Superconductor-antiferromagnet-superconductor π Josephson junction based on an antiferromagnetic barrier

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We show theoretically that π Josephson junctions may be constructed by use of antiferromagnetic (AF) metals between superconducting electrodes. We argue that the AF magnetic ordering introduces the energy difference of electrons in a Cooper pair due to the effect of the exchange field varying in space. Such an energy difference is quadratic in the amplitude of exchange field and this is sufficient to change the behavior of a Josephson junction from 0 to π junction if the width of the AF metal is big enough. The advantage of using an AF barrier instead of a ferromagnetic one is that it does not suppress Cooper pairing in superconducting electrodes as much as the ferromagnet barrier does. However, to reach π-junction regime the AF metal should be a clean one with the electron mean free path bigger than the junction width.

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Initially a ferromagnetic barrier between superconductors was proposed to produce a S-F-S Josephson junction with phase difference π in the ground state [1]. This proposal was realized experimentally by Ryazanov *et al.* [2] by using a CuNi alloy. The π junction can be used as a battery to generate phase differences in superconducting circuits or as a storage of digital information. This is due to the fact that the shortening of the superconducting electrodes by superconducting wire results in the spontaneous supercurrent circulating in the loop formed by the junction and the wire.

The idea behind the use of a ferromagnetic metal as a barrier in Josephson junction in order to change the sign of the critical current was based on the following arguments. The exchange field h inherent to ferromagnets splits energies of electrons with opposite spins in Cooper pair by the amount $2\mu h$, where μ is the magnetic moment of conducting electrons in the ferromagnetic metal. As a result, the Cooper pair moving in the x direction with the Fermi velocity v_F acquires the phase difference $\varphi_f = 2\mu h d/\hbar v_F$, while the pair moving in the opposite direction acquires the phase difference $-\varphi_f$. Here d is the width of the ferromagnetic barrier between superconducting electrodes, and for simplicity we take in the following $\hbar = 1$ and $\mu = 1$. As a result, due to the presence of the exchange field, the Josephson current between the superconducting electrodes is modified from $\propto \sin\varphi$ to $\propto (1/2)[\sin(\varphi+\varphi_f)+\sin(\varphi-\varphi_f)] = \sin\varphi\cos\varphi_f$. It changes sign as the product hd increases. Correspondingly, the Josephson junction switches periodically from a 0 to π junction as long as d is smaller than the electron mean free path.

Here we will show that the ferromagnetic barrier in the Josephson junction may be replaced by that of a metallic antiferromagnet. Such a barrier affects superconducting electrodes much less than the ferromagnetic one. Indeed, the destruction effect of AF exchange field on the Cooper pairing is not so strong as that of ferromagnets because the average exchange field acting on the conducting electrons, due to magnetic moments, vanishes in the AF. Although the first order effect in exchange field h is absent in the AF, the second order effects in h still change the behavior of Cooper pairs entering the AF metal. They result in the splitting of the Cooper electron energies in the AF metal by the amount ϵ_a which is quadratical in h. Such a splitting may be sufficient to produce an oscillatory dependence of the critical current on the exchange field, if the AF width d is smaller than the electron mean free path ℓ_s. We will show in the following that the transverse spiral exchange field $\mathbf{h}_e(\mathbf{r})$ with the wave vector $\mathbf{q}$ along the z axis,

$$\mathbf{h}_e(\mathbf{r}) = 2h[\cos(\mathbf{q}\cdot\mathbf{r}), \sin(\mathbf{q}\cdot\mathbf{r}), 0], \tag{1}$$

splits the energies of the Cooper pair electrons inside the AF by a value $\epsilon_a \approx (v_F q/2)[(4h^2/v_F^2 q^2 + 1)^{1/2} - 1]$ resulting in the acquired phase difference $\varphi_a = 2\epsilon_a d/v_F$ and thus in the additional oscillating factor $\cos\varphi_a$ in the critical current. This result is correct when $v_F q \ll \epsilon_F$ (ϵ_F is the Fermi energy), and this limit will be considered in the following. It was predicted previously in Ref. [3] that in this limit and at $h \ll v_F q$ the energy difference $\epsilon_a \approx h^2/(v_F q)$ of the Cooper pair electrons in clean superconducting antiferromagnets results in the formation of a nonuniform FFLO-like state below the Neel temperature T_N when $T_N < \epsilon_a$. In fact, formation of nonuniform state in bulk AF superconductors and of the phase difference φ_a in the Josephson system S-AF-S are both consequences of the energy splitting ϵ_a of Cooper pair electrons in an antiferromagnetically ordered system.

Obviously, the AF coordinate dependent exchange field results in the net phase difference φ_a only if the electron mean free path ℓ is bigger than d. Hence, sufficiently clean antiferromagnets may be used to produce a π junction. The condition $\ell \geqslant d$ is a disadvantage of AF barrier at low h in comparison with a ferromagnetic one, but the advantage is that superconductivity in the electrodes is damaged much weaker. One can hope that a rich variety of antiferromagnetic metals provides a broad choice of parameters h, q to fulfill the condition $\ell_s \geqslant d$ and obtain the phase difference needed for π junction. In the following we will discuss rare earth metals with spiral structure like Ho metal with a very strong exchange field acting on conduction electrons. We will consider also antiferromagnetic metals with localized

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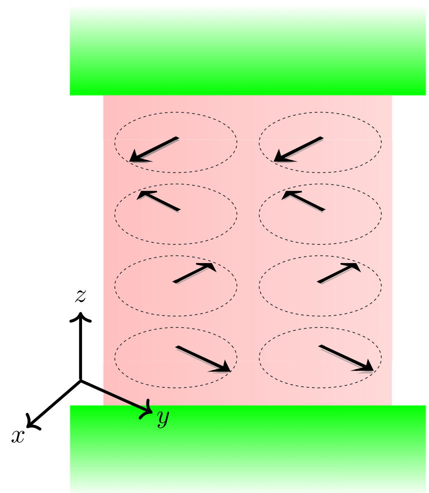

FIG. 1. The S-AF-S Josephson junction with spiral antiferromagnet between two BCS superconductors.

alternating magnetic moments like molibdenites $MMoS_8$, see Refs. [4–6] and borocarbides MNi_2B_2C (M is rare earth), where $h \ll v_F q$ since $q = \pi/a$ is large [7–9]. Here a is the distance between magnetic moments. The phase transition into antiferromagnetic phase in such crystals occurs at quite low Neel temperatures of the order 10–30 K which indicates not very strong alternating exchange AF field acting on conducting electrons. This allows us to give quantitative treatment of effect of AF ordering on the Josephson current. We note that molibdenites and borocarbide metals become superconducting and thus one can use them as a barrier in Josephson junction only at temperatures higher than their superconducting critical temperatures.

We consider S-AF-S Josephson junction made of alternating BCS superconductor (S), normal state metal with antiferromagnetic spiral ordering (AF) and another BCS superconductor, see Fig. 1. The length of the AF part along the z axis is d, and its boundaries are at $z = -d/2$ and $z = +d/2$. We will use the Bogolyubov-de Gennes equation to describe this system, and we find the Josephson current through the junction at a given phase difference φ of the superconductors S. Previously a similar approach was used for S-F-S junction [1,10,11], and the formation of a π junction was successfully predicted in that way [1,10]. We will ignore the scattering of electrons due to impurities, i.e., we assume that the mean free path of electrons in AF $\ell \geqslant d$. We will follow notations and derivations of the Josephson current in the S-N-S junction presented in the book of Kopnin [12]. The Hamiltonian of the conducting electrons in the AF is

$$\hat{\mathcal{H}} = \mathcal{H}_e - \mathcal{H}_{ex}, \quad \mathcal{H}_e = \int d^3\mathbf{r}\hat{\psi}^+_\alpha(\mathbf{r})(\epsilon(\mathbf{p}) - \epsilon_F)\hat{\psi}_\alpha(\mathbf{r}),$$
$$\mathcal{H}_{ex} = 2\hat{\psi}^+_\alpha(\mathbf{r})h_i(\mathbf{r})(\sigma_i)_{\alpha\beta}\hat{\psi}_\beta(\mathbf{r}). \tag{2}$$

Here $\epsilon(\mathbf{p}) = \mathbf{p}^2/2m$ and m is the electron mass, while α,β are electron spin indexes and σ_i are the Pauli matrices, $\sigma_\pm = \sigma_x \pm i\sigma_y$. We consider transverse spiral spin ordering which induces exchange field $h_e(z)$ given by Eq. (1). In the Bogolyubov-de Gennes approach the wave functions $u(\mathbf{r})$ and $v(\mathbf{r})$ for electrons and holes, respectively, are determined by the equations

$$\mathcal{H}U(\mathbf{r}) + \Delta V(\mathbf{r}) = EU(\mathbf{r}), \tag{3}$$

$$\mathcal{H}^* V(\mathbf{r}) + \Delta^* U(\mathbf{r}) = -EV(\mathbf{r}), \tag{4}$$

$$\mathcal{H}^* = \mathcal{H}_e - \mathcal{H}^*_{ex},$$
$$\mathcal{H}^*_{ex} = 2h[\sigma_x \cos(qz) - \sigma_y \sin(qz)] = h(\sigma_+ e^{iqz} + \sigma_- e^{-iqz}), \tag{5}$$

where Δ is the superconducting order parameter, h is the amplitude of the spiral exchange field, and E is the electron energy. In a normal AF metal we take $\Delta = 0$. We consider a quasiparticle with a momentum parallel to the z axis and with up and down spins. The wave function of the quasiparticle is

$$U(\mathbf{r}) = \int dk_x dk_y e^{ik_x x + ik_y y}[U_\uparrow(z)\eta_\uparrow + U_\downarrow(z)\eta_\downarrow] \tag{6}$$

and similar for the hole wave function. Here $\eta_\uparrow$ and $\eta_\downarrow$ are spin functions for up and down spins, respectively. We obtain equations for the type 1 quasiparticles in the Fourier representation $U_\alpha(z) = \int dk_z \exp(ik_z z)U_\alpha(k_z)$:

$$\mathcal{H}_e(k_z)U_\uparrow(k_z) - hU_\downarrow(k_z + q) = \epsilon U_\uparrow(k_z), \tag{7}$$

$$\mathcal{H}_e(k_z + q)U_\downarrow(k_z + q) - hU_\uparrow(k_z) = \epsilon U_\downarrow(z), \tag{8}$$

$$\mathcal{H}_e(k_z) = k_z^2/2m - \epsilon_F, \tag{9}$$

and similar equations with reversed spins and signs of q for the type 2 quasiparticles. Here $\epsilon = E - E_{xy}$, where E_{xy} is the quasiparticle kinetic energy in the x,y plane, while E is that of motion along the z axis. In the following we will use quasiparticle z-axis momentum accounted from the Fermi vector, $k = k_z - k_F$. We will see that important wave vectors contributing to the Josephson current are well below k_F at $h \ll \epsilon_F$. First we get results in the quasiclassical approximation assuming $q \ll k_F$ and later we will show corrections to this approximation. The wave function of the type 1 quasiparticles is a superposition of spin up and down contributions (the spiral exchange field does not conserve spin):

$$U_1(z) = e^{i(\epsilon+\epsilon_a)z/v_z}\left[\eta_\uparrow + \frac{h}{v_z q + \epsilon_a}e^{iqz}\eta_\downarrow\right], \tag{10}$$

with the dispersion relations for electrons in the AF

$$v_z k - \epsilon = -v_z q/2 \pm [(v_z q/2)^2 + h^2]^{1/2}. \tag{11}$$

Here and in the following we account only for energies $\epsilon \ll v_z q$, ignoring contributions from high energies $\epsilon \approx 2v_z q$. Typically they lay well above Δ, while the main contribution to the Josephson current at low temperatures comes from the Andreev bound states with energies $|\epsilon|$ below Δ. From Eq. (11) we see that k is indeed small, $k \ll k_F$ as was assumed early, at $h \ll \epsilon_F$ and $\epsilon \ll \Delta$. Then $v_z k - \epsilon = \epsilon_a$, where

$$\epsilon_a(v_z) \approx \frac{v_z q}{2}\left[\left(\frac{4h^2}{v_z^2 q^2} + 1\right)^{1/2} - 1\right]. \tag{12}$$

To obtain results beyond the quasiclassical approach we need to renormalize $v_z q$ as $v_z q(1 + q/2mv_z)$. For the hole wave function V_α the quasiclassical equations have the form

$$-iv_z\partial_z V_\uparrow(z) - he^{iqz}V_\downarrow(z) = -\epsilon V_\uparrow(z), \quad (13)$$

$$-iv_z\partial_z V_\downarrow(z) - he^{-iqz}V_\uparrow(z) = -\epsilon V_\downarrow(z). \quad (14)$$

We find

$$V_1(z) = e^{-i(\epsilon+\epsilon_a)z/v_z}\left[\eta_\uparrow - \frac{h}{v_z q + \epsilon_a}e^{-iqz}\eta_\downarrow\right]. \quad (15)$$

Solutions for the equations (13), (14) with reversed spins and opposite signs of q provide another set of quasiparticles of the type 2, (U_2, V_2), for which all spins in solutions for (U_1, V_1) should be reversed and $\epsilon_a(v_z)$ replaced by $-\epsilon_a(v_z)$:

$$U_2(z) = e^{i(\epsilon-\epsilon_a)z/v_z}\left[\eta_\downarrow - \frac{h}{v_z q - \epsilon_a}e^{-iqz}\eta_\uparrow\right], \quad (16)$$

$$V_2(z) = e^{-i(\epsilon-\epsilon_a)z/v_z}\left[\eta_\downarrow + \frac{h}{v_z q - \epsilon_a}e^{iqz}\eta_\uparrow\right]. \quad (17)$$

The next step is to find the Andreev bound states for electrons and holes inside the antiferromagnet. We denote the phase difference between top and bottom superconductors by φ. For electrons moving up in the top superconductor the functions $U_\alpha(z)$ and $V_\alpha(z)$ for the energies $\epsilon < \Delta$ decay as

$$U_1 = e^{-\lambda_S z}\left[\eta_\uparrow + \frac{h}{v_z q + \epsilon_a}e^{iqd/2}\eta_\downarrow\right]U_0\exp(i\varphi/4), \quad (18)$$

$$V_1 = e^{-\lambda_S z}\left[\eta_\uparrow - \frac{h}{v_z q + \epsilon_a}e^{-iqd/2}\eta_\downarrow\right]V_0\exp(-i\varphi/4), \quad (19)$$

$$\lambda_S = \frac{\sqrt{|\Delta|^2-\epsilon^2}}{v_z}, \quad U_0, V_0 = \frac{1}{\sqrt{2}}\left(1 \pm i\frac{\sqrt{|\Delta|^2-\epsilon^2}}{\epsilon}\right)^{1/2}. \quad (20)$$

Here we assume conservation of spin at the boundary AF-S. We find the wave functions of the type 1 quasiparticle moving up in the AF region and the corresponding reflected quasiparticle moving down as

$$U_1(z) = A\exp\left(i\frac{(\epsilon+\epsilon_a)z}{v_z}\right)\left[\eta_\uparrow + \frac{h\exp(iqz)}{v_z q + \epsilon_a}\eta_\downarrow\right], \quad (21)$$

$$V_1(z) = AR_1\exp\left(-i\frac{(\epsilon+\epsilon_a)z}{v_z}\right)\left[\eta_\uparrow - \frac{h\exp(-iqz)}{v_z q + \epsilon_a}\eta_\downarrow\right], \quad (22)$$

and similar for the type 2 solution. Here A is the normalization factor and R is the reflection coefficient. Continuity of the wave function across the boundary between AF and the top superconductor for the type 1 quasiparticle leads to the relations

$$A\exp[i(\epsilon+\epsilon_a(v_z))d/2v_z] = U_0\exp(-\lambda_s d/2 + i\varphi/4),$$

$$AR_1\exp[-i(\epsilon+\epsilon_a(v_z))d/2v_z] = V_0\exp(-\lambda_s d/2 - i\varphi/4).$$

These equations give the reflection coefficient for the type 1 quasiparticles:

$$R_1 = (V_0/U_0)\exp(i\epsilon d/v_z)\exp[-i(\varphi-\varphi_a)/2], \quad (23)$$

$$\varphi_a = 2\epsilon_a(v_z)d/v_z = 2\left[\left(h^2/v_z^2 + q^2/4\right)^{1/2} - q/2\right]d. \quad (24)$$

Continuity at the bottom interface gives for the same reflection coefficient the expression

$$R_1 = (U_0/V_0)\exp(-i\epsilon d/v_z)\exp[i(\varphi-\varphi_a)/2]. \quad (25)$$

Comparing these relations to those for normal junction S-N-S (obtained at $h = 0$) we see that the phase difference is renormalized due to the AF exchange field by the value $-\varphi_a(v_z)$ for the type 1 states and by the value $+\varphi_a(v_z)$ for the type 2 states. The spectra of the Andreev bound states are also renormalized in the same way. Equating R_1 given by Eqs. (23) and (25) we obtain the energies of bound states of the type 1 quasiparticles as

$$\epsilon(v_z) = \pm|\omega_z|\left[\frac{\varphi-\varphi_a}{2} \mp \arcsin\frac{\epsilon}{|\Delta|} + \pi\left(\ell \pm \frac{1}{2}\right)\right], \quad (26)$$

where $-\pi/2 < \arcsin(\epsilon/|\Delta|) < \pi/2$ and $\omega_z = d/v_z$, while ℓ is an integer. Here upper signs are for $v_z > 0$ and lower signs are for $v_z < 0$. For the type 2 quasiparticles we get Eq. (26) with opposite sign of φ_a.

Let us now consider the Josephson current in S-AF-S junction. The current is given as

$$I = -\frac{ie}{m}\sum_n [f_n u_n^*(\mathbf{r})\nabla u_n(\mathbf{r}) + (1-f_n)v_n(\mathbf{r})\nabla v_n^*(\mathbf{r}) - \text{c.c.}],$$

where n labels various quantum states and $f_n = 1/(\exp\epsilon_n/T + 1)$ is the Fermi-Dirac distribution function. The quantum number n describes states belonging to various quantum states k_x, k_y, and $k_z(\epsilon)$ within the area S of the junction. Applying the semiclassical approximation, we calculate the derivatives only of the rapidly varying functions $\exp(i\mathbf{k}_F\cdot\mathbf{r})$. At low temperatures one needs to account for the Andreev states with energies $\epsilon < |\Delta|$.

In S-N-S junction the conditions of a small or large width of the normal region depends on whether ω_z is large or small in comparison with $|\Delta|$. Due to the condition $-\pi/2 < \arcsin(\epsilon/|\Delta|) < \pi/2$ at $d < \xi_0$ only $\ell = -1$ for $v_z > 0$ and $\ell = 0$ for $v_z < 0$ remain as solutions. For a junction with AF barrier such a short-junction definition becomes meaningless due to the renormalization of the phase difference by the term φ_a. At $h \gg |\Delta|$ the phase φ_a can be large in comparison with φ even for $d \ll \xi_0 = v_F/\Delta$. Hence, we are not necessarily limited to the values $\ell = -1$ for $v_z > 0$ and $\ell = 0$ for $v_z < 0$ as in the case without the AF exchange field (see, e.g., Ref. [12]). As a result, the number of states contributing to the Josephson current depends on φ_a. Hence, the simple result (at $T = 0$)

$$I_z = \frac{\pi|\Delta|}{eR_N}\sin\frac{\varphi}{2} \quad (27)$$

of Ref. [12] for point contact S-N-S is valid in the case of the AF barrier only when $\varphi_a(v_F) \ll 1$.

To find the Josephson current in long junctions, $d \gg \xi_0$, we follow derivations in Ref. [12]. We account first for the type 1 states. It follows from expression (3.39) of Ref. [12] that the dependence of the Josephson current on the phase difference is renormalized by $\varphi_a(v_z)$:

$$I_z = -\sum_{v_z>0} \frac{v_z e}{2d} \sum_{\ell=-\ell_0}^{\ell_0} \tanh \frac{|\omega_z|[\varphi - \varphi_a(v_z) - \pi + 2\pi\ell]}{4T}. \quad (28)$$

Here $\ell_0 = |\Delta|/(\pi|\omega_z|) \gg 1$. For the type 2 states we obtain a similar expression but with an opposite sign of φ_a. Summing them up and replacing summation over ℓ by summation over Matsubara frequencies, as described in Ref. [12], we obtain

$$I_z = 8Te \sin\varphi \sum_{v_z>0} \cos[\varphi_a(v_z)] \exp(-2\pi T d/v_z). \quad (29)$$

At low temperatures $v_F/2\pi T d \ll 1$ the Josephson current via AF barrier is

$$I = I_c \sin\varphi \cos\varphi_a, \quad \varphi_a = qd[(\alpha^2+1)^{1/2} - 1], \quad (30)$$

$$\alpha = 2h/v_F q, \quad (31)$$

$$I_c = \frac{4v_F}{eR_N} d \exp(-d/\xi), \quad \xi = v_F/2\pi T, \quad (32)$$

where R_N is the resistance of the AF barrier in the normal state. The factor $\cos(\varphi_a)$ oscillates and changes sign as the product hd increases resulting in a 0- or in a π-junction.

Next we will discuss the range of parameters when our approach is valid. At $q = 0$ equation (30) provides result for clean ferromagnetic barrier, $\varphi_f = 2hd/v_F$. It was shown in Ref. [10] that such a result for S-F-S junction holds if $h\tau_s \gg 1$ or $d \ll \ell = v_F\tau_s$, where $\tau_s = \ell_s/v_F$ is the impurity scattering time. If $h\tau_s \ll 1$ the oscillatory dependence of the Josephson current on the width d vanishes on the characteristic length $(D/h)^{1/2}$, where $D = v_F^2\tau_s/3$ is the diffusion coefficient. These conditions are due to the fact that the accumulation of phase on the length L is $2hL/v_F$, while the dispersion of the phase accumulation is of the order of hDL/v_F. For a coordinate dependent AF exchange field the situation is more complicated. The energy difference ϵ_a is now taken at distances $r \gg 2\pi/q$ and thus the electron mean free path should be bigger than $2\pi/q$. In addition, as in the case of a ferromagnet, one needs $\epsilon_a d/v_F \gg 1$. Thus the electron mean free path should be bigger than both v_F/ϵ_a and $2\pi/q$.

Now we will discuss whether known antiferromagnets may be useful as a candidate for the Josephson π-junction S-AF-S. Two families of metallic antiferromagnets are well studied: rare earth metals and crystals like rare earth borocarbides. Let us consider first the AF with a short period of the exchange field and thus small α. It is the case of borocarbide antiferromagnets MNi_2B_2C with a square lattice of alternating magnetic moments M such as Er. They have large $q = \pi/a$, where $a \approx 4$ Å is the distance between magnetic atoms (see Fig. 2). We may estimate the value of h by assuming that the antiferromagnetic ordering and thus the Neel temperature T_N is determined predominantly by RKKY coupling of spins mediated by the conduction electrons. Then for erbium borocarbide $T_N \approx h^2/\epsilon_F$, while the Fermi energy ϵ_F is of the same order as $v_F q$. Hence, the corresponding energy splitting of Cooper pair electrons is of the order of T_N. To reach π junction one needs $d \approx \pi v_F/T_N$ of the order 2×10^{-4} cm and the electron mean free path of the size or bigger than this value (we use the estimates $T_N = 5$ K and $v_F = 5\times10^7$ cm/s). Smaller d and ℓ may be needed if antiferromagnet crystals with larger T_N are used. Up to now we do not have precise information on the electron mean free path in borocarbide crystals and at this stage cannot conclude whether to use them in S-AF-S π junctions is realistic or not.

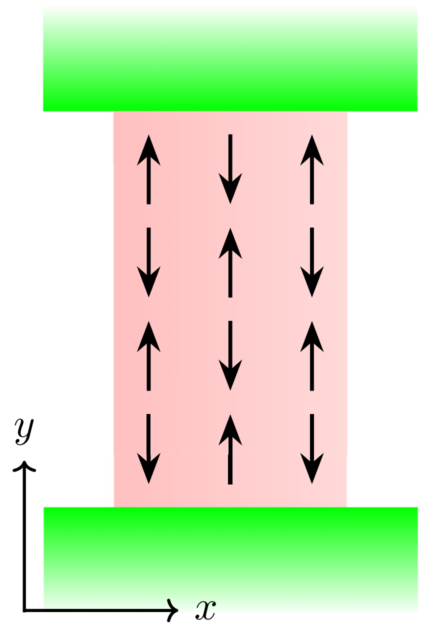

FIG. 2. The S-AF-S Josephson junction with metallic antiferromagnet between two BCS superconductors. The magnetic structure of the antiferromagnet is a square lattice of alternating up and down spins as in rare earth borocarbides.

Next we discuss the possibility to use rare earth metals with a larger period of helicoidal exchange field [13–15]. Information on Ho is most available. The Neel temperature of Ho is $T_N = 133$ K. Below T_N and down to the temperature 20 K the magnetic structure is a spiral with the wave vector $q = 2\pi/6c$ along the c axis, where $c = 5.6$ Å is the distance between the magnetic moments along this axis [14]. Below 20 K a conic structure with a ferromagnetic component along the c axis was observed in crystals, though it was absent in thin films [16]. According to the estimates in Ref. [17] in a conic phase, the ferromagnetic component of the exchange field along the c axis, h_f, is about 1.1 eV, while the amplitude h_s of the spiral field is about 7 eV (the observed cone angle is $\approx 80^\circ$). The amplitude of the exchange field in the spiral phase between 20 K and 50 K is also about h_s. Electron parameters in Ho metal were estimated as $v_F = 10^8$ cm/s and $\epsilon_F = 7.7$ eV. Thus the value $\alpha = 2h/\hbar v_F q = 10$ is large in comparison with unity. As a result, the energy splitting of Cooper pair electrons in such spiral phase, $\epsilon_a \approx 2h - v_F q \approx 13.6$ eV, is close to the Fermi energy. Then our results cannot be used to describe this situation because the assumed conditions $h, v_F q \ll \epsilon_F$ are not fulfilled.

In conclusion, we have shown that the exchange field in antiferromagnetic clean metals results in oscillating dependence of the critical current on the junction width and the amplitude of the exchange field. Thus the AF barriers may be used to transform 0 junctions into π junctions. We discussed S-AF-S junctions made of borocarbide crystals MNi_2B_2C with rare earth ions M. In this class we see that a barrier with the width of the order micron is needed to reach a π junction and the mean electron scattering path should be of the same size. For Ho metal as an AF barrier our analytical results cannot be used and less crude numerical methods should be applied to make definite conclusions.

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