frac{1}{q^2+(2k_i-q_z)^2}\right], \tag{15}$$

$$E_{f\pm} = \frac{\hbar^2}{2m}[(\mathbf{k}+\mathbf{q})^2+(k_i\pm q_z)^2]. \tag{16}$$

B. Linked cluster expansion

We use the linked cluster expansion, and retain only terms that link the two metal slabs. Only terms with even powers of V_{LR} are nonzero.

(i) In second order we get

$$E_2 = -\frac{1}{2}\int_0^\beta d\tau\, e^{i\tau\omega_n}\langle T_\tau V_{LR}(\tau)V_{LR}(0)\rangle \tag{17}$$

$$= \frac{1}{2\beta}\sum_{\mathbf{q},iq_m} e^{-4qd}\Lambda_L(q,iq_m)\Lambda_R(q,i\omega_n-iq_m) \tag{18}$$

$$= \frac{1}{2}\left(\frac{e^2}{2\varepsilon_0 A L^2}\right)^2\sum_{\mathbf{q}} q^2 e^{-4dq}\sum_{k_i,k_f,p_i,p_f} I^2(k_f,k_i)I^2(p_f,p_i)\sum_{\mathbf{pk}\sigma}\frac{n_{ki}n_{pi}(1-n_{kf})(1-n_{pf})-n_{kf}n_{pf}(1-n_{ki})(1-n_{pi})}{E_{ki}+E_{pi}-E_{kf}-E_{pf}-i\omega_n}. \tag{19}$$

(ii) Third order perturbation theory adds an electron bubble diagram on one side of the gap, or on the other:

$$\Omega_3 = \frac{1}{2\beta}\sum_{\mathbf{q},iq_m} e^{-4qd}\left[\Lambda_L(q,iq_m)\left(\Lambda_R(q,i\omega_n-iq_m)^2+\frac{U_R^2}{q^2+q_z^2}\right)+\Lambda_R(q,i\omega_n-iq_m)\left(\Lambda_L(q,iq_m)^2+\frac{U_L^2}{q^2+q_z^2}\right)\right]. \tag{20}$$

These series are defining the surface dielectric function we derived earlier [37]:

$$\varepsilon_R(q) = 1+\Lambda(q,iq_m)+\frac{1}{L}\sum_{q_x}\frac{U_R^2(q,q_z;iq_m)}{(q^2+q_z^2)[1-\chi(q,q_z;iq_m)]},$$

$$E_{LR} = \frac{1}{2A\beta}\sum_{iq_m,\mathbf{q}} e^{-4dq}\left(1-\frac{1}{\varepsilon_L(q,iq_m)}\right)\left(1-\frac{1}{\varepsilon_R(q,i\omega_n-iq_m)}\right). \tag{21}$$

When we consider fourth-order perturbation theory, there is a term with four powers of V_{LR}, divided by 4!. The perturbation series needs a coupling constant integral [38], which gives the final result

$$E_{LR}(i\omega_n) = -\frac{1}{2A\beta}\sum_{iq_m,\mathbf{q}} \ln[1-\phi(\mathbf{q},iq_m)], \tag{22}$$

$$\phi(\mathbf{q},iq_m) = e^{-4dq}\left(1-\frac{1}{\varepsilon_L(q,iq_m)}\right)\left(1-\frac{1}{\varepsilon_R(q,i\omega_n-iq_m)}\right). \tag{23}$$

The term with a logarithm is identical to the ground state energy in [39], after setting $i\omega_n = 0$.

C. Energy transfer

The rate of energy transfer is obtained by taking the retarded limit of $i\omega_n \to \omega + i\delta$, then the imaginary part as $\omega = 0$. This step is done after the evaluation of the summation over iq_m. Define a spectral function for the inverse dielectric function,

$$\left(1-\frac{1}{\varepsilon_L(q,iq_m)}\right) = \int \frac{dx}{2\pi}\frac{A_L(\mathbf{q},x)}{iq_m - x}, \tag{24}$$

$$\left(1-\frac{1}{\varepsilon_R(q,i\omega_n-iq_m)}\right) = \int \frac{dy}{2\pi}\frac{A_R(\mathbf{q},y)}{i\omega_n - iq_m - y}, \tag{25}$$

then we evaluate

$$\frac{1}{\beta}\sum_{iq_m}\phi(\mathbf{q},iq_m) = e^{-2qd}\int\frac{dx}{2\pi}A_L(\mathbf{q},x)\int\frac{dy}{2\pi}A_R(\mathbf{q},y)\frac{1+n_B(x)+n_B(y)}{i\omega_n - x - y}.$$

The series in Eq. (22) is

$$E_{LR} = \frac{1}{2A\beta}\sum_{iq_m,\mathbf{q}}\left[\phi + \frac{1}{2}\phi^2 + \frac{1}{3}\phi^3\cdots\right], \tag{26}$$

$$= \frac{1}{A}\sum_{\mathbf{q}} e^{-2qd}\int\frac{dx}{2\pi}A_L(\mathbf{q},x)\int\frac{dy}{2\pi}A_R(\mathbf{q},y)\frac{1+n_B(x)+n_B(y)}{i\omega_n - x - y}$$
$$\times\left[1 + 2e^{-2qd}G_L(q,x)G_R(q,y) + 3e^{-4qd}G_L^2(q,x)G_R^2(q,y)+\cdots\right], \tag{27}$$

$$E_{LR} = \frac{1}{A}\sum_{\mathbf{q}} e^{-2qd}\int\frac{dx}{2\pi}A_L(\mathbf{q},x)\int\frac{dy}{2\pi}A_R(\mathbf{q},y)\frac{1+n_B(x)+n_B(y)}{(i\omega_n - x - y)[1-e^{-2qd}G_L(q,x)G_Rq,(y)]^2},$$

$$G_j(q,x) = \int\frac{dx'}{2\pi}\frac{A_j(\mathbf{q},x')}{x-x'}. \tag{28}$$

Next (i) take $i\omega_n \to \omega + i\delta$, (ii) take the imaginary part, and (iii) set $\omega = 0$. The energy transfer per unit area is x (or y)

$$\left(\frac{dE_{LR}}{dt}\right) = \frac{2\pi}{\hbar A}\sum_{\mathbf{q}} e^{-4qd}\int\frac{dx}{2\pi}xA_L(\mathbf{q},x)\int\frac{dy}{2\pi}A_R(\mathbf{q},y)\frac{[1+n_B(x)+n_B(y)]\delta(x+y)}{[1-e^{-4qd}G_L(q,x)G_R(q,y)]^2}.$$

In thermal equilbrium, the factor

$$[1+n_B(x)+n_B(y)]\delta(x+y) = 0. \tag{29}$$

It is not zero if one side has a different temperature than the other. So let $T_L = T + \Delta T/2$, $T_R = T - \Delta T/2$. Expand the above expression in the small quantity $\Delta T/T$,

$$\left(\frac{dE_{LR}}{dt}\right) = \frac{2\pi\Delta T}{\hbar A k_B T^2}\sum_{\mathbf{q}} e^{-4qd}\int\frac{dx}{2\pi}x^2A_L(\mathbf{q},x)n_B(x)\int\frac{dy}{2\pi}A_R(\mathbf{q},y)n_B(y)\frac{\delta(x+y)}{[1-e^{-4qd}G_L(q,x)G_R(q,y)]^2}, \tag{30}$$

$$= \frac{2\pi\Delta T}{\hbar A k_B T^2}\sum_{\mathbf{q}} e^{-2qd}\int\frac{dx}{(2\pi)^2}x^2A_L(\mathbf{q},x)A_R(\mathbf{q},-x)n_B(x)[1+n_B(x)]\frac{1}{[1-e^{-2qd}G_L(q,x)G_R(q,-x)]^2}. \tag{31}$$

The factor of $n_B(x)[1+n_B(x)]$ confines $x \sim k_B T$ to small values of energy. The spectral functions are linear in x at small values,

$$A_j(\mathbf{q},x) \approx x B_j(\mathbf{q}), \tag{32}$$

$$\int dx\, x^4 n_B(x)[1+n_B(x)] = \frac{4\pi^4}{15}(k_B T)^5, \tag{33}$$

$$\left(\frac{dE_{LR}}{dt}\right) = \frac{2\pi^3}{15\hbar}(k_B \Delta T)(k_B T)^3 H(d), \tag{34}$$

$$H(d) = \int \frac{d^2q}{(2\pi)^2} e^{-4qd} \frac{B_L(q)B_R(q)}{[1-e^{-4qd}G_L(q)G_R(q)]^2}, \tag{35}$$

$$G_j(q) = \left(1 - \frac{1}{\varepsilon_j(q)}\right). \tag{36}$$

The function $B_j(q)$ is quite complicated. The energy transfer by electron-electron interactions goes as $\sim T^3\Delta T$. The units of (dE_{LR}/dt) are watts per area. Equation (35) is similar to an equation in [15] for energy transfer by photons, although in their case the factors of B_L, B_R, G_L, G_R are different.

III. SURFACE PLASMON INDUCED ENERGY TRANSFER

Now we consider the other case of energy transfer using photonic modes in the gap between the metals. The surface plasmon energy itself has no term, that I have yet located, that directly transfers energy from one surface to the other. The ground state energy from surface plasmons (SP) is derived from the Brillouin form of the electromagnetic energy [35,36],

$$\mathcal{E} = \frac{1}{2}\int d^3r\left[g(\omega)E^2 + \frac{1}{\mu_0}B^2\right], \tag{37}$$

$$= \sum_{\mathbf{q}} \hbar\omega(q)[a^\dagger_{\mathbf{q}}a_{\mathbf{q}} + 1/2], \tag{38}$$

$$g(\omega) = \frac{\partial[\omega\varepsilon(\omega)]}{\partial\omega}, \tag{39}$$

where the surface plasmons are derived from the dielectric function. The average value of the occupation number is

$$\langle a^\dagger_{\mathbf{q}}a_{\mathbf{q}}\rangle = \frac{1}{e^{\hbar\omega(q)/k_B\bar{T}} - 1}, \tag{40}$$

where $\bar{T}$ is the average temperature in the system. This term has no feature of sending heat from one interface to the other. The Bose-Einstein occupation number has the average temperature of the two sides. The energy transfer is a second-order process, shown in Fig. 1.

A. Vector potential

There are six possible electromagnetic modes in the vacuum gap [40]: symmetric and antisymmetric versions of (i) surface plasmons, (ii) transverse magnetic wave guide modes, and (iii) transverse electric wave guide modes. We are interested in small vacuum gaps, on the order of nanometers. In this case, the symmetric surface plasmon has a frequency of $\omega_q = qv_p$, $v_p^2 = cd\omega_p$, where $2d$ is the gap width. The other five modes have a frequency $\sim O(\omega_p)$, where ω_p is the plasma frequency of the metal. These other modes have a smaller contribution to the energy transfer when the air gap is very small.

The electric and magnetic fields depend upon the quantization of the vector potential in the gap, in terms of raising ($a^\dagger_{\mathbf{q}}$) and lowering ($a_{\mathbf{q}}$) operators. The form for the symmetric surface plasmon is

$$A_{x,y}(\mathbf{r}) = \sum_{\mathbf{q}} B_0 q_{x,y} g(z)(a_{\mathbf{q}}\xi + a^\dagger_{\mathbf{q}}\xi^*), \tag{41}$$

$$A_z(\mathbf{r}) = \sum_{\mathbf{q}} B_1 f_1(z)(a_{\mathbf{q}}\xi - a^\dagger_{\mathbf{q}}\xi^*), \tag{42}$$

$$\xi = \exp[i(\mathbf{q}\cdot\rho - \omega_q t)], \tag{43}$$

$$g(z) = \sinh(pz), \quad |z| < d, \tag{44}$$

$$= \pm\sinh(pd)\exp[-\gamma(|z|-d)], \quad |z| > d, \tag{45}$$

$$f_1(z) = \cosh(pz), \quad |z| < d, \tag{46}$$

$$= \frac{\cosh(pd)}{\varepsilon}\exp[-\gamma(|z|-d)], \quad |z| > d, \tag{47}$$

$$B_1 = -i\frac{q^2}{p}B_0, \tag{48}$$

$$p^2 = q^2 - \frac{\omega^2}{c^2}, \quad \gamma^2 = q^2 - \varepsilon(\omega)\frac{\omega^2}{c^2}. \tag{49}$$

Note that these equations satisfy $\nabla\cdot\mathbf{A} = 0$. The quantization is done using Eq. (37) following the method in [41,42]. The electric field ($\mathbf{E} = -\partial\mathbf{A}/\partial t$) and magnetic fields ($\mathbf{B} = \nabla\times\mathbf{A}$) are

$$(B_x,B_y) = \sum_{\mathbf{q}} B_0(q_y,-q_x)(a_{\mathbf{q}}\xi + a^\dagger_{\mathbf{q}}\xi^*)\frac{\omega_q^2}{c^2p}f_2(z), \tag{50}$$

$$f_2(z) = \cosh(pz), \quad |z| < d, \tag{51}$$

$$= \cosh(pd)\exp[-\gamma(|z|-d)], \quad |z| > d, \tag{52}$$

$$E_{x,y} = -i\sum_{\mathbf{q}} B_0 q_{x,y}\omega_q g(z)(a_{\mathbf{q}}\xi - a^\dagger_{\mathbf{q}}\xi^*), \tag{53}$$

$$E_z = -\sum_{\mathbf{q}} B_0\frac{q^2\omega_q}{p}f_1(z)(a_{\mathbf{q}}\xi + a^\dagger_{\mathbf{q}}\xi^*). \tag{54}$$

The magnetic field $B_\mu(z)$ is symmetric in z. The dispersion relation for this mode is

$$0 = \frac{\gamma}{\varepsilon} + p\tanh(pd). \quad (55)$$

For small gap openings $\alpha \equiv \omega_p d/c < 1$, the mode frequency is approximately

$$\omega_q^2 \approx \frac{(qd)^2\omega_p^2}{\sqrt{(qd)^2+\alpha^2}(1+\sqrt{(qd)^2+\alpha^2})}. \quad (56)$$

The frequency goes as $\omega_q = qv_p$ when $qd < \alpha$ and becomes $\omega_q = \omega_p$ when $qd > \alpha$. We can now evaluate the integrals in Eq. (37) and then set the result equal to Eq. (38). On the right-hand side of Eq. (37) we neglect terms such as aa and $a^\dagger a^\dagger$, as explained in [41,42]:

$$\hbar\omega_q = A\varepsilon_0 B_0^2 \frac{\omega_q^2 q^2}{p^2} I, \quad (57)$$

$$I = 2d\frac{\omega_q^2}{c^2} + \frac{2q^2}{p}\sinh(pd)\cosh(pd) + \frac{\cosh^2(pd)}{\gamma}\left[\frac{\omega_q^2}{c^2} + \frac{gq^2}{\varepsilon^2}\right] + \frac{gp^2}{\gamma}\sinh^2(pd),$$

$$B_0 = \frac{p}{q\omega_q}\sqrt{\frac{\hbar\omega_q}{A\varepsilon_0 I}} \equiv \frac{b_0}{\sqrt{A}}. \quad (58)$$

B. Current-current correlations

Consider the transfer of electron energy using current-current correlation functions. The interaction in each metal is

$$V_{R,L} = -\int d^3r\, \mathbf{j}_{L,R}(\mathbf{r})\cdot\mathbf{A}_{L,R}(\mathbf{r}). \quad (59)$$

The fourth-order ground state energy $\sim(V_L+V_R)^4$. The term we want is

$$E_{GS,4} = \frac{3!}{4!}\int_0^\beta d\tau_1 e^{i\omega_n\tau_1}\int_0^\beta d\tau_2 \times \int_0^\beta d\tau_3 \langle T_\tau V_L(\tau_1)V_L(\tau_2)V_R(\tau_3)V_R(0)\rangle, \quad (60)$$

$$= \frac{1}{2}\int d^3r_1\cdots d^3r_4 \int_0^\beta d\tau_1 e^{i\omega_n\tau_1}\int_0^\beta d\tau_2 \times \int_0^\beta d\tau_3 \Pi_{L,\mu\nu}(\mathbf{r}_1,\mathbf{r}_2;\tau_1-\tau_2) \times \Pi_{R,\lambda,\delta}(\mathbf{r}_3,\mathbf{r}_4;\tau_3)\mathcal{D}_{\mu\lambda}(\mathbf{r}_1,\mathbf{r}_3;\tau_1-\tau_3)\mathcal{D}_{\nu\delta}(\mathbf{r}_2,\mathbf{r}_4;\tau_2), \quad (61)$$

$$\mathcal{D}_{\mu\nu}(\mathbf{r}_1,\mathbf{r}_2;\tau_1-\tau_2) = -\langle T_\tau A_\mu(\mathbf{r}_1,\tau_1)A_\nu(\mathbf{r}_2,\tau_2)\rangle, \quad (62)$$

$$\Pi_{\mu\nu}(\mathbf{r}_1,\mathbf{r}_2;\tau_1-\tau_2) = -\langle T_\tau j_\mu(\mathbf{r}_1,\tau_1)j_\nu(\mathbf{r}_2,\tau_2)\rangle. \quad (63)$$

Evaluating the $\int d\tau$ in the usual way gives

$$E_{GS,4} = \frac{1}{2}\int d^3r_1\cdots d^3r_4 Q(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4;i\omega_n), \quad (64)$$

$$Q(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4;i\omega_n) = \frac{1}{\beta}\sum_{iq_m}\Pi_{L,\mu\nu}(\mathbf{r}_1,\mathbf{r}_2;iq_m)\Pi_{R,\lambda,\delta}(\mathbf{r}_3,\mathbf{r}_4;i\omega_n-iq_m) \times \mathcal{D}_{\mu\lambda}(\mathbf{r}_1,\mathbf{r}_3;i\omega_n-iq_m)\mathcal{D}_{\nu\delta}(\mathbf{r}_2,\mathbf{r}_4;iq_m). \quad (65)$$

The current operator for fermions is derived from the Dirac equation in Sakurai [43] in the section on "Gordon Decomposition." It contains a spin rotation term, which is not found in the current operator for spin zero bosons:

$$j_\mu = \frac{e}{2m}\{\psi^\dagger[p_\mu + i(\vec{p}\times\vec{\sigma})_\mu]\psi + \text{H.c.}\}, \quad (66)$$

where σ are the Pauli matrices. In the nonrelativistic limit, for one of our two coupled slabs, it is

$$j_\mu(\mathbf{r},\tau) = \frac{1}{\Omega}\sum_{\mathbf{q}} e^{-i\mathbf{q}\cdot\rho}\sum_{\mathbf{k}}\Lambda_\mu(z)C^\dagger_{\mathbf{k}+\mathbf{q},k_f,s'}C_{\mathbf{k},k_i,s}e^{-\tau(E_i-E_f)}, \quad (67)$$

$$E_i = \frac{\hbar^2}{2m}\left(k^2+k_i^2\right), \quad (68)$$

$$E_f = \frac{\hbar^2}{2m}\left[(k+\mathbf{q})^2+k_f^2\right], \quad (69)$$

$$\Lambda_\mu = \frac{e\hbar}{2m}\hat{\phi}_f^\dagger\mathcal{L}_\mu\hat{\phi}_i, \quad (70)$$

$$\vec{\mathcal{L}} = s_f s_i\{\mathcal{I}(\mathbf{k}+\mathbf{k}') + i\hat{x}\sigma_z(k_y-k_y') - i\hat{y}\sigma_z(k_x-k_x') + i\hat{z}[\sigma_y(k_x-k_x')-\sigma_x(k_y-k_y')]\} + s_f c_i k_i[-i\mathcal{I}\hat{z} - \hat{x}\sigma_y + \hat{y}\sigma_x] + s_i c_f k_f[i\mathcal{I}\hat{z} - \hat{x}\sigma_y + \hat{y}\sigma_x], \quad (71)$$

$$s_i = \sin[k_i(|z|-d)],\quad s_f = \sin[k_f(|z|-d)], \quad (72)$$

$$c_i = \cos[k_i(|z|-d)],\quad c_f = \cos[k_f(|z|-d)], \quad (73)$$

where $\mathbf{k}$ is the two dimensional wave vector parallel to the surfaces of the slab, $\mathbf{k}' = \mathbf{k}+\mathbf{q}$, while $k_{i,f}$ are the wave vectors in the z direction. The spinor states are $\hat{\phi} = (1,0)$ for spin up and $(0,1)$ for spin down. $\mathcal{I}$ is the two dimensional identity matrix and σ_μ are the Pauli matrices. The current-current correlation function is

$$\Pi_{\mu\nu}(\mathbf{r}_1,\mathbf{r}_2;iq_m) = \frac{1}{\Omega^2}\sum_{\mathbf{k},\mathbf{q},k_i,k_f}\sum_{ss'} e^{i\mathbf{q}\cdot(\rho_2-\rho_1)}\Lambda_\mu(z_1)\Lambda_\nu(z_2) \times \frac{n_F(E_i)-n_F(E_f)}{E_i-E_f-iq_m}. \quad (74)$$

At this point it is convenient to perform the integral over dz for each $j\cdot A$ term. Since the currents are only in the metal, we use the vector potential in this region. Define

$$\mathcal{L} = \int_d^\infty dz \sum_\mu \mathcal{A}_\mu(z)\mathcal{L}_\mu(z)e^{-\gamma(z-d)}, \quad (75)$$

$$\mathcal{A}_\mu = B_0 \sinh(pd)q_\mu, \quad \mu = (x,y), \quad (76)$$

$$= B_0 \sinh(pd)i\frac{q^2}{\gamma}, \quad \mu = z. \quad (77)$$

Renormalizing the integrals to $z = 0$, we have

$$\int_0^\infty dz\, e^{-\gamma z}\sin(k_i z)\sin(k_f z) = \gamma V(k_i,k_f), \quad (78)$$

$$\int_0^\infty dz\, e^{-\gamma z}(s_i c_f k_f + s_f c_i k_i) = \gamma^2 V(k_i,k_f), \quad (79)$$

$$\int_0^\infty dz\, e^{-\gamma z}(s_i c_f k_f - s_f c_i k_i) = k_- k_+ V(k_i,k_f), \quad (80)$$

$$V(k_i,k_f) = \frac{1}{2}\left(\frac{1}{\gamma^2+k_-^2} - \frac{1}{\gamma^2+k_+^2}\right). \quad (81)$$

The final integral is

$$\mathcal{L} = B_0 \sinh(pq)V(k_i,k_f)\left\{\mathcal{I}\left[\gamma \mathbf{q}\cdot(2\mathbf{k}+\mathbf{q}) - \frac{q^2}{\gamma}(k_+k_-)\right] + \varepsilon\frac{\omega^2}{c^2}(q_x\sigma_y - q_y\sigma_x)\right\}. \quad (82)$$

The summation over final spins and the average over initial spins for one metal gives

$$\frac{1}{2}\left(\frac{e\hbar}{2m}\right)^2 \sum_{ss'} \hat{\phi}_f^\dagger \mathcal{L}\hat{\phi}_i\hat{\phi}_i^\dagger\mathcal{L}^\dagger\hat{\phi}_f. \quad (83)$$

Since the summation $\sum_s \hat{\phi}_i\hat{\phi}^\dagger = \mathcal{I}$, then the above result is

$$\frac{1}{2}\left(\frac{e\hbar}{2m}\right)^2 \mathrm{Tr}\{\mathcal{L}\mathcal{L}^\dagger\} = B_0^2\sinh^2(pq)V(k_i,k_f)^2\left(\frac{e\hbar}{2m}\right)^2\Xi, \quad (84)$$

$$\Xi = \left[\gamma\mathbf{q}\cdot(2\mathbf{k}+\mathbf{q}) - \frac{q^2}{\gamma}k_-k_+\right]^2 + q^2\left(\varepsilon\frac{\omega^2}{c^2}\right)^2, \quad (85)$$

where we assume that $\mathbf{q}_1 = -\mathbf{q}_2$. The photon Green's function for the SP is

$$D_{\mu\nu}(\mathbf{r}_1,\mathbf{r}_2,iq_m) = \sum_{\mathbf{q}} \mathcal{A}_\mu(z_1)\mathcal{A}_\nu(z_2)\frac{2\omega_q}{(iq_m)^2-\omega_q^2}e^{i\mathbf{q}\cdot(\rho_1-\rho_2)}. \quad (86)$$

Now we evaluate Eq. (65). We define $\mathcal{S}$ as the summation over iq_m of the important factors

$$\mathcal{S} = \frac{1}{\beta}\sum_{iq_m}\frac{1}{\Delta_L - iq_m}\frac{1}{\Delta_R + iq_m - i\omega_n} \times \frac{(2\omega_q)^2}{[(iq_m)^2-\omega_q^2][(iq_m-i\omega_n)^2-\omega_q^2]}, \quad (87)$$

$$\Delta_L = E_{Li} - E_{Lf}, \quad \Delta_R = E_{Ri} - E_{Rf}. \quad (88)$$

The fourth-order ground state energy is

$$E_{GS,4} = \left(\frac{e\hbar}{2m}\right)^4 \sum_{\mathbf{q}} b_0^4 \sinh^4(pq)U_1(q)U_2(q)S, \quad (89)$$

$$U_1(q) = \frac{1}{AL^2}\sum_{\mathbf{k}_1k_{1i}k_{1f}} \Xi V^2(k_{1i},k_{1f})[n_F(E_{1i}) - n_F(E_{1f})], \quad (90)$$

where b_0 is defined in Eq. (58). The dimensional units of $(e\hbar b_0/m)$ is energy times volume. The summation over iq_m in $\mathcal{S}$ is evaluated [38], giving the result

$$-\mathcal{S} = \frac{(2\omega_q)^2}{\Delta_L+\Delta_R-i\omega_n}\left[\frac{n_B(-\Delta_R)}{[\Delta_R^2-\omega_q^2][(i\omega_n-\Delta_R)^2-\omega_q^2]} - \frac{n_B(\Delta_L)}{[\Delta_L^2-\omega_q^2][(i\omega_n-\Delta_L)^2-\omega_q^2]}\right] + n_B(\omega_q)T_1 + [1+n_B(\omega_q)]T_2, \quad (91)$$

$$T_1 = \frac{2\omega_q}{[\Delta_L-\omega_q][\Delta_R+\omega_q-i\omega_n][i\omega_n-2\omega_q]i\omega_n} + \frac{2\omega_q}{[\Delta_R+\omega_q][\Delta_L-\omega_q-i\omega_n][i\omega_n+2\omega_q]i\omega_n}, \quad (92)$$

$$T_2 = \frac{2\omega_q}{[\Delta_R-\omega_q][\Delta_L+\omega_q-i\omega_n][i\omega_n-2\omega_q]i\omega_n} + \frac{2\omega_q}{[\Delta_L+\omega_q][\Delta_R-\omega_q-i\omega_n][i\omega_n+2\omega_q]i\omega_n}. \quad (93)$$

The next steps are to (i) set $i\omega_n \to \omega + i\varepsilon$, (ii) take the imaginary part, and (iii) set $\omega = 0$. We are interested in energy transfer, so are interested in energy terms that are of order $O(k_BT)$. This rules out δ functions such as $\delta(0)$, $\delta(\omega \pm 2\omega_q)$, and $\delta(\Delta_{R,L} \pm \omega_q)$. The only term that can transfer thermal energy is the first term:

$$\mathrm{Im}\{\mathcal{S}\} = -\pi\frac{(2\omega_q)^2[1+n_B(\Delta_L)+n_B(\Delta_R)]}{[\Delta_L^2-\omega_q^2][\Delta_R^2-\omega_q^2]}\delta(\Delta_L+\Delta_R). \quad (94)$$

The above expression is a zero in thermal equilibrium when $\Delta_L + \Delta_R = 0$. Energy transfer can occur when $T_L \neq T_R$. Assume the temperature difference is small: $T_L = T + \Delta T/2$, $T_R = T - \Delta T/2$. Expand the above expression in powers of ΔT and keep the first nonzero term:

$$1 + n_B(\Delta_L) + n_B(\Delta_R) \approx n_B(\Delta_L)n_B(\Delta_R)\frac{\Delta T}{2k_BT^2}(\Delta_R - \Delta_L). \quad (95)$$

Next we add the electron occupation numbers in Eq. (74):

$$[n_F(E_{Li}) - n_F(E_{Lf})][n_F(E_{Ri}) - n_F(E_{Rf})]n_B(\Delta_L)n_B(\Delta_R) = [1-n_F(E_{Lf})]n_F(E_{Li})[1-n_F(E_{Rf})]n_F(E_{Ri}). \quad (96)$$

Next we insert another integral over energy. The energy transfer per area is $\Delta_L = -\Delta_R$:

$$\delta(\Delta_L + \Delta_R) = \int dE\, \delta(\Delta_L - E)\delta(\Delta_R + E), \quad (97)$$

$$\left(\frac{\partial E}{\partial t}\right) = \frac{2\pi \Delta T}{\hbar k_B T^2 A} \sum_{\mathbf{q}} (\hbar\omega_q)^2 \left(\frac{e\hbar}{m} b_0 \sinh(pd)\right)^4 \times \int dE \frac{E^2}{[E^2 - (\hbar\omega_q)^2]^2} F_L(E) F_R(-E),$$

$$F_L(E) = \frac{1}{2AL^2} \sum_{\mathbf{k}k_i k_f} \Xi V(k_i,k_f)^2 \delta(\Delta_L - E) \times [1 - n_F(E_{Lf})] n_F(E_{Li}). \quad (98)$$

We now provide a series of steps to show that this term is small compared to the electron-electron term in the prior section. The integral over $\int dE$ confines values of $E \sim \mu + O(k_B T)$, where μ is the chemical potential. Write $F_L(E)$ as

$$F_L(E) = \frac{m}{2\pi^3} \int d^2k\, dk_i dk_f\, \Xi V^2 \delta\big(q^2 + 2\mathbf{k}\cdot\mathbf{q} + k_f^2 - k_i^2 + q_E^2\big) n_F(E_i)[1 - n_F(E_i - E)],$$

$$q_E^2 = \frac{2mE}{\hbar^2}, \quad (99)$$

The factor $\mathbf{q}\cdot\mathbf{k} = qk\cos(\theta)$ in the δ function eliminates the δ function when performing the angular integral,

$$\int_0^{2\pi} d\theta\, \delta[A + B\cos(\theta)] = \frac{2}{\sqrt{B^2 - A^2}}, \quad (100)$$

$$= \frac{2}{\sqrt{4q^2k^2 - \big(q^2 + k_f^2 - k_i^2 + q_E^2\big)}}. \quad (101)$$

The integral $\int k\, dk = (m/\hbar^2)\int d\varepsilon_k = (m/\hbar^2)\int d\xi$, where $\xi = \varepsilon_k - \mu$, where μ is the chemical potential. The integral over $\int d\xi$ is dominated by the occupation factors, and limits values of $\xi \sim k_B T$:

$$\int d\xi\, n_F(E_i)[1 - n_F(E_i - E)] = E n_B(E), \quad (102)$$

$$F_L(E) \approx \frac{m^2 E n_B(E)}{\pi^3 \hbar^4} \int dk_i dk_f \times \frac{\Xi V^2(k_i,k_f)}{\sqrt{4q^2\big(k_F^2 - k_i^2\big) - \big(q^2 + k_f^2 - k_i^2 + q_E^2\big)^2}}$$

$$\Xi = \left[\frac{K_= k_-}{\gamma}\varepsilon\frac{\omega^2}{c^2} + \gamma q_E^2\right]^2 + q^2\left(\varepsilon\frac{\omega^2}{c^2}\right)^2. \quad (103)$$

Combining the factors $E n_B(E)$ from $F_l F_R$ gives the integral

$$\int dE\, E^4 n_B(E) n_B(-E) = (k_B T)^5 \frac{\pi^4}{120}. \quad (104)$$

The latter integral is identical to the one in electron-electron interactions. However, it ignores the factors of $[(\hbar\omega_q)^2 - E^2]^2$ in the formula for dE/dt, and the factors of E in Ξ. The former we neglect since $\hbar\omega_q \gg E \sim k_B T$. The factors in Ξ actually make this surface plasmon contribution much smaller than the one from electron-electron interactions. The terms in Ξ with $O(1/c^2)$ are very small since they are basically $O(1/mc^2)$, while the terms with q_E^2 have another factor of $(k_B T/\mu)$ and are small. So we conclude that the surface plasmon contribution to the tunneling of heat between two metals is negligible for small gap sizes compared to the near field terms from electron-electron interactions.

IV. DISCUSSION

We have calculated the tunneling of heat between two parallel metal surfaces. Two processes were evaluated: (i) electron-electron interactions between low energy charge fluctuations and (ii) the exchange of energy by a virtual process using surface plasmon polaritons. We find that the electron-electron process provides the larger energy transfer when the gap sizes are few nanometers.

The experiments of Chen's group [25] measured the heat transfer between two insulators, or an insulator and a semiconductor. In these cases the heat is carried by phonons [29]. The tunneling of heat occurs by optical phonon to optical phonon, because of the long range of electric fields they generate. We showed earlier that both bulk and surface phonons contribute to these forces [44]. The experiments in Refs. [1,20,22–24,26] were between two metals. Our theory applies to the experiments with small air gaps [23,24]. Chen's group [25] measured the tunneling between a metal and an insulator, and our prior theory [27,28] describes this case.

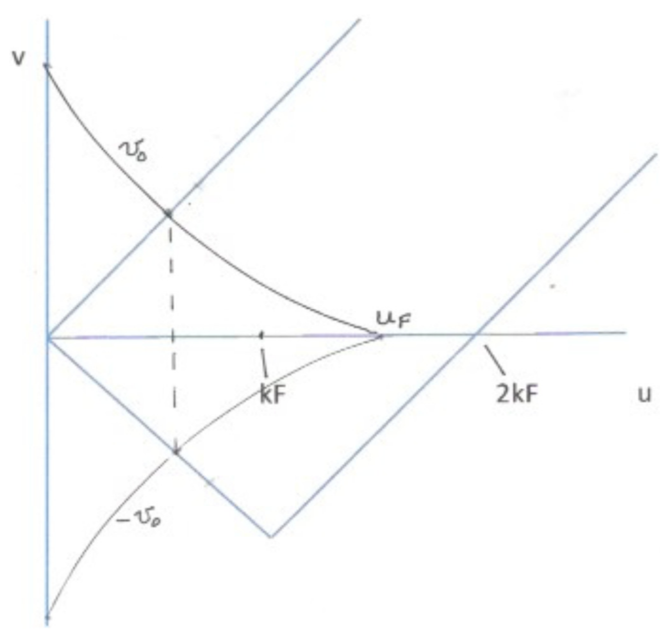

FIG. 2. Integration limits for $\int du\, dv$ are the diamond-shaped area. The vertical dashed line is at u_0.

APPENDIX

1. Evaluation of Im{$\Lambda(q,\omega)$}

We write in SI units

$$\Lambda(\mathbf{q},\omega) = \frac{e^2 q}{\varepsilon_0 A L^2}\sum_{k_i,k_f,\mathbf{k}} n_F(E_i) I^2(q;k_i,k_f)\left[\frac{1}{E_i - E_f - \hbar\omega} + \frac{1}{E_i - E_f + \hbar\omega}\right].$$

Set $\omega \to \omega + i\delta$ and take twice the imaginary part, which is called $A_\Lambda(q,\omega)$:

$$A_\Lambda(\mathbf{q},\omega) = -\frac{2\pi e^2 q}{\varepsilon_0 A L^2}\sum_{k_i,k_f,\mathbf{k}} n_F(E_i) I^2(q;k_i,k_f)[\delta(E_i - E_f - \hbar\omega) - \delta(E_i - E_f + \hbar\omega)].$$

Write the argument of the δ functions as $\hbar^2(q^2 + 2\mathbf{q}\cdot\mathbf{k} + k_f^2 - k_i^2 \pm q_0^2)/2m$, where $q_0^2 = 2m\omega/\hbar$. First evaluate the integral $\int d^2k$:

$$\int \frac{d^2k}{(2\pi)^2} n_F(E_i)[\delta(E_i - E_f - \hbar\omega) - \delta(E_i - E_f + \hbar\omega)]$$
$$= \frac{2m}{(4\pi q\hbar)^2}\left[\sqrt{4q^2(k_F^2 - k_i^2) - A_+^2} - \sqrt{4q^2(k_F^2 - k_i^2) - A_-^2}\right], \tag{A1}$$

$$A_\pm = A \pm q_0^2, \quad A = q^2 + k_f^2 - k_i^2. \tag{A2}$$

We are interested in energy transfer at thermal energies, which involves small values of ω. So we expand the above expression in powers of q_0^2 and retain only the first term,

$$A_\Lambda(\mathbf{q},\omega) \approx \omega B_\lambda(q), \tag{A3}$$

$$B_\Lambda(q) = \frac{e^2 m^2}{\pi\varepsilon_0 q\hbar^3} J(q), \tag{A4}$$

$$J(q) = \frac{1}{L^2}\sum_{k_i,k_f} I^2(q;k_i,k_f)\frac{A}{\sqrt{4q^2(k_F^2 - k_i^2) - A^2}}. \tag{A5}$$

We evaluate the integral $J(q)$ by making a change of variables $u = (k_f + k_i)$, $v = (k_f - k_i)$. The new limits are

$$2\int_0^{k_F} dk_i \int_0^\infty dk_f = \int_0^{k_F} du \int_{-u}^{u} dv + \int_{k_F}^\infty du \int_{u-2k_F}^{u} dv. \tag{A6}$$

Other limits are imposed by the requirement that the argument of the square root must be positive:

$$0 < 4q^2k_F^2 - q^4 - \left(k_f^2 - k_i^2\right)^2 - 2q^2\left(k_i^2 + k_f^2\right), \tag{A7}$$

$$0 < 4q^2k_F^2 - (q^2 + u^2)(q^2 + v^2), \tag{A8}$$

$$0 < (u^2 + q^2)\left(v_0^2 - v^2\right), \tag{A9}$$

$$v_0^2 = \frac{4q^2k_F^2}{q^2 + u^2} - q^2. \tag{A10}$$

Define $u_F^2 = 4k_F^2 - q^2$, and $u_F > u$ for $v_0^2 > 0$. This results in the integration limits shown in Fig. 2,

$$\int_0^{u_0} du \int_{-u}^{u} dv + \int_{u_0}^{u_F} du \int_{-v_0}^{v_0} dv, \tag{A11}$$

$$u_0 = \sqrt{q(2k_F - q)}, \tag{A12}$$

and u_0 is where $u_0 = v_0(u_0)$,

$$J(q) = \frac{q^2}{\pi^2}\left[\int_0^{u_0} du \int_0^{u} dv + \int_{u_0}^{u_F} du \int_0^{v_0} dv\right]\frac{1}{\sqrt{4q^2k_F^2 - (q^2 + u^2)(q^2 + v^2)}}\left[\frac{1}{q^2 + u^2} - \frac{1}{q^2 + v^2}\right]^2. \tag{A13}$$

The integrand is symmetric in u and v. We use that feature to change the integration limits to

$$J(q) = \frac{q^2}{\pi^2}\int_0^{u_F} du \int_0^{v_0} dv \frac{1}{\sqrt{4q^2k_F^2 - (q^2 + u^2)(q^2 + v^2)}}\left[\frac{1}{(q^2 + u^2)^2} - \frac{1}{(q^2 + v^2)(q^2 + u^2)}\right]. \tag{A14}$$

Then let $v = v_0 \sin(\phi)$,

$$J(q) = \frac{q^2}{\pi^2}\int_0^{u_F} du \int_0^{\pi/2} d\phi \frac{1}{(q^2+u^2)^{3/2}}\left[\frac{1}{q^2+u^2} - \frac{1}{q^2+v_0^2\sin^2(\phi)}\right] \tag{A15}$$

$$= \frac{q^2}{2\pi}\int_0^{u_F} du\left[\frac{1}{(q^2+u^2)^{5/2}} - \frac{1}{2k_F q^2(q^2+u^2)}\right] \tag{A16}$$

$$= \frac{1}{2\pi q^2}\left[\frac{1}{3}(2+y^2)\sqrt{1-y^2} - y\,\tan^{-1}\left(\frac{\sqrt{1-y^2}}{y}\right)\right], \tag{A17}$$

$$B_\Lambda(q) = \frac{\hbar}{\pi(a_0 q)\varepsilon(q)}\left[\frac{1}{3}(2+y^2)\sqrt{1-y^2} - y\,\tan^{-1}\left(\frac{\sqrt{1-y^2}}{y}\right)\right], \tag{A18}$$

where $y = q/2k_F$. The result goes as $O(q^{-3})$.

2. Evaluation of Im{$U_b(q,\omega)$}

In Eq. (14) we change variables in the terms $n_F(E_{f\pm})$ so they become $n_F(E_i)$:

$$U_b = \frac{e^2}{\varepsilon_0\Omega}\sum_{\mathbf{k},k_i} n_F(E_i)\left\{\frac{1}{q^2+(2k_i+q_z)^2}\left[\frac{1}{E_i-E_{f+}-\hbar\omega-i\eta} + \frac{1}{E_i-E_{f+}+\hbar\omega+i\eta}\right]\right.$$
$$\left. + \frac{1}{q^2+(2k_i-q_z)^2}\left[\frac{1}{E_i-E_{f-}-\hbar\omega-i\eta} + \frac{1}{E_i-E_{f-}+\hbar\omega+i\eta}\right]\right\}. \tag{A19}$$

Take minus two times the imaginary part to get the spectral function:

$$A_{Ub}(q,\omega) = \frac{2\pi e^2}{\varepsilon_0\Omega}\sum_{\mathbf{k},k_i} n_F(E_i)\left\{\frac{1}{q^2+(2k_i+q_z)^2}[\delta(E_i-E_{f+}-\hbar\omega) - \delta(E_i-E_{f+}+\hbar\omega)]\right.$$
$$\left. + \frac{1}{q^2+(2k_i-q_z)^2}[\delta(E_i-E_{f-}-\hbar\omega) - \delta(E_i-E_{f-}+\hbar\omega)]\right\}. \tag{A20}$$

The first integral has the form

$$\int \frac{d^2k}{(2\pi)^2}\delta(E_i-E_{f+}-\hbar\omega) = \frac{2m}{\hbar^2}\int \frac{d^2k}{(2\pi)^2}\delta\left[q^2+2qk\,\cos(\theta)+q_z(q_z+2k_i)+q_0^2\right]$$
$$= \frac{m}{\pi^2\hbar^2}\int \frac{k\,dk}{\sqrt{4q^2k^2-\left(A_+ + q_0^2\right)^2}} \tag{A21}$$

$$= \frac{m}{4\pi^2\hbar^2 q^2}\sqrt{4q^2\left(k_F^2-k_i^2\right)-\left(A_+ + q_0^2\right)^2}, \tag{A22}$$

$$A_+ = q^2 + q_z(q_z+2k_i), \quad q_0^2 = \frac{2m\omega}{\hbar}. \tag{A23}$$

We expand this for small value of $q_0^2 \propto \omega$, and retain only the first term:

$$A_{Ub}(q,\omega) \approx \omega B_{Ub}(q), \tag{A24}$$

$$B_{Ub}(q) = \frac{4\hbar}{a_0\varepsilon(q)}\tilde{J}(q), \tag{A25}$$

$$\tilde{J}(q) = \int_0^{k_F} dk_i\left[\frac{1}{\sqrt{4q^2k_F^2-Q^2(q^2+\xi_+^2)}}\frac{q^2+q_z\xi_+}{q^2+\xi_+^2} + \frac{1}{\sqrt{4q^2k_F^2-Q^2(q^2+\xi_-^2)}}\frac{q^2-q_z\xi_-}{q^2+\xi_-^2}\right], \tag{A26}$$

$$Q^2 = q^2+q_z^2, \quad \xi_\pm = 2k_i \pm q_z. \tag{A27}$$

Change integration variables from k_i to $\xi_\pm$:

$$\tilde{J}(q) = \frac{1}{2Q}\int_{q_z}^{2k_F+q_z} d\xi_+ \frac{q^2+q_z\xi_+}{(q^2+\xi_+^2)\sqrt{V^2-\xi_+^2}} + \frac{1}{2Q}\int_{-q_z}^{2k_F-q_z} d\xi_- \frac{q^2-q_z\xi_-}{(q^2+\xi_-^2)\sqrt{V^2-\xi_-^2}}, \tag{A28}$$

$$V^2 = \frac{4q^2k_F^2}{Q^2} - q^2. \tag{A29}$$

In the second integral change $\xi_- \to -\xi_+$ and combine the integrals,

$$\tilde{J}(q) = \frac{1}{2Q}\int_{q_z-2k_F}^{2k_F+q_z} d\xi \frac{q^2+q_z\xi}{(q^2+\xi^2)\sqrt{V^2-\xi^2}}. \tag{A30}$$

There are two possible limits to the integration. One is the one shown, $q_z + 2k_F < \xi < q_z - 2k_F$, and the second is $-V < \xi < V$. The latter is binding, since $V^2 < (2k_F \pm q_z)^2$ leads to $0 < (Q^2 \pm 2k_F q_z)^2$:

$$\tilde{J}(q) = \frac{1}{2Q}\int_{-V}^{V} d\xi \frac{q^2+q_z\xi}{(q^2+\xi^2)\sqrt{V^2-\xi^2}}. \tag{A31}$$

The term in the integrand $q_x\xi$ averages to zero. Change integration variables $\xi = V\ \sin(\phi)$,

$$\tilde{J}(q) = \frac{1}{2Q}\int_{-\pi/2}^{\pi/2} d\phi \frac{q^2}{q^2+V^2\sin^2(\phi)} \tag{A32}$$

$$= \frac{\pi}{4k_F}, \tag{A33}$$

$$B_{Ub} = \frac{\pi\hbar}{a_0 k_F \varepsilon(q)} = \frac{\pi\hbar}{4y^2 E_F(a_0k_F)}. \tag{A34}$$

The result for the Lindhard/RPA dielectric function is well known:

$$B_\chi = \frac{e^2m^2}{\pi\varepsilon_0 q^3} = \frac{\hbar}{4y^3E_Fk_Fa_0}. \tag{A35}$$

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