

Shape of the magnetoroton at $\nu = 1/3$ and $\nu = 7/3$ in real samples

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We revisit the theory of the collective neutral excitation mode in the fractional quantum Hall effect at Landau level filling fractions $\nu = 1/3$ and $\nu = 7/3$. We include the effect of finite thickness of the two-dimensional electron gas and use extensive exact diagonalizations in the torus geometry. In the lowest Landau level the collective gapped mode, i.e., the magnetoroton always merges in the continuum in the long-wavelength limit. In the second Landau level the mode is well defined only for wave vectors smaller than a critical value and disappears in the continuum beyond this point. Its curvature near zero momentum is opposite to that of the LLL. It is well separated from the continuum even at zero momentum, and the gap of the continuum of higher-lying states is twice the collective mode gap at $k = 0$. The shape of the dispersion relation survives a perturbative treatment of Landau level mixing.

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I. INTRODUCTION

Quantum liquids display very special collective modes that dominate their low-energy long-wavelength behavior. In the case of liquid helium-4 the ground state has broken gauge symmetry associated with number conservation, and as a consequence there is a phonon branch of excitations with no gap. While the gapless nature of the mode is dictated by general requirements, here the Goldstone theorem, its shape for finite wave vector is nonuniversal. Remarkably it has a nontrivial minimum dubbed the roton. This roton state has been studied experimentally in great detail, and Feynman [1] has developed the so-called single-mode approximation (SMA) that captures in a neat way its nature.

Two-dimensional (2D) electronic systems under a strong magnetic field exhibit the fractional quantum Hall effect (FQHE) at low enough temperatures. The most prominent of these states of matter happens for filling factor $\nu = 1/3$ of the lowest Landau level [2] (LLL). It has been observed for conventional semiconductor artificial devices, quantum wells, and heterostructures, as well as in atomically 2D systems like monolayer and bilayer graphene. The ground state of the 2D electrons for Landau level filling factor $\nu = 1/3$ is adequately described by the Laughlin wave function. This state has no broken symmetry and is a prime example of topological order. The incompressibility that is responsible for the macroscopic phenomenology of the state also leads to gapped collective neutral excitations. The lowest-lying density mode can also be described by a Landau-level adapted single-mode approximation [3,4] (SMA), and it features also a minimum energy as a function of wave vector. This minimum is called the magnetoroton. In addition to the SMA its mere existence has been confirmed by exact diagonalization of small systems, and there are trial wave functions constructed with composite-fermion states. In the composite fermion approach the magnetoroton is the lowest-energy particle-hole excitation between effective composite fermion Landau levels [5,6]. Other wave functions have also been proposed [7–9]. Other FQHE states have more complex neutral excitations [10,11]. Inelastic light scattering has been used to probe the collective mode and is partly explained by existing theories [12–16]. One intriguing suggestion is the existence of a two-roton bound state [16–18] as a possible lowest-lying state for small wave

vectors. This question has proven difficult to answer mainly because of limitations of exact diagonalization to very small systems.

In this paper we study the Laughlin state at $\nu = 7/3$ and obtain the neutral excitation spectrum by using large-scale exact diagonalizations on the torus geometry with spin-polarized electrons. The transport phenomenology of this fraction has been known for some time to be analogous to that of $\nu = 1/3$ as theoretically expected by uplifting the Laughlin wave function in the second Landau level [19] (LL). Here we find that there are very strong finite size effects, obscuring the spectrum structure up to $N_e = 10$ electrons. However nonzero thickness of the 2D electron gas (2DEG) resuscitates the familiar magnetoroton, provided one reaches large enough systems with $N_e \geq 11$, and uses the torus geometry. For small width $w/\ell \lesssim 1$ (where ℓ is the magnetic length $\sqrt{\hbar/eB}$) the system may be compressible, but for $w/\ell = 2 - 3$ we observe the clear signature of the MR mode. However it has now a different structure with regard to its LLL counterpart: It is well defined only for wave vectors $k\ell \lesssim 1.8$ and enters the continuum beyond this value. For $k \approx 0$ there are two gaps leading again to a well-defined mode in the long-wavelength limit. Contrary to the LLL case the curvature of the dispersion relation is upwards close to $k = 0$, and there is a secondary maximum in addition to a roton minimum. There is no clear limiting behavior when $k \rightarrow \infty$ which is in line with the fact that in this limit the magnetoroton is expected to become a quasihole-quasielectron pair, and their size is probably very large [20,21]. It may very well be that the collective mode at small wave vector is not continuously connected to the quasiparticle-quasihole mode expected at larger wave vector. The shape of the dispersion relation of MR mode is essentially unaffected by Landau level mixing effects, at least in a perturbative treatment.

In Sec. II we present several models to take into account the finite thickness of the 2DEG in the Coulomb interaction potential. In Sec. III we use exact diagonalization on the torus geometry to study the LLL magnetoroton. Section IV is devoted to the shape of the MR at $\nu = 7/3$; it contains an extensive discussion of finite-size effects with the torus and sphere geometry. Landau-level mixing is treated perturbatively in Sec. V. Finally Sec. VI contains a discussion of experimental results on the MR shape and our conclusions.

II. MODEL INTERACTIONS FOR FINITE-WIDTH EFFECTS

The wave functions of electrons in 2DEG samples have a finite extent in the z direction perpendicular to the plane of the electron gas. However the 2D nature of the motion means that excitations in this z direction are energetically forbidden: The z motion is frozen in its ground state. One can thus make the approximation that electronic wave functions factorizes. If we call ϕ_0 the ground state for the z coordinate then the electron interactions can be written as:

$$V(\mathbf{x} - \mathbf{y}) = \frac{e^2}{\epsilon} \int dz_1 dz_2 \frac{|\phi(z_1)|^2 |\phi(z_2)|^2}{\sqrt{(\mathbf{x} - \mathbf{y})^2 + (z_1 - z_2)^2}}. \quad (1)$$

This modified form of the potential can then be written in second quantized form projected onto the LLL or the second Landau level. The Fourier transform of the interaction potential has now the following form:

$$\tilde{V}(q) = \frac{2\pi e^2}{\epsilon q} \int dz_1 dz_2 |\phi(z_1)|^2 |\phi(z_2)|^2 e^{-q|z_1 - z_2|}, \quad (2)$$

where we see that the bare Coulomb potential $2\pi e^2/q$ is multiplicatively changed by a form factor depending upon the shape of the z wave function. 2DEGs come mostly in two varieties: heterostructures and quantum wells. In the first case the potential felt by electrons in the z direction perpendicular to the 2D plane is approximately triangular, and a trial wave function has been proposed by Fang and Howard:

$$\phi_{FH}(z) = \frac{2}{(2b)^{3/2}} z e^{-bz/2}, \quad (3)$$

where the parameter b can be determined variationally as a function of the junction parameters including the electronic density [22]. In quantum wells (QW) the corresponding wave function is the usual eigenstate for a square well with some finite width d :

$$\phi_{SQ}(z) = \sqrt{\frac{2}{d}} \cos\left(\frac{\pi z}{d}\right). \quad (4)$$

It has also been suggested that one can use a simple ad hoc modification of the potential at short distances. Zhang and Das Sarma have proposed the substitution:

$$\frac{1}{r} \rightarrow \frac{1}{\sqrt{r^2 + a^2}}, \quad (5)$$

where the cutoff a has the dimension of a length and captures the finite-width effect. From a practical point of view it has been noted that a simple Gaussian wave function reproduces correctly the LL-projected Hamiltonian [23]:

$$\phi_{GS}(z) = \frac{1}{(w\sqrt{2\pi})^{1/2}} e^{-z^2/4w^2}. \quad (6)$$

This is the model we use in this paper: We quote the width w without extra subscript when it corresponds to the Gaussian model while we add the name of the model wave function as a subscript otherwise. The effect of the finite width has been studied notably by exact diagonalization of small systems [23–25]. It is known that wide quantum wells have a tendency to stabilize quantum Hall states in the second Landau level mainly based on overlap calculations with model

states [24–26]. If the width of the well becomes very large, then the electronic density may prefer to form two layers of charge giving rise to an effective two-component system whose physics is now quite distinct [27]. In this work we consider only the single-component case which is realized for not too thick samples. It is known that there are samples with density small enough to stay in the one-component regime but with values w/ℓ up to 5 to 6 for $\nu = 1/3$. Previous studies have shown that the activation gap decreases with w but the FQHE regime survives [28–32]. For example references ([27,30]) have studied a system with density $n = 6.4 \times 10^{-10} \text{ cm}^2$ and a width $w_{SQ} = 75 \text{ nm}$ which leads to a fractional quantum Hall state at $\nu = 7/3$ with $w_{SQ}/\ell = 3.2$ while retaining single-component physics.

III. THE LLL MAGNETOROTON

To study the FQHE with the modified potential Eq. (1) we use the torus geometry [33]: Magnetic translation symmetries allow us to classify eigenstates by a two-dimensional conserved quasimomentum \mathbf{K} living in a discrete Brillouin zone with only N^2 points where N is the GCD of the number of electrons N_e and the number of flux quanta N_ϕ through the system. We fix the filling factor $N_e/N_\phi = 1/3$. On a rectangular cell with periodic boundary conditions we have:

$$\mathbf{K}^2 = (2\pi s/L_x)^2 + (2\pi t/L_y)^2, \quad (7)$$

where s, t are integers running from 0 to N . The many-body eigenstates can then be plotted against the dimensionless momentum $k\ell \equiv |\mathbf{K}|\ell$. We use an aspect ratio $L_x/L_y = 0.9$ in this work since the physics is only weakly dependent of this value even in the second LL. For zero width we recover the well-known shape of the magnetoroton mode [3,4]. For small wave vectors its energy rises and disappears into the continuum. He and Platzman have argued [16] that there is a crossing of levels close to $k = 0$ and that a state with a two-roton character becomes lower in energy. Present data does not shed any light on this issue. The magnetoroton appears clearly in Fig. 1. The ground state is at $\mathbf{K} = 0$ and is isolated

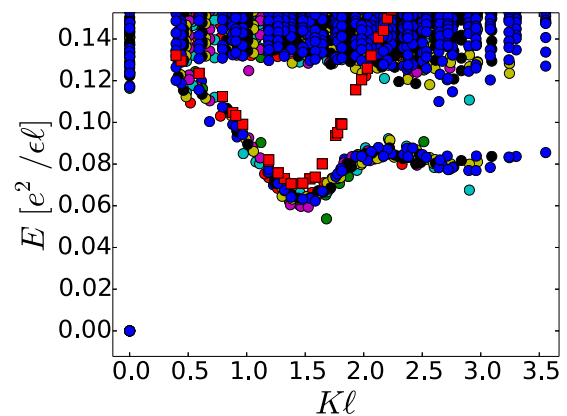


FIG. 1. Spectra for systems of $N_e = 5$ up to $N_e = 12$ electrons at filling $\nu = 1/3$ in a rectangular cell with aspect ratio $L_x/L_y = 0.9$. No finite width is assumed. The magnetoroton branch is clearly defined for all momenta. There is a clear minimum at wave vector $K\ell \approx 1.5$. Finite-size effects are negligible. The biggest system corresponds to blue points. The red square points are the SMA values.

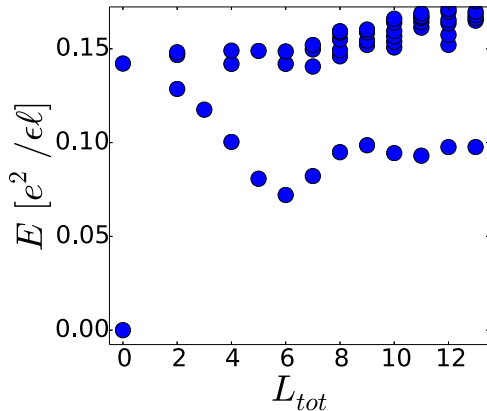


FIG. 2. Sphere spectrum for $N_e = 13$ electrons in the LLL with pure Coulomb interaction. The magnetoroton branch is well defined and extends up to $L = N_e = 13$. The shape of the magnetoroton is similar to the torus case in Fig. 1.

by a gap from all excitations: This gap is larger than the typical level spacing to the finite number of particles. The magnetoroton branch is well defined and is rather insensitive to finite-size effects. When including finite-width effects the overall shape does not change, notably the $\mathbf{K} = 0$ behavior remains but the energy scale is lowered. This is the case for realistic wave functions included in the modified potential Eq. (1), be it Gaussian, square well, or Fang-Howard, provided that the charge distribution has the same variance, spectra are very similar and differ only in quantitative details. To get a trial wave function to describe the MR, Girvin, MacDonald, and Platzman have proposed to adapt an idea due to Feynman. One creates a density excitation by acting upon the ground state with a density operator of definite momentum $\hat{\rho}_{\mathbf{K}}$ and one projects the resulting state into the LLL:

$$|\Psi_{SMA}(\mathbf{K})\rangle \equiv \mathcal{P}_{LLL} \hat{\rho}_{\mathbf{K}} |\Psi_0\rangle. \quad (8)$$

These trial states give a successful estimate of the energy of the MR in the LLL case: see Fig. 1 where we have plotted the SMA energy estimates in red squares. The small wave vector behavior and the MR minimum are correctly reproduced. In the CF theory it is possible to build excitonic states in which a composite fermion is raised from the lowest CF Landau level into the first excited CF Landau level. This also gives a satisfactory description of the MR mode. We note that sphere calculations also reveal the presence of the MR mode. In Fig. 2 we plot the eigenenergies of $N_e = 13$ electrons at $\nu = 1/3$ as a function of the total angular momentum L_{tot} . The MR branch is also prominent and extends up to $L_{tot} = N_e$ as predicted by CF theory on the sphere.

If we consider a realistic potential with finite width we find that the shape of the MR mode is unchanged for the cases of Fang-Howard, square well, and Gaussian models. However this is not the case with the Zhang-Das Sarma proposal [34] which has a qualitative impact in the LLL. For example with $a = 4\ell$ the LLL MR is split off from the continuum at $K = 0$: see Fig. 3. This example shows that some of the MR features are nonuniversal: There is no reason why the collective mode branch should merge in the continuum at small wave vectors.

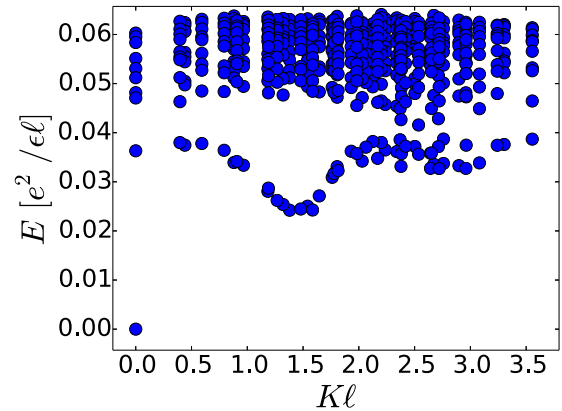


FIG. 3. Torus spectrum for $N_e = 12$ electrons in the LLL. The aspect ratio is 0.9. The finite width is now captured by the Zhang-Das Sarma modification of the Coulomb potential with parameter $a = 4\ell$. This has a big impact on the MR shape. Notably it does not enter the continuum in the long-wavelength limit.

IV. SECOND LANDAU LEVEL

We now turn to the study of the $\nu = 7/3$ state. The first approach to the FQHE physics in the second Landau level is to use the appropriate orbital wave functions and fully neglect Landau level mixing. This is an approximation which is certainly less good than in the LLL but has the merit of mapping

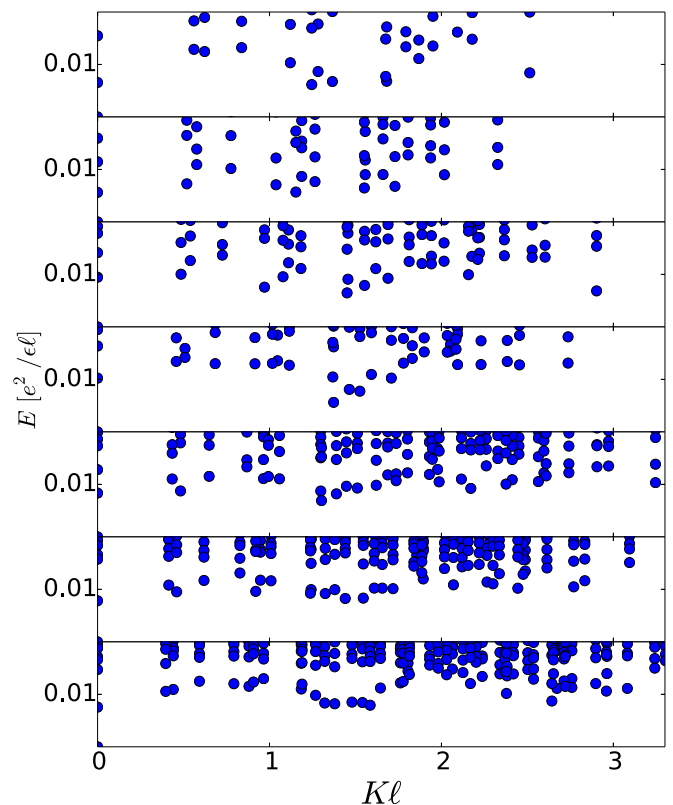


FIG. 4. Evolution of the torus spectrum when increasing the number of particles. From top to bottom $N_e = 6$ to 12. The cell is rectangular with aspect ratio 0.9, and we use a finite-width potential $w/\ell = 3$. The collective mode branch is discernible for $N_e = 10$ and fully developed only for $N_e = 12$.

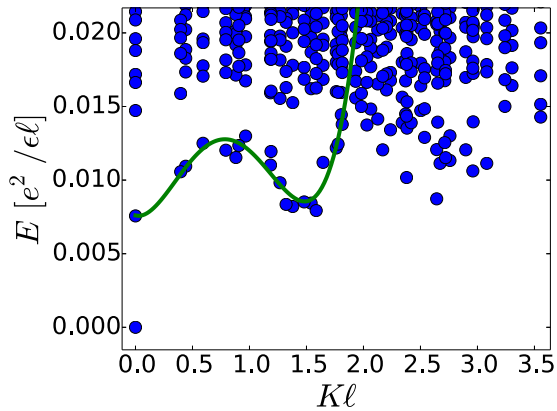


FIG. 5. Spectrum of $N_e = 12$ electrons on a rectangular cell with aspect ratio $L_x/L_y = 0.9$ for $\nu = 7/3$. The width is taken to be Gaussian $w/\ell = 3$. The magnetoroton branch is clearly defined only for momenta $\lesssim 1.8\ell^{-1}$. The solid green line is a fit by a quartic polynomial. The dispersion relation has a maximum and a minimum.

the problem on the very same Hamiltonian as in the LLL with renormalized matrix elements. This is the point of view we adopt in this section with the added finite-width effects. We will consider perturbative treatment of LL mixing in Sec. V. For small systems it is known [33] that the system is probably gapless in the zero-width case at filling factor $\nu = 7/3$. While the study of larger system sizes up to $N_e = 12$ does not allow us to strengthen this conclusion, we find that nonzero width gives rise to a well-defined collective mode well separated from the continuum. We observe that the MR branch appears beyond approximately 10 electrons as can be seen in Fig. 4.

We plot a typical spectrum in Fig. 5 for $N_e = 12$ electrons in a rectangular cell with a Gaussian wave function width $w/\ell = 3$. The MR mode now is definitely separated from the continuum at $\mathbf{K} = 0$ and has one maximum and one minimum before seemingly entering the continuum for $k\ell \approx 1.8$. There is an obvious limitation in torus calculations which is the dramatic discretization of momenta in Eq. (7). One way to overcome this is to perform calculations in an oblique cell with varying angle, allowing overlaps in momenta definition. This can be seen in Fig. 6 where we have used a set of unit cells interpolating between a square and a hexagonal cell. One can see more clearly the dispersion relation of the MR mode. The features that we observe in the single rectangular cell of Fig. 5 stand out clearly. Concerning the spectrum at zero wave vector, our data show that the *second* excited state has an energy gap which is very close to twice the first energy gap, so the continuum of states is likely to be a two-particle continuum made of the MR excitations. By using the ground state wave function obtained by exact diagonalization for $\nu = 7/3$ one can obtain the SMA states in the second LL. The results for $N_e = 12$ electrons are plotted in Fig. 7 with red symbols. The overall shape of the collective mode is well reproduced by the SMA while now the energies are too high by a factor of 30–50%. We note that the SMA works only if we use the ground state from exact diagonalization in Eq. (8): Indeed using the Laughlin wave function in the second Landau level is much less satisfactory [35]. An important quantity that can be derived easily from the SMA wave function is the LL-projected

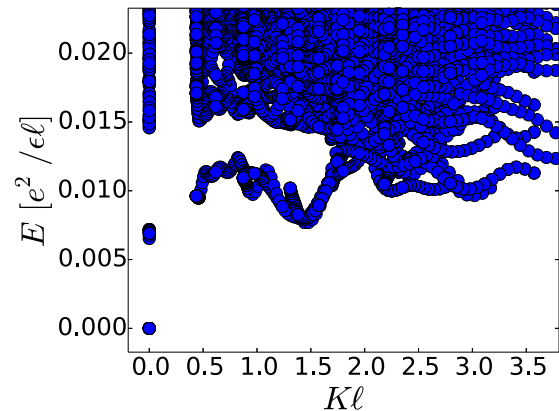


FIG. 6. Excitation spectrum for $N_e = 11$ electrons in an oblique cell with equal sides and an angle that interpolates between square and hexagonal cell. Spectra for all shapes are plotted in this figure: This is a way to obtain more points in the dispersion relation of the collective mode. The potential assumes a Gaussian width $w/\ell = 3$.

static structure factor which can be defined through the guiding center coordinates \mathbf{R}_i :

$$S_0[\mathbf{q}] = \sum_{i < j} \langle \exp i\mathbf{q}(\mathbf{R}_i - \mathbf{R}_j) \rangle. \quad (9)$$

It can be used to reveal the fluid or crystalline character of the system. When evaluated in the $\nu = 1/3$ state it is almost perfectly isotropic: see the three-dimensional plot Fig. 8(a). For the largest system studied here this quantity is also isotropic for $\nu = 7/3$ with no evidence of incipient charge-density wave order: see Fig. 8(b).

The composite fermion wave functions are also able to reproduce the MR dispersion accurately in the LLL but only in the sphere or disk geometry. In Scarola *et al.* [36,37] there is a calculation of the MR in the second LL which has a shape similar to what we observe, but these results are obtained in the case of zero width. It would be interesting to compare CF calculations of the MR mode including finite-width effects.

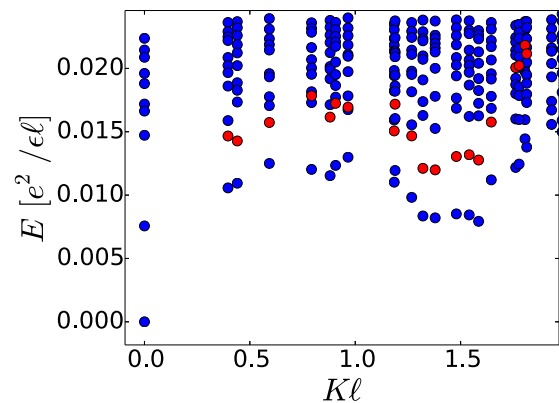


FIG. 7. Spectrum of $N_e = 12$ electrons on a rectangular cell with aspect ratio $L_x/L_y = 0.9$ for $\nu = 7/3$ and Gaussian width $w/\ell = 3$. Blue points comes from exact diagonalization while red points are results of the SMA applied to the exact ground state. The shape of the collective mode is correctly reproduced but the energies are much higher.

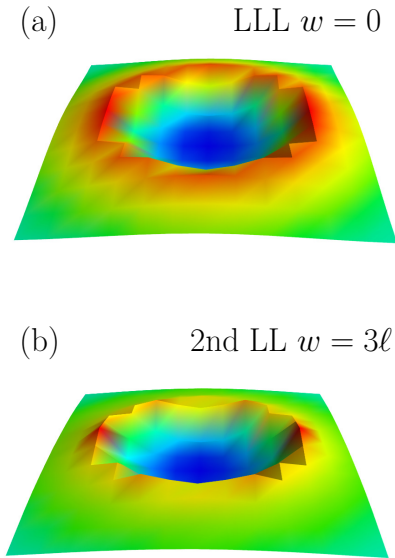


FIG. 8. The guiding center structure factor $S_0(\mathbf{q})$ for $N_e = 12$ electrons in a rectangular cell with aspect ratio $L_x/L_y = 0.9$ for the $\nu = 1/3$ state (a) in the LLL and zero width. In the second LL (b) with a Gaussian width $w/\ell = 3$ the isotropy is almost as good as in the LLL.

Not all these MR features are seen in the spherical geometry [38–40] for the same sizes which means that the torus finite-size effects are different from those observed in the spherical geometry. In the sphere case energy levels are classified by the total angular momentum and the MR branch is obvious in the LLL, extending up to $L = N_e$ as explained by the exciton picture of composite fermions. The LLL finite size effects are very small. Indeed the shape of the MR branch is obvious even for small systems. When going to larger systems it is possible to observe oscillations at large momenta: see Fig. 2. On the sphere geometry there is no intrinsic way to treat the z extent of the electronic wave function so one has to use the interaction matrix elements computed in the planar case. This adds an uncontrolled bias that will disappear only in the thermodynamic limit. A sample calculation for $N_e = 13$ in the second LL is presented in Fig. 9. Some features are in agreement with the torus results. Notably the curvature of the MR mode is downwards at small momentum. The mode is very close or merges with the higher-lying continuum for $L \approx 7-8$. This corresponds to the characteristic wave vector $k\ell \approx L/R \approx 1.7$, where R is the sphere radius in agreement with the torus result. One may interpret the states at higher L values as a quasiparticle-quasihole branch since it terminates also at $L = N_e$ as predicted by the exciton CF picture. The most serious discrepancy is the lack of low-lying states at $L = 1$ and accordingly there is no state at $L = 0$ candidate to come from the smooth continuation of the MR branch at small angular momentum. The low angular momentum states are those with the largest spatial wavelength and it is certainly those states that are most sensitive to the curvature of the sphere. This means that one needs much larger spheres than toruses to capture the $\nu = 7/3$ physics. Note that torus geometry for zero width in the LLL case $\nu = 1/3$ gives results consistent with the sphere

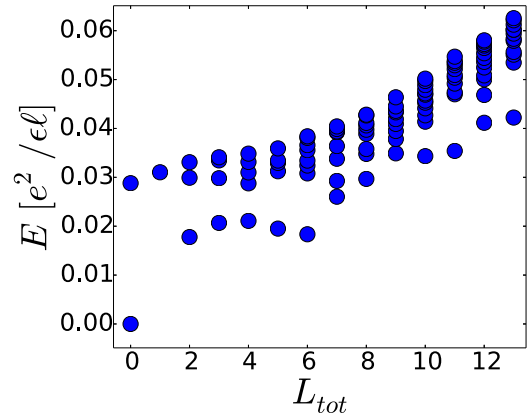


FIG. 9. Sphere spectrum for $N_e = 13$ electrons in the second LL using planar pseudopotentials with finite width modeled after a square well of width $w/\ell = 3$. The MR mode has a downward curvature for small enough momenta as observed in the torus geometry. There are no states at low energy for $L = 1$ as in the LLL case and at variance with the LLL it is not clear that the branch extends to a nontrivial value at $L = 0$.

results [23]: The large wave-vector limit of the MR branch is approximately $0.09e^2/\epsilon\ell$ in Fig. 1.

The finite-size dependences of the gaps one can define in the spectra above are very irregular. This is the case in the LLL but to a lesser degree. So it is difficult to give an estimate of gaps even for finite width. We just quote that for bigger sizes the $K = 0$ gaps seem to stabilize and are $\approx 0.017e^2/\epsilon\ell$ for $w = 2\ell$ and $\approx 0.0075e^2/\epsilon\ell$ for $w = 3\ell$. As is the case for the LLL, the gaps are smaller with wider wells. For smaller widths $w/\ell \lesssim 1$ the MR branch is not well defined, and it is not possible to give an estimate of the $K = 0$ for MR gaps. The gaps we estimate are defined through the dispersion relation of the neutral collective mode. As such they are not directly related to the quasiparticle-quasihole gap that governs the activated law of the longitudinal resistance.

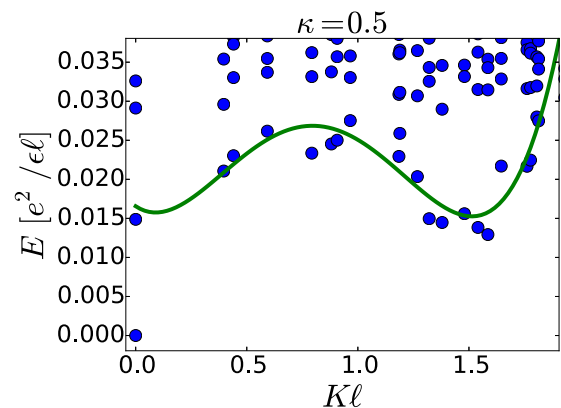


FIG. 10. Torus spectrum for $N_e = 12$ electrons in the second LL. Aspect ratio is 0.9. Finite-width effects are modeled by a square well $w_{SQ} = 4\ell$. The LL mixing is treated as a perturbation with $\kappa = 0.5$. Close-up on the MR mode: its shape is essentially unchanged with respect to the zero LL-mixing case.

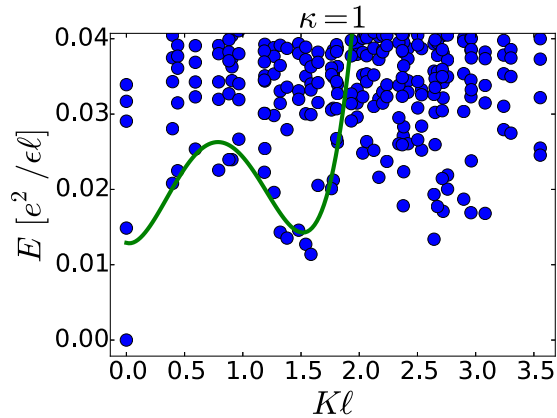


FIG. 11. Torus spectrum for $N_e = 12$ electrons in the 2nd LL. Aspect ratio is 0.9. Finite-width effects are modeled by a square well $w_{SQ} = 4\ell$. Here we display the full Brillouin zone for LL mixing parameter $\kappa = 1$, the largest value of Landau level mixing in perturbation theory.

V. LANDAU LEVEL MIXING

For comparison with experiments at $\nu = 7/3$ it is important to know how and if the MR dispersion relation is changed by the inclusion of Landau level mixing, i.e., by virtual transitions of electrons towards the occupied LLL and unoccupied $N > 1$ LLs. This is an effect which is stronger in the second LL than in the LLL since the magnetic field is weaker and LL spacing is smaller and hence worth studying. The strength of these effects is characterized by the ratio of the typical interaction energy and the cyclotron energy $\kappa = (e^2/\epsilon\ell)/(\hbar\omega_c)$, where $\omega_c = eB/m$ is the cyclotron frequency. Bishara and Nayak [41] have shown that integrating out virtual transitions to the LLL and higher levels with $N > 1$ can be done in perturbation theory in the parameter κ . At first order in κ this procedure generates additional two-body and three-body interactions:

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & \mathcal{H}_{\text{Coulomb}} + \kappa \sum_{i < j} \sum_m \delta V_m^{(2)} \mathcal{P}_{ij}^{(m)} \\ & + \kappa \sum_{i < j < k} \sum_m \delta V_m^{(3)} \mathcal{P}_{ijk}^{(m)}, \end{aligned} \quad (10)$$

where $\mathcal{H}_{\text{Coulomb}}$ is the Coulomb Hamiltonian including finite-width effects projected onto the $N = 1$ LL, the two-body and three-body pseudopotentials $\delta V_m^{(2)}, \delta V_m^{(3)}$ have been computed with a square well wave function in the z direction [42–44], the operators $\mathcal{P}_{ij}^{(m)}$ (resp. $\mathcal{P}_{ijk}^{(m)}$) are projectors onto the state of relative angular momentum m for two-body (resp. three-body) states. These operators can be translated on the torus geometry and treated by exact diagonalization following Ref. [45]. It should be noted that the three-body interactions generate a Hamiltonian matrix which is now much less sparse and there is thus a huge computational overhead. We have repeated the

diagonalization of the largest system with 12 electrons for $\kappa = 0.5$ and $\kappa = 1$ computing five eigenvalues in each momentum sector instead of 10. The lowest-lying energies are shown in Figs. 10 and 11. We find that while all energies are shifted by a κ -dependent value the MR dispersion relation remains essentially unaltered. Notably the characteristic wave vectors and gaps undergo only small changes for $\kappa \lesssim 1$, a value beyond which it is not clear one can trust perturbation theory. So the main findings of the present work are robust to LL mixing at least within the scope of perturbation theory.

VI. CONCLUSIONS

By torus exact diagonalization of the Coulomb interaction in the second Landau level for filling factor $\nu = 7/3$ we have obtained the dispersion relation of the collective mode—the magnetoroton—expected from a liquid state with Laughlin-like correlations. The observation of this mode requires both very large systems with more than 10 electrons and also a finite width of the electron gas. Even if this width reduces the gap scale as is the case in the LLL we find that the MR mode becomes discernible only when $w/\ell \gtrsim 1$. The Laughlin-like physics at $\nu = 7/3$ is also seen in the guiding center structure factor which is almost isotropic. The mode at long wavelength stays definitely below a higher-lying continuum of states. The gap from the ground state to the continuum is almost exactly twice the MR gap, probably indicating the two-particle nature of these states. While the MR minimum is at the same wave vector as in the LLL case there is also a maximum of the dispersion at $k\ell \approx 0.8$. The MR mode disappears in the continuum for $k\ell \approx 1.8$. These features are captured correctly by the SMA and are resilient to Landau level mixing.

Several experiments have probed the MR in the LLL by inelastic light scattering [12–14,46–48] as well as phonon absorption [49,50]. Detailed studies of the MR dispersion relation at $\nu = 1/3$ are in quantitative agreement with theoretical calculations [47,51] contrary to the magnetotransport gaps. Some details of the excited states beyond the lowest-lying MR branch are seen experimentally [51,52]. Recent works have started the study of the second Landau level physics where several phases are in competition beyond the FQHE liquids [53–57]. The collective mode shape we observe from exact diagonalization and SMA calculations should be accessible to inelastic light scattering provided one uses a wide enough quantum well. Strictly speaking we cannot exclude that this shape is also correct for small width where finite-size effects are more problematic. The existence of multiple critical points with vanishing derivatives in the dispersion should appear as several points with enhanced density of states.

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