Phase diagram of UCoGe

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The temperature-pressure phase diagram of ferromagnetic superconductor UCoGe includes four phase transitions. They are between the paramagnetic and the ferromagnetic states with the subsequent transition in the superconducting ferromagnetic state and between the normal and the superconducting states after which the transition to the superconducting ferromagnetic state has to occur. Here we have developed the Landau theory description of the phase diagram and established the specific ordering arising at each type of transition. The phase transitions to the ferromagnetic superconducting state are inevitably accompanied by the emergence of screening currents. The corresponding magnetostatics considerations allow for establishing the significant difference between the transition from the ferromagnetic to the ferromagnetic superconducting state and the transition from the superconducting to the ferromagnetic superconducting state.

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I. INTRODUCTION

The superconductivity (SC) in the uranium ferromagnetic (FM) compounds UGe_2 and URhGe discovered more than a decade ago [1,2] and more recently in the related compound UCoGe [3] is still the subject of quite active investigations (see recent experimental [4] and theoretical [5] reviews and references therein). The existence of superconducting states at temperatures far below the Curie temperature and very high upper critical fields in these materials does not leave doubts that here we deal with the triplet superconductivity, such as it is in the superfluid ^{3}He. This is also confirmed by the measurements of the Knight shift on the nucleus of ^{59}Co, which is proved unchanged in the superconducting state [6].

One of many peculiar properties of UCoGe is that the ferromagnetism in this compound is suppressed by pressure, whereas the superconductivity arising at small pressures inside of the ferromagnetic state continues to exist at high pressures in the paramagnetic state. The pressure-temperature phase diagram shown in Fig. 1 has been established first in the paper [7] and then confirmed in many subsequent studies (see, for instance, the last one [8]). The phase transition from the paramagnetic to the ferromagnetic state and following it the phase transition to the ferromagnetic superconducting state at low pressures and the phase transition from the normal to the superconducting state at high pressures are firmly registered. Whereas the phase transition from the superconducting to the ferromagnetic superconducting state shown in the Fig. 1 by the dashed line is still not confirmed experimentally.

A theoretical phase diagram description has been proposed recently by Cheung and Raghu [9]. Making use of the numerical calculations applied to the minimal Landau model of a neutral ferromagnetic superfluid state with one component order parameter for each spin up-up and spin down-down Cooper pair states they were able to reproduce the general structure of the UCoGe phase diagram and to predict a first-order phase transition near the boundary between the normal phase and the ferromagnetic superconducting phase.

Here I reconsider the same problem making use of the analytical calculations applied to the same minimal model for neutral ferromagnetic or nonmagnetic superfluid states. The results of Ref. [9] were confirmed.

A phase transition of the normal metallic to the superconducting state has its own specific properties different from the properties of a phase transition in the neutral Fermi liquid to the superfluid state. So, in the last part of the paper, I will discuss the significant difference between the two transitions, namely, between the phase transition from the ferromagnetic normal state to the ferromagnetic superconducting state and the transition from the superconducting state to the ferromagnetic superconducting state. This difference arises due to the essentially different screenings of the magnetic moment at these two transitions. In the latter case the screening is complete, and instead of a bulk phase transition, there is the gradual formation of the Meissner state as it occurs in a superconductor of the second kind under an external magnetic field smaller than H_{c1}.

II. MODEL

The triplet-pairing superconducting state order parameter is given by the complex spin vector [10],

$$\mathbf{d}(\mathbf{k},\mathbf{r}) = \frac{1}{2}[-\Delta^{\uparrow}(\mathbf{k},\mathbf{r})(\hat{x}+i\hat{y}) + \Delta^{\downarrow}(\mathbf{k},\mathbf{r})(\hat{x}-i\hat{y})] + \Delta^{0}(\mathbf{k},\mathbf{r})\hat{z}, \quad (1)$$

where $\Delta_{\uparrow}(\mathbf{k},\mathbf{r})$, $\Delta_{\downarrow}(\mathbf{r},\mathbf{k},\mathbf{r})$, and $\Delta_0(\mathbf{k},\mathbf{r})$ are the amplitudes of spin up, spin down, and zero spin of the superconducting order parameter depending on the Cooper pair center of gravity coordinate $\mathbf{r}$ and the momentum $\mathbf{k}$ of pairing electrons. In the tetragonal ferromagnets with an easy axis along the $\hat{z}$ direction there are only two superconducting states A and B with different critical temperatures [11]. The general form of the order parameter for the A state in a two-band spin-up spin-down superconducting ferromagnet,

$$\Delta_A^{\uparrow}(\mathbf{k},\mathbf{r}) = \hat{k}_x\eta_x^{\uparrow}(\mathbf{r}) + i\hat{k}_y\eta_y^{\uparrow}(\mathbf{r}),$$
$$\Delta_A^{\downarrow}(\mathbf{k},\mathbf{r}) = \hat{k}_x\eta_x^{\downarrow}(\mathbf{r}) + i\hat{k}_y\eta_y^{\downarrow}(\mathbf{r}), \quad (2)$$
$$\Delta_A^{0}(\mathbf{k},\mathbf{r}) = \hat{k}_z\eta_z^{0}(\mathbf{r})$$

depends on the five complex amplitudes $\eta_x^{\uparrow}$, $\eta_y^{\uparrow}$, $\eta_x^{\downarrow}$, $\eta_y^{\downarrow}$, and η_z^0, which obey coupled Ginzburg-Landau equations, derived in the linear approximation in the papers [5,12].

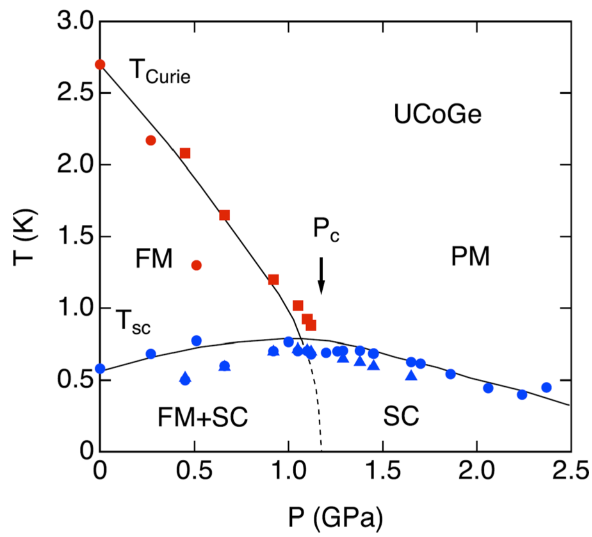

FIG. 1. Temperature-pressure phase diagram of UCoGe. Notations FM, SC, and PM used for ferromagnetic, superconducting, and paramagnetic phases correspondingly [4].

$\hat{k}_i = k_i/|\mathbf{k}|$, $i = x,y,z$ are the projections of the unit $\hat{\mathbf{k}}$ vector on the coordinate axis.

The order parameter of the paramagnetic superconducting state [5,13] in an orthorhombic metal looks like the order parameter of the superfluid ^{3}He-B phase [10],

$$\Delta^{\uparrow}(\mathbf{k},\mathbf{r}) = -\hat{k}_x\eta_x(\mathbf{r}) + i\hat{k}_y\eta_y(\mathbf{r}),$$
$$\Delta^{\downarrow}(\mathbf{k},\mathbf{r}) = \hat{k}_x\eta_x(\mathbf{r}) + i\hat{k}_y\eta_y(\mathbf{r}), \quad (3)$$
$$\Delta^{0}_{A}(\mathbf{k},\mathbf{r}) = \hat{k}_z\eta_z(\mathbf{r}).$$

To avoid excessive difficulties the authors of Ref. [9] considered the minimal model for the superconducting ferromagnetic state with the order parameter,

$$\Delta^{\uparrow}(\mathbf{k},\mathbf{r}) = \hat{k}_x\eta_{\uparrow}(\mathbf{r}),$$
$$\Delta^{\downarrow}(\mathbf{k},\mathbf{r}) = \hat{k}_x\eta_{\downarrow}(\mathbf{r}). \quad (4)$$

The corresponding simplest order parameter for the paramagnetic superconducting state looks like the order parameter for the recently discovered polar state of superfluid ^{3}He [14],

$$\Delta^{\uparrow}(\mathbf{k},\mathbf{r}) = -\hat{k}_x\eta(\mathbf{r}),$$
$$\Delta^{\downarrow}(\mathbf{k},\mathbf{r}) = \hat{k}_x\eta(\mathbf{r}). \quad (5)$$

In neglect of the interactions of electron charges with magnetic fields created by the magnetization one can write following Ref. [9] the gradient-independent Landau free-energy density as

$$F = \alpha M^2 + \beta M^4 + \alpha_1(|\eta_{\uparrow}|^2 + |\eta_{\downarrow}|^2) + \gamma_1 M(|\eta_{\uparrow}|^2 - |\eta_{\downarrow}|^2)$$
$$+ \gamma_2(\eta_{\uparrow}\eta_{\downarrow}^{\star} + \eta_{\uparrow}^{\star}\eta_{\downarrow}) + B(|\eta_{\uparrow}|^2 + |\eta_{\downarrow}|^2)^2$$
$$+ C(|\eta_{\uparrow}|^2 - |\eta_{\downarrow}|^2)^2, \quad (6)$$

where M is the density of the magnetic-moment component along the easy axis,

$$\alpha = \alpha_0(T - T_c), \quad \alpha_1 = \alpha_{10}(T - T_{sc0}), \quad (7)$$

$T_c(P)$ is the pressure-dependent Curie temperature, and $T_{sc0}(P)$ is the formal critical temperature of the superconducting transition in the single band (say just spin-up) case. The phenomenological treatment does not allow for fixing the pressure dependences of these critical temperatures and the other coefficients in Eq. (6). In what follows we will assume that the pressure dependences $T_c(P)$ and $T_{sc0}(P)$ qualitatively correspond to the phase diagram with the intersection of the phase-transition lines shown in Fig. 1.

One can note that the symmetry also allows the following interaction $i\gamma_3 M(\eta_{\uparrow}\eta_{\downarrow}^{\star} - \eta_{\uparrow}^{\star}\eta_{\downarrow})$ between the superconducting and the magnetic order parameters [13], but the general enough microscopic calculations [5,12] do not confirm the existence of this term.

In general, the free-energy fourth-order terms actually have forms different from $B(|\eta_{\uparrow}|^2 + |\eta_{\downarrow}|^2)^2 + C(|\eta_{\uparrow}|^2 - |\eta_{\downarrow}|^2)^2$ used in Ref. [9]. If the normal-state Green's functions are diagonal in the band indices (in our case they are spin-up and spin-down indices) Wick's decoupling does not produce any mixing terms between the band order parameters (see, for instance, Ref. [15]). In this case the fourth-order terms with respect to the superconducting order parameters are

$$\beta_1(|\eta_{\uparrow}|^4 + |\eta_{\downarrow}|^4) + \tilde{\beta}_1 M(|\eta_{\uparrow}|^4 - |\eta_{\downarrow}|^4). \quad (8)$$

If in the normal state there is the band mixing interaction this leads to the emergence of the additional terms,

$$\beta_2|\eta_{\uparrow}|^2|\eta_{\downarrow}|^2 + \beta_3[(\eta_{\uparrow}\eta_{\downarrow}^{\star})^2 + (\eta_{\uparrow}^{\star}\eta_{\downarrow})^2]$$
$$+ \beta_4(|\eta_{\uparrow}|^2 + |\eta_{\downarrow}|^2)(\eta_{\uparrow}\eta_{\downarrow}^{\star} + \eta_{\uparrow}^{\star}\eta_{\downarrow})$$
$$+ \tilde{\beta}_4 M(|\eta_{\uparrow}|^2 - |\eta_{\downarrow}|^2)(\eta_{\uparrow}\eta_{\downarrow}^{\star} + \eta_{\uparrow}^{\star}\eta_{\downarrow}). \quad (9)$$

One can show that the additional fourth-order terms do not introduce a qualitative modification in the phase diagram. So, we will work with the same free-energy density as in Ref. [9],

$$F = \alpha M^2 + \beta M^4 + \alpha_1(|\eta_{\uparrow}|^2 + |\eta_{\downarrow}|^2) + \gamma_1 M(|\eta_{\uparrow}|^2 - |\eta_{\downarrow}|^2)$$
$$+ \gamma_2(\eta_{\uparrow}\eta_{\downarrow}^{\star} + \eta_{\uparrow}^{\star}\eta_{\downarrow}) + \beta_1(|\eta_{\uparrow}|^4 + |\eta_{\downarrow}|^4) + \beta_2|\eta_{\uparrow}|^2|\eta_{\downarrow}|^2. \quad (10)$$

III. PHASE TRANSITIONS IN NEUTRAL FERMI LIQUIDS

At low pressures the system first passes from the paramagnetic to the ferromagnetic state and then from the ferromagnetic state to the ferromagnetic superconducting state. We begin with consideration for these phase transitions and then discuss the high-pressure transitions from the normal to the superconducting state and from the superconducting state to the ferromagnetic superconducting state and the transition from the normal to the ferromagnetic superconducting state.

A. Phase transition from the paramagnetic to the ferromagnetic state

The second-order transition from the paramagnetic to the ferromagnetic state occurs at $T = T_{\text{Curie}}(P)$. Below this temperature the magnetic moment acquires the finite value, and a superconducting ordering is absent

$$M^2 = [M_0(T)]^2 = -\frac{\alpha_0(T - T_c(P))}{2\beta}, \quad \eta_{\uparrow} = \eta_{\downarrow} = 0. \quad (11)$$

B. Phase transition from the ferromagnetic state to the superconducting ferromagnetic state

At the subsequent phase transition the superconducting order parameter amplitudes $\eta_\uparrow,\eta_\downarrow$ appear, and the magnetic moment acquires a magnitude of $M = M_0 + m$. Accepting for certainty that the coefficient $\gamma_2 = -|\gamma_2|$ is negative, we see from Eq. (10) that the phase difference between the superconducting order parameters is absent

$$\eta_\uparrow = \eta_1 e^{i\varphi}, \quad \eta_\downarrow = \eta_2 e^{i\varphi}. \tag{12}$$

Here, η_1 and η_2 are the modules of the superconducting order parameters. Thus, one can rewrite the free-energy density (10) as

$$F = \alpha M^2 + \beta M^4 + \alpha_1(\eta_1^2 + \eta_2^2) + \gamma_1 M(\eta_1^2 - \eta_2^2) - 2|\gamma_2|\eta_1\eta_2 + \beta_1(\eta_1^4 + \eta_2^4) + \beta_2\eta_1^2\eta_2^2. \tag{13}$$

The minimization of the free-energy density (13) with respect to η_1, η_2, and m yields the equations,

$$\alpha_1\eta_1 + \gamma_1(M_0+m)\eta_1 - |\gamma_2|\eta_2 + 2\beta_1\eta_1^3 + \beta_2\eta_1\eta_2^2 = 0, \tag{14}$$

$$\alpha_1\eta_2 - \gamma_1(M_0+m)\eta_2 - |\gamma_2|\eta_1 + 2\beta_1\eta_2^3 + \beta_2\eta_1^2\eta_2 = 0, \tag{15}$$

$$2\alpha m + 12\beta M_0^2 m + 12\beta M_0 m^2 + 4\beta m^3 + \gamma_1(\eta_1^2 - \eta_2^2) = 0. \tag{16}$$

Here, we have taken into account that M_0 is the minimum of free energy at $\eta_1 = \eta_2 = 0$ and omitted the fourth-order terms. The corresponding linear equations for η_1,η_2,

$$(\alpha_1 + \gamma_1 M_0)\eta_1 - |\gamma_2|\eta_2 = 0, \tag{17}$$

$$-|\gamma_2|\eta_1 + (\alpha_1 - \gamma_1 M_0)\eta_2 = 0 \tag{18}$$

are not coupled with a linear equation for m. Equating the determinant of this system to zero and taking into account Eq. (11) we obtain the equation,

$$T_{sc} = T_{sc0} + \frac{\sqrt{[\gamma_1(M_0(T_{sc})]^2 + \gamma_2^2}}{\alpha_{10}} \tag{19}$$

for the temperature T_{sc} of the transition to the superconducting ferromagnetic state. We will not write the explicit formula for T_{sc} in view of its cumbersome shape. Let us only note that according to this equation the pressure decrease in the Curie temperature $T_c(P)$ causes the increase in the superconducting transition temperature $T_{sc}(P)$ although this is not the only reason for the $T_{sc}(P)$ pressure dependence.

The linear equation with respect to m gives

$$m \cong -\frac{\gamma_1(\eta_1^2 - \eta_2^2)}{8\beta M_0^2}. \tag{20}$$

So, m is proved to be of the next order of smallness in comparison with $\eta_1 \propto \eta_2 \propto \sqrt{T_{sc} - T}$. Substitution of Eq. (20) to Eqs. (14) and (15) gives the equations of the third order with respect to the amplitudes η_1,η_2. An analytic solution of this system is possible only at the negligibly small coefficient of $|\gamma_2|$. In this case at $\gamma_1 > 0$ we obtain

$$\eta_2^2 \cong -\frac{\alpha_{10}}{2\beta_1 - \frac{\gamma_1^2}{8\beta M_0^2}}\left(T - T_{sc0} - \frac{\gamma_1 M_0}{\alpha_{10}}\right), \tag{21}$$

$$\eta_1 \cong \frac{|\gamma_2|}{\alpha_1 + \gamma_1 M_0}\eta_2. \tag{22}$$

This description of the second-order phase transition from the ferromagnetic to the ferromagnetic superconducting state is valid at the assumption $m \ll M_0$. However, at the pressure enhancement the Curie temperature and the critical temperature of the superconducting transition (see Fig. 1) approach each other, and the value of M_0 gets smaller, and according to Eq. (20) the value of m increases. One can expect the turning of the second-order transition into the first-order transition such that the order parameters η_1,η_2,m undergo finite jumps from zero to the finite values at temperatures higher than the critical temperature given by Eq. (19). Indeed, this type of behavior was established in Ref. [9] by the numerical solution of nonlinear equations for the order parameter components at close enough values of T_{Curie} and T_{sc}.

C. Phase transitions from the normal to the ferromagnetic superconducting state

To establish the whole phase diagram one must consider the phase transition from the normal nonmagnetic state to the superconducting state. The free-energy density (13) minimization with respect to η_1,η_2,M yields

$$\alpha_1\eta_1 + \gamma_1 M\eta_1 - |\gamma_2|\eta_2 + 2\beta_1\eta_1^3 + \beta_2\eta_1\eta_2^2 = 0, \tag{23}$$

$$\alpha_1\eta_2 - \gamma_1 M\eta_2 - |\gamma_2|\eta_1 + 2\beta_1\eta_2^3 + \beta_2\eta_1^2\eta_2 = 0, \tag{24}$$

$$2\alpha M + 4\beta M^3 + \gamma_1(\eta_1^2 - \eta_2^2) = 0. \tag{25}$$

At $\alpha > 0$ there are two types of solutions for these equations such that

$$\eta_1 = \eta_2, \quad M = 0, \tag{26}$$

and

$$\eta_1 \neq \eta_2, \quad M \neq 0. \tag{27}$$

In the first case the transition to the ferromagnetic superconducting state occurs by means of two consecutive phase transitions: the phase transition from the normal state to the nonmagnetic superconducting state followed at lower temperatures by the transition to the ferromagnetic superconducting state. In the second case the phase transition to the ferromagnetic superconducting state occurs directly from the normal state. We consider these situations separately.

1. Two consecutive phase transitions from the normal to the ferromagnetic superconducting state

The solution (26) is realized at large enough positive α when the formation of a ferromagnetic state is not energetically profitable. In this case the common magnitude of the superconducting amplitudes is

$$\eta^2 = -\frac{\alpha_1 - |\gamma_2|}{2\beta_1 + \beta_2}. \tag{28}$$

At positive sum $2\beta_1 + \beta_2 > 0$ this phase transition is of the second order and occurs at

$$T_{sc} = T_{sco} + \frac{|\gamma_2|}{\alpha_{10}}, \tag{29}$$

that coincides with Eq. (19) at $M_0 = 0$.

To pass in the ferromagnetic superconducting state the system must undergo one more phase transition. At this transition the magnetization M spontaneously appears, and the superconducting order parameter amplitudes acquire the deviations from the value given by Eq. (28),

$$\eta_1 = \eta + \delta_1, \quad \eta_2 = \eta + \delta_2. \tag{30}$$

The free energy acquires the following form:

$$\begin{aligned} F =& \alpha M^2 + \beta M^4 + \alpha_1(\delta_1^2 + \delta_2^2) + \gamma_1 M[2\eta(\delta_1 - \delta_2) \\ &+ \delta_1^2 - \delta_2^2] - 2|\gamma_2|\delta_1\delta_2 \\ &+ \beta_1[6\eta^2(\delta_1^2 + \delta_2^2) + 4\eta(\delta_1^3 + \delta_2^3) + \delta_1^4 + \delta_2^4] \\ &+ \beta_2[\eta^2(\delta_1^2 + \delta_2^2 + 4\delta_1\delta_2) + 2\eta\delta_1\delta_2(\delta_1 + \delta_2) + \delta_1^2\delta_2^2]. \end{aligned} \tag{31}$$

Here we have taken into account that η is the minimum of free energy at $M = \delta_1 = \delta_2 = 0$ and omitted the zero-order terms with respect to M,δ_1,δ_2. The order parameters are determined from the conditions of the free-energy minimum,

$$\frac{\partial F}{\partial \delta_1} = 0, \quad \frac{\partial F}{\partial \delta_2} = 0, \quad \frac{\partial F}{\partial M} = 0. \tag{32}$$

One can easily check that in linear approximation the equations for $(\delta_1 - \delta_2)$ and M,

$$\begin{aligned} [\alpha_1 + |\gamma_2| + (6\beta_1 - \beta_2)\eta^2](\delta_1 - \delta_2) + 2\gamma_1\eta M = 0, \\ \gamma_1\eta(\delta_1 - \delta_2) + \alpha M = 0 \end{aligned} \tag{33}$$

are decoupled from the equation for $(\delta_1 + \delta_2)$. Hence, the latter combination is on the next order of smallness in comparison with

$$M \propto (\delta_1 - \delta_2) \propto \sqrt{T_{scM} - T}. \tag{34}$$

Here T_{scM} is the critical temperature of transition from the superconducting to the superconducting ferromagnetic state which is determined from the equation given by the equality to zero of the determinant of the system (33),

$$[\alpha_1 + |\gamma_2| + (6\beta_1 - \beta_2)\eta^2]\alpha - 2[\gamma_1\eta]^2 = 0. \tag{35}$$

2. *Direct phase transition from the normal to the ferromagnetic superconducting state*

The second-type solution (27) is realized at small enough positive α. The analytical treatment is possible in neglect of third-order term $4\beta M^3$ in Eq. (25), then

$$M \cong -\frac{\gamma_1(\eta_1^2 - \eta_2^2)}{2\alpha}. \tag{36}$$

Passing to the sum and the difference of Eqs. (23) and (24) and using Eq. (36) we come to the equations,

$$\eta\left[\alpha_1 - |\gamma_2| - 2\frac{\gamma_1^2}{\alpha}\delta^2 + 2\beta_1(\eta^2 + 3\delta^2) + \beta_2(\eta^2 - \delta^2)\right] = 0, \tag{37}$$

$$\delta\left[\alpha_1 + |\gamma_2| - 2\frac{\gamma_1^2}{\alpha}\eta^2 + 2\beta_1(3\eta^2 + \delta^2) - \beta_2(\eta^2 - \delta^2)\right] = 0, \tag{38}$$

where

$$\eta = \frac{1}{2}(\eta_1 + \eta_2), \quad \delta = \frac{1}{2}(\eta_1 - \eta_2). \tag{39}$$

The solution of these equations at $\eta \neq 0$, $\delta \neq 0$ is

$$\eta^2 = \frac{1}{2}\frac{\alpha_1\alpha}{\gamma_1^2 - 4\beta_1\alpha} + \frac{1}{2}\frac{|\gamma_2|\alpha}{\gamma_1^2 - (2\beta_1 + \beta_2)\alpha}, \tag{40}$$

$$\delta^2 = \frac{1}{2}\frac{\alpha_1\alpha}{\gamma_1^2 - 4\beta_1\alpha} - \frac{1}{2}\frac{|\gamma_2|\alpha}{\gamma_1^2 - (2\beta_1 + \beta_2)\alpha}. \tag{41}$$

Thus, at direct transition from the normal to the ferromagnetic superconducting state the order parameter components M,η_1,η_2 undergo the finite jumps. This is the phase transition of the first order.

At a phase transition the free energy is not changed, that gives the equation for the phase-transition temperature,

$$\begin{aligned} F =& \alpha M^2 + \beta M^4 + \alpha_1(\eta_1^2 + \eta_2^2) + \gamma_1 M(\eta_1^2 - \eta_2^2) \\ &-2|\gamma_2|\eta_1\eta_2 + \beta_1(\eta_1^4 + \eta_2^4) + \beta_2\eta_1^2\eta_2^2 = 0. \end{aligned} \tag{42}$$

We solve this equation with the assumption that the coefficient $|\gamma_2|$ is negligibly small. Then at small enough values of α one can use the approximate expressions,

$$\eta^2 \approx \delta^2 \approx \frac{\alpha\alpha_1}{2\gamma_1^2}, \quad M \approx -2\frac{\gamma_1}{\alpha}\eta^2. \tag{43}$$

Substituting them to Eq. (42) we obtain

$$\eta^4\left[\alpha_1 - \frac{\gamma_1^2}{\alpha} + 4\beta\right] = 0. \tag{44}$$

So, at small enough positive α the phase-transition temperature is given by

$$T_{sc} \approx T_{sco} + \frac{\gamma_1^2}{\alpha\alpha_{10}}, \tag{45}$$

that exceeds the critical temperature of the second-order transition given by Eq. (29).

D. Phase diagram

The analytic derivation performed at several not strongly restrictive assumptions leads to the conclusion that the direct phase transition from the normal to the ferromagnetic superconducting state is of the first order. This confirms the statement numerically established in Ref. [9]. On the other hand, as we have pointed out in Sec. III B, when the temperatures of phase transitions to the normal ferromagnetic and to the superconducting ferromagnetic states are close

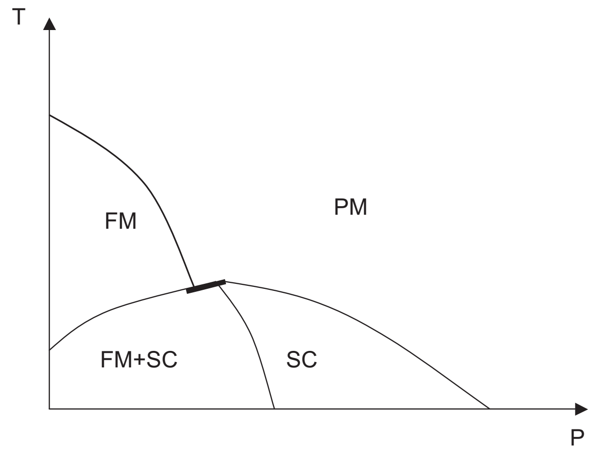

FIG. 2. Schematic temperature-pressure phase diagram of UCoGe in the frame of a neutral Fermi-liquid model. Notations FM, SC, and PM used for ferromagnetic, superconducting, and paramagnetic phases correspondingly. The thin and thick lines are the lines of the second- and the first-order transitions correspondingly.

to each other, the transition from the ferromagnetic to the ferromagnetic superconducting state is of the first order.

Thus, the simple phase diagram with an intersection of ferromagnetic and superconducting phase-transition lines, such as is drawn in Fig. 1, cannot be realized at least in the frame of the model under consideration. Near the intersection of the critical lines of the ferromagnetic and the superconducting transitions there is a piece of the transition of the first order as is shown in Fig. 2 by the thick line. The pressure interval where this transition takes place can be quite small. The presence of the first-order transition seems to be in correspondence with the sharp drop in resistivity at the superconducting phase transition in this pressure interval found in the paper [8].

IV. MAGNETOSTATICS

The authors of Ref. [9] discussed the phase transition between the SC state and the FM + SC state in the neutral Fermi liquid. The situation is changed in a charged Fermi liquid because the magnetization in the superconducting state is inevitably accompanied by the screening currents. The emergence of the superconducting state in the ferromagnetic state of UCoGe takes place at finite magnetization, whereas the arising of the ferromagnetic state in the superconducting state of UCoGe is accompanied by the smooth increasing of magnetization from zero to the finite value. This determines the difference between the transition from the ferromagnetic to the superconducting ferromagnetic state and the transition from the superconducting to the superconducting ferromagnetic state which we discuss here.

Let us consider a cylindrical sample of radius R with an axis parallel to the easy magnetization axis. A phase transition to the superconducting ferromagnetic state is accompanied by the appearance of supercurrents. The corresponding London equation for magnetic induction is

$$\mathrm{curl}\,\mathbf{B} = \frac{4\pi \mathbf{j}}{c} = -\frac{\mathbf{A}}{\delta^2} + 4\pi\,\mathrm{curl}\,\mathbf{M}, \tag{46}$$

where δ is the London penetration depth. The contribution to the current due to the term $c\,\mathrm{curl}\,\mathbf{M}$ is nonvanishing only in the surface layer with thickness on the order of coherence length $\xi \ll \delta \ll R$ [10], hence, the small enough magnetic field has to decay in the sample volume,

$$\mathbf{B}(r) = \mathbf{B}(R)\exp\left(-\frac{R-r}{\delta}\right), \tag{47}$$

where the surface magnetic field is determined by the magnetic moment created by the supercurrents flowing in the surface layer,

$$\mathbf{B}(R) = 4\pi\mathbf{M}. \tag{48}$$

In UCoGe according to the phase diagram drawn in Fig. 2 the formation of a ferromagnetic superconducting state occurs: (i) from the normal ferromagnetic state and (ii) from the nonmagnetic normal state either through the two consecutive phase transitions of the second order $N \to$ SC and then SC $\to$ FM + SC or directly by means of the first-order transition.

A. Magnetostatics below transition from the ferromagnetic to the ferromagnetic superconducting state

Just below the temperature of the phase transition from the ferromagnetic to the ferromagnetic superconducting state discussed in Sec. III B the field at the surface is

$$\mathbf{B}(R) = 4\pi\left(M_0 - \frac{\gamma_1(\eta_1^2 - \eta_2^2)}{8\beta M_0^2}\right). \tag{49}$$

According to the experimental results reported in Refs. [16–18] this field at ambient pressure is larger than the lower critical field H_{c1} in UCoGe. So, the complete field screening is not realized, and the phase transition occurs directly for the superconducting mixed state. The vortex cores occupy the small part of the sample volume, and almost the whole volume is in the superconducting state with the order parameter given by Eqs. (21) and (22). The specific heat jump at the phase transition to the superconducting state at the ambient pressure has the finite value,

$$\Delta C \cong \frac{\alpha_{10} T_{sc}}{2\beta_1 - \frac{\gamma_1^2}{8\beta M_0^2}}. \tag{50}$$

Here, we have neglected the temperature dependence of $M_0(T)$, just taking it as the constant $M_0 = M_0(T_{sc})$. The specific heat jump at this transition has been registered [4].

B. Magnetostatics below the transition from the superconducting to the ferromagnetic superconducting state

Another situation takes place at the transition from the superconducting to the ferromagnetic superconducting state. In this case discussed in Sec. III C the surface field is determined by the small magnetic moment arising at the transition in the magnetic superconducting state,

$$\mathbf{B}(R) \cong 4\pi M \propto \sqrt{T_{scM} - T}. \tag{51}$$

This field is certainly smaller than the lower critical field in the well-developed superconducting state with superconducting

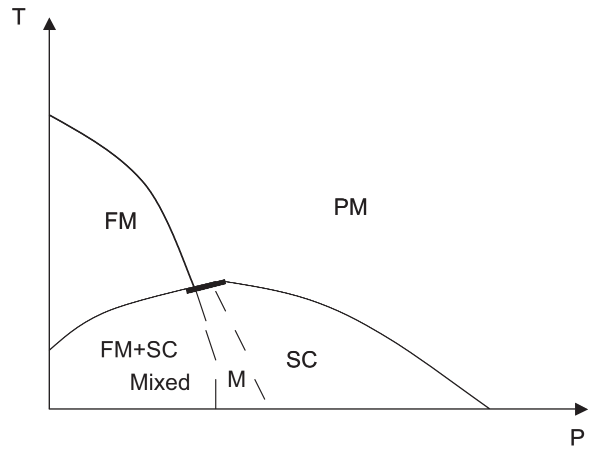

FIG. 3. Schematic temperature-pressure phase diagram of UCoGe taking into account the effect of supercurrents screening. Notations FM, SC, and PM used for ferromagnetic, superconducting, and paramagnetic phases correspondingly. The thin and thick lines are the lines of the second- and the first-order transitions correspondingly. The right dashed line is the imaginary line of the transition between the nonmagnetic and the ferromagnetic Meissner superconducting states which is not a phase transition in the bulk of the sample. The left dashed line is the line of H_{c1} dividing the Meissner and the mixed ferromagnetic superconducting states.

density $n_s \propto \eta^2 \propto (T_{sc} - T)$,

$$H_{c1} = 2\pi \mu_b n_s \ln \frac{\delta}{\xi} \propto (T_{sc} - T). \tag{52}$$

The transition to the ferromagnetic superconducting state is characterized by the emergence of magnetic-moment M and the magnetic part of the superconducting ordering $\sim(\delta_1 - \delta_2)$. But the supercurrents completely screen this magnetism in the bulk of the material. The gradual increase in magnetization from zero to some finite value is not accompanied by a bulk phase transition as it is in the process of the Meissner state formation in a superconductor of the second kind under an external magnetic field smaller than H_{c1}.

The pressure decrease stimulates ferromagnetism. Hence, at low temperatures and low pressures the magnetization will exceed the lower critical field, and a sample passes to the ferromagnetic superconducting mixed state. So, instead of a phase transition between the superconducting and the ferromagnetic superconducting state one can expect just the transition between the Meissner and the mixed superconducting states.

Thus, there is no bulk phase transition at all. The ferromagnetic superconducting Meissner state exists in the region between the two dashed lines shown in Fig. 3 The actual position of the H_{c1} line is subject to experimental determination. It can be in principle much more to the left than it is drawn in Fig. 3 and as more to the right such that near the first-order transition the two dashed lines can be merged to one line.

V. CONCLUSION

The Landau theory allows for establishing specific properties of phase transformations in the anisotropic ferromagnetic superconducting material UCoGe. It was found that the phase transition from the ferromagnetic to the ferromagnetic superconducting state at ambient pressure is characterized by the appearance of a superconducting part of the order parameter whereas the ferromagnetic component does undergo insignificant changes. However, at higher pressures this transition can turn to the transition of the first order.

It was proven that the direct phase transition from the nonmagnetic normal state to the ferromagnetic superconducting state is of the first order and exists in a small pressure interval. Out of this interval the transition to the ferromagnetic superconducting state occurs by means of two consecutive phase transitions of the second order from the normal to the nonmagnetic superconducting state and then from the nonmagnetic superconducting state to the ferromagnetic superconducting state. The model phase diagram in a neutral Fermi liquid acquires the shape shown in Fig. 2.

The phase diagram modification introduced by screening currents in a charged Fermi liquid is shown in Fig. 3. The magnetic moment at the phase transition from the ferromagnetic to the ferromagnetic superconducting state is just partially screened by the superconducting currents. Whereas at the phase transition from the superconducting state to the ferromagnetic superconducting state this screening is complete which shades the manifestations of a bulk phase transition. The ferromagnetic mixed superconducting state occurs only at lower pressures where the spontaneous magnetic moment exceeds the lower critical field. The position of the H_{c1} line is subject to experimental determination.

[1] S. S. Saxena, P. Agarval, K. Ahilan, F. M. Grosche, R. K. W. Hasselwimmer, M. J. Steiner, E. Pugh, I. R. Walker, S. R. Julian, P. Monthoux, G. G. Lonzarich, A. Huxley, I. Sheikin, D. Braithwaite, and J. Flouquet, Nature (London) **406**, 587 (2000).

[2] D. Aoki, A. Huxley, E. Ressouche, D. Braithwaite, J. Flouquet, J.-P. Brison, E. Lhotel, and C. Paulsen, Nature (London) **413**, 613 (2001).

[3] N. T. Huy, A. Gasparini, D. E. de Nijs, Y. Huang, J. C. P. Klaasse, T. Gortenmulder, A. de Visser, A. Hamann, T. Gorlach, and H. v. Lohneysen, Phys. Rev. Lett. **99**, 067006 (2007).

[4] D. Aoki and J. Flouquet, J. Phys. Soc. Jpn. **83**, 061011 (2014).

[5] V. P. Mineev, Usp. Fiz. Nauk **187**, 129 (2017) [Phys.-Usp. **60**(2), (2017)], arXiv:1605.07319.

[6] T. Hattori, K. Karube, Y. Ihara, K. Ishida, K. Deguchi, N. K. Sato, and T. Yamamura, Phys. Rev. B **88**, 085127 (2013).

[7] E. Slooten, T. Naka, A. Gasparini, Y. K. Huang, and A. de Visser, Phys. Rev. Lett. **103**, 097003 (2009).

[8] G. Bastien, D. Braithwaite, D. Aoki, G. Knebel, and J. Flouquet, Phys. Rev. B **94**, 125110 (2016).

[9] A. K. C. Cheung and S. Raghu, Phys. Rev. B **93**, 134516 (2016).

[10] V. P. Mineev and K. V. Samokhin, *Intoduction to Unconventional Superconductivity* (Gordon and Breach Science Publishers, Australia, Canada, 1999).

[11] V. P. Mineev, Phys. Rev. B **66**, 134504 (2002).

[12] V. P. Mineev, Phys. Rev. B **90**, 064506 (2014).

[13] V. P. Mineev, *Coexistence of Triplet Superconductivity and Itinerant Ferromagnetism*, AIP Conf. Proc. No. 1134 (AIP, New York, 2009), p. 68.

[14] V. V. Dmitriev, A. A. Senin, A. A. Soldatov, and A. N. Yudin, Phys. Rev. Lett. **115**, 165304 (2015).

[15] M. E. Zhitomirsky and V.-H. Dao, Phys. Rev. B **69**, 054508 (2004).

[16] K. Deguchi, E. Osaki, S. Ban, N. Tamura, Y. Simura, T. Sakakibara, I. Satoh, and N. K. Sato, J. Phys. Soc. Jpn. **79**, 083708 (2010).

[17] C. Paulsen, D. J. Hykel, K. Hasselbach, and D. Aoki, Phys. Rev. Lett. **109**, 237001 (2012).

[18] D. J. Hykel, C. Paulsen, D. Aoki, J. R. Kirtley, and K. Hasselbach, Phys. Rev. B **90**, 184501 (2014).