Topological phase transitions in line-nodal superconductors

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Fathoming interplay between symmetry and topology of many-electron wave functions has deepened our understanding of quantum many-body systems, particularly after the discovery of topological insulators. Topology of electron wave functions often enforces and protects emergent gapless excitation, and symmetry is intrinsically tied to the topological protection of the excitations. Namely, unless the symmetry is broken, the topological nature of the excitations is intact. We show intriguing phenomena of interplay between symmetry and topology in three-dimensional topological phase transitions associated with line-nodal superconductors. More specifically, we discover an exotic universality class out of topological line-nodal superconductors. The order parameter of broken symmetries is strongly correlated with underlying line-nodal fermions, and this gives rise to a large anomalous dimension in sharp contrast to that of the Landau-Ginzburg theory. Remarkably, hyperscaling violation and emergent relativistic scaling appear in spite of the presence of nonrelativistic fermionic excitation. We also propose characteristic experimental signatures around the phase transitions, for example, a linear phase boundary in a temperature-tuning parameter phase diagram, and discuss the implication of recent experiments in pnictides and heavy-fermion systems.

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I. INTRODUCTION

Superconductivity is one of the most intriguing quantum many-body effects in condensed-matter systems: Electrons form Cooper pairs whose Bose-Einstein condensation becomes an impetus of the striking characteristics of superconductors (SCs), for example, the Meissner effect and zero resistivity [1]. The pair formation suppresses gapless fermionic excitation, and only the superconducting order parameter becomes important in conventional SCs. However, in unconventional SCs, fermionic excitation is not fully suppressed generically and remains gapless, so the order parameter and fermions coexist and reveal an intriguing unconventional nature [2–4].

The fermionic excitation in unconventional SCs is often protected and classified by its topological nature. One convenient way to characterize topological nature is the Berry phase (or flux) of the Bogoliubov–de Gennes (BdG) Hamiltonian. In the literature [5–7], the structure of the BdG Hamiltonian has been extensively studied and is applied to weakly correlated systems. Proximity effects between topologically different phases (or defects in momentum space) have been investigated and experimentally tested, focusing on a search for novel excitation such as Majorana modes [8,9].

Here, we focus on a class of unconventional SCs whose topological nature is protected by a symmetry. Namely, unless the symmetry is broken, the topologically protected nodal structure is intact. In this class, change of the topology and spontaneous breaking of the symmetry appear concomitantly at quantum-critical points, and thus an intriguing interplay between symmetry and topology is expected. Therefore, topological phase transitions around the class of unconventional SCs become a perfect venue to investigate the interplay between topology and symmetry. In two dimensions (2D), Sachdev and co-workers have investigated a similar class of the transition in the context of d-wave SCs [10–12]. They found the universality class of the phase transitions at which the point nodes of d-wave SCs disappear is that of the Higgs-Yukawa theory, i.e., the theory with relativistic fermions and bosons in 2D.

A richer structure exists in three spatial dimensions (3D). Line nodes are available in 3D in addition to point nodes. Point and line nodes are obviously not homeomorphic and thus are topologically different. The effective phase space of line-nodal excitation is qualitatively distinct from that of order-parameter fluctuation, as shown by codimension analysis in literature [5–7]. Thus, concomitant appearance of symmetry breaking and change of topology in line-nodal SCs has us expect an exotic universality class of the topological transitions.

Abundant experiments in phase transitions inside superconducting domes with line-nodal excitation is another motivation for the current work. Inside superconducting domes, tuning parameters such as pressure, impurity, and doping often invoke phase transitions, inducing so-called coexistence regions [13–16]. Interestingly, many of these systems, in particular, pnictides and heavy-fermion systems, are suggested to have line-nodal excitations [16–27]. For example, the recent experiment in $Ba_{0.65}Rb_{0.35}Fe_2As_2$ shows transitions from nodeless SCs to line-nodal SCs by tuning pressure, and various intriguing characters are reported, such as insensitivity of superconducting temperature to pressure in spite of clear transition in the SC gap structure [28–30]. Thus, it is imperative to deepen our understanding in quantum phase transitions inside superconducting domes with line nodes.

We investigate topological phase transitions with a concomitant appearance of symmetry breaking and change of topology in line-nodal SCs, which often appear in side superconducting domes. We discover an exotic universality class out of the interplay between symmetry and topology. Furthermore, we apply our theoretical results to experiments and discuss direct relation with recent experiments in pnictides and heavy-fermion systems.

II. MODEL AND ANALYSIS

A. Symmetry and phases

Topological line-nodal SCs protected by a symmetry maintain their nodal structure unless the protecting symmetry is broken. Therefore, adjacent symmetry-broken phases may be described by representations of the symmetry. For example, the polar phase with line nodes, A phase with point nodes, and nodeless B phase in liquid ^{3}He are described by investigating symmetry representations of $\mathrm{SO}(3)_L \times \mathrm{SO}(3)_S \times \mathrm{U}(1)_\phi$. Below, we take the group $\mathcal{G} = C_{4v} \times \mathcal{T} \times \mathcal{P}$, one of the common lattice groups in line-nodal SC experiments (here $\mathcal{P}$ and $\mathcal{T}$ are for particle-hole and time-reversal symmetries), as a prototype. Its generalization to other groups is straightforward.

It is well understood in the literature [5] that the SC model with the symmetry group $\mathcal{G}$,

$$H_0 = \sum_{\boldsymbol{k}} \Psi_{\boldsymbol{k}}^\dagger [h(\boldsymbol{k})\tau^z + \Delta(\boldsymbol{k})\tau^x]\Psi_{\boldsymbol{k}}, \tag{1}$$

has line nodes protected by $\mathcal{T}$ symmetry. A four-component spinor $\Psi_{\boldsymbol{k}}^\dagger = (\psi_{\boldsymbol{k}}^\dagger, i\sigma^y\psi_{-\boldsymbol{k}}^T)$ with $\psi_{\boldsymbol{k}}^\dagger = (c_{\boldsymbol{k},\uparrow}^*, c_{\boldsymbol{k},\downarrow}^*)$ is introduced, and the particle-hole (spin) space Pauli matrices $\tau^{x,y,z}$ $(\sigma^{x,y,z})$ are used. The τ^z term describes a normal-state spectrum $h(\boldsymbol{k}) = \epsilon(\boldsymbol{k}) - \mu + \alpha\vec{l}(\boldsymbol{k})\cdot\vec{\sigma}$, and the τ^x term describes a pairing term $\Delta(\boldsymbol{k}) = [\Delta_s + \Delta_t\vec{d}(\boldsymbol{k})\cdot\vec{\sigma}]$. Energy dispersion $\epsilon(\boldsymbol{k}) = -2t[\cos(k_x) + \cos(k_y) + \cos(k_z)]$ is introduced with spin-orbit coupling strength α. The orbital axis of the pairing and spin-orbit terms are identical $\vec{d}(\boldsymbol{k}) = \vec{l}(\boldsymbol{k}) = (\sin(k_x), \sin(k_y), 0)$, which usually maximizes T_c [31,32]. The pairing amplitudes $\{\Delta_s, \Delta_t\}$ are chosen to be real and positive without losing generality because of the $\mathcal{T}$ symmetry. As illustrated in Fig. 1(a), the system exhibits two topological

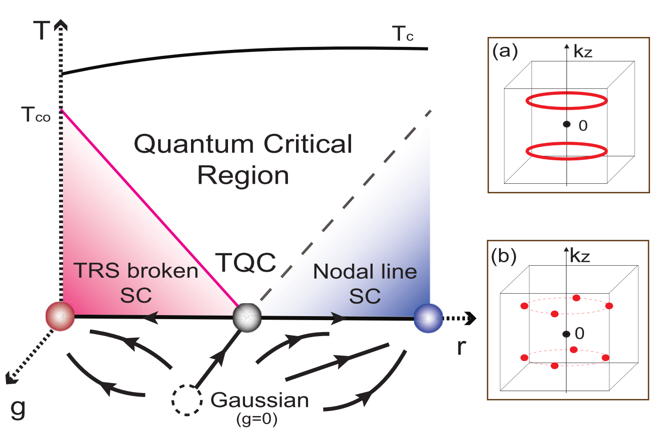

FIG. 1. Phase diagram and renormalization group (RG) flow. Three axes are for temperature (T), the tuning parameter (r), and the coupling between order-parameter and line-node fermions (g). In the r-T plane, the critical region is parametrically wider than that of the conventional ϕ^4 theory. In the r-g plane, the RG flow is illustrated by arrows. The "Gaussian" fixed point has Laudau mean-field theory's critical exponents due to the upper critical dimension. Once the coupling g turns on, the Gaussian fixed point becomes destabilized and RG flows go into "TQC." At $T = 0$, the left (red) sphere is for the ordered phase, and the right (blue) sphere is for the disordered phase. T_c is for the superconducting dome temperature, and T_{co} is for critical temperature of the symmetry-breaking order parameter. (a) Nodal lines in momentum space in the symmetric phase are illustrated at $k_z = \pm k_z^*$ in addition to the zero point $\boldsymbol{k} = \boldsymbol{0}$ (black dot). (b) Nodal points in momentum space in a symmetry-broken phase (eight nodal points).

TABLE I. C_{4v} representations for topological phase transitions. For simplicity, $\mathcal{T}$-broken and spin-singlet representations are only illustrated. The first column is for representations. The second column is the matrix structure in the Nambu space. The third column is for continuum representations near nodal lines. The last column is for the numbers of the nodal points in each representation.

Rep.	Lattice $[\mathcal{F}_s(\boldsymbol{k})M^s]$	Continuum	No.
A_1	τ^y	τ^y	0
A_2	$\sin(k_x)\sin(k_y)[\cos(k_x) - \cos(k_y)]\tau^y$	$\sin(4\theta)\tau^y$	16
B_1	$[\cos(k_x) - \cos(k_y)]\tau^y$	$\cos(2\theta)\tau^y$	8
B_2	$\sin(k_x)\sin(k_y)\tau^y$	$\sin(2\theta)\tau^y$	8
E	$\sin(k_x)\sin(k_z)\tau^y$,	$\cos(\theta)\tau^y\mu^z$,	4
	$\sin(k_y)\sin(k_z)\tau^y$	$\sin(\theta)\tau^y\mu^z$	

line nodes separated in momentum space protected by the $\mathcal{T}$ symmetry.

It is obvious that $\mathcal{T}$-symmetry-breaking superconductivity (the term with τ^y) changes nodal structure, so order-parameter representations for topological phase transitions can be illustrated as in Table I. Group-theory analysis guarantees coupling terms between order parameters and fermionic excitation,

$$H_{\psi-\phi} = \sum_s \phi_s \sum_{\boldsymbol{k}} \Psi_{\boldsymbol{k}}^\dagger \mathcal{F}_s(\boldsymbol{k}) M^s \Psi_{\boldsymbol{k}}.$$

s is for representations (and multiplicity) and $\mathcal{F}_s\, M^s$ are illustrated in Table I. For detail of this classification, see **A** in the Supplemental Material [33]. Note that $s = E$ is a two-dimensional representation, so the corresponding order parameter $(\phi_{s=E})$ has two components.

Two topologically different cases exist. First, momentum independence of A_1 representation makes fermion spectrum gapped completely, the so-called is pairing. In the ^{3}He context, this phase corresponds to the weakly $\mathcal{T}$-broken analog of the B phase. Second, the order parameters in A_2, B_1, B_2, and E representations leaves point nodes due to angular dependence. Nodal points appear when $\mathcal{F}_s(\mathbf{k})$ has zeros on line nodes and is, in fact, Weyl nodes. This phase corresponds to the A phase in ^{3}He.

B. Mean-field theory and renormalization group

Armed with understanding of adjacent symmetry-broken phases, we consider topological phase transitions. Standard mean-field theory (MFT) with on-site interaction $-u(\Psi^\dagger\tau^y\Psi)^2$ gives a mean-field free-energy density of "isotropic" A_1 representation order parameter (is pairing),

$$\mathcal{F}_{MF}(\phi) = (r - r_c + T)\phi^2 + k_f|\phi|^3 + \cdots, \tag{2}$$

where $u = 1/r$ is used. Coefficients of each term are scaled to be one and $\cdots$ is for higher-order terms. Notice that the unusual $|\phi|^3$ term appears whose presence is solely from line-nodal fermions manifested by k_f. It also guarantees the phase transition is continuous and makes the usual ϕ^4 term irrelevant. Furthermore, the order-parameter critical exponent becomes significantly different from one of the Landau MFT (which only contains bosonic degrees of freedom), $\langle\phi\rangle \sim (r - r_c)$, giving $\beta = 1$, which already suggests a universality class.

We now investigate quantum criticality around the continuous phase transitions. For simplicity, we omit the subscript s and introduce one real scalar field ϕ to describe the order parameter. Its generalization to the E representation with two scalar fields is straightforward.

In the phenomenological Landau-Ginzburg theory, order-parameter fluctuation near quantum phase transitions may be described by

$$S_\phi = \int_{x,\tau} \frac{1}{2}(\partial_\tau \phi)^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{r}{2}\phi^2 + \frac{\lambda}{4!}\phi^4. \quad (3)$$

Of course, this action is not complete in our systems and necessary to be supplemented by the corrections from fermions. Without the coupling between the order parameter and fermions, the critical theory $\mathcal{S}_\phi$ with $r = r_c$ is well understood, the so-called the ϕ^4 theory: In 3D, it is at the upper critical dimension. Thus, the Landau MFT works well up to logarithmic correction, and hyperscaling is satisfied. Below, we show that the coupling to fermions significantly changes low-energy physics and induce a universality class.

The total action with fermions is

$$S_c = S_\phi + S_\psi, \quad S_\psi = \int_{x,\tau} \Psi^\dagger(\partial_\tau + \mathcal{H}_0)\Psi + g\int_\tau H_{\psi-\phi}.$$

A coupling constant g characterizes strength of the coupling between fermions and bosons, and the Hamiltonian density $\mathcal{H}_0$ is introduced ($H_0 = \int_x \Psi^\dagger \mathcal{H}_0 \Psi$).

Near phase transitions, low-energy and momentum degrees of freedom become important, so we only need the low-energy continuum theory of the BdG Hamiltonian Eq. (1) near nodes and obtain

$$\mathcal{H}_0(\boldsymbol{k}) \approx v_z \delta k_z \mu^z \tau^z + v_\perp \delta k_\perp \tau^x, \quad (4)$$

where the momentum is $\boldsymbol{k} = ((k_f + \delta k_\perp)\cos(\theta_k),(k_f + \delta k_\perp)\sin(\theta_k),k_z^*\mu^z + \delta k_z)$. Here $\mu^z = \pm 1$ represents the "which-line-node" index and the effective parameters $\{v_z, v_\perp\}$ are the functions of the microscopic parameters. The low-energy fermion spectrum (say, $\mu^z = +1$) without the fermion-boson coupling is

$$\epsilon_0(\delta k_z,\delta k_\perp,\theta_k) = \pm\sqrt{(v_z\delta k_z)^2 + (v_\perp \delta k_\perp)^2}. \quad (5)$$

One parameter, the angle $0 \leqslant \theta_k \leqslant 2\pi$, characterizes zero energy states, so a nodal line exists in momentum space.

Density of states near zero energy vanishes linearly in ϵ, $\mathcal{D}_f(\epsilon) \sim k_f|\epsilon|$ in a sharp contrast to ones of Fermi surfaces ($\sim \epsilon^0$), nodal points ($\sim \epsilon^2$), and order parameters ($\sim \epsilon^2$). It is clear that phase space of nodal-line fermion excitation is different from that of fluctuation of the order parameter. Such a difference in the phase spaces of bosons and fermions is a consequence of the codimension mismatch.

The coupling term is also written in terms of low-energy degrees of freedoms,

$$g\int_x H_{\psi-\phi} \approx g\int_{\boldsymbol{k},\omega,\boldsymbol{q},\Omega} \phi_{\boldsymbol{q},\Omega}\mathcal{F}(\theta_{\boldsymbol{k}})\Psi^\dagger_{\boldsymbol{k}+\boldsymbol{q},\omega+\Omega}\mathcal{M}\Psi_{\boldsymbol{k},\omega},$$

the so-called the Yukawa coupling. We use Shankar's decomposition of fermion operators around the line node, $\Psi_{\mathbf{k}} \approx \Psi(\delta k_z,\delta k_\perp,\theta_k;\mu^z)$.

The standard large-N_f analysis is performed by introducing N_f copies of fermion flavors coupled to the boson ϕ. The lowest-order boson self-energy $\Sigma_b(\Omega,\boldsymbol{q})$ can be obtained by the usual bubble diagram. For A_1 representation, the boson self-energy is

$$\Sigma_b(\Omega,\boldsymbol{q}) = N_f g^2 \int_{\boldsymbol{k},\omega} \mathrm{Tr}[\tau^y G_{f,0}(\omega,\boldsymbol{k})\tau^y G_{f,0}(\omega+\Omega,\boldsymbol{q}+\boldsymbol{k})],$$

where $G^{-1}_{f,0}(\omega,\boldsymbol{k}) = -i\omega + \mathcal{H}_0(\boldsymbol{k})$ is the bare fermion propagator. Notice that the integration is over fermionic momentum and frequency; thus, the main contribution comes from line-nodal fermions. Basically, the momentum integration can be replaced with energy integration with $\mathcal{D}_f(\epsilon) \sim k_f|\epsilon|$. The integration is straightforward (see **C.1** in [33]) and we find

$$\delta\Sigma_b(\Omega,\boldsymbol{q}) = \mathcal{C}(k_f N_f)\sqrt{\Omega^2 + v_z^2 q_z^2 + v_\perp^2 q_\perp^2}\, el[\rho(\Omega,\boldsymbol{q})],$$

with $\delta\Sigma_b \equiv \Sigma_b(\Omega,\boldsymbol{q}) - \Sigma_b(0,\boldsymbol{0})$ and $\mathcal{C} = \frac{g^2}{4\pi v_\perp v_z}$. The complete elliptic integral $el[x]$ and variable $\rho(\Omega,\boldsymbol{q}) = 1/(1 + \frac{\Omega^2 + v_z^2 q_z^2}{v_\perp^2 q_\perp^2})$ are used. The elliptic integral is well defined in all ranges of momentum and frequency; thus, as the lowest-order approximation, one can treat the integral as a constant since $1 \leqslant el[x] < 2$.

Two remarks follow. First, the linear dependence in momentum and frequency can be understood by power counting with the linear fermionic density of states. Second, the boson propagator contains the factor $N_f k_f$. Thus, one can understand the large N_f analysis as an expansion with the $\frac{1}{N_f k_f}$ factor. The presence of k_f already suggests suppression of infrared divergences in loop calculations (see below).

The modified boson action is

$$S_\phi^{eff} = \int_{\boldsymbol{q},\Omega} \frac{|\phi_{\boldsymbol{q},\Omega}|^2}{2}[\tilde{r} + \boldsymbol{q}^2 + \Omega^2 + \delta\Sigma_b(\Omega,\boldsymbol{q}))] + \cdots,$$

with $\tilde{r} = r + \Sigma_b(0,\boldsymbol{0})$. The self-energy manifestly dominates over the bare terms at long wavelengths; thus, the bare terms may be ignored near the critical point ($\tilde{r} = 0$) and the boson propagator becomes $G_b(\Omega,\boldsymbol{q}) \to \delta\Sigma_b(\Omega,\boldsymbol{q})^{-1}$.

The backreaction of the bosons to fermions is obtained by the fermion self-energy,

$$\Sigma_f(\omega,\boldsymbol{k}) = g^2 \int_{\Omega,\boldsymbol{q}} \tau^y G_f(\omega+\Omega,\boldsymbol{k}+\boldsymbol{q})\tau^y G_b(\Omega,\boldsymbol{q}).$$

Straightforward calculation shows that the corrections to the parameters of the bare fermion action Eq. (4) have the structure

$$\frac{\delta\Sigma_f(\omega,\boldsymbol{k})}{\delta\epsilon^a} \propto \frac{1}{N_f\, k_f} \times (\Lambda - \mu), \quad (6)$$

where $\epsilon^a = (\omega, v_\perp\delta k_\perp, v_z k_z)$, and Λ and μ are the ultraviolet (UV) and infrared (IR) cutoffs. k_f is the largest momentum scale, $k_f \gg \Lambda \gg \mu$ in this work. The same cutoff dependence in the vertex correction is found (omitted here; see **C** in [33] for details). The absence of the infrared divergences indicates that our one-loop calculation is *exact* in terms of divergence structure and associated critical exponents [34]. The control parameter of our calculation is not just $1/N_f$, but $1/N_f k_f$, as shown above. Here k_f is the Fermi momentum of the normal state, which is the largest scale in the condensed-matter system. Thus, the control parameter of the calculation $N_f k_f$ is expected to be large enough even in the limit of $N_f \to 1$, and hence we argue the critical exponent to be correct at the finite N_f.

Two remarks follow. First, the momentum integration captures order-parameter fluctuation, so it may be replaced with energy integration with density of states, $\mathcal{D}(\epsilon) \sim \epsilon^2$. Next, the cutoff dependence is a result of the large N_f expansion with k_f, as discussed before. The absence of the infrared divergence indicates that perturbation theory works well. Thus, fermions and bosons become basically decoupled at low energy. In a renormalization-group sense, this indicates that the vertex operator is irrelevant at low energy. For other representations, nodal points survive the symmetry-breaking transition and become the point nodes. Despite the presence of the nodal points, the coupling between the order parameter and the fermion becomes effectively zero, and the nodal points become "cold spots" of the transition. Thus, the gapless excitations from the nodal points do not affect the low-energy dynamics of the fermion and bosons, and this is thoroughly checked in **B** and **C** in [33].

The critical theory associated with topological line-nodal SCs is

$$\frac{\mathcal{S}_\phi^c}{N_f k_f} = \int_{\Omega,\boldsymbol{q}} \sqrt{\Omega^2 + v_z^2 q_z^2 + v_\perp^2 q_\perp^2}\,\mathcal{R}(\rho(\Omega,\boldsymbol{q}))\frac{|\phi|^2}{2}, \tag{7}$$

setting $\tilde{r} = 0$. $\mathcal{R}(\rho(\Omega,\boldsymbol{q}))$ is an order one nonzero positive well-defined function to characterize representations (see **C.1** in [33]). Therefore, critical exponents do not depend on $\mathcal{R}(x)$. We omit the ϕ^4 term, which is justified below.

Let us list the striking characteristics of our critical theory. First, the damping term, $k_f|\Omega|$, at $\boldsymbol{q} = 0$ exists. The presence of the damping term appears due to the absence of the Ward identity in our systems in a sharp contrast to line-nodal normal semimetal with the Coulomb interaction. Its form is the same as the Hertz-Millis theory of antiferromagnetic transitions, but momentum dependence is also linear, so the dynamic critical exponent is relativistic ($z = 1$).

Moreover, the anomalous dimension of the order parameter is large ($\eta_\phi = 1$), so the scaling dimension of the order parameter is $[\phi] = \frac{d+z-2+\eta_\phi}{2} = \frac{3}{2}$. This is completely different from one of the Landau theory (ϕ^4 theory) at the upper critical dimension ($d = 3$ with $z = 1$). Due to the large anomalous dimension, the correlation length behaves $\xi^{-1} \sim |r - r_c|$, so $\nu = 1$. Also, the anomalous dimension makes the ϕ^4 coupling irrelevant, $[\lambda] < 0$. So our critical theory is stable, which becomes a sanity check of the MFT in Eq. (2).

The susceptibility exponent is $\gamma = 1$, and the Fisher equality is satisfied $(2 - \eta_\phi)\nu = \gamma$. Basically decoupled fermions and bosons contribute to specific heat independently, $C_v \sim a_f T^2 + a_b T^3$. The first term is from line-nodal fermions, and the second term is from order-parameter fluctuations with $d/z = 3$ (see **D** in [33]).

The hyperscaling is *violated* even in 3D. If not, one would get the order-parameter critical exponent, β [$\langle\phi\rangle \sim (r_c - r)^\beta$] by the scaling relation, $\beta = \frac{(d+z-2+\eta)\nu}{2} = \frac{3}{2}$. However, we already observe $\beta = 1$ in our MFT in Eq. (2), and also the perturbative calculation in our critical theory gives (see **E** in [33])

$$\tilde{r} + \Sigma_b(0,\mathbf{0};T) - \Sigma_b(0,\mathbf{0};T=0) \sim \tilde{r} + T,$$

TABLE II. Critical theories of QCP in three spatial dimensions ($d = 3$). The first row includes critical exponents ($\Omega \sim q^z$, $\xi^{-1} \sim |r - r_c|^\nu$, $\chi_\phi \sim |r - r_c|^{-\gamma}$, and $[\phi] = \frac{d+z-2+\eta}{2}$). "HS" is for hyperscaling. Both Higgs-Yukawa and ϕ^4 theory are at the upper critical dimension, so the exponents are ones of the Landau MFT. Both quadratic band touching quantum-critical point (QBT-QCP) and nodal-line QCP have wider quantum-critical region $\nu = 1$ with large anomalous dimension $\eta = 1$ obtained by large N_f analysis.

QCP in 3D	z	ν	β	γ	η	HS
ϕ^4 Theory [35]	1	$\frac{1}{2}$	$\frac{1}{2}$	1	0	O
Higgs-Yukawa [35,36]	1	$\frac{1}{2}$	$\frac{1}{2}$	1	0	O
QBT-QCP [37,38]	2	1	2	1	1	O
Hertz-Millis [40,41]	2 or 3	$\frac{1}{2}$	$\frac{1}{2}$	1	0	X
Nodal line QCP	1	1	1	1	1	X

giving the critical temperature scaling, $T_{co} \sim |r_c - r| = \tilde{r}$, which gives a qualitatively wider quantum-critical region than the one of the Landau MFT, $T_{co,L} \sim \sqrt{r_c - r}$. The hyperscaling violation indicates that the Yukawa coupling is *dangerously* irrelevant. In Table II, we compare our critical theory with other critical theories in 3D [35–41] in terms of critical exponents and hyperscaling applicability.

We remark that our low-energy theory has a larger symmetry than one of the original system, namely $U(1)$ rotational symmetry not the original C_{4v}. Thus, k_f is independent of the angle $\theta_{\boldsymbol{k}}$. This is an artifact of the linearization approximation, but it is not difficult to see that the universality class is not modified by inclusion of symmetry-breaking terms down to C_{4v} unless a singular fermion spectrum such as nesting appears.

This is because the codimension mismatch is the key of linear dependence of momentum and frequency in the boson self-energy with the presence of k_f and the absence of IR divergence in the fermion self-energy. Thus, all critical exponents are the same as ones of Eq. (7). This is also consistent with previous literature on quantum criticality [42,43]. We also explicitly show the linear dependence without the linearized fermion dispersion approximation in the Supplemental Material [33].

III. DISCUSSION AND CONCLUSION

Our theoretical results can be directly applied to quantum phase transitions beneath superconducting domes. We provide an additional smoking-gun signature of line-nodal SCs. Namely, the linear phase boundary $T_{co} \sim (r_c - r)$, from hyperscaling violation, between two different SCs identifies the presence of line nodes. Interestingly, the linearlike shape phase boundary between B phase and C phase in UPt_3 was reported [29]. Based on the suggested gap structures with the b directional magnetic field, one can analyze one nodal line in C phase becomes nodal points in B phase, which is perfectly well suited to our phase transition. Several other heavy-fermion systems, for example, UCoGe, are also suggested to have a linear phase boundary between two different SCs [20,21], and one of SCs at least is suggested to have line nodes, though further thorough investigation is necessary.

Furthermore, direct measurement of critical exponents is possible. In our universality class, order parameter is strongly correlated in the sense that it has a large anomalous dimension, so its direct measurement shows qualitatively different behaviors from Landau MFTs. In particular, the fluctuation of the $\mathcal{T}$-breaking order parameters has been extensively studied in a context of chiral SCs [44–49]. There are several concrete experimental methods to measure the fluctuation, namely, the spin-polarized muon scattering [2,49,50]. From our critical exponents, we obtain the change in the distribution $\delta\sigma$ of internal magnetic fields relative to the $\mathcal{T}$-symmetric phase is $\delta\sigma(r,T) \propto \langle\phi(r,T)\rangle$. Then, our scaling analysis gives $\delta\sigma(r,T) \propto (r_c - r)\,\mathcal{F}(\frac{T}{r-r_c})$ with a scaling function $\mathcal{F}$. Thus, the $\mathcal{T}$-breaking signal is qualitatively different from that of the Landau MFT result $\delta\sigma_L(r_c,T=0) \propto \sqrt{r_c - r}$.

The hyperscaling violation and associated fermion-boson decoupling indicate that fermions and order parameters contribute separately to the scaling properties of physical quantities. Since phase space of order-parameter fluctuation is qualitatively smaller than that of fermions, most physical quantities are mainly dominated by weakly interacting fermions with a band structure determined by an order-parameter condensation. For example, the penetration depth is mainly determined by the weakly interacting fermions. As discussed in the literature [51,52], superconducting density at zero temperature, $\Delta\lambda^{-2}(r) = a_1(r - r_c) + a_2\langle\phi^2\rangle$, has a nonuniversal linear term, and the hyperscaling violation gives $\langle\phi^2\rangle \sim (r - r_c)^2$. Thus, the nonuniversal term always dominates over the order-parameter contribution. Therefore, in spite of the presence of quantum criticality, the penetration depth is mainly determined by the fermionic excitation, $\Delta\lambda^{-2}(T) \sim T$ in a symmetric phase with nodal-line excitation, and $\Delta\lambda^{-2}(T) \sim e^{-\langle\phi\rangle/T} \sim e^{-(r_c-r)/T}$ in a symmetry-broken phase without nodal excitation (A_1 representation). More detailed discussion on physical quantities will appear in future works.

Furthermore, the fermion-boson decoupling indicates that the superconducting temperature, which forms superconducting domes, is insensitive to the onset of topological phase transitions. The vertex correction without the infrared divergence implies that the order-parameter fluctuation does not modify the low-energy properties of fermions, which are related to T_c. Also, we perform the mean-field-type analysis by integrating out fermions [53,54] and obtain qualitatively different behavior, $T_c(r) \sim T_c(r_c) + (r - r_c)^3$ for $r < r_c$, in sharp contrast to $T_c(r) \sim T_c(r_c) + (r - r_c)$ in the conventional Landau-Ginzburg theory. Roughly speaking, the cubic dependence can be understood by the absolute cubic term in Eq. (2) (see **F** in [33]). These calculations indicate that T_c is qualitatively more insensitive to tuning parameters than one of the Landau-Ginzburg theory.

We use such insensitivity to explain puzzles of recent experiments in topological phase transitions in pnictides and heavy-fermion systems. For example, $Ba_{0.65}Rb_{0.35}Fe_2As_2$ shows a gap-structure changing phase transition from nodeless SCs to nodal-line SCs varying with pressure [28]. It is found that T_c is insensitive to pressure in spite of the gap-structure change. A similar puzzle also appears in Yb-doped $CeCoIn_5$ and we find no anomaly in T_c in terms of the symmetry-breaking scenario. We believe such insensitivity can be understood as the consequence of our universality class, $T_c(r) \sim T_c(r_c) + (r - r_c)^3$.

In conclusion, we have described topological phase transitions associated with line-nodal SCs where topology and symmetry reveal intriguing interplay phenomena. We find that quantum criticality naturally appears and its universality class of the transition shows characteristics such as emergent relativistic scaling, hyperscaling violation, and an unusually wide quantum-critical region. We also apply our theoretical results to recent experiments and predict scaling forms of physical quantities. In particular, we provide a plausible explanation of insensitivity of T_c in the experiments. Our results can also be applied to topological phase transitions out of normal nodal ring semimetals naturally if the chemical potential is fixed to be zero. Future theoretical studies should include more comprehensive treatment of perturbations of critical points such as disorder, finite temperature, and magnetic-field effects. Quantitative comparison with experiments would be also desirable.

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