On the Nambu fermion-boson relations for superfluid ^{3}He

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Superfluid ^{3}He is a spin-triplet ($S = 1$), p-wave ($L = 1$) BCS condensate of Cooper pairs with total angular momentum $J = 0$ in the ground state. In addition to the breaking of U(1) gauge symmetry, separate spin or orbital rotation symmetry is broken to the maximal subgroup $SO(3)_S \times SO(3)_L \to SO(3)_J$. The fermions acquire mass $m_F \equiv \Delta$, where Δ is the BCS gap. There are also 18 bosonic excitations: 4 Nambu-Goldstone modes and 14 massive amplitude Higgs modes. The bosonic modes are labeled by the total angular momentum $J \in \{0,1,2\}$, and parity under particle-hole symmetry $c = \pm 1$. For each pair of angular momentum quantum numbers J, J_z, there are two bosonic partners with $c = \pm 1$. Based on this spectrum, Nambu proposed a sum rule connecting the fermion and boson masses for BCS-type theories, which for ^{3}He-B is $M_{J,+}^2 + M_{J,-}^2 = 4m_F^2$ for each family of bosonic modes labeled by J, where $M_{J,c}$ is the mass of the bosonic mode with quantum numbers (J,c). The Nambu sum rule (NSR) has recently been discussed in the context of Nambu-Jona-Lasinio models for physics beyond the standard model to speculate on possible partners to the recently discovered Higgs boson at higher energies. Here, we point out that the Nambu fermion-boson mass relations are not exact. Corrections to the bosonic masses from (i) leading-order strong-coupling corrections to BCS theory, and (ii) polarization of the parent fermionic vacuum lead to violations of the sum rule. Results for these mass corrections are given in both the $T \to 0$ and $T \to T_c$ limits. We also discuss experimental results, and theoretical analysis, for the masses of the $J^c = 2^\pm$ Higgs modes and the magnitude of the violation of the NSR.

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I. INTRODUCTION

One of the key features of spontaneous symmetry breaking in condensed matter and quantum field theory is the emergence of new elementary quanta: phonons in crystalline solids, magnons in ferromagnets, the Higgs and gauge bosons of the standard model. In the latter example, spontaneous symmetry breaking (SSB) in the BCS theory of superconductivity played an important role in theoretical models for the mass spectrum of elementary particles [1–3]. In BCS superfluids, the binding of fermions into Cooper pairs leads to an energy gap Δ in the fermion spectrum, i.e., fermions in the broken symmetry phase (Bogoliubov quasiparticles) acquire a mass $m_F = \Delta$, while condensation of Cooper pairs leads to the breaking of global U(1) gauge symmetry, the generator being particle number. The latter also implies that the Bogoliubov fermions are no longer particle number (fermion "charge") eigenstates, but coherent superpositions of normal-state particles and holes. Charge conservation is ensured by an additional contribution to the charge current, a collective mode of the broken symmetry phase. This massless bosonic excitation of the phase of condensate amplitude [4,5] is the Nambu-Goldstone (NG) mode associated with broken U(1) symmetry, and is manifest as a phonon in neutral superfluid ^{3}He.

II. THE NAMBU MASS RELATIONS

The development by Nambu and Jona-Lasinio (NJL) of a dynamical theory for the masses of elementary particles [1] was influenced by the BCS theory of superconductivity, and particularly the contributions of Bogoliubov [5], Valatin [6], and Anderson [2,7] on the excitation spectrum of fermions and the collective excitations (bosonic) associated with broken gauge symmetry [8]. BCS-type theories, including the NJL theory, imply a connection between the masses of the emergent fermionic and bosonic excitations. In the case of conventional BCS theory, there are two bosonic modes: the phase mode and the amplitude mode with mass $M_H = 2\Delta$. The phase mode, discussed independently by Anderson and Bogoliubov, is the massless NG mode ($M_{NG} = 0$), while the amplitude mode is the Higgs boson of BCS theory [3,9]. This doubling of the bosonic spectrum reflects a discrete symmetry under charge conjugation c (i.e.. "particle $\leftrightarrow$ hole" symmetry) of the symmetry unbroken fermionic vacuum [10,11], and is characteristic of spontaneous symmetry breaking of the BCS type, including BCS systems with more complex symmetry-breaking phase transitions. In particular, the amplitude (phase) mode has even (odd) parity with respect to charge conjugation [10]. Furthermore, the masses of the fermions and the bosons obey the sum rule $M_{NG}^2 + M_H^2 = (2m_F)^2$.

Nambu argued that similar sum rules apply to a broad class of BCS-type theories, from nuclear structure and QCD to exotic pairing in condensed matter systems, that exhibit complex symmetry breaking [12]. The ground state of superfluid ^{3}He provides the paradigm. Superfluid ^{3}He-B is a condensate of p-wave ($L = 1$), spin-triplet ($S = 1$) Cooper pairs with total angular momentum $J = 0$. Thus, in addition to the breaking of U(1), the symmetry of the normal quantum liquid with respect to separate spin or orbital rotations is broken to the maximal subgroup $SO(3)_S \times SO(3)_L \to SO(3)_J$. The fermion spectrum is isotropic and gapped with mass determined by the

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binding energy of Cooper pairs $m_F = \Delta$. However, there are now 18 bosonic excitations: 4 NG modes and 14 massive Higgs modes. The bosonic modes are organized into six multiplets labeled by J^c: total angular momentum, $J \in \{0,1,2\}$, and parity under charge conjugation (particle $\leftrightarrow$ hole), $c = \pm 1$.[1] For each J there are $2J + 1$ degenerate states with angular momentum projection $m = -J, \ldots, +J$, and for each pair of values of J,m there are two bosonic modes with $c = \pm 1$.

The $J = 0$ modes are the NG mode associated with broken U(1) symmetry ($J^c = 0^-$) and the Higgs mode ($J^c = 0^+$), which has the same quantum numbers as the B-phase vacuum, i.e., the condensate of ground-state Cooper pairs. There are six $J = 1$ modes: three NG modes ($J^c = 1^+$) corresponding to the degeneracy of the B-phase ground state with respect to *relative* spin-orbit rotations, and three Higgs modes ($J^c = 1^-$) with masses $M_{1,-} = 2\Delta$ [13]. Finally, there are 10 modes with $J = 2$, all of which are Higgs modes with masses $M_{2,\pm} < 2\Delta$, with original calculations giving $M_{2,+} = \sqrt{\frac{2}{5}}\,2\Delta$ and $M_{2,-} = \sqrt{\frac{3}{5}}\,2\Delta$ [14–17]. Nambu noted that all three multiplets obey a sum rule connecting the masses of the conjugate bosonic modes and the fermionic mass [12]

$$M_{J,+}^2 + M_{J,-}^2 = (2m_F)^2, \quad J \in \{0,1,2\} \tag{1}$$

and suggested that such fermion-boson relations are generic to BCS-type NJL models in which both fermion and boson excitations originate from interactions between massless progenitor fermions and spontaneous symmetry breaking (see also Ref. [18]). Nambu further speculated that these fermion-boson mass relations reflected a hidden supersymmetry in class of BCS-NJL models [12], and in the case of of conventional s-wave, spin-singlet BCS superconductivity was able to construct a supersymmetric representation for the static part of the effective Hamiltonian H_s, and identify the superalgebra as $\mathtt{su(2/1)}$. The fermion operators in Nambu's construction factorize H_s, and provide ladder operators connecting the fermionic and bosonic sectors of the spectrum, and generate the fermion-boson mass relations: $M_{NG} = 0$, $m_F = \Delta$, and $M_H = 2\Delta$ [19].[2]

Recently, Volovik and Zubkov argued that the Nambu sum rule (NSR) for ^{3}He-B follows from the *symmetry* of the B-phase vacuum and the quantum numbers (J,m) (cf. Sec. 2.2 of Ref. [20]). Based on the NSR for a NJL-type theory of top quark condensation, the authors suggest the possibility that there may be a partner to the Higgs boson with a mass of 125 GeV, e.g., a Higgs partner near 270 GeV [18,20], analogous to the Higgs partners for the $J = 2$ bosonic spectrum of ^{3}He-B. Here, we point out that estimates of the mass of a Higgs partner based on such sum rules may be imprecise because the NSR is generally violated. The origins of the violation of the NSR contain detailed information about the parent fermionic vacuum. While one might expect that the masses for the J multiplets to be protected by the residual symmetry of the broken symmetry vacuum state, it is generally not the case. As a result, the NSR is not exact, particularly for BCS-type theories with multiplets of NG and Higgs bosons with quantum numbers that are distinct from that of the broken symmetry vacuum state. We discuss the violations of the NSR for the case of ^{3}He-B in two limits: (i) time-dependent Ginzburg-Landau (TDGL) theory appropriate for $T \lesssim T_c$ and (ii) a dynamical theory for coupled fermionic and bosonic excitations of ^{3}He-B within the BCS theory for p-wave, spin-triplet pairing (i.e., one-loop approximation to the self-energy) for temperature $T \to 0$. In particular, interactions between the progenitor fermions, combined with vacuum polarization, lead to mass shifts of the Higgs modes whose quantum numbers differ from the broken symmetry vacuum state, e.g., the $J^c = 1^\pm$ and $J^c = 2^\pm$ modes of ^{3}He-B, and thus to violations of the Nambu sum rule. Explicit results for these mass corrections are derived in both the $T \to 0$ and $T \to T_c$ limits.

In Secs. III and IV we introduce a Lagrangian for the bosonic modes of a spin-triplet, p-wave BCS condensate based on a time-dependent extension of Ginzburg-Landau theory (TDGL). This allows us to identify and calculate the bosonic spectrum for ^{3}He, and to quantify strong-coupling corrections to the bosonic masses in the limit $T \to T_c$. In particular, strong-coupling feedback (i.e., next-to-leading-order loop corrections) leads to mass shifts, and thus violations of the NSR. We also obtain a formula for the mass of the $J^c = 2^-$ mode in the GL limit that could provide a direct determination of the GL strong-coupling parameter β_1 from measurements of the $J^c = 2^-$ mode via ultrasound spectroscopy.

At low temperatures, strong-coupling feedback corrections are suppressed. However, in Sec. V we show that vacuum polarization and four-fermion interactions, in both the particle-hole (Landau) and the particle-particle (Cooper) channels, lead to substantial mass corrections for $T \ll T_c$, and in some cases strong violations of the NSR. We discuss experimental measurements for the masses of the $J^c = 2^\pm$ modes, and compare the observed mass shifts with theoretical calculations of the polarization corrections to the masses from interactions in the Landau and Cooper channels.

III. GINZBURG-LANDAU FUNCTIONAL

We start from a Ginzburg-Landau (GL) functional applicable to p-wave, spin-triplet pairing beyond the weak-coupling BCS limit, and use this formulation to obtain an effective Lagrangian for the bosonic fluctuations of superfluid ^{3}He-B in the strong-coupling limit. The GL theory for superfluid ^{3}He was developed by several authors [21–23]. We follow the notation Ref. [24] which provides the bridge between the GL theory and the microscopic theory of leading-order strong-coupling effects. The order parameter is identified with the mean-field pairing self-energy $\hat{\Delta}(\mathbf{p})$, which is a 2×2 matrix of the spin components of the pairing amplitude. For p-wave, spin-triplet condensates the order parameter is symmetric in spin space $\hat{\Delta}(\mathbf{p}) = (i\sigma_\alpha\sigma_y)\,A_{\alpha i}\,(\hat{\mathbf{p}})_i$, and parametrized by a 3×3 complex matrix $A_{\alpha i}$, that transforms as a vector with respect to index $\alpha = \{x',y',z'\}$ under spin rotations, and, separately, as a vector with respect to index $i = \{x,y,z\}$ under

[1] Modes with $J \geqslant 3$ are also possible if we include subdominant pairing interactions with orbital angular momenta $\ell \geqslant 3$, even if the ground state is $\ell = 1$, $S = 1$, and $J = 0$ [39].

[2] A similar analysis for ^{3}He-B should be possible, but the construction of the ladder operators and the identification of the superalgebra for a supersymmetric representation of the Hamiltonian for the B phase of ^{3}He is a future challenge.

orbital rotations. This representation for the order parameter provides us with a basis for an irreducible representation of the maximal symmetry group of normal ^{3}He, $\mathsf{G} = \mathrm{SO}(3)_S \times \mathrm{SO}(3)_L \times \mathrm{U}(1)_N \times P \times T$. The GL free-energy functional is then constructed from products of $A_{\alpha i}$ and its derivatives $\partial_j A_{\alpha i}$ that are invariant under G. The general form for the GL functional for the condensation energy and gradient energy is

$$\mathscr{F}[A] = \int_V dV\,\{U(A) + W(\partial A)\}, \tag{2}$$

where

$$\begin{aligned} U = {} & \alpha(T)\mathrm{Tr}(AA^\dagger) + \beta_1|\mathrm{Tr}(AA^T)|^2 + \beta_2[\mathrm{Tr}(AA^\dagger)]^2 \\ & + \beta_3\mathrm{Tr}[AA^T(AA^T)^*] + \beta_4\mathrm{Tr}[(AA^\dagger)^2] \\ & + \beta_5\mathrm{Tr}[AA^\dagger(AA^\dagger)^*] \end{aligned} \tag{3}$$

are the six invariants for the condensation energy density, and

$$W = K_1\partial_j A_{\alpha i}\partial_j A^*_{\alpha i} + K_2\partial_i A_{\alpha i}\partial_j A^*_{\alpha j} + K_3\partial_j A_{\alpha i}\partial_i A^*_{\alpha j} \tag{4}$$

are the three second-order invariants for the gradient energy.

Weak-coupling BCS theory can be formulated at all temperatures in terms of a stationary functional of $\hat{\Delta}(\mathbf{p})$ [25,26], which depends on material parameters of the parent fermionic ground state: $N(0) = k_f^3/2\pi^2 v_f p_f$ is the single-spin quasiparticle density of states at the Fermi surface, expressed in terms of the Fermi velocity v_f, Fermi momentum, and Fermi wave number $p_f = \hbar k_f$. The GL limit of the weak-coupling functional can be expressed in the form of Eqs. (3) and (4) with the following material parameters:

$$\alpha(T) = \frac{1}{3}N(0)(T/T_c - 1), \quad \beta_1^{\mathrm{wc}} \equiv \frac{7\zeta(3)}{240}\frac{N(0)}{(\pi k_B T_c)^2}, \tag{5}$$

$$2\beta_1^{\mathrm{wc}} = -\beta_2^{\mathrm{wc}} = -\beta_3^{\mathrm{wc}} = -\beta_4^{\mathrm{wc}} = +\beta_5^{\mathrm{wc}} = -2\beta_{\mathrm{wc}}. \tag{6}$$

Strong-coupling corrections to the weak-coupling GL β parameters based on the leading-order expansion of Rainer and Serene [24] were calculated and reported in Ref. [27] for quasiparticle scattering that is dominated by ferromagnetic spin fluctuation exchange. The results for the strong-coupling corrections to the weak coupling β_i^{wc} are extrapolated to all pressures as shown in Fig. 1, with $p = 0$ bar corresponding to weak coupling.

The weak-coupling form of the gradient energy in Eq. (4) is similarly obtained with the gradient coefficients given by

$$K_1^{\mathrm{wc}} = K_2^{\mathrm{wc}} = K_3^{\mathrm{wc}} = \frac{1}{5}N(0)\,\xi_{\mathrm{GL}}^2\,, \tag{7}$$

$$\xi_{\mathrm{GL}} = \sqrt{\frac{7\zeta(3)}{12}}\,\frac{\hbar v_f}{2\pi k_B T_c}. \tag{8}$$

The Cooper pair correlation length ξ_{GL} varies from $\xi_{\mathrm{GL}} \simeq 650$ Å at $p = 0$ bar to $\xi_{\mathrm{GL}} \simeq 150$ Å at $p = 34$ bar.

The Balian-Werthamer (BW) state defined by

$$A^{\mathrm{BW}}_{\alpha i} = \frac{\Delta}{\sqrt{3}}\,e^{i\varphi}R[\vec{\vartheta}]_{\alpha i}\,, \tag{9}$$

where $R[\vec{\vartheta}]$ is an orthogonal matrix, minimizes the GL functional for $\Delta^2 = -\alpha(T)/2\beta_B$, with $\beta_B = \beta_{12} + \frac{1}{3}\beta_{345}$, in the weak-coupling limit $\beta_B^{\mathrm{wc}} = \frac{5}{6}\beta^{\mathrm{wc}}$, and for all pressures $P < P_{\mathrm{PCP}} \approx 21$ bar. Note that the amplitude of the order parameter Δ is fixed at the minimum of the effective potential. However, the phase φ and the orthogonal matrix $R[\vec{\vartheta}]$, parametrized by a rotation angle ϑ about an axis of rotation $\hat{\mathbf{n}}$, define the degeneracy space of the B phase. In particular,

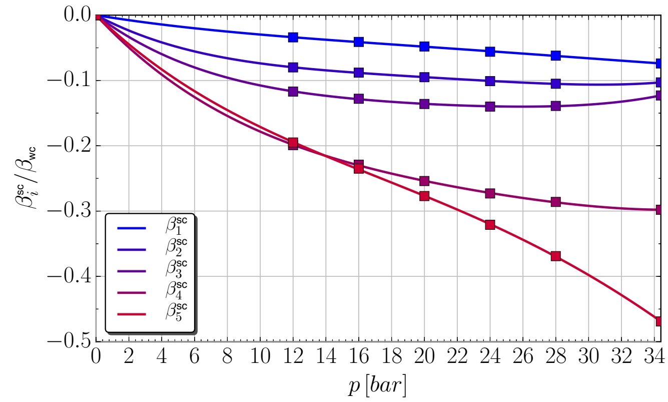


FIG. 1. Strong-coupling corrections ($\beta_i^{\mathrm{sc}} \equiv \beta_i - \beta_i^{\mathrm{wc}}$) to the GL β parameters interpolated from the results of Ref. [27] (data squares). The β_i^{sc} are extrapolated below $P = 12$ bar to weak coupling ($\beta_i^{\mathrm{sc}} = 0$) at $p = 0$ bar.

$$B_{\alpha i} \equiv \frac{\Delta}{\sqrt{3}}\,\delta_{\alpha i}, \tag{10}$$

corresponding to pairs with $L = 1$, $S = 1$ and $J = 0$ is degenerate with states obtained by any relative rotation $R[\vec{\vartheta}]$ of the spin and orbital coordinates combined with a gauge transformation $e^{i\varphi}$. Since the GL functional defined by Eqs. (3) and (4) is invariant under these operations, we can use the $J = 0$ BW state as the reference ground state.

IV. TIME-DEPENDENT GL THEORY

Bosonic excitations of the BW ground state are represented by space-time fluctuations of the pairing amplitude: $\mathscr{D}_{\alpha i}(\mathbf{r},t) = A_{\alpha i}(\mathbf{r},t) - B_{\alpha i}$. The potential energy for these fluctuations is obtained by expanding the GL functional to second order in the fluctuations $\mathscr{D}(\mathbf{r},t)$: $\mathscr{U}[\mathscr{D}] = \mathscr{F}[A] - \mathscr{F}[B]$ [28,29]. Time-dependent fluctuations $\dot{\mathscr{D}}_{\alpha i} = \partial_t\mathscr{D}_{\alpha i}$ lead to an additional invariant $\mathscr{K} = \tau\int_V dV\,\dot{\mathscr{D}}_{\alpha i}\dot{\mathscr{D}}^*_{\alpha i}$, where τ is the effective inertia for Cooper pair fluctuations.[3] The Lagrangian for the bosonic excitations $\mathscr{L} = \mathscr{K} - \mathscr{U}$ takes the form

$$\mathscr{L} = \int dV\left\{\tau\,\mathrm{Tr}\{\dot{\mathscr{D}}\dot{\mathscr{D}}^\dagger\} - \alpha\,\mathrm{Tr}\{\mathscr{D}\mathscr{D}^\dagger\} - \sum_{p=1}^{5}\beta_p u_p(\mathscr{D}) - \sum_{l=1}^{3}K_l\,v_l(\partial\mathscr{D}) - (\eta_{\alpha i}\mathscr{D}^*_{\alpha i} + \eta^*_{\alpha i}\mathscr{D}_{\alpha i})\right\}. \tag{11}$$

[3] We have omitted the invariant that is first order in time derivatives $\mathrm{Tr}\{\dot{\mathscr{D}}\mathscr{D}^\dagger\}$−H.c. This invariant is odd under charge conjugation, and thus has a small, but non zero, prefactor only because of the weak violation of particle-hole symmetry of the normal fermionic vacuum.

TABLE I. Irreducible tensor representations $\{t^{(J,M)}_{ij}\}$ of $\mathrm{SO(3)}_J$ for $J \leqslant 2$, and corresponding spherical harmonics $\mathscr{Y}_{JM}(\hat{\mathbf{p}})$. The base unit vectors $\mathbf{e}^{(0)}_i = \hat{\mathbf{z}}_i$, $\mathbf{e}^{(+)}_i = -\frac{1}{\sqrt{2}}(\hat{\mathbf{x}}_i + i\hat{\mathbf{y}}_i)$, and $\mathbf{e}^{(-)}_i = +\frac{1}{\sqrt{2}}(\hat{\mathbf{x}}_i - i\hat{\mathbf{y}}_i)$ are orthonormal: $\mathbf{e}^{(\mu)*}\cdot\mathbf{e}^{(\nu)} = \delta_{\mu\nu}$.

J	M	$t^{(J,M)}_{ij}$	$\mathscr{Y}_{Jm}(\hat{\mathbf{p}})$
0	0	$\frac{1}{\sqrt{3}}\delta_{ij}$	1
	$+1$	$\sqrt{3}\,\varepsilon_{ijk}\mathbf{e}^{(+)}_k$	$-\sqrt{\frac{3}{2}}\,\hat{\mathbf{p}}_+$
1	0	$\sqrt{3}\,\varepsilon_{ijk}\mathbf{e}^{(0)}_k$	$+\sqrt{3}\,\hat{\mathbf{p}}_z$
	-1	$\sqrt{3}\,\varepsilon_{ijk}\mathbf{e}^{(-)}_k$	$+\sqrt{\frac{3}{2}}\,\hat{\mathbf{p}}_-$
	$+2$	$\mathbf{e}^{(+)}_i\mathbf{e}^{(+)}_j$	$+\sqrt{\frac{15}{8}}\,\hat{\mathbf{p}}^2_+$
	$+1$	$\sqrt{\frac{1}{2}}\big(\mathbf{e}^{(0)}_i\mathbf{e}^{(+)}_j + \mathbf{e}^{(+)}_i\mathbf{e}^{(0)}_j\big)$	$-\sqrt{\frac{15}{2}}\,\hat{\mathbf{p}}_z\hat{\mathbf{p}}_+$
2	0	$\sqrt{\frac{3}{2}}\big(\mathbf{e}^{(0)}_i\mathbf{e}^{(0)}_j - \frac{1}{3}\delta_{ij}\big)$	$+\sqrt{\frac{5}{4}}(3\hat{\mathbf{p}}^2_z - 1)$
	-1	$\sqrt{\frac{1}{2}}\big(\mathbf{e}^{(0)}_i\mathbf{e}^{(-)}_j + \mathbf{e}^{(-)}_i\mathbf{e}^{(0)}_j\big)$	$+\sqrt{\frac{15}{2}}\,\hat{\mathbf{p}}_z\,\hat{\mathbf{p}}_-$
	-2	$\mathbf{e}^{(-)}_i\mathbf{e}^{(-)}_j$	$+\sqrt{\frac{15}{8}}\,\hat{\mathbf{p}}^2_-$

The terms $u_p(\mathscr{D})$ are the effective potentials corresponding to fluctuations $\mathscr{D}$ relative to the BW ground state, which to quadratic order in $\mathscr{D}$ are given in Eqs. (A1)–(A5) of the Appendix. The terms $w_l(\partial\mathscr{D})$ are obtained from Eq. (4) with $A \to \mathscr{D}$, and the last pair of terms in Eq. (11) represent an effective external source potential for Cooper pair fluctuations.

The Euler-Lagrange equations

$$\frac{\delta\mathscr{L}}{\delta\mathscr{D}^*_{\alpha i}} - \frac{\partial}{\partial t}\frac{\delta\mathscr{L}}{\delta\dot{\mathscr{D}}^*_{\alpha i}} - \frac{\partial}{\partial x_j}\frac{\delta\mathscr{L}}{\delta\partial_j\mathscr{D}^*_{\alpha i}} = 0 \tag{12}$$

reduce to field equations for the Cooper pair fluctuations,

$$\tau\ddot{\mathscr{D}}_{\alpha i} - |\alpha|\mathscr{D}_{\alpha i} + \Delta^2\sum_{a=1}^{5}\beta_a\frac{\partial\bar{u}_a}{\partial\mathscr{D}^*_{\alpha i}} - \sum_{a=1}^{3}K_a\,\partial_k\frac{\partial\bar{v}_a}{\partial[\partial_k\mathscr{D}^*_{\alpha i}]} = \eta_{\alpha i}. \tag{13}$$

The field equations reduce to coupled equations for pair fluctuation modes of wavelength $\mathbf{q}$: $\mathscr{D}_{\alpha i}(\mathbf{r},t) \to \mathscr{D}_{\alpha i}(\mathbf{q};t)e^{i\mathbf{q}\cdot\mathbf{r}}$. Furthermore, the BW ground state is invariant under joint spin and orbital rotations. Thus, the $\mathbf{q}=0$ bosonic excitations can be labeled by the quantum numbers J, and $m \in \{-J,\ldots,+J\}$ for the total angular momentum and its projection along a fixed quantization axis $\hat{\mathbf{z}}$. The dynamical equations for the bosonic modes decouple when expressed in terms of spherical tensors that form bases for representations of $\mathrm{SO(3)}_J$ for $J = 0,1,2$,

$$\mathscr{D}_{\alpha i}(\mathbf{r},t) = \sum_{J,m} D_{J,m}(\mathbf{r},t)\,t^{(J,m)}_{\alpha i}, \tag{14}$$

where the set of nine spherical tensors defined in Table I (i) span the space of rank-two tensors, (ii) form irreducible representations of $\mathrm{SO(3)}_J$, and (iii) satisfy the orthonormality conditions

$$\mathrm{Tr}\{\hat{t}^{(J,m)\dagger}\,\hat{t}^{(J',m')}\} = \delta_{J,J'}\,\delta_{m,m'}. \tag{15}$$

TABLE II. Bosonic mode spectrum for the B phase of ^{3}He. The masses of the modes are given for weak coupling in the GL limit. AH designates amplitude Higgs modes.

Mode	Symmetry	Mass	Name
$D^{(+)}_{0,m}$	$J=0$, c $=+1$	2Δ	Amplitude
$D^{(-)}_{0,m}$	$J=0$, c $=-1$	0	Phase mode
$D^{(+)}_{1,m}$	$J=1$, c $=+1$	0	NG spin-orbit modes
$D^{(-)}_{1,m}$	$J=1$, c $=-1$	2Δ	AH spin-orbit modes
$D^{(+)}_{2,m}$	$J=2$, c $=+1$	$\sqrt{\frac{8}{5}}\Delta$	2^+ AH modes
$D^{(-)}_{2,m}$	$J=2$, c $=-1$	$\sqrt{\frac{12}{5}}\Delta$	2^- AH modes

In the absence of a perturbation that breaks the rotational symmetry of the ground state, there are $(2J+1)$ degenerate modes with spin J. There is, in addition, a doubling of the bosonic modes related to the discrete symmetry of the normal fermionic ground state under charge conjugation. Thus, the full set of quantum numbers for the bosonic spectrum is $\{J,m,\mathrm{c}\}$ where $\mathrm{c}=\pm1$ is the parity of the bosonic mode under charge conjugation. The parity eigenstates are the linear combinations (i.e., real and imaginary amplitudes)[4]

$$D^{(\mathrm{c})}_{J,m} = (D_{J,m} + \mathrm{c}\,D^{\dagger}_{J,m})/2. \tag{16}$$

The sources can also be expanded in this basis: $\eta_{\alpha i} = \sum_{J,m,\mathrm{c}}\eta^{(\mathrm{c})}_{J,m}\,t^{(J,m)}_{\alpha i}$. The equations for the 18 bosonic modes then decouple into three doublets labeled by J,c, each of which is $(2J+1)$-fold degenerate as shown in Table II.

The equations of motion for the 18 bosonic modes are obtained by projecting out the $J,m,$c components of Eq. (13). In the limit $\mathbf{q}=0$, the modes decouple into three doublets labeled by J,c, each of which is $(2J+1)$-fold degenerate. The dispersion of the bosonic modes can be calculated perturbatively to leading order in $(v_f|\mathbf{q}|/\Delta)^2$. Thus, the resulting equations of motion can be expressed as

$$\partial^2_t D^{(\mathrm{c})}_{J,m} + \omega^{(\mathrm{c})}_{J,m}(\mathbf{q})^2\,D^{(\mathrm{c})}_{J,m} = \frac{1}{\tau}\eta^{(\mathrm{c})}_{J,m}, \tag{17}$$

$$\text{where}\quad \omega^{(\mathrm{c})}_{J,m}(\mathbf{q}) = \sqrt{M^2_{J,\mathrm{c}} + \big(c^{(\mathrm{c})}_{J,|m|}|\mathbf{q}|\big)^2} \tag{18}$$

is the dispersion relation for bosonic excitations with quantum numbers $\{J,m,\mathrm{c}\}$ and $M_{J,\mathrm{c}}$ is the corresponding excitation energy at $\mathbf{q}=0$, i.e., the mass. For $\mathbf{q}\neq0$, the degeneracy of the bosonic spectrum is partially lifted, i.e., the velocities $c^{(\mathrm{c})}_{J,|m|}$ give rise to a dispersion splitting that depends on $|m|$, with quantization axis $\mathbf{q}$ [30,31].

A. $J = 0$ modes

The masses and velocities of the bosonic modes obtained from the TDGL Lagrangian in the weak-coupling limit are summarized in Table II. The $J=0$ modes correspond to the two bosonic modes that are present for any BCS

[4]Note that the parity of the modes is defined relative to that of the BW ground state, which is defined as c $=+1$.

condensate of Cooper pairs, i.e., excitations of the phase $D^{(-)}_{0,0}$ and amplitude $D^{(+)}_{0,0}$, with the *same internal symmetry as the condensate* of Cooper pairs. The $J^c = 0^-$ mode is the Anderson-Bogoliubov (AB) phase mode. In particular, if we consider *only* fluctuations of the phase of the BW ground state $A_{\alpha i} = B_{\alpha i}\, e^{i\vartheta(\mathbf{r},t)} \approx B_{\alpha i}[1 + i\vartheta(\mathbf{r},t)]$, then $D^{(-)}_{0,0} = i\Delta\, \vartheta(\mathbf{r},t)$. This is the massless NG mode corresponding to the broken U(1) symmetry, with the dispersion relation $\omega^{(-)}_{0,0} = c_{0,0}|\mathbf{q}|$. Within the TDGL theory, the AB mode propagates with velocity $c_{0,0} = \sqrt{(K_1 + \frac{1}{2}K_{23})/\tau}$. In the weak-coupling limit for the effective action derived by bosonization of the fermionic action, the velocity is $c_{0,0} = v_f/\sqrt{3}$ [32], showing that the bosonic excitation energies are determined by the properties of the underlying fermionic vacuum, in this case the group velocity of normal-state fermionic excitations at the Fermi surface. However, this result for the velocity of the NG phase mode is further renormalized by coupling of the phase fluctuations to dynamical fluctuations of the underlying fermionic vacuum which are absent from the bosonic action based on the TDGL Lagrangian of Eq. (11). This coupling leads to $c_{0,0} \to c_1 + (c_0 - c_1)\,\mathscr{Y}(T/T_c)$, where c_1 (c_0) is the first (zero) sound velocity of the interacting *normal* Fermi liquid and $\mathscr{Y}(T/T_c)$ measures the dynamical response of the condensate. In particular, $\mathscr{Y} \to 0$ ($\mathscr{Y} \to 1$) for $T \to 0$ ($T \to T_c$). This remarkable result shows that the velocity of the NG phase mode is renormalized to the hydrodynamic sound velocity of normal ^{3}He at $T = 0$, and that the $J = 0$, c $= -1$ NG mode is manifest in superfluid ^{3}He as longitudinal sound [33–35].

The partner to the NG phase mode is the $J^c = 0^+$ "amplitude" mode. This is the Higgs boson of superfluid ^{3}He, i.e., the bosonic excitation of the condensate with the *same internal symmetry* ($L = 1$, $S = 1$, $J = 0$, c $= +1$) as condensate of Cooper pairs that comprise the ground state [3]. For this reason, the renormalizations of the $J^c = 0^+$ bosonic mass and the mass of fermionic excitations of the $J^c = 0^+$ BW state are equivalent; thus, $M_{0,+} = 2m_F$, where $m_F = \Delta$ is the renormalized fermionic mass in the dispersion relation for fermionic excitations $E^2_{\mathbf{p}} = m^2_F + v^2_f(p - p_f)^2$. This allows us to fix the effective inertia of the bosonic fluctuations in the TDGL Lagrangian of Eq. (11) for the BW ground state as $\tau = \beta_{\mathrm{B}} \equiv \beta_{12} + \frac{1}{3}\beta_{345}$. Thus, the Nambu sum rule $M^2_{0,-} + M^2_{0,+} = 4m^2_F$ is obeyed for the $J = 0$ modes. However, strong-coupling corrections to the TDGL Lagrangian lead to violations of the Nambu sum rule for bosonic excitations with $J \neq 0$.

B. Violations of the Nambu sum rule for $J \neq 0$

In addition to the NG mode associated with broken U(1) symmetry, there are 3 NG modes associated with spontaneously broken *relative* spin-orbit rotation symmetry $\mathrm{SO}(3)_S \times \mathrm{SO}(3)_L \to \mathrm{SO}(3)_J$. These NG modes reflect the degeneracy of the BW ground state with respect to relative spin-orbit rotations $\mathrm{SO}(3)_{L-S}$, whose generators form a vector representation of SO(3). Thus, the corresponding NG modes are the $J^c = 1^+$ modes, which are spin-orbit waves with excitation energies $\omega_{1,m} = c_{1,m}|\mathbf{q}|$, and velocities $c_{1,0} = \frac{1}{5}v_f$ and $c_{1,\pm1} = \frac{2}{5}v_f$ in the weak-coupling limit [32]. The velocities are also renormalized in the limit $T \to 0$ by the coupling to dynamical fluctuations of the underlying fermionic vacuum.[5]

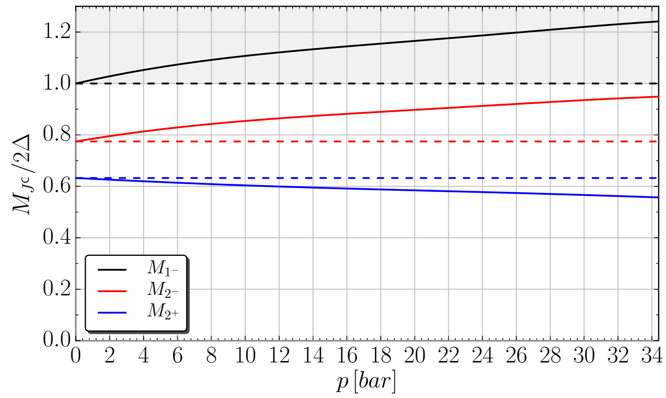


FIG. 2. Strong-coupling corrections to the bosonic masses obtained from the TDGL theory for the GL β parameters shown in Fig. 1. The dashed lines correspond to the weak-coupling values for the masses.

The partners to these NG modes are the $J^c = 1^-$ Higgs modes with mass

$$M_{1,-} = 2\Delta\left(\frac{-\beta_1 + \frac{1}{3}(\beta_4 - \beta_{35})}{\beta_{12} + \frac{1}{3}\beta_{345}}\right)^{\frac{1}{2}}, \tag{19}$$

which reduces to $M^{\mathrm{wc}}_{1,-} = 2\Delta$ in the weak-coupling limit for the GL β parameters [Eqs. (6)]. However, in the strong-coupling limit the masses of the $J^c = 1^-$ modes deviate from $2m_F$, which implies a violation of the NSR for the $J = 1$ bosonic modes. Theoretical calculations of the strong-coupling β parameters predict that the $J^c = 1^-$ Higgs modes are pushed to energies above the pair-breaking edge of 2Δ, as shown in Fig. 2. This opens the possibility for the $J^c = 1^-$ modes to decay into unbound fermion pairs. Thus, we expect the $J^c = 1^-$ modes are at best resonances with finite lifetime.

For $J = 2$ there are two fivefold multiplets of Higgs modes with masses

$$M_{2,+} = 2\Delta\left(\frac{\frac{1}{3}\beta_{345}}{\beta_{12} + \frac{1}{3}\beta_{345}}\right)^{\frac{1}{2}}, \tag{20}$$

$$M_{2,-} = 2\Delta\left(\frac{-\beta_1}{\beta_{12} + \frac{1}{3}\beta_{345}}\right)^{\frac{1}{2}}. \tag{21}$$

Equation (21) provides a fifth observable that might be used to determine GL β parameters from independent experiments in the GL regime [36]. In the weak-coupling limit with β_i given by Eqs. (6), the masses reduce to $M^{\mathrm{wc}}_{2,+} = \sqrt{\frac{8}{5}}\Delta$ and

[5] The weak breaking of *relative* spin-orbit rotation symmetry by the nuclear dipolar interaction present in the normal-state partially lifts the degeneracy of the $J^c = 1^+$ NG modes, endowing the $m = 0$ mode with a very small mass determined by the nuclear dipole energy. This is the "light Higgs" scenario discussed by Zavjalov *et al.* [63]. See Sec. VII D.

$M_{2,-}^{\mathrm{wc}} = \sqrt{\frac{12}{5}}\Delta$. Thus, the $J^{\mathrm{c}} = 2^{\pm}$ Higgs modes obey the NSR in the weak-coupling limit of the TDGL theory [12,18,20].

However, the NSR is violated by strong-coupling corrections to the Higgs masses, shown in Fig. 2 as a function of pressure for the strong-coupling β parameters shown in Fig. 1. The asymmetry in the mass corrections for $M_{2,\pm}$ leads to a sizable violation of the NSR at high pressures: $\sum_{\mathrm{c}} M_{2,\mathrm{c}}^2/4m_F^2 - 1 \approx 20\%$ at $p = 34$ bar. The violations of the NSR have the following origin: The strong-coupling Lagrangian for the bosonic fluctuations (11) and (A1)–(A5) depends on the symmetry of the mode; thus, the strong-coupling renormalization of the Higgs masses depends on J^{c}. For the $J = 0^+$ mode, the strong-coupling renormalization of the mass is the same as that of the $J = 0^+$ ground-state amplitude Δ, and thus the fermion mass, in which case the NSR is satisfied even with strong-coupling corrections. However, for modes with $J \neq 0$, the renormalization of the mass of the Higgs mode is a different combination of the strong coupling β's than that which renormalizes Δ, leading to violations of the NSR.

V. BEYOND TDGL THEORY

The TDGL theory is limited in its applicability because it is based on an effective action with *only* bosonic degrees of freedom. However, the parent state of a BCS condensate is the Fermi-liquid ground state ("fermionic vacuum"). In order to calculate effects on the bosonic spectrum arising from "back-action" of the fermionic vacuum, we require a dynamical theory that includes both fermion and bosonic degrees of freedom.

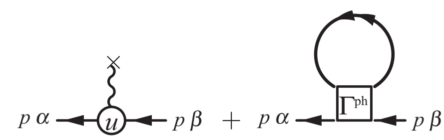
Microscopic formulations of the theory of collective excitations in superfluid ^{3}He-B were developed on the basis of mean-field kinetic equations in Ref. [37], Kubo theory in Refs. [15,38], a functional integral formulation of the hydrodynamic action in Ref. [17], and quasiclassical transport theory in Refs. [39–42]. We highlight the coupling between bosonic and fermionic degrees of freedom that lead to mass shifts of the Higgs modes. Results for the mass shifts of the $J^{\mathrm{c}} = 2^{\pm}$ Higgs modes reported in Ref. [39] are interpreted here in terms of interactions that result from polarization of the fermionic vacuum by the creation of a bosonic mode that has different symmetry than that of the unpolarized vacuum. The Higgs modes with different parities, $\mathrm{c} = \pm 1$, also polarize the fermionic vacuum in different channels, activating different interactions and leading to different mass shifts. Thus, the violation of the $J = 2$ NSR is directly related to the vacuum polarization mass shifts for the two charge-conjugation partners of the $J = 2$ multiplet.

A. Particle-hole self-energy

For an interacting Fermi system, the two-body interaction between isolated ^{3}He atoms is renormalized to effective interactions between low-energy fermionic quasiparticles that are well-defined excitations within a low-energy band near the Fermi surface, $|\varepsilon| \leqslant \hbar\Omega_c \ll E_f$, and thus a shell in momentum space, $\delta p \leqslant \hbar\Omega_c/v_f$.

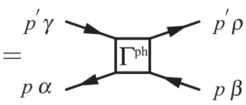
A disturbance of the vacuum state from that of an isotropic Fermi sea, e.g., by a perturbation that couples to the quasiparticle states in the vicinity of the Fermi surface, generates a polarization of the fermionic vacuum, and a corresponding self-energy correction to the energy of a fermionic quasiparticle. The leading-order correction is given by the combined external field $u_{\alpha\beta}(p)$ plus mean-field (one-loop) interaction energy associated with a particle-hole excitation of the fermionic vacuum state

$$\Sigma_{\alpha\beta}(p) = \quad (22)$$



The interaction between fermionic quasiparticles shown in Eq. (22) is represented by a four-point vertex that sums bare two-body interactions to all orders involving all possible intermediate states of high-energy fermions. The vertex that determines the leading-order quasiparticle self-energy Γ^{ph} defines the forward-scattering amplitude for particle and hole pairs (Landau channel) scattering within the low-energy shell near the Fermi surface

$$\Gamma^{\mathrm{ph}}_{\alpha\beta;\gamma\rho}(p,p') =$$



$$= \Gamma^{(s)}(p,p')\delta_{\alpha\gamma}\delta_{\beta\rho} + \Gamma^{(a)}(p,p')\vec{\sigma}_{\alpha\gamma}\cdot\vec{\sigma}_{\beta\rho}, \tag{23}$$

with amplitudes $\Gamma^{(s)}(p,p')$ for spin-independent scattering, $\Gamma^{(a)}(p,p')$, representing the spin-dependent "exchange" scattering amplitude. The fermion propagator in the presence of the external perturbation $u_{\alpha\beta}(p)$ is represented by

$$G_{\alpha\beta}(p) = \langle \psi_\alpha(p)\bar{\psi}_\beta(p)\rangle \equiv \alpha \overset{+p \qquad\quad +p}{\longleftarrow\!\!\!\!\longleftarrow\!\!\!\!-\!\!\!-} \beta, \tag{24}$$

where $p = (\mathbf{p},\varepsilon_n)$ is the four-momentum, $\varepsilon_n = (2n+1)\pi T$ is the fermion Matsubara energy, and α and β are the initial- and final-state spin projections defining the fermion propagator.

For ^{3}He quasiparticles and pairs confined to a low-energy band near the Fermi surface, the vertex function, which varies slowly with $|\mathbf{p}|$ in the neighborhood of the Fermi surface, can be evaluated with $\mathbf{p} = p_f\hat{p}$, $\varepsilon_n \to 0$ and $\mathbf{p}' = p_f\hat{p}'$, $\varepsilon_n' \to 0$ within the low-energy band $|\varepsilon_n|,|\varepsilon_n'| \leqslant \hbar\Omega_c$. In the same limit, we approximate the momentum-space integral as $\int \frac{d^3p'}{(2\pi)^3}(\ldots) \to \int \frac{d\Omega_{\hat{p}'}}{4\pi}N(0)\int d\xi_{\mathbf{p}'}(\ldots)$. The resulting vertex part reduces to functions of the relative momenta $A^{(s,a)}(\hat{p},\hat{p}') = 2N(0)\Gamma^{(s,a)}(p_f\hat{p},\varepsilon=0;\,p_f\hat{p}',\varepsilon'=0)$, where $N(0)$ is the density of states at the Fermi level and $\xi_{\mathbf{p}} = v_f(|\mathbf{p}| - p_f)$ is the quasiparticle excitation energy in the low-energy band near the Fermi surface. Rotational invariance implies that the vertex part can be expanded in terms of basis functions of the irreducible representations of $\mathrm{SO}(3)_L$, i.e., spherical harmonics $\{Y_{\ell,m}(\hat{p})\,|\, m = -\ell\ldots+\ell\}$, defined on the Fermi surface,

$$A^{(s,a)}(\hat{p},\hat{p}') = \sum_{\ell} A_\ell^{(s,a)} \sum_{m=-\ell}^{+\ell} Y_{\ell,m}(\hat{p})\,Y_{\ell,m}^*(\hat{p}'), \tag{25}$$

where the sum is over relative angular momentum channels $\ell \geqslant 0$. The resulting spin-independent $[\Sigma(\hat{p})]$ and exchange $[\vec{\Sigma}(\hat{p})]$ self-energies defined on the low-energy bandwidth of the interaction are given by

$$\Sigma(\hat{p}) = \Sigma_{\text{ext}}(\hat{p}) + \int \frac{d\Omega_{\hat{p}'}}{4\pi} A^{(s)}(\hat{p},\hat{p}')\, T \sum_{\varepsilon_{n'}}{}' \, g(\hat{p}',\varepsilon_n'), \tag{26}$$

$$\vec{\Sigma}(\hat{p}) = \vec{\Sigma}_{\text{ext}}(\hat{p}) + \int \frac{d\Omega_{\hat{p}'}}{4\pi} A^{(a)}(\hat{p},\hat{p}')\, T \sum_{\varepsilon_{n'}}{}' \, \vec{g}(\hat{p}',\varepsilon_n'), \tag{27}$$

where g and $\vec{g}$ are the scalar and spin-vector components of the quasiclassical propagator obtained by integration over the momentum shell $-\Omega_c \leqslant v_f \delta p \leqslant \Omega_c$ near the Fermi surface, $\int d\xi_{\mathbf{p}}\, G_{\alpha\beta}(p) \equiv g_{\alpha\beta}(\hat{p},\varepsilon_n) = g(\hat{p},\varepsilon_n)\,\delta_{\alpha\beta} + \vec{g}(\hat{p},\varepsilon_n) \cdot \vec{\sigma}_{\alpha\beta}$. Note that the Matsubara sum $\sum'$ is restricted to $|\varepsilon_n'| \leqslant \hbar\Omega_c$, and the self-energies vanish for the undisturbed Fermi sea.

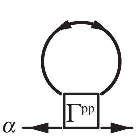
B. Particle-particle self-energy

The Cooper instability results from repeated scattering of fermion pairs with zero total momentum (Cooper channel) that leads to the formation of bound fermion pairs. Unbounded growth of the particle-particle amplitude is regulated by the formation of a new ground state, defined in terms of a macroscopic amplitude

$$F_{\alpha\beta}(p) = \langle \psi_\alpha(p)\psi_\beta(-p) \rangle \equiv \alpha \quad \beta, \tag{28}$$

for a condensate of fermion pairs with zero center-of-mass energy and momentum. The condensate and interaction in the Cooper channel also generates an associated mean field

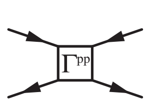
$$\Delta_{\alpha\beta}(p) = +p\,\alpha \quad -p\,\beta \tag{29}$$



$$= -T \sum_{\varepsilon_n'} \int \frac{d^3p'}{(2\pi)^3} \Gamma^{\text{pp}}_{\alpha\beta;\gamma\rho}(p,p')\, F_{\gamma\rho}(p'),$$

where

$$\Gamma^{\text{pp}}_{\alpha\beta;\gamma\rho}(p,p') = \tag{30}$$



$$= \Gamma^{(0)}(p,p')(i\sigma_y)_{\alpha\beta}(i\sigma_y)_{\gamma\rho} + \Gamma^{(1)}(p,p')(i\vec{\sigma}\sigma_y)_{\alpha\beta} \cdot (i\sigma_y\vec{\sigma})_{\gamma\rho} \tag{31}$$

is the four-fermion vertex that is irreducible in the particle-particle channel, expressed in terms of the spin-singlet ($S = 0$), even-parity and spin-triplet ($S = 1$), odd-parity pairing interactions $\Gamma^{(0)}(p,p')$ and $\Gamma^{(1)}(p,p')$, respectively. Thus, the pairing self-energy separates into singlet and triplet components

$$\Delta_{\alpha\beta}(p) = d(p)\,(i\sigma_y)_{\alpha\beta} + \vec{d}(p) \cdot (i\vec{\sigma}\sigma_y)_{\alpha\beta}. \tag{32}$$

Fermion pairs with binding energy $|\Delta| < \Omega_c$ are confined to a low-energy band near the Fermi surface $|\varepsilon| \leqslant \hbar\Omega_c \ll E_f$, and a shell in momentum space $\delta p \leqslant \hbar\Omega_c/v_f$. Thus, the particle-particle irreducible vertex, which varies slowly on with $|\mathbf{p}|$ in the neighborhood of the Fermi surface, can also be evaluated with $\mathbf{p} = p_f\hat{p}$, $\varepsilon_n \to 0$ and $\mathbf{p}' = p_f\hat{p}'$, $\varepsilon_n' \to 0$. Thus, Γ^{pp} reduces to even- and odd-parity functions of the relative momenta $V^{(S)}(\hat{p},\hat{p}') = 2N(0)\Gamma^{(S)}(p_f\hat{p},\varepsilon = 0; p_f\hat{p}',\varepsilon' = 0)$, and rotational invariance of the normal-state fermionic vacuum implies

$$\begin{aligned} V^{\binom{0}{1}}(\hat{p},\hat{p}') &= -\sum_{\ell}^{\binom{\text{even}}{\text{odd}}} v_\ell \sum_{m=-\ell}^{+\ell} Y_{\ell,m}(\hat{p})\, Y^*_{\ell,m}(\hat{p}') \\ &= -\sum_{\ell}^{\binom{\text{even}}{\text{odd}}} (2\ell+1)\, v_\ell\, P_\ell(\hat{p}\cdot\hat{p}'), \end{aligned} \tag{33}$$

where the sum is over all even (odd) orbital angular momentum channels, $\ell \geqslant 0$, for spin-singlet (spin-triplet) pair scattering, and $-v_\ell$ is the pairing interaction ("coupling constant") in the orbital angular momentum channel ℓ.[6] The singlet $[d(\hat{p})]$ and triplet $[\vec{d}(\hat{p})]$ self-energies are given by

$$d(\hat{p}) = -\int \frac{d\Omega_{\hat{p}'}}{4\pi} V^{(0)}(\hat{p},\hat{p}')\; T \sum_{\varepsilon_{n'}}{}' \, f(\hat{p}',\varepsilon_n'), \tag{34}$$

$$\vec{d}(\hat{p}) = -\int \frac{d\Omega_{\hat{p}'}}{4\pi} V^{(1)}(\hat{p},\hat{p}')\; T \sum_{\varepsilon_{n'}}{}' \, \vec{f}(\hat{p}',\varepsilon_n'), \tag{35}$$

where $f_{\alpha\beta}(\hat{p},\varepsilon_n) \equiv \int d\xi_{\mathbf{p}}\, F_{\alpha\beta}(p) = f(\hat{p},\varepsilon_n)\,(i\sigma_y)_{\alpha\beta} + \vec{f}(\hat{p},\varepsilon_n) \cdot (i\vec{\sigma}\sigma_y)_{\alpha\beta}$ is the quasiclassical pair propagator expressed in terms of the anomalous singlet and triplet components f and $\vec{f}$.

The breaking of U(1) symmetry by pair condensation implies mixing of normal-state particle and hole states. Particle-hole coherence is accommodated by introducing Nambu spinors $\Psi = (\psi_\uparrow, \psi_\downarrow, \psi_\uparrow^\dagger, \psi_\downarrow^\dagger)$ or, equivalently, by a 4×4 Nambu matrix propagator in the combined particle-hole and spin space. In the quasiclassical limit, the Nambu propagator is represented by the diagonal and off-diagonal quasiclassical propagators $\hat{g}$ and $\hat{f}$ and their conjugates $\hat{g}'$ and $\hat{f}'$:

$$\widehat{g} = \begin{pmatrix} g + \vec{g}\cdot\vec{\boldsymbol{\sigma}} & f\, i\sigma_y + \vec{f}\cdot i\vec{\boldsymbol{\sigma}}\sigma_y \\ f'\, i\sigma_y + \vec{f}'\cdot i\sigma_y\vec{\boldsymbol{\sigma}} & g' - \vec{g}'\cdot\sigma_y\vec{\boldsymbol{\sigma}}\sigma_y \end{pmatrix}, \tag{36}$$

where g ($\vec{g}$) is the spin scalar (vector) component of the fermion propagator, while f ($\vec{f}$) is the spin singlet (triplet) component of the anomalous pair propagator. The lower row of the Nambu matrix represents the conjugate propagators $\hat{\underline{g}}$ and $\hat{\underline{f}}$, which are related to $\hat{g}$ and $\hat{f}$ by the combination of fermion antisymmetry and particle-hole conjugation symmetries (cf.

[6] $v_\ell > 0$ corresponds to an *attractive* interaction in channel ℓ.

Appendix A 2). Similarly, the quasiparticle and pairing self-energies are organized into a 4×4 Nambu matrix

$$\widehat{\Sigma} = \begin{pmatrix} \Sigma + \vec{\Sigma} \cdot \vec{\boldsymbol{\sigma}} & d\, i\sigma_y + \vec{d} \cdot i\vec{\boldsymbol{\sigma}}\sigma_y \\ d'\, i\sigma_y + \vec{d}' \cdot i\sigma_y\vec{\boldsymbol{\sigma}} & \Sigma' - \vec{\Sigma}' \cdot \sigma_y\vec{\boldsymbol{\sigma}}\sigma_y \end{pmatrix}, \quad (37)$$

with the corresponding symmetry relations connecting the conjugate self-energies to Σ, $\vec{\Sigma}$, d, and $\vec{d}$. This doubling of the fermionic and bosonic degrees of freedom, which is forced by the breaking of global U(1) symmetry, is the origin of the doublets of bosonic modes labeled by parity under charge conjugation $\mathrm{c} = \pm 1$ in BCS-type theories.

VI. EILENBERGERS' EQUATIONS

The quasiparticle and anomalous pair propagators and self-energies, organized into 4×4 Nambu matrices, obey Gorkov's equations [43]. Eilenberger transformed Gorkov's equations into a matrix transport-type equation for the quasiclassical propagator and self-energy [44]

$$[i\varepsilon_n\widehat{\tau}_3 - \widehat{\Sigma}(\hat{p},\mathbf{R})\,,\widehat{g}] + i\hbar\mathbf{v}_{\hat{p}} \cdot \boldsymbol{\nabla}\widehat{g} = 0. \quad (38)$$

In contrast to Gorkov's equation, which is a second-order differential equation with a unit source term originating from the fermion anticommutation relations, Eilenberger's equation is a homogeneous, first-order differential equation describing the evolution of the quasiclassical propagator along classical trajectories defined by the Fermi velocity $\mathbf{v}_{\hat{p}} = v_f\,\hat{p}$. The form of Eilenberger's equation in Eq. (38) governs the *equilibrium* propagator, including inhomogeneous states described by an external potential or a spatially varying mean pairing self-energy $\widehat{\Delta}(\hat{p},\mathbf{R})$, but must be supplemented by the normalization condition [44]

$$\widehat{g}(\hat{p},\varepsilon_n;\mathbf{R})^2 = -\pi^2\,\widehat{1}, \quad (39)$$

which restores the constraint on the spectral weight implied by the source term in Gorkov's equation. For the spatially homogeneous ground state of superfluid ^{3}He-B Eilenberger's equation reduces to

$$[i\varepsilon_n\widehat{\tau}_3 - \widehat{\Sigma}(\hat{p})\,,\widehat{g}_0] = 0, \quad (40)$$

and the homogeneous self-energy $\widehat{\Sigma} \equiv \widehat{\Delta}(\hat{p})$ is defined by the mean-field pairing self-energy for the ^{3}He ground state

$$\widehat{\Delta}(\hat{p}) = \begin{pmatrix} 0 & \vec{\Delta}(\hat{p}) \cdot i\vec{\sigma}\sigma_y \\ \vec{\Delta}(\hat{p}) \cdot i\sigma_y\vec{\sigma} & 0 \end{pmatrix}, \quad (41)$$

where $\vec{\Delta}(\hat{p}) = \Delta\,\hat{p}$ is the $J = 0$ BW order parameter. Here and after we denote the equilibrium spin-triplet order parameter as $\vec{\Delta}$ and reserve $\vec{d}$ for the nonequilibrium fluctuations of the spin-triplet order parameter. The 4×4 matrix order parameter for the BW state satisfies $\widehat{\Delta}(\hat{p})\widehat{\Delta}(\hat{p}) = -|\Delta|^2\,\widehat{1}$. Thus, the equilibrium propagator for the BW state is given by

$$\widehat{g}_0(\hat{p},\varepsilon_n) = -\pi\frac{i\varepsilon_n\widehat{\tau}_3 - \widehat{\Delta}(\hat{p})}{\sqrt{\varepsilon_n^2 + |\Delta|^2}}. \quad (42)$$

Note that the diagonal component of $\widehat{g}_0$ is odd in frequency. This implies that the diagonal (fermionic) self-energies $\Sigma(\hat{p})$ and $\vec{\Sigma}(\hat{p})$, evaluated with Eq. (42), vanish in equilibrium. However, if the ground state is perturbed, e.g., by a bosonic fluctuation of the Cooper pair condensate, the fermionic self-energy, in general, no longer vanishes.

In equilibrium, the anomalous self-energy reduces to the self-consistency equation ("gap equation") for the spin-triplet order parameter

$$\vec{\Delta}(\hat{p}) = -\frac{\pi}{\beta}\sum_{\varepsilon_n}{}' \int\frac{d\Omega_{\hat{p}}}{4\pi}\,V^{(1)}(\hat{p},\hat{p}')\,\frac{\vec{\Delta}(\hat{p}')}{\sqrt{\varepsilon_n^2 + |\vec{\Delta}(\hat{p}')|^2}}. \quad (43)$$

The linearized gap equation defines the instability temperatures for Cooper pairing with orbital angular momentum ℓ:

$$\frac{1}{v_\ell} = \pi T_{c_\ell}\sum_{\varepsilon_n}{}'\frac{1}{|\varepsilon_n|} \equiv K(T_{c_\ell}) \quad (44)$$

for attractive interactions $v_\ell > 0$. The function $K(T)$ is a digamma function of argument $\hbar\Omega_c/2\pi T \gg 1$, in which case

$$K(T) \equiv \pi T\sum_{\varepsilon_n}{}'\frac{1}{|\varepsilon_n|} \simeq \ln\left(\frac{2e^{\gamma_{\mathrm{E}}}}{\pi}\frac{\hbar\Omega_c}{T}\right), \quad (45)$$

where $\gamma_{\mathrm{E}} \simeq 0.577\,21$ is Euler's constant. This function plays a central role in regulating the log divergence of frequency sums in the Cooper channel. In ^{3}He, the p-wave pairing channel is the dominant attractive channel; the f-wave channel is also attractive, but subdominant, i.e., $0 < T_{c_3} < T_{c_1} \equiv T_c$.

The anomalous self-energy in the p-wave channel also determines the mass (gap), $m_F = \Delta$, of fermionic excitations of the Balian-Werthamer phase. In particular, the p-wave projection of Eq. (43) reduces to the BCS gap equation

$$\ln(T/T_c) = 2\pi T\sum_{n\geqslant 0}^{\infty}\left(\frac{1}{\sqrt{\varepsilon_n^2 + \Delta^2}} - \frac{1}{\varepsilon_n}\right). \quad (46)$$

Note that both the pairing interaction v_1 and cutoff Ω_c in Eq. (43) have been eliminated in favor of the transition temperature by regulating the log-divergent sum using Eq. (45) and the linearized gap equation for T_c, Eq. (44).

The Balian-Werthamer state, which has an isotropic gap in the fermionic spectrum, is maximally effective in using states near the Fermi surface for pair condensation. As a result, the B phase is stable down to $T = 0$ in spite of the attractive f-wave pairing interaction [45]. Nevertheless, subdominant f-wave pairing plays an important role in the bosonic excitation spectrum of the B phase. In particular, p-wave, spin-triplet Higgs modes with $J = 2$ polarize the B-phase vacuum. The $J = 2$ polarization couples to f-wave, spin-triplet Cooper pair fluctuations with $J = 2$, leading to mass corrections to the $J^{\mathrm{c}} = 2^{\pm}$ Higgs modes. In the following, we derive the dynamical equations for the bosonic modes including the polarization terms from the f-wave pairing channel, and self-energy corrections from the Landau channel.

VII. DYNAMICAL EQUATIONS

In order to describe the nonequilibrium response, or fluctuations, relative to homogeneous equilibrium, we must generalize the low-energy quasiparticle and Cooper pair propagators to functions of two time (τ_1,τ_2) or frequency $(\varepsilon_{n_1},\varepsilon_{n_2})$ variables. Specifically, we must include the dependence on the global time coordinate $\Upsilon = (\tau_1 + \tau_2)/2$ or, equivalently, the total Matsubara energy ω_m, in addition to the relative time difference $\tau_1 - \tau_2$ or corresponding fermion Matsubara energy ε_n. Thus, the $\xi_{\mathbf{p}}$-integrated quasiclassical propagator generalizes to $\widehat{g}(\hat{p},\varepsilon_n) \to \widehat{g}(\hat{p},\varepsilon_n;\mathbf{q},\omega_m)$, where $\mathbf{q}$ is the total momentum or wave vector for a Fourier mode associated with the center-of-mass coordinate $\mathbf{R}$.

The space-time dynamics of the coupled system of fermionic and bosonic excitations of the broken symmetry ground state is encoded in the Keldysh propagator [46], which is obtained here by analytic continuation to the real energy axes, e.g., $i\varepsilon_n \to \varepsilon + i0^+$ followed by $i\omega_m \to \omega + i0^+$. Thus,

$$\frac{1}{\beta}\sum_{\varepsilon_n} \widehat{g}(\varepsilon_n;\omega_m) \xrightarrow[i\omega_m \to \omega + i0^+]{} \int_{-\infty}^{+\infty} \frac{d\varepsilon}{4\pi i}\, \widehat{g}^K(\varepsilon;\omega), \quad (47)$$

where $\widehat{g}^K(\hat{p},\varepsilon;\mathbf{q},\omega)$ is the real energy and frequency-dependent Keldysh propagator. The Keldysh propagator determines the response to any space-time–dependent excitation. For example, the particle current is given by

$$\mathbf{J} = N(0)\int \frac{d\Omega_{\hat{p}}}{4\pi} \int \frac{d\varepsilon}{4\pi i}\,(\mathbf{v_p})\mathrm{Tr}\{\widehat{\tau}_3 \widehat{g}^K(\hat{p},\varepsilon;\mathbf{q},\omega)\}. \quad (48)$$

The off-diagonal Nambu components of the Kelysh propagator determine the bosonic modes of the interacting fermionic and bosonic systems. The spin-triplet bosonic excitations are obtained from the anomalous triplet propagator $\vec{f}^K$ and the self-consistent solution for the anomalous self-energy obtained by analytic continuation of Eq. (35):

$$\vec{d}(\hat{p};\mathbf{q},\omega) = -\int \frac{d\Omega_{\hat{p}}}{4\pi}\, V^{(1)}(\hat{p},\hat{p}') \int \frac{d\varepsilon}{4\pi i}\, \vec{f}^K(\hat{p},\varepsilon;\mathbf{q},\omega). \quad (49)$$

To calculate the Keldysh propagator $\widehat{g}^K$, we generalize Eilenberger's transport equation for the two-time/frequency nonequilibrium Matsubara propagator

$$[i\varepsilon\widehat{\tau}_3 - \widehat{\Sigma}]\circ \widehat{g} - \widehat{g}\circ[i\varepsilon\widehat{\tau}_3 - \widehat{\Sigma}] + i\mathbf{v}_{\hat{p}}\cdot\nabla\widehat{g} = 0, \quad (50)$$

where the $A\circ B(\varepsilon_{n_1},\varepsilon_{n_2}) \equiv \frac{1}{\beta}\sum_{n_3} A(\varepsilon_{n_1},\varepsilon_{n_3})\, B(\varepsilon_{n_3},\varepsilon_{n_2})$ is a convolution in Matsubara energies. For the two-frequency, nonequilibrium propagator the normalization condition is also a convolution product in Matsubara frequencies

$$\widehat{g}\circ\widehat{g} \equiv \frac{1}{\beta}\sum_{\varepsilon_{n_3}} \widehat{g}(\varepsilon_{n_1},\varepsilon_{n_3})\widehat{g}(\varepsilon_{n_3},\varepsilon_{n_2}) = -\pi^2\beta\delta_{\varepsilon_{n_1},\varepsilon_{n_2}}\widehat{1}. \quad (51)$$

If we express the full propagator as a correction to the equilibrium propagator [Eq. (42)]

$$\widehat{g}(\hat{p},\mathbf{q};\varepsilon_{n_1},\varepsilon_{n_2}) = \widehat{g}_0(\hat{p},\varepsilon_{n_1})\,\beta\delta_{\varepsilon_{n_1},\varepsilon_{n_2}} + \delta\widehat{g}(\hat{p},\mathbf{q};\varepsilon_{n_1},\varepsilon_{n_2}), \quad (52)$$

then to linear order in $\delta\widehat{g}$ the normalization condition for the correction to the propagator becomes after setting $\varepsilon_{n_1} = \varepsilon_n + \omega_m$, $\varepsilon_{n_2} = \varepsilon_n$, and $\delta\widehat{g}(\hat{p},\mathbf{q};\varepsilon_{n_1},\varepsilon_{n_2}) \equiv \delta\widehat{g}(\hat{p},\mathbf{q};\varepsilon_n,\omega_m)$:

$$\widehat{g}_0(\varepsilon_n + \omega_m)\delta\widehat{g}(\varepsilon_n,\omega_m) + \delta\widehat{g}(\varepsilon_n,\omega_m)\widehat{g}_0(\varepsilon_n) = 0. \quad (53)$$

The bosonic modes of the interacting Fermi superfluid are obtained from the linearized dynamical equations for the fluctuations of the anomalous self energy $\delta\hat{\Delta} = \hat{\Delta} - \hat{\Delta}_0$, where the equilibrium self-energy $\hat{\Delta}_0$ is defined by off-diagonal mean-field pairing self-energy for the ^{3}He ground state [Eqs. (41) and (43)]. These fluctuations are coupled to fluctuations of the fermionic self-energy $\delta\hat{\Sigma}$. The coupled dynamical equations for the components of $\delta\widehat{\Sigma}(\hat{p};\mathbf{q},\omega)$ are obtained solving the nonequilibrium Eilenberger equation (50) for $\widehat{g}$ to linear order in the self-energy fluctuations $\delta\widehat{\Sigma}$. The linearized nonequilibrium Eilenberger equation becomes

$$\{i(\varepsilon_n + \omega_m)\widehat{\tau}_3 - \widehat{\Delta}(\hat{p})\}\,\delta\widehat{g} - \delta\widehat{g}\,\{i\varepsilon_n\widehat{\tau}_3 - \widehat{\Delta}(\hat{p})\} - \mathbf{v}_{\hat{p}}\cdot\mathbf{q}\;\delta\widehat{g}$$
$$+ \widehat{g}_0(\hat{p},\varepsilon_n + \omega_m)\,\delta\widehat{\Sigma} - \delta\widehat{\Sigma}\,\widehat{g}_0(\hat{p},\varepsilon_n) = 0. \quad (54)$$

The normalization conditions (39) and (53), combined with Eq. (42), provide a direct method of inverting Eq. (54) for the nonequilibrium quasiclassical propagator

$$\delta\widehat{g} = \left(\frac{-1}{\pi^2 D_+^2 + (\mathbf{v}_{\hat{p}}\cdot\mathbf{q})^2}\right)$$
$$\times [D_+\{\widehat{g}_0(\varepsilon_n + \omega_m)\,\delta\widehat{\Sigma}\,\widehat{g}_0(\varepsilon_n) + \pi^2\,\delta\widehat{\Sigma}\}$$
$$+ \mathbf{v}_{\hat{p}}\cdot\mathbf{q}\,\{\delta\widehat{\Sigma}\,\widehat{g}_0(\varepsilon_n) - \widehat{g}_0(\varepsilon_n + \omega_m)\,\delta\widehat{\Sigma}\}], \quad (55)$$

where $D_+(\varepsilon_n,\omega_m) = D(\varepsilon_n + \omega_m) + D(\varepsilon_n)$, and

$$D(\varepsilon_n) \equiv \frac{-1}{\pi}\sqrt{\varepsilon_n^2 + |\Delta|^2} \quad (56)$$

is the denominator of the equilibrium propagator.

To calculate the mass spectrum of the bosonic modes, we need only the $\mathbf{q} = 0$ propagators, in which case

$$\delta\widehat{g} = \frac{-1}{\pi^2}\frac{1}{D_+}\{\widehat{g}_0(\varepsilon_n + \omega_m)\,\delta\widehat{\Sigma}\,\widehat{g}_0(\varepsilon_n) + \pi^2\,\delta\widehat{\Sigma}\}. \quad (57)$$

In zero magnetic field, spin-singlet bosonic fluctuations, if they exist, do not couple to spin-triplet bosonic fluctuations. However, we must retain fluctuations of the fermionic self-energy, thus the form of the fluctuation self-energy becomes

$$\delta\widehat{\Sigma} = \begin{pmatrix} \Sigma + \vec{\Sigma}\cdot\vec{\boldsymbol{\sigma}} & \vec{d}\cdot i\vec{\boldsymbol{\sigma}}\sigma_y \\ \vec{d}'\cdot i\sigma_y\vec{\boldsymbol{\sigma}} & \Sigma' - \vec{\Sigma}'\cdot\sigma_y\vec{\boldsymbol{\sigma}}\sigma_y \end{pmatrix}, \quad (58)$$

where the conjugate spin-triplet order-parameter amplitudes are related by $\vec{d}'(\hat{p};\mathbf{q},\omega_m) = \vec{d}(\hat{p};-\mathbf{q},-\omega_m)^*$ (see Appendix). The linear combinations

$$\vec{d}^{(\pm)}(\hat{p};\mathbf{q},\omega_m) \equiv \vec{d}(\hat{p};\mathbf{q},\omega_m) \pm \vec{d}'(\hat{p};\mathbf{q},\omega_m) \quad (59)$$

have charge-conjugation parities $\mathrm{c} = \pm 1$; the dynamical equations for bosonic modes then separate into charge-conjugation doublets with opposite parity. The bosonic modes of Cooper pairs also couple to the fluctuations of the fermionic self-energy, in both the spin scalar and vector channels

$$\Sigma^{(\pm)}(\hat{p};\mathbf{q},\omega_m) \equiv \Sigma(\hat{p};\mathbf{q},\omega_m) \pm \Sigma'(\hat{p};\mathbf{q},\omega_m), \quad (60)$$

$$\vec{\Sigma}^{(\pm)}(\hat{p};\mathbf{q},\omega_m) \equiv \vec{\Sigma}(\hat{p};\mathbf{q},\omega_m) \pm \vec{\Sigma}'(\hat{p};\mathbf{q},\omega_m). \quad (61)$$

Note that the exchange and conjugation symmetry relations for the diagnoal self-energies [Eqs. (A10) and (A11) and (A18) and (A19)] imply that the fermionic self-energies $\Sigma^{(\pm)}$ and $\vec{\Sigma}^{(\pm)}$ are also even (odd) with respect to charge-conjugation parity $\mathrm{c} = \pm 1$.

The dynamical equations for the spin-triplet bosonic modes are obtained from the off-diagonal *and* diagonal components of

$\delta\widehat{g}$ in Eq. (55), the self-consistency equations for the leading-order mean-field self-energies [Eqs. (26), (27), and (35)]. Two response functions are obtained from the propagator in Eq. (57) that determine the bosonic and fermionic self-energies

$$\gamma(i\omega_m) = -\frac{1}{\beta}\sum_{\varepsilon_n}\Big[\frac{1}{D(\varepsilon_n)} + \frac{1}{D(\varepsilon_n+\omega_m)}\Big], \tag{62}$$

$$\lambda(i\omega_m) = \frac{2}{\pi^2\beta}\sum_{\varepsilon_n}\frac{|\Delta^2|}{D_+(\varepsilon_n,\omega_m)D(\varepsilon_n)D(\varepsilon_n+\omega_m)}. \tag{63}$$

The Matsubara sum defining $\gamma(i\omega_m)$ is log divergent, regulated by the cutoff Ω_c. The frequency dependence of γ can be neglected since it gives a negligible correction of order $(\omega_m/\Omega_c)^2 \ll 1$. Thus,

$$\frac{1}{2}\gamma = \frac{\pi}{\beta}\sum_{\varepsilon_n}{}' \frac{1}{\sqrt{\varepsilon_n^2+|\Delta|^2}} = \frac{1}{v_1}, \tag{64}$$

where the latter equality follows from the equilibrium gap equations (44)–(46). The function $\lambda(i\omega_m)$ is defined by a convergent Matsubara sum. Analytic continuation to real frequencies of Eq. (63) in the manner of Eq. (47) yields

$$\lambda(\omega) \equiv |\Delta|^2\,\bar{\lambda}(\omega) = |\Delta|^2\int_{|\Delta|}^{\infty}\frac{d\varepsilon}{\sqrt{\varepsilon^2-|\Delta|^2}}\frac{\tanh\left(\frac{\beta\varepsilon}{2}\right)}{\varepsilon^2-\omega^2/4}, \tag{65}$$

which is the Tsuneto function with $\omega \to \omega + i0^+$ defining the retarded (causal) response [47]. For $|\omega| < 2|\Delta|$, $\lambda(\omega)$ is real and defines the nonresonant frequency response of the condensate, while for $|\omega| > 2|\Delta|$, $\mathrm{Im}\lambda(\omega) \neq 0$ is the spectral density of unbound fermion pairs. In the $T = 0$ limit, with $x = \omega/2|\Delta|$,

$$\lambda(\omega) = \begin{cases} \frac{\sin^{-1}(x)}{x\sqrt{1-x^2}}, & |x| < 1 \\ \frac{1}{2x\sqrt{x^2-1}}\Big[\ln\Big|\frac{\sqrt{x^2-1}-x}{\sqrt{x^2-1}+x}\Big| + i\pi\,\mathrm{sgn}(x)\Big], & |x| > 1. \end{cases} \tag{66}$$

Thus, analytic continuation to real frequencies for the $\mathbf{q} = 0$ limit leads to the following dynamical equations for the spin-triplet bosonic modes of the B-phase ground state [39–42]

$$\vec{d}^{(-)}(\hat{p};\omega) = -\int\frac{d\Omega_{p'}}{4\pi}\,V^{(1)}(\hat{p},\hat{p}')\bigg\{\Big[\frac{1}{2}\gamma + \frac{1}{4}(\omega^2-4|\Delta|^2)\bar{\lambda}(\omega)\Big] \times \vec{d}^{(-)}(\hat{p}';\omega) + \bar{\lambda}(\omega)\vec{\Delta}(\hat{p}')\,[\vec{\Delta}(\hat{p}')\cdot\vec{d}^{(-)}(\hat{p}';\omega)] - \frac{1}{2}\omega\,\bar{\lambda}(\omega)\,\vec{\Delta}(\hat{p}')\,\Sigma^{(+)}(\hat{p}';\omega)\bigg\}, \tag{67}$$

$$\vec{d}^{(+)}(\hat{p};\omega) = -\int\frac{d\Omega_{p'}}{4\pi}\,V^{(1)}(\hat{p},\hat{p}')\bigg\{\Big[\frac{1}{2}\gamma + \frac{1}{4}\omega^2\bar{\lambda}(\omega)\Big] \times \vec{d}^{(+)}(\hat{p}';\omega) - \bar{\lambda}(\omega)\vec{\Delta}(\hat{p}')[\vec{\Delta}(\hat{p}')\cdot\vec{d}^{(+)}(\hat{p}';\omega)] + \frac{i}{2}\omega\,\bar{\lambda}(\omega)\,\vec{\Delta}(\hat{p}')\times\vec{\Sigma}^{(+)}(\hat{p}';\omega)\bigg\}. \tag{68}$$

Note that the equations of motion for the bosonic fluctuations of the order parameter couple to the fermionic self-energies *linearly* in the frequency ω, and that only the even orbital parity fermionic fluctuations contribute in the $\mathbf{q} = 0$ limit.

For the moment we omit pairing fluctuations in higher angular momentum channels, i.e., set $v_\ell = 0$ for $\ell \geqslant 3$. We then expand the spin-triplet order-parameter amplitudes $\vec{d}^{(\pm)}(\hat{p})$ in terms of the p-wave basis $d^{(\pm)}_\alpha(\hat{p}) = \mathscr{D}^{(\pm)}_{\alpha i}\,\hat{p}_i$, where $\mathscr{D}^{(\pm)}_{\alpha i}$ is equivalent to the bivector representation of the order parameter discussed in the context of the TDGL theory for the bosonic modes. For the B-phase ground state with total angular momentum $J = 0$, i.e., $\vec{\Delta}(\hat{p}) = \Delta\hat{p}$ or, equivalently, $A_{\alpha i} = \Delta/\sqrt{3}\,\delta_{\alpha i}$, Eqs. (67) and (68) can be solved by expanding the pairing fluctuations in spherical tensors that define bases for the representations of the residual symmetry group of the B phase $H = \mathrm{SO}(3)_J$, with total angular momentum $J = 0,1,2$,

$$\mathscr{D}^{(\pm)}_{\alpha i} = \sum_{J=0,1,2}\;\sum_{m=-J,+J} D^{(\pm)}_{J,m}\,t^{(J,m)}_{\alpha i}. \tag{69}$$

Note that time-dependent fluctuations of the fermionic self-energy, e.g., $\omega\Sigma^{(+)}(\hat{p};\omega)$, appear as "source" terms in the equations of motion for the order-parameter collective modes.

A. Nambu-Goldstone and Higgs modes with c = −1

In the case of the modes with parity $\mathrm{c} = -1$ we can express

$$\Sigma^{(+)}(\hat{p};\omega) = \sum_J^{\text{even}}\sum_m \Sigma^{(+)}_{J,m}(\omega)\,\hat{p}_i\,t^{(J,m)}_{ij}\,\hat{p}_j. \tag{70}$$

Note that only self-energy fluctuations of even J couple to the bosonic modes with $\mathrm{c} = -1$. Equation (67) then decouples into the dynamical equations for bosonic mode amplitudes with total angular momentum J. In particular, the equation for dynamical fluctuations with $J^{\mathrm{c}} = 0^-$ is given by

$$\omega^2\,D^{(-)}_{0,0} = 2\,|\Delta|\,\omega\,\Sigma^{(+)}_{0,0}. \tag{71}$$

In the simplest case, the $J = 0$ contribution to the fermionic self-energy represents a fluctuation in the chemical potential, i.e., $\Sigma^{(+)}_{0,0}(\omega) = 2\delta\mu(\omega)$, and as discussed earlier the pairing fluctuation with $J^{\mathrm{c}} = 0^-$ represents time-dependent fluctuations of the phase of the B-phase ground state, i.e., $D^{(-)}_{0,0} = 2i|\Delta|\,\vartheta(\omega)$. This is the massless Anderson-Bogoliubov mode, which in the time domain for $\mathbf{q} = 0$ obeys the Josephson phase relation $\hbar\partial_t\vartheta = -2\delta\mu$. As we show below, this result is unrenormalized by interactions between fermions in either particle-hole or particle-particle channels.

Projecting out the pairing fluctuations with $J^c = 1^-$ from Eq. (67) yields

$$(\omega^2 - 4|\Delta|^2)\, D^{(-)}_{1,m} = 0. \tag{72}$$

This is a quite remarkable result: the $J^c = 1^-$ pairing fluctuations do not couple to fluctuations in the fermion self-energy. Furthermore, neither d-wave, spin-singlet, nor f-wave spin-triplet pairing fluctuations couple the $J^c = 1^-$ modes, which implies that the mass of $J^c = 1^-$ Higgs modes, $M_{1,-} = 2\Delta$, is unrenormalized by interactions to leading order in the expansion.

By contrast, the $J^c = 2^-$ modes obey the following dynamical equations:

$$\left[\omega^2 - \frac{12}{5}|\Delta|^2\right] D^{(-)}_{2,m} = \frac{4}{5}\,|\Delta|\,\omega\,\Sigma^{(+)}_{2,m}. \tag{73}$$

In the absence of fermion interactions in the particle-hole channel $\Sigma^{(+)}_{2,m}(\omega)$ represents *external* stress fluctuations $u^{(+)}_{2,m}(\omega)$ that couple directly to the $J^c = 2^-$ bosonic modes. In this case, the mass of the this Higgs mode is equal to the weak-coupling TDGL result $M_{2,-} = \sqrt{12/5}\,\Delta$, but now extended to all temperatures. However, the weak-coupling result for the mass of the $J^c = 2^-$ Higgs mode is renormalized by fermionic interactions. Qualitatively, this is expected given that external stress fluctuations couple directly to the $J^c = 2^-$ pairing fluctuations. Excitation of a $J^c = 2^-$ Higgs boson *polarizes* the $J = 0$ fermionic vacuum, inducing a fermionic self-energy correction of the same symmetry that couples back to generate a mass correction to the $J^c = 2^-$ Higgs modes. The polarization correction to the Higgs mass is encoded in Eq. (A24) in the limit $\mathbf{q} = 0$, which can be expressed as

$$\begin{aligned}\Sigma^{(+)}(\hat{p};\omega) &= u^{(+)}(\hat{p};\omega)\\ &\quad + \int \frac{d\Omega_{\hat{p}'}}{4\pi}\, F^s(\hat{p},\hat{p}')\Big\{-\lambda(\omega)\,\Sigma^{(+)}(\hat{p}';\omega)\\ &\quad + \frac{1}{2}\bar{\lambda}(\omega)\,\omega\,\vec{\Delta}(\hat{p}')\cdot\vec{d}^{(-)}(\hat{p}';\omega)\Big\},\end{aligned} \tag{74}$$

where $u^{(+)}(\hat{p};\omega)$ represents unrenormalized external forces coupling to excitations of ^{3}He-B, and we have expressed the dynamical self-energy in terms of the spin-symmetric particle-hole irreducible interaction $F^s(\hat{p},\hat{p}')$ (cf. Appendix A 3). Projecting out the amplitudes with $J = 0,2$ defined in Eq. (70) gives

$$\begin{aligned}&\left[1 + F^s_0\,\lambda(\omega)\right]\Sigma^{(+)}_{0,0}(\omega)\\ &\quad = u_{0,0}(\omega) + F^s_0\,\lambda(\omega)\left(\frac{\omega}{2|\Delta|}\right) D^{(-)}_{0,0}(\omega),\end{aligned} \tag{75}$$

$$\begin{aligned}&\left[1 + \frac{1}{5}F^s_2\,\lambda(\omega)\right]\Sigma^{(+)}_{2,m}(\omega)\\ &\quad = u_{2,m}(\omega) + \frac{1}{5}F^s_2\,\lambda(\omega)\left(\frac{\omega}{2|\Delta|}\right) D^{(-)}_{2,m}(\omega).\end{aligned} \tag{76}$$

The key result shown in Eqs. (75) and (76) is that excitation of pairing fluctuations $D^{(-)}_{J,m}(\omega)$ polarizes the condensate and generates internal*internal* stresses that are proportional to (i) interactions in the particle-hole channel $F^s_{2,0}$, (ii) the time derivative of the bosonic mode amplitudes $\omega\, D^{(-)}_{J,m}(\omega)$, and (iii) the dynamical response of the condensate $\lambda(\omega)$, even in the absence of bulk external forces, i.e.. $u_{J,m}(\omega) = 0$. In the case of the $J^c = 0^-$ mode, combining Eq. (71) with $\Sigma^{(+)}_{0,0}(\omega)$ now given by Eq. (75) still yields the *unrenormalized* dynamical equation for excitation of the Anderson-Bogoliubov phase mode

$$\omega^2\, D^{(-)}_{0,0} = 2\,|\Delta|\,\omega\, u^{(+)}_{0,0}. \tag{77}$$

The interaction F^s_0 drops out because the polarization induced by the $J^c = 0^-$ bosonic mode has the same rotational symmetry as the vacuum state.

However, in the case of the $J^c = 2^-$ Higgs modes, combining Eq. (73) with $\Sigma^{(+)}_{2,m}(\omega)$ given by Eq. (76) yields

$$D^{(-)}_{2,m} = \frac{\frac{4}{5}\,|\Delta|\,\omega\, u^{(+)}_{2,m}(\omega)}{\left[\omega^2 - \frac{12}{5}|\Delta|^2 + \lambda(\omega)\,\frac{3}{25}\,F^s_2\,(\omega^2 - 4|\Delta|^2)\right]}, \tag{78}$$

which has a pole at $\omega = M_{2,-}$, the renormalized mass of the $J^c = 2^-$ mode. Before discussing the quantitative effect of the Landau interaction on the $J^c = 2^-$ Higgs mass, we consider the effect of interactions in the Cooper channel.

B. f-wave interactions in the Cooper channel

Theoretical models for fermionic interactions in the particle-particle (Cooper) channel based on exchange of long-lived ferromagnetic spin fluctuations predict p-wave spin-triplet pairing with subdominant attraction in the f-wave Cooper channel, including a strong subdominant f-wave attractive interaction at high pressures [48,49]. The masses of the $J^c = 2^-$ modes are sensitive to fermionic interactions in the particle-particle channel, the most relevant being the f-wave, spin-triplet channel. Pairing fluctuations in the Cooper channel couple to the p-wave, spin-triplet modes with $J = 2$ leading to renormalization of the mass of the $J^c = 2^\pm$ Higgs modes. Note that f-wave pairing fluctuations do not couple to the $J = 0,1$ bosonic modes.

The generalization of Eqs. (73) and (76) to include the f-wave pairing channel in the dynamics of the $J^c = 2^-$ modes is obtained from Eqs. (67) and (74) by retaining both p-wave and f-wave pairing amplitudes

$$d^{(-)}_\alpha(\hat{p}) = \mathscr{D}^{(-)}_{\alpha i}\,\hat{p}_i + \mathscr{F}^{(-)}_{\alpha;ijk}\,\hat{p}_i\,\hat{p}_j\,\hat{p}_k, \tag{79}$$

where $\mathscr{D}^{(-)}_{\alpha i}$ is a second-rank tensor under the residual symmetry group of the B-phase $\mathrm{SO}(3)_J$, representing p-wave, spin-triplet fluctuations with odd charge-conjugation parity [Eq. (69)], and $\mathscr{F}^{(-)}_{\alpha;ijk}$ is a fourth-rank tensor with f-wave orbital symmetry, and thus is completely symmetric and traceless in any pair of the orbital indices (ijk). Spin-triplet, f-wave pairing fluctuations couple only to the $J = 2$ p-wave, triplet modes. Thus, for pure $J = 2$, $S = 1$, $\ell = 3$ fluctuations,

$$\begin{aligned}\mathscr{F}^{(-)}_{\alpha;ijk} &= \frac{5}{9}\Big\{\big(\delta_{\alpha i}\mathscr{F}^{(-)}_{jk} + \delta_{\alpha j}\mathscr{F}^{(-)}_{ik} + \delta_{\alpha k}\mathscr{F}^{(-)}_{ij}\big)\\ &\quad - \frac{2}{5}\big(\mathscr{F}^{(-)}_{\alpha i}\delta_{jk} + \mathscr{F}^{(-)}_{\alpha j}\delta_{ik} + \mathscr{F}^{(-)}_{\alpha k}\delta_{ij}\big)\Big\},\end{aligned} \tag{80}$$

where by contraction

$$\mathscr{F}^{(-)}_{jk} \equiv \tfrac{3}{7}\,\mathscr{F}^{(-)}_{\alpha;\alpha jk} \tag{81}$$

is a rank-two, traceless, and symmetric $J = 2$ tensor. In particular, we can expand $\mathscr{F}^{(-)}_{ij}$ in the $J = 2$ base tensors

$$\mathscr{F}^{(-)}_{ij} = \sum_{m=-2}^{+2} F^{(-)}_{2,m}\, t^{(2,m)}_{ij}. \tag{82}$$

The $J = 2^-$ gap distortion is determined by both the p- and f-wave $J = 2$ tensors

$$\vec{\Delta}(\hat{p}) \cdot \vec{d}^{(-)}(\hat{p};\omega) = \Delta\left(\mathscr{D}^{(-)}_{ij} + \mathscr{F}^{(-)}_{ij}\right)\hat{p}_i\,\hat{p}_j, \tag{83}$$

and thus the $J = 2^-$ component of the fermionic self-energy induced by the $J^c = 2^-$ Higgs modes [cf. Eq. (76)] becomes

$$\left(1 + \frac{1}{5}F^s_2\,\lambda(\omega)\right)\Sigma^{(+)}_{2,m}(\omega) = u_{2,m}(\omega) + \frac{1}{5}F^s_2\,\lambda(\omega)\left(\frac{\omega}{2|\Delta|}\right)\left[D^{(-)}_{2,m}(\omega) + F^{(-)}_{2,m}(\omega)\right]. \tag{84}$$

The $J^c = 2^-$ amplitudes satisfy coupled time-dependent gap equations obtained by projecting out the p- and f-wave components of Eq. (67) where v_ℓ are the pairing interactions in orbital angular momentum channel, $\ell = 1,3,\ldots$. The p-wave interaction is the dominant attractive channel. The relevant measure of the strength of the subdominant f-wave pairing interaction is

$$x_3^{-1} \equiv \left(\frac{1}{v_1} - \frac{1}{v_3}\right)^{-1} = \ln(T_{c_3}/T_c)^{-1}, \tag{85}$$

where $x_3^{-1} < 0$ ($x_3^{-1} > 0$) for attractive (repulsive) f-wave pairing. The latter equality, valid for attractive f-wave pairing, is obtained from Eq. (44) for the eigenvalue spectrum of the linearized gap equation, with T_c the p-wave transition temperature and T_{c_3} the f-wave *instability temperature* for subdominant f-wave pairing.

Projecting out the $\ell = 1$, $J = 2$ component of Eq. (67), which generalizes Eq. (73), leads to

$$\left[\omega^2 - \tfrac{12}{5}|\Delta|^2\right] D^{(-)}_{2,m} + \tfrac{8}{5}|\Delta|^2\, F^{(-)}_{2,m} = \tfrac{4}{5}\,|\Delta|\,\omega\,\Sigma^{(+)}_{2,m}. \tag{86}$$

Projecting out the $\ell = 3$ amplitudes from Eq. (67) gives

$$\begin{aligned}&\left[x_3 + \frac{1}{4}\bar{\lambda}(\omega)(\omega^2 - 4|\Delta|^2)\right]\\ &\quad\times d^{(3,-)}_\alpha(\hat{p};\omega) + 7\int\frac{d\Omega_{\hat{p}'}}{4\pi}\,P_3(\hat{p}\cdot\hat{p}')\\ &\quad\times\Big\{\bar{\lambda}(\omega)\,\Delta_\alpha(\hat{p}')\,\vec{\Delta}(\hat{p}')\cdot\vec{d}^{(-)}(\hat{p}';\omega)\\ &\quad= \frac{1}{2}\bar{\lambda}(\omega)\,\omega\,\Delta_\alpha(\hat{p}')\,\Sigma^{+}(\hat{p}';\omega)\Big\}.\end{aligned} \tag{87}$$

The $J = 2$ components of Eq. (87) are obtained by contracting with $\hat{p}_\alpha$ to obtain an equation for $F^{(-)}(\hat{p}) \equiv \mathscr{F}^{(-)}_{ij}\,\hat{p}_i\,\hat{p}_j$, then evaluating the angular average using the addition theorem for the Legendre polynomials $(\hat{p}\cdot\hat{p}')\,P_3(\hat{p}\cdot\hat{p}') = \frac{1}{7}\{4\,P_4(\hat{p}\cdot\hat{p}') + 3\,P_2(\hat{p}\cdot\hat{p}')\}$ to obtain

$$\left[\bar{x}_3 + \tfrac{1}{4}\left(\omega^2 - \tfrac{8}{5}|\Delta|^2\right)\right]F^{(-)}_{2,m} + \tfrac{3}{5}|\Delta|^2\,D^{(-)}_{2,m} = \tfrac{3}{10}\,|\Delta|\,\omega\,\Sigma^{+}_{2,m}, \tag{88}$$

where $\bar{x}_3 \equiv x_3/\bar{\lambda}(\omega)$. Eliminating the fermionic self-energy between Eqs. (86) and (88) gives the subdominant f-wave, $J^c = 2^-$ amplitude in terms of the dominant p-wave, $J^c = 2^-$:

$$\left[\bar{x}_3 + \tfrac{1}{4}(\omega^2 - 4|\Delta|^2)\right]F^{(-)}_{2,m} = \tfrac{3}{8}(\omega^2 - 4|\Delta|^2)\,D^{(-)}_{2,m}. \tag{89}$$

The *total* $J^c = 2^-$ Higgs amplitude, the sum of the p- and f-wave amplitudes $H^{(-)}_{2,m}(\omega) \equiv D^{(-)}_{2,m} + F^{(-)}_{2,m}$, that polarizes the fermionic vacuum [Eq. (76)] is governed by the dynamical equation obtained by combining Eqs. (86) and (89). This gives the retarded propagator for the $J^c = 2^-$ Higgs mode

$$H^{(-)}_{2,m} = \frac{\tfrac{4}{5}\,|\Delta|\,\omega\,u^{(+)}_{2,m}(\omega)\left[1 + \tfrac{5}{8}\,x_3^{-1}\,(\omega^2 - 4|\Delta|^2)\bar{\lambda}(\omega)\right]}{\left[\omega^2 - \tfrac{12}{5}|\Delta|^2 + \lambda(\omega)\,(\omega^2 - 4|\Delta|^2)\left(\tfrac{3}{25}\,F^s_2\, + (\omega/2|\Delta|)^2\,x_3^{-1}\right)\right]}. \tag{90}$$

The renormalized $J^c = 2^-$ Higgs mass is obtained from the pole of the propagator in Eq. (90). In the limit $T \to T_c^-$ the Tsuneto function scales as $\lambda(\omega = M_{2,-}) \propto \Delta(T)/T_c \to 0$. Thus, the $J^c = 2^-$ Higgs mass scales to the the weak-coupling TDGL result at T_c:

$$M_{2^-} \approx \sqrt{\frac{12}{5}}\Delta(T)\left[1 + \frac{\pi}{10}\sqrt{\frac{5}{2}}\,\frac{\Delta(T)}{T_c}\,\left(F^s_2/5 + x_3^{-1}\right)\right]. \tag{91}$$

However, the leading-order correction to the mass $\propto \Delta(T)/T_c \sim (1 - T/T_c)^{\frac{1}{2}}$ onsets rapidly below T_c. Thus, mass renormalization becomes significant, of order F_2^2 or x_3^{-1}, for $T \to 0$. For weak interactions in both the Landau and Cooper channels, $|F^s_2| \ll 1$ and $|x_3^{-1}| \ll 1$, at $T = 0$ the renormalized mass obtained from the pole of the $J^c = 2^-$ propagator in Eq. (90) is

$$M_{2^-} \approx \sqrt{\frac{12}{5}}\Delta\left[1 + a\left(F^s_2/5 + x_3^{-1}\right)\right], \tag{92}$$

where $a = \frac{1}{\sqrt{6}}\arcsin\left(\sqrt{\frac{3}{5}}\right) \approx 0.362$. The Landau channel interaction F^s_2 obtained from measurements of the zero sound velocity ranges from $F^s_2 \approx 0.5$ at $P = 0$ bar to $F^s_2 \approx 1.0$ at $P = 34$ bar, although earlier measurements reported $F^s_2 \approx -0.5$ at $p = 0$ bar [50].

The f-wave interaction in the Cooper has been determined from measurements of the mass of the $J^c = 2^-$, $m = 0$ Higgs mode based on resonant absorption of longitudinal zero sound. These experiments yield results ranging from $x_3^{-1} \approx 0.0$ at $p = 0$ bar to $x_3^{-1} \approx -0.5$ at $p = 14$ bar (cf. Fig. 50 in Ref. [51]). Determinations of the mass of the $J^c = 2^-$, $m = \pm 1$ Higgs modes based on transverse sound

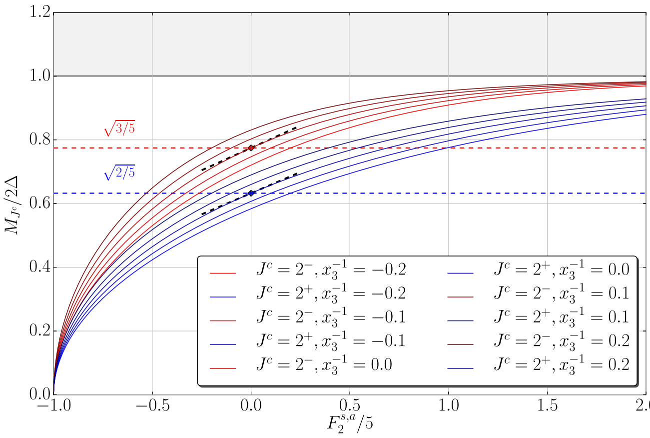


FIG. 3. Masses of the $J^c = 2^{\pm}$ Higgs modes vs $\ell = 2$ particle-hole ($F_2^{s,a}$) and f-wave pairing (x_3^{-1}) interactions at $T = 0$. The perturbative results [Eqs. (92) and (113)] for $x_3^{-1} = 0$ are shown as the dashed black lines.

propagation and acoustic Faraday rotation by Lee *et al.* [35,52], as well as more recent measurements by Collett *et al.* [53] yield attractive f-wave interactions of similar magnitude. The f-wave interaction in the Cooper channel also contributes to the nonlinear nuclear magnetic susceptibility for the B phase [54]. Analysis of magnetic susceptibility measurements of Hoyt *et al.* [55] yields a stronger, but subdominant, attractive f-wave interaction with $x_3^{-1} \simeq -1.75$ ($T_{c_3}/T_c \simeq 0.56$) at low pressure [56].

Figure 3 shows the mass of the $J = 2^c = 2^-$ Higgs mode as a function of F_2^s for various values of the f-wave pairing interaction x_3^{-1} obtained from numerical solution for the pole of the propagator $H_{2,m}^{(-)}$ in Eq. (90). Note that "repulsive" interactions in either channel ($F_2^{s,a} > 0$ or $x_3^{-1} > 0$) push the mass above the weak-coupling result towards the mass of unbound fermion pairs, while "attractive interactions" soften the mode. In particular, $M_{2,-} \to 0$ for $F_2^s/5 \to -1$, signaling a dynamical instability of the ground state. The soft mode is the dynamical signature of the Pomeranchuk instability of the underlying fermionic vacuum [57].

C. Nambu-Goldstone and Higgs modes with c = +1

In the case of the bosonic modes with parity c = +1 the fermion self-energy that couples to these modes is expressed in terms of the momentum-dependent exchange field $\vec{\Sigma}^{(+)}(\hat{p};\omega)$. Equation (68) decouples into the dynamical equations for bosonic mode amplitudes with total angular momentum J, with orbital angular momentum $\ell = 1$, $D_{J,m}^{(+)}$ and $\ell = 3$, $F_{J,m}^{(+)}$. The self-energy fluctuations originating from the exchange contribution to the quasiparticle interaction are even under $\hat{p} \to -\hat{p}$; thus, only fluctuations with even J couple to the bosonic modes for c = +1. To obtain the dynamical equations for the J^+ modes, it is convenient to introduce

$$\vec{G}^{(+)}(\hat{p};\omega) = \vec{\Delta}(\hat{p}) \times \vec{\Sigma}^{(+)}(\hat{p};\omega)/|\vec{\Delta}(\hat{p})|. \tag{93}$$

For the $J = 0^+$ ground state $\vec{G}^{(+)}(\hat{p};\omega) = \hat{p} \times \vec{\Sigma}^{(+)}$ is a vector under spin rotations, *odd* under $\hat{p} \to -\hat{p}$ and enters Eq. (68) acting as an effective source field for Cooper pair fluctuations with c = +1.

It is sufficient to retain only the $\ell = 0$ and 2 contributions to the particle-hole exchange interaction $F^a(\hat{p},\hat{p}') = F_0^a + F_2^a\, P_2(\hat{p}\cdot\hat{p})$, in which case we can express the vector components of the quasiparticle exchange field in terms $\ell = 0$ and 2 spherical tensors

$$\Sigma_\gamma^{(+)}(\hat{p};\omega) = \Sigma_\gamma^{(0)} + \Sigma_{\gamma:\alpha\beta}^{(2)}\, \hat{p}_\alpha \hat{p}_\beta, \tag{94}$$

where $\Sigma_{\gamma:\alpha\beta}^{(2)}$ is traceless and symmetric in the indices α,β. The vector function $\vec{G}^{(+)}(\hat{p};\omega)$ by construction contains only p- and f-wave orbital components $\vec{G}^{(+)}(\hat{p};\omega) = \vec{G}^{(1)}(\hat{p};\omega) + \vec{G}^{(3)}(\hat{p};\omega)$, with $G_\gamma^{(\ell)}(\hat{p}) = \langle (2\ell+1)\, P_\ell(\hat{p}\cdot\hat{p}')\, G_\gamma^{(+)}(\hat{p}')\rangle_{\hat{p}'}$, where $\langle\ldots\rangle_{\hat{p}} \equiv \int d\Omega_{\hat{p}}/4\pi(\ldots)$. Equivalently, the p-wave contribution is defined by a second-rank tensor under joint spin and orbital rotations,

$$G_{\gamma i}^{(1)} = \langle 3\,\hat{p}_i\, G_\gamma^{(+)}(\hat{p})\rangle_{\hat{p}} = \varepsilon_{\alpha i\gamma}\, \Sigma_\gamma^{(0)} + \frac{2}{5}\varepsilon_{\alpha\beta\gamma}\, \Sigma_{\gamma:\beta i}^{(2)} \tag{95}$$

$$= G_{\gamma i}^{(1,0)} + G_{\gamma i}^{(1,1)} + G_{\gamma i}^{(1,2)}, \tag{96}$$

where the second equation is the reduction in terms of $J = 0,1,2$ tensors. The $J = 0$ component is defined by the trace, which is easily seen to vanish, i.e., $G_{\alpha i}^{(1,0)} \equiv 0$. The $J = 1$ components can be expressed in terms of an axial vector

$$G_{\alpha i}^{(1,1)} = \varepsilon_{\alpha i\nu}\, G_\nu^{(1,1)} \quad \text{with} \quad G_\nu^{(1,1)} = \Sigma_\nu^{(0)} - \tfrac{1}{5}\, \Sigma_{\gamma:\gamma\nu}^{(2)}. \tag{97}$$

Finally, the $J = 2$ components are determined by the traceless, symmetric tensor

$$G_{\alpha i}^{(1,2)} = \tfrac{1}{5}\big(\varepsilon_{\alpha\beta\gamma}\, \Sigma_{\gamma:\beta i}^{(2)} + \varepsilon_{i\beta\gamma}\, \Sigma_{\gamma:\beta\alpha}^{(2)}\big), \tag{98}$$

which can be expanded in the basis of $J = 2$ tensors

$$G_{\alpha i}^{(1,2)} = \sum_{m=-2}^{+2} G_{2,m}\, t_{\alpha i}^{(2,m)}. \tag{99}$$

These contributions to the exchange field couple to the bosonic mode amplitudes with quantum numbers J,m and c = +1, represented by second- and fourth-rank tensors that are the c = +1 complements of those in Eq. (79):

$$d_\alpha^{(+)}(\hat{p}) = \mathscr{D}_{\alpha i}^{(+)}\, \hat{p}_i + \mathscr{F}_{\alpha;ijk}^{(+)}\, \hat{p}_i \hat{p}_j \hat{p}_k\,, \tag{100}$$

where the spin-triplet, p-wave order-parameter fluctuations are expanded in the basis of tensors with $J = 0,1,2$,

$$\mathscr{D}_{\alpha i}^{(+)} = \sum_{J=0,1,2} \sum_{m=-J}^{J} D_{J,m}^{(+)}\, t_{\alpha i}^{(J,m)}, \tag{101}$$

and similarly for spin-triplet, f-wave fluctuations with $J = 2^+$, $\mathscr{F}_{\alpha i}^{(+)} = \sum_{m=-2}^{+2} F_{2,m}^{(+)}\, t_{\alpha i}^{(2,m)}$ where $\mathscr{F}_{\alpha i}^{(+)} = \frac{3}{7}\mathscr{F}_{\gamma:\gamma\alpha i}$.

The equation governing the $J^c = 0^+$ mode is

$$(\omega^2 - 4|\Delta|^2) D_{0,0}^{(+)} = 0. \tag{102}$$

This is the dynamical equation for the Higgs mode with the *exact* quantum numbers of the B-phase vacuum state. As a result, there is no coupling to the $J^{c}=0^{+}$ mode via acoustic or magnetic fluctuations.[7]

The $J^{c}=1^{+}$ modes are Nambu-Goldstone modes associated with broken *relative* spin-orbit rotation symmetry. It is convenient to express these mode amplitudes in the Cartesian representation $D^{(+,1)}_{\alpha}=\frac{1}{2}\varepsilon_{\alpha\beta\gamma}\,\mathscr{D}^{(+)}_{\beta\gamma}$. Projecting out these amplitudes from Eq. (68) yields

$$i\omega\,D^{(+,1)}_{\alpha}=2\Delta\,\frac{1+\frac{1}{15}\lambda(\omega)\,F^{a}_{2}}{1-\frac{2}{45}\lambda(\omega)^{2}\,F^{a}_{0}\,F^{a}_{2}}\left(-\frac{\gamma\hbar}{2}\,H_{\alpha}(\omega)\right),\quad(103)$$

where $H_{\alpha}(\omega)$ is the Fourier component of the time-dependent external magnetic field and γ is the gyromagnetic ratio of ^{3}He. Exchange interactions renormalize the coupling of the $J^{c}=1^{+}$ modes to an external field, but the massless NG mode is protected by the continuous degeneracy of the BW ground state with respect to *relative* spin-orbit rotations. At finite wavelength these excitations correspond spin waves mediated by $J^{c}=1^{+}$ NG modes of the Cooper pairs with dispersion given by $\omega=c_{m}\,q$, where c_{m} are the spin-wave velocities in ^{3}He-B. See Sec. VII D discussion of weak symmetry-breaking perturbations on the $J^{c}=1^{+}$ modes.

The $J^{c}=2^{+}$ excitations obey the dynamical equations

$$\left[\omega^{2}-\tfrac{8}{5}|\Delta|^{2}\right]D^{(+)}_{2,m}=\tfrac{8}{5}|\Delta|^{2}\,F^{(+)}_{2,m}-i\omega(2\Delta)\,G_{2,m}.\quad(104)$$

In the absence of fermion interactions in the particle-particle channel, the f-wave amplitude vanishes, $F^{(+)}_{2,m}\equiv 0$. And, if we also ignore fermionic interactions in the particle-hole channel, then $G_{2,m}(\omega)$ represents an *external* field that couples to directly to the $J^{c}=2^{+}$ modes. In this case, the mass of this Higgs mode is equal to the weak-coupling result $M_{2,+}=\sqrt{8/5}\,\Delta$. However, $M_{2,+}$ is renormalized by fermionic interactions in both the particle-particle and particle-hole channels. Just as in the case for the $J^{c}=2^{-}$ modes excitation of a $J^{c}=2^{+}$ Higgs boson *polarizes* the $J=0^{+}$ fermionic vacuum and introduces a fermionic self-energy correction with the same symmetry that couples back to generate a mass correction to the $J^{c}=2^{+}$ Higgs modes.

In addition, pairing interactions in the spin-triplet, f-wave channel lead to dynamical excitations of the B-phase vacuum with spin $J^{c}=2^{+},m$, i.e., $F^{(+)}_{2,m}$, which mixes with the spin-triplet, p-wave modes of the same symmetry. We obtain the dynamical equation for the $F^{(+)}_{2,m}$ amplitudes by projecting out the f-wave orbital components of Eq. (68) to obtain

$$\begin{aligned}\left[4\bar{x}_{3}+\left(\omega^{2}-\tfrac{12}{5}|\Delta|^{2}\right)\right]F^{(+)}_{2,m}+\tfrac{12}{5}|\Delta|^{2}\,D^{(+)}_{2,m}\\=+i\omega(2\Delta)\,G_{2,m},\end{aligned}\quad(105)$$

where $\bar{x}_{3}\equiv x_{3}/\bar{\lambda}(\omega)$. Note that we have used the identity $\hat{p}\cdot\vec{G}(\hat{p})=\hat{p}\cdot\vec{G}^{(1)}(\hat{p})+\hat{p}\cdot\vec{G}^{(3)}(\hat{p})\equiv 0$ to express the source term in Eq. (105) in terms of the p-wave, $J=2$ component of $\vec{G}(\hat{p})$, i.e., $G^{(3,2)}_{\gamma i}=-G^{(1,2)}_{\gamma i}$. Eliminating $G_{2,m}$ from Eqs. (104) and (105) gives the f-wave, $J^{c}=2^{+}$ amplitude in terms of the corresponding dominant p-wave amplitude

$$[4\bar{x}_{3}+(\omega^{2}-4|\Delta|^{2})]\,F^{(+)}_{2,m}=-(\omega^{2}-4|\Delta|^{2})\,D^{(+)}_{2,m}.\quad(106)$$

[7]However, the process of two-phonon absorption and excitation of the $J^{c}=0^{+}$ is not forbidden.

The polarization corrections to the $J^{c}=2^{+}$ Higgs mass are obtained from Eqs. (104), (106), and (A26) in the limit $\mathbf{q}=0$, which can be expressed as

$$\begin{aligned}&\vec{\Sigma}^{(+)}(\hat{p};\omega)\\&=\vec{h}^{(+)}(\hat{p};\omega)+\int\frac{d\Omega_{\hat{p}'}}{4\pi}\,F^{a}(\hat{p},\hat{p}')\\&\quad\times\Big\{-\lambda(\omega)\left(\vec{\Sigma}^{(+)}(\hat{p}';\omega)-\hat{p}'[\hat{p}'\cdot\vec{\Sigma}^{(+)}(\hat{p}';\omega)]\right)\\&\quad-\left(\frac{\omega}{2\Delta}\right)\lambda(\omega)\,\hat{p}'\times\vec{d}^{(+)}(\hat{p}';\omega)\Big\},\end{aligned}\quad(107)$$

where $\vec{h}^{(+)}(\hat{p};\omega)$ represents the external field coupling to fermionic excitations via the magnetic moment of the ^{3}He nucleus, and $F^{a}(\hat{p},\hat{p}')$ represents the spin-dependent exchange interaction in ^{3}He [cf. Eq. (23), the paragraph preceding Eq. (25), and Eq. (A28)]. Note that Eq. (A28) has been inverted and used to express $\vec{\Sigma}^{(+)}(\hat{p};\omega)$ in terms of the Landau interaction $F^{a}(\hat{p},\hat{p}')$. For $c=+1$ bosonic excitations, the coupling of the fermionic self-energy fluctuations is determined by the p-wave, $J=0,1,2$ components of $\vec{G}^{(+)}(\hat{p};\omega)$ in Eq. (96). Fluctuations of $G^{(1)}_{\gamma i}$ with $J=0$ vanish by symmetry as $\vec{G}^{(+)}(\hat{p};\omega)$ is purely transverse with respect to $\hat{p}$. Fluctuations with $J=1$ are defined by the $\ell=0,2$ orbital components of $\vec{\Sigma}^{(+)}(\hat{p};\omega)$ in Eq. (97), while the $J=2$ components are defined by Eq. (98).

The dynamical equation for $G^{(1,1)}_{\alpha i}=\varepsilon_{\alpha i\gamma}(\Sigma^{(0)}_{\gamma}-\frac{1}{5}\Sigma^{(2)}_{\nu:\nu\gamma})$ is constructed from the equations for the $\ell=0$ and 2 exchange fields

$$\left(1+\frac{2}{3}\lambda F^{a}_{0}\right)\Sigma^{(0)}_{\gamma}=h_{\gamma}-i\frac{2}{3}\left(\frac{\omega}{2\Delta}\right)\lambda\,F^{a}_{0}\,\frac{1}{2}\varepsilon_{\alpha\beta\gamma}\,\mathscr{D}^{(1,1)}_{\alpha\beta},\quad(108)$$

$$\left(1+\frac{1}{15}\lambda F^{a}_{2}\right)\Sigma^{(2)}_{\nu:\nu\gamma}=i\frac{1}{3}\left(\frac{\omega}{2\Delta}\right)\lambda\,F^{a}_{2}\,\frac{1}{2}\varepsilon_{\alpha\beta\gamma}\,\mathscr{D}^{(1,1)}_{\alpha\beta},\quad(109)$$

which shows that $G^{(1,1)}_{\alpha i}$ couples *only* to the $\ell=1$, $J^{c}=1^{+}$ bosonic modes, thus leading to Eq. (103) for these NG modes.

The fermionic self-energy that couples to the $J^{c}=2^{+}$ bosonic modes is determined by the $\ell=2$ components of the exchange field defined by $G^{(1,2)}_{\alpha i}$ in Eq. (98). The equation of motion for the $(2,m)$ components is then

$$\begin{aligned}&\left(1+\frac{1}{5}\lambda F^{a}_{2}\right)G_{2,m}\\&=h^{(1,2)}_{2,m}-i\left(\frac{\omega}{2\Delta}\right)\frac{3}{25}\lambda\,F^{a}_{2}\,\{2F^{(+)}_{2,m}-D^{(+)}_{2,m}\},\end{aligned}\quad(110)$$

where $h^{(1,2)}_{2,m}$ are the components of a generalized, momentum-dependent, external magnetic field that couples to fermionic and bosonic excitations with $J=2$ via the nuclear spin. Combining Eqs. (104), (106), and (110) we obtain the response function for the $J^{c}=2^{+}$ Higgs amplitude

$H^{(+)}_{2,m} = D^{(+)}_{2,m} + F^{(+)}_{2,m}$:

$$H^{(+)}_{2,m} = \frac{-i\omega(2|\Delta|)\, h^{(1,2)}_{2,m}(\omega)}{\left(\omega^2 - \frac{8}{5}|\Delta|^2\right) + \lambda(\omega)(\omega^2 - 4|\Delta|^2)\left(\frac{2}{25}\, F^a_2 + (\omega/2|\Delta|)^2\, x_3^{-1}\right)}. \tag{111}$$

The renormalized mass of the $J^{\mathrm{c}} = 2^+$ Higgs mode is obtained from the pole of the propagator in Eq. (111); M_{2^+} scales to the the weak-coupling TDGL result for $T \to T_c^-$,

$$M_{2^+} \approx \sqrt{\frac{8}{5}}\Delta(T)\left[1 + \frac{\pi}{4}\sqrt{\frac{3}{5}}\,\frac{\Delta(T)}{T_c}\left(F^a_2/5 + x_3^{-1}\right)\right], \tag{112}$$

with the leading-order correction developing rapidly below T_c. For weak interactions $|F^s_2| \ll 1$ and $|x_3^{-1}| \ll 1$, the vacuum polarization correction at $T = 0$ can also be calculated perturbatively:

$$M_{2^+} \approx \sqrt{\frac{8}{5}}\Delta\left[1 + b\left(F^a_2/5 + x_3^{-1}\right)\right], \tag{113}$$

where $b = \frac{3}{2\sqrt{6}} \arcsin(\sqrt{\frac{2}{5}}) \approx 0.419$. Note that the $\ell = 2$ exchange interaction F^a_2 is reported by Halperin and Varoquaux [51] to vary between $F^a_2 \approx -0.88$ at $P = 0$ bar and $F^a_2 \approx -0.01$ at $P = 32$ bar. Figure 3 shows the mass of the $J^{\mathrm{c}} = 2^+$ Higgs mode as a function of the the $\ell = 2$ exchange interaction F^a_2 for various values of the f-wave interaction x_3^{-1} obtained from numerical solution for the pole of the propagator $H^{(+)}_{2,m}$ in Eq. (111). Repulsive interactions push the mass above the weak-coupling result. Attractive f-wave and exchange interactions reduce the mass; the f-wave interaction is less effective for strong ferromagnetic exchange $F^a_2/5 \to -1$, for which $M_{2^+} \to 0^+$, as is clear from the equation for M_{2^+} defined by the pole of Eq. (111). In this limit, the soft mode is dominated by the Pomeranchuk instability of the underlying fermionic vacuum. Nevertheless, for fixed $F^a_2/5 > -1$ $M_{2^+} \to 0^+$ as $T_{c_3} \to T_c$ ($x_3^{-1} \to -\infty$).

The charge-conjugation parity of the bosonic modes with the same orbital, spin, and total angular momentum quantum number is reflected dramatically in the polarization corrections to the masses of the Higgs modes. The $J^{\mathrm{c}} = 2^-$ modes couple to a quadrupolar excitation of the fermionic vacuum, leading to a mass shift from the interaction F^s_2 in the spin-symmetric particle-hole channel, which is generally repulsive except possibly near $p = 0$ bar [51]. By contrast, excitation of the $J^{\mathrm{c}} = 2^+$ modes is coupled to a quadrupolar spin polarization, and thus has a polarization correction to its mass from the interaction F^a_2 in the antisymmetric (exchange) particle-hole channel; this interaction is expected to be attractive at all pressures. In addition, both $J^{\mathrm{c}} = 2^\pm$ Higgs modes couple to f-wave pairing fluctuations with the same J and parity c. In this case, the asymmetry in the mass shifts for $J^{\mathrm{c}} = 2^\pm$ originates from $(\omega/2|\Delta|)^2\, x_3^{-1}$. Thus, the aysmmetry in the weak-coupling mass spectrum, i.e., $\sqrt{12/5}\Delta$ versus $\sqrt{8/5}\Delta$, leads to additional asymmetry in the polarization corrections from the f-wave interactions in the Cooper channel. These trends are shown explicitly by the perturbative results in Eqs. (92) and (113). Figure 4 summarizes the magnitude of the corrections to the NSR for a range of interactions in the Landau and Cooper channels. The violation of the NSR onsets rapidly below T_c, with deviations of order 20%–30% for the fermionic interactions characteristic of normal ^{3}He.

Excitation of the $J^{\mathrm{c}} = 2^+,m$ modes typically occurs through weakly coupled channels at finite wavelength, $q \neq 0$, as coupling via an external field with symmetry $h^{(1,2)}_{2,m}$ is not easily realized. Koch and Wölfle showed that the weak violation of particle-hole symmetry by the normal-state fermionic vacuum lifts a selection rule that otherwise prohibits the coupling of the $J^{\mathrm{c}} = 2^+$ Higgs modes to density and mass current fluctuations [58]. Thus, the $J^{\mathrm{c}} = 2^+,m$ modes can be excited by density and mass current channels, albeit with a coupling that is reduced by the factor $\zeta \approx k_{\mathrm{B}}T_c/E_f \ll 1$, the measure of the asymmetry of the spectrum of particle and hole excitations of the normal fermionic vacuum at $\varepsilon \approx k_{\mathrm{B}}T_c$ [41]. This coupling leads to resonant excitation of the $J^{\mathrm{c}} = 2^+$ Higgs mode by absorption of zero-sound phonons. Indeed, ultrasound absorption spectroscopy provided the first detection of the Higgs mode in a BCS condensate [59,60]. The definitive identification of the absorption resonance as the $J^{\mathrm{c}} = 2^+$ Higgs mode was made by Avenel *et al.* who observed the fivefold Zeeman splitting of the zero-sound absorption resonance in an applied magnetic field [61].

Acoustic spectroscopy provides precision measurements of the mass of the $J^{\mathrm{c}} = 2^+$ Higgs mode. The magnitude of the polarization correction to the the $J^{\mathrm{c}} = 2^+$ Higgs mass for $T \to 0$ is measured to be $\delta M_{2^+} \approx -0.19\,\Delta$, as shown in Fig. 5, indicating that the interactions giving rise to the mass shift are net attractive. The data are from Ref. [60] for a pressure of $p = 13$ bar (yellow diamonds), and from Ref. [61] for pressures $p = 0.8$–3.5 bar (red squares). Also shown are theoretical results for the polarization correction calculated as a function of temperature. In this case, we assumed the

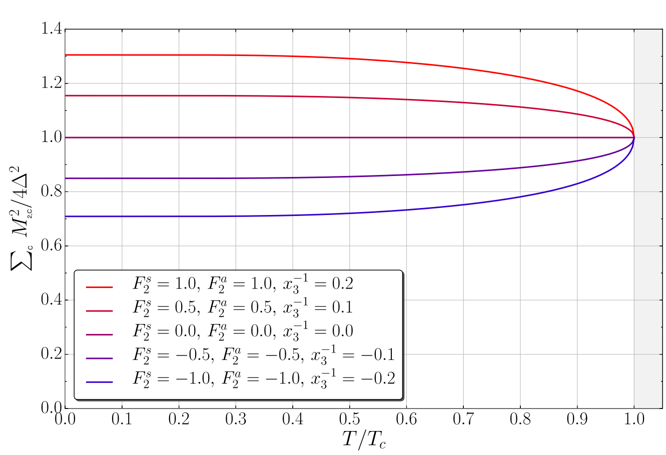


FIG. 4. Deviation of the Nambu sum from polarization corrections to the the $J = 2$ Higgs modes of ^{3}He-B for a range of interactions in both the Landau and Cooper channels.

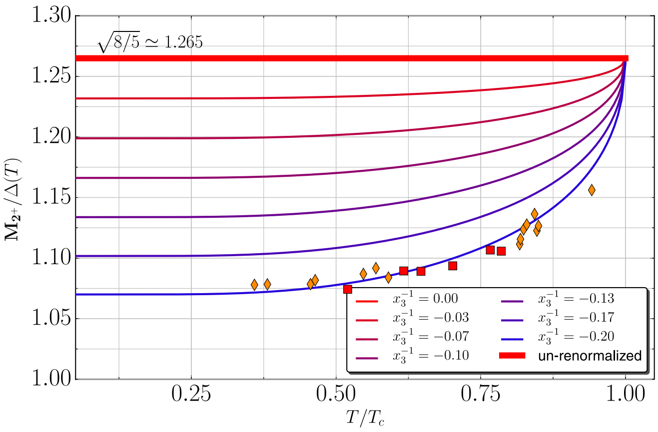


FIG. 5. $J^c = 2^+$ Higgs mass. The data are for a pressure of $p =$ 13 bar (yellow diamonds) [60] and for $p = 0.8$–3.5 bar (red squares) [61]. Theoretical calculations of the mass are for $F_2^a = -0.88$ and values of the f-wave interaction in the Cooper channel given in the legend.

most attractive estimate for the exchange interaction $F_2^a = -0.88$ [51], which accounts for only half of the measured value of δM_{2^+}. An attractive f-wave interaction $x_3^{-1} \approx -0.2$ in the Cooper channel provides the additional polarization correction. If we use the weaker value of $F_2^a \simeq -0.37$ reported by the *Helium-Three Calculator* [50] for $p = 13$ bar we obtain a correspondingly stronger attractive f-wave interaction $x_3^{-1} \simeq -0.35$. An attractive f-wave interaction of similar magnitude $x_3^{-1} \simeq -0.33$ at $p \approx 4.3$ bar is also inferred from an analysis of acoustic Faraday rotation of transverse sound that is mediated by the $J^c = 2^-$ Higgs mode [35,52]. Analysis of recent acoustic Faraday rotation measurements, outside the regime of the linear Zeeman splitting of the energy levels of the $J^c = 2^+, m$ modes, reports comparable or smaller values: $x_3^{-1} \approx -0.4$ to $x_3^{-1} \approx -0.2$ [53,62]. A complete and systematic determination of the relevant interactions in the Landau and Cooper channels is possible from the combined measurements of the masses of the $J^c = 2^\pm$ modes using longitudinal and transverse sound spectroscopy, combined with measurements of the velocities of zero sound, first sound, and the magnetic susceptibilities in both the normal and superfluid phases of ^{3}He.

D. Light Higgs modes in the $J^c = 1^+$ sector

The $J^c = 1^+$ mode amplitudes can be related to the parameters of the degeneracy space of *relative* spin and orbital rotations, i.e., $\mathscr{R}[\vartheta\mathbf{n}] \in \mathrm{SO}(3)_{L-S}$, where $\mathbf{n}$ is the axis of rotation, defined by polar and azimuthal angles, and a third variable being angle of rotation ϑ. The angles define massless NG modes reflecting the spontaneous breaking of *separate* symmetries under spin and orbital rotations, i.e.. $\mathrm{SO}(3)_L \times \mathrm{SO}(3)_S$. The $J^c = 1^+$ multiplet provides a "light Higgs" extension of the standard model in particle physics [63]. The light Higgs scenario works as follows: In ^{3}He separate invariance under spin and orbital rotations is broken by the nuclear dipole-dipole interaction, which acts as weak symmetry-breaking perturbation with an energy scale of order $V_D \sim 10^{-7}$ K per particle compared to the characteristic two-body interaction energy of order $V \sim 1$ K. The dipolar energy lifts the degeneracy with respect to separate spin and orbital rotations, which renders the $J^c = 1^+$ multiplet a triplet of "pseudo-Nambu-Goldstone modes" in which one or more of the NG modes acquires a mass from the weak symmetry-breaking field. Long-wavelength excitations of the axis of rotation $\mathbf{n}$ remain gapless; however, excitations of the rotation angle ϑ acquire a mass gap $M_{\mathrm{LH}}/\hbar = \Omega_{\mathrm{B}} \simeq 10$ kHz $\ll 2\Delta/\hbar \simeq 100$ MHz, where Ω_{B} is the longitudinal NMR resonance frequency of ^{3}He-B. An external magnetic field further lifts the degeneracy of the remaining zero-mass NG modes which split into an optical magnon with mass $M_{\mathrm{opt}} = \hbar\gamma B$ and a massless acoustic magnon. A direct detection of the light Higgs boson in ^{3}He-B was recently achieved by measuring the decay of optical magnons created by magnetic pumping (a magnon BEC). A sharp threshold for decay of optical magnons to a pair of light Higgs modes was observed by tuning the mass of the optical magnons on resonance, i.e., $M_{\mathrm{opt}} = \hbar\gamma B \geqslant 2M_{\mathrm{LH}} = 2\hbar\Omega_{\mathrm{B}}$ [63].

VIII. SUMMARY AND OUTLOOK

Mass generation based on spontaneous symmetry breaking and the introduction of an internal symmetry (particle-hole symmetry in BCS theory) implies a connection between the masses of the fermion and boson excitations of the broken symmetry vacuum state, and a hidden supersymmetry in the class of BCS-NJL theories [12,19]. The Nambu sum rule, inspired in part by the bosonic spectrum of ^{3}He-B, however, is not protected against symmetry-breaking perturbations to the broken symmetry vacuum state, including polarization of the vacuum state by excitation of a Higgs boson with symmetry distinct from that of the vacuum. For the case of ^{3}He-B, we show that corrections to the weak-coupling BCS theory and fermionic interactions combined with vacuum polarization by the Higgs fields, lead to corrections to the masses of the Higgs modes, and in general a violation of the NSR. Our results, as well as other effects of weak perturbations like the nuclear dipolar energy, the Zeeman energy and weak violations of particle-hole symmetry, highlight the roles of symmetry-breaking perturbations.

Current research in topological condensed matter addresses the transport properties and spectrum of fermionic excitations confined near surfaces, interfaces, and edges of topological insulators and topological superconductors. Relatively recent theoretical work has shown how supersymmetry can also emerge at the boundary of topological superfluids [64]. The B phase of superfluid ^{3}He is the realization of a three-dimensional time-reversal invariant topological superfluid, with a spectrum of helical Majorana fermions confined on any bounding surface. Thus, a frontier in topological quantum fluids is the role of confinement as a symmetry-breaking perturbation on the bosonic spectrum of confined ^{3}He-B, and the possible signatures of the surface spectrum of Majorana fermions in the bosonic modes of confined ^{3}He-B. New studies of the effects of confinement and symmetry-breaking perturbations on both the bulk and surface bosonic and fermionic excitations of topological superfluids will hopefully shed new light on the connection between spontaneous symmetry breaking, hidden supersymmetry, and topology of the broken symmetry vacuum state in topological superfluids.

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APPENDIX

1. TDGL effective potentials

The potentials that enter the TDGL functional that determine the masses of the bosonic modes are given by $u_p = \Delta^2\,\bar{u}_p$:

$$\bar{u}_1 = \tfrac{4}{3}\mathrm{Tr}\{\mathscr{D}\}\mathrm{Tr}\{\mathscr{D}^*\} + \mathrm{Tr}\{\mathscr{D}\mathscr{D}^{\mathrm{tr}}\} + \mathrm{Tr}\{\mathscr{D}\mathscr{D}^{\mathrm{tr}}\}^*, \tag{A1}$$

$$\bar{u}_2 = 2\,\mathrm{Tr}\{\mathscr{D}\mathscr{D}^\dagger\} + \tfrac{1}{3}(\mathrm{Tr}\{\mathscr{D}\} + \mathrm{Tr}\{\mathscr{D}^*\})^2, \tag{A2}$$

$$\begin{aligned}\bar{u}_3 = {} & \tfrac{1}{3}(\mathrm{Tr}\{\mathscr{D}\mathscr{D}^{\mathrm{tr}}\} + \mathrm{Tr}\{\mathscr{D}\mathscr{D}^{\mathrm{tr}}\}^*) \\ & + \tfrac{2}{3}(\mathrm{Tr}\{\mathscr{D}\mathscr{D}^\dagger\} + \mathrm{Tr}\{\mathscr{D}\mathscr{D}^*\}),\end{aligned} \tag{A3}$$

$$\bar{u}_4 = \tfrac{4}{3}\mathrm{Tr}\{\mathscr{D}\mathscr{D}^\dagger\} + \tfrac{1}{3}(\mathrm{Tr}\{\mathscr{D}^2\} + \mathrm{Tr}\{\mathscr{D}^2\}^*), \tag{A4}$$

$$\begin{aligned}\bar{u}_5 = {} & \tfrac{1}{3}(\mathrm{Tr}\{\mathscr{D}\mathscr{D}^\dagger\} + \mathrm{Tr}\{\mathscr{D}\mathscr{D}^\dagger\}^*) \\ & + \tfrac{1}{3}(\mathrm{Tr}\{\mathscr{D}^{\mathrm{tr}}\mathscr{D}\} + \mathrm{Tr}\{\mathscr{D}^{\mathrm{tr}}\mathscr{D}\}^*) + \tfrac{2}{3}\mathrm{Tr}\{\mathscr{D}\mathscr{D}^*\}.\end{aligned} \tag{A5}$$

Note that these potentials are defined relative to the BW ground state, and thus invariant only under $\mathrm{SO}(3)_J \times T$.

2. Symmetry relations

The components of the 4×4 Nambu propagator are related by fundamental symmetries with respect to (i) permutation exchange symmetry and (ii) conjugation symmetry. These symmetries imply the following relations between the components of the quasiclassical propagator.

a. Exchange symmetry

$$g'(\hat{p},\epsilon_n;\mathbf{q},\omega_m) = +g(-\hat{p},-\epsilon_n;\mathbf{q},\omega_m), \tag{A6}$$

$$\vec{g}'(\hat{p},\epsilon_n;\mathbf{q},\omega_m) = +\vec{g}(-\hat{p},-\epsilon_n;\mathbf{q},\omega_m), \tag{A7}$$

$$f(\hat{p},\epsilon_n;\mathbf{q},\omega_m) = +f(-\hat{p},-\epsilon_n;\mathbf{q},\omega_m), \tag{A8}$$

$$\vec{f}(\hat{p},\epsilon_n;\mathbf{q},\omega_m) = -\vec{f}(-\hat{p},-\epsilon_n;\mathbf{q},\omega_m), \tag{A9}$$

as well as for the mean-field self-energies

$$\Sigma'(\hat{p};\mathbf{q},\omega_m) = +\Sigma(-\hat{p};\mathbf{q},\omega_m), \tag{A10}$$

$$\vec{\Sigma}'(\hat{p};\mathbf{q},\omega_m) = +\vec{\Sigma}(-\hat{p};\mathbf{q},\omega_m), \tag{A11}$$

$$d(\hat{p};\mathbf{q},\omega_m) = +d(-\hat{p};\mathbf{q},\omega_m), \tag{A12}$$

$$\vec{d}(\hat{p};\mathbf{q},\omega_m) = -\vec{d}(-\hat{p};\mathbf{q},\omega_m). \tag{A13}$$

Note that Eqs. (A12) and (A13) reflect the fact that spin-singlet Cooper pairs have even parity, while spin-triplet pairs are odd parity.

b. Conjugation symmetry

The conjugation symmetry relations follow from complex conjugation of the two-point functions

$$g'(\hat{p},\epsilon_n;\mathbf{q},\omega_m) = +g(-\hat{p},\epsilon_n;-\mathbf{q},\omega_m)^*, \tag{A14}$$

$$\vec{g}'(\hat{p},\epsilon_n;\mathbf{q},\omega_m) = +\vec{g}(-\hat{p},\epsilon_n;-\mathbf{q},\omega_m)^*, \tag{A15}$$

$$f'(\hat{p},\epsilon_n;\mathbf{q},\omega_m) = +f(-\hat{p},\epsilon_n;-\mathbf{q},\omega_m)^*, \tag{A16}$$

$$\vec{f}'(\hat{p},\epsilon_n;\mathbf{q},\omega_m) = -\vec{f}(-\hat{p},\epsilon_n;-\mathbf{q},\omega_m)^*, \tag{A17}$$

$$\Sigma'(\hat{p};\mathbf{q},\omega_m) = +\Sigma(-\hat{p};-\mathbf{q},\omega_m)^*, \tag{A18}$$

$$\vec{\Sigma}'(\hat{p};\mathbf{q},\omega_m) = +\vec{\Sigma}(-\hat{p};-\mathbf{q},\omega_m)^*, \tag{A19}$$

$$d'(\hat{p};\mathbf{q},\omega_m) = +d(-\hat{p};-\mathbf{q},\omega_m)^*, \tag{A20}$$

$$\vec{d}'(\hat{p};\mathbf{q},\omega_m) = -\vec{d}(-\hat{p};-\mathbf{q},\omega_m)^*. \tag{A21}$$

3. Dynamical equations

$$\begin{aligned}\vec{dg}^{(-)}(\hat{p};\mathbf{q},\omega) = \int \frac{d\Omega_{p'}}{4\pi}\, V^{(1)}(\hat{p},\hat{p}') \Bigg\{ \left[\frac{1}{2}\gamma + \frac{1}{4}[\omega^2 - \eta'^2 - 4|\vec{\Delta}(\hat{p}')|^2]\bar{\lambda}(\hat{p}')\right] \vec{d}^{(-)}(\hat{p}') + \bar{\lambda}(\hat{p}')\vec{\Delta}(\hat{p}')\,[\vec{\Delta}(\hat{p}')\cdot\vec{d}^{(-)}(\hat{p}')] \\ - \frac{1}{2}\eta'\,\bar{\lambda}(\hat{p}')\,\vec{\Delta}(\hat{p}')\,\Sigma^{(-)}(\hat{p}') - \frac{1}{2}\omega\,\bar{\lambda}(\hat{p}')\,\vec{\Delta}(\hat{p}')\,\Sigma^{(+)}(\hat{p}') \Bigg\},\end{aligned} \tag{A22}$$

$$\begin{aligned}\vec{d}^{(+)}(\hat{p};\mathbf{q},\omega) = \int \frac{d\Omega_{p'}}{4\pi}\, V^{(1)}(\hat{p},\hat{p}') \Bigg\{ \left[\frac{1}{2}\gamma + \frac{1}{4}(\omega^2 - \eta'^2)\bar{\lambda}(\hat{p}')\right] \vec{d}^{(+)}(\hat{p}') - \bar{\lambda}(\hat{p}')\vec{\Delta}(\hat{p}')[\vec{\Delta}(\hat{p}')\cdot\vec{d}^{(+)}(\hat{p}')] \\ + \frac{i}{2}\eta'\,\bar{\lambda}(\hat{p}')\,\vec{\Delta}(\hat{p}')\times\vec{\Sigma}^{(-)}(\hat{p}') + \frac{i}{2}\omega\,\bar{\lambda}(\hat{p}')\,\vec{\Delta}(\hat{p}')\times\vec{\Sigma}^{(+)}(\hat{p}') \Bigg\},\end{aligned} \tag{A23}$$

$$\Sigma^{(+)}(\hat{p};\mathbf{q},\omega) = \Sigma^{(+)}_{\text{ext}}(\hat{p}) + \int \frac{d\Omega_{p'}}{4\pi}\, A^s(\hat{p},\hat{p}')\Bigg[\left(\frac{\omega^2}{\omega^2-\eta'^2}\right)(1-\lambda(\hat{p}'))\,\Sigma^{(+)}(\hat{p}') + \left(\frac{\omega\eta'}{\omega^2-\eta'^2}\right)(1-\lambda(\hat{p}'))\Sigma^{(-)}(\hat{p}') + \frac{1}{2}\omega\,\bar{\lambda}(\hat{p}')\,\vec{\Delta}(\hat{p}')\cdot\vec{d}^{(-)}(\hat{p}')\Bigg], \tag{A24}$$

$$\Sigma^{(-)}(\hat{p};\mathbf{q},\omega) = \Sigma^{(-)}_{\text{ext}}(\hat{p}) + \int \frac{d\Omega_{p'}}{4\pi}\, A^s(\hat{p},\hat{p}')\Bigg[\left(\frac{\omega\eta'}{\omega^2-\eta'^2}\right)(1-\lambda(\hat{p}'))\Sigma^{(+)}(\hat{p}') + \left\{1+\left(\frac{\eta'^2}{\omega^2-\eta'^2}\right)(1-\lambda(\hat{p}'))\right\}\Sigma^{(-)}(\hat{p}') + \frac{1}{2}\eta'\,\bar{\lambda}(\hat{p}')\,\vec{\Delta}(\hat{p}')\cdot\vec{d}^{(-)}(\hat{p}')\Bigg], \tag{A25}$$

$$\vec{\Sigma}^{(+)}(\hat{p};\mathbf{q},\omega) = \vec{\Sigma}^{(+)}_{\text{ext}}(\hat{p}) + \int \frac{d\Omega_{p'}}{4\pi}\, A^a(\hat{p},\hat{p}')\Bigg[\left(\frac{\omega^2}{\omega^2-\eta'^2}\right)(1-\lambda(\hat{p}'))\,\vec{\Sigma}^{(+)}(\hat{p}') + \bar{\lambda}(\hat{p}')(\vec{\Delta}(\hat{p}')\cdot\vec{\Sigma}^{(+)}(\hat{p}'))\vec{\Delta}(\hat{p}') + \left(\frac{\omega\eta'}{\omega^2-\eta'^2}\right)(1-\lambda(\hat{p}'))\vec{\Sigma}^{(-)}(\hat{p}') - \frac{i}{2}\omega\,\bar{\lambda}(\hat{p}')\,\vec{\Delta}(\hat{p}')\times\vec{d}^{(+)}(\hat{p}')\Bigg], \tag{A26}$$

$$\vec{\Sigma}^{(-)}(\hat{p};\mathbf{q},\omega) = \vec{\Sigma}^{(-)}_{\text{ext}}(\hat{p}) + \int \frac{d\Omega_{p'}}{4\pi}\, A^a(\hat{p},\hat{p}')\Bigg[\left\{1+\left(\frac{\eta'^2}{\omega^2-\eta'^2}\right)(1-\lambda(\hat{p}'))\right\}\vec{\Sigma}^{(-)}(\hat{p}') - \bar{\lambda}(\hat{p}')(\vec{\Delta}(\hat{p}')\cdot\vec{\Sigma}^{(-)}(\hat{p}'))\vec{\Delta}(\hat{p}') + \left(\frac{\omega\eta'}{\omega^2-\eta'^2}\right)(1-\lambda(\hat{p}'))\vec{\Sigma}^{(+)}(\hat{p}') - \frac{i}{2}\eta'\,\bar{\lambda}(\hat{p}')\,\vec{\Delta}(\hat{p}')\times\vec{d}^{(+)}(\hat{p}')\Bigg], \tag{A27}$$

where $\eta' \equiv \mathbf{v}_{\hat{p}'}\cdot\mathbf{q}$ and the Tsuneto function, $\lambda(\hat{p}') \equiv \lambda(\eta',\omega;|\Delta(\hat{p}')|)$, for $\mathbf{q}\neq 0$ is given by Eq. (62) of Ref. [40]. The particle-particle interaction vertex in the spin-triplet channel is parametrized by an interaction parameter v_ℓ for each odd-parity angular momentum channel, as in Eq. (33). In the case of the particle-hole interaction vertex, the functions $A^{s,a}(\hat{p},\hat{p}')$ are the forward scattering amplitudes for spin-independent (A^s) and spin-exchange (A^a) scattering of quasiparticles with momenta near the Fermi surface. These amplitudes are related to the the Landau interactions $F^{s,a}(\hat{p},\hat{p}')$ by the integral equation

$$A^{s,a}(\hat{p},\hat{p}') = F^{s,a}(\hat{p},\hat{p}') - \int \frac{d\Omega_{p''}}{4\pi} F^{s,a}(\hat{p},\hat{p}'')\,A^{s,a}(\hat{p}'',\hat{p}'). \tag{A28}$$

The standard parametrization of the Landau interaction function in terms of the Landau parameters is $F^{s,a}(\hat{p},\hat{p}') = \sum_{\ell\geqslant 0} F^{s,a}_\ell\, P_\ell(\hat{p}\cdot\hat{p}')$.

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