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Detecting topological superconductivity using low-frequency doubled Shapiro steps

Jay D. Sau and F. Setiawan
Condensed Matter Theory Center and Joint Quantum Institute, Department of Physics, University of Maryland, College Park, Maryland 20742, USA

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The fractional Josephson effect has been observed in many instances as a signature of a topological superconducting state containing zero-energy Majorana modes. We present a nontopological scenario which can produce a fractional Josephson effect generically in semiconductor-based Josephson junctions, namely, a resonant impurity bound state weakly coupled to a highly transparent channel. We show that the fractional ac Josephson effect can be generated by the Landau-Zener processes which flip the electron occupancy of the impurity bound state. The Josephson effect signature for Majorana modes become distinct from this nontopological scenario only at low frequency. We prove that a variant of the fractional ac Josephson effect, namely, the low-frequency doubled Shapiro steps, can provide a more reliable signature of the topological superconducting state.

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Superconductors supporting Majorana zero modes (MZMs) [1–4] at defects provide one of the simplest examples of topological superconductors (TSs) [5,6]. In fact, a number of proposals [7–12] to realize such MZMs have met with considerable success [13–18]. Such systems containing MZMs are particularly interesting [19–25] because of the topologically degenerate Hilbert space and non-Abelian statistics associated with them that make such MZMs useful for realizing topological quantum computation [26]. While preliminary evidence for MZMs in the form of a zero-bias conductance peak have already been observed [13–18,27–31], confirmatory signatures of the topological nature of MZMs are still lacking.

The zero-bias conductance peak provides evidence for the existence of zero-energy end modes which can arise not only from TSs but also from a variety of nontopological features associated with the details of the end of the system [32–35]. In contrast, the topological invariant of a TS, being a bulk property, is not affected by the details of the potential at the end. The topological invariant of a one-dimensional TS can be determined from the change in the fermion parity of the Josephson junction (JJ) [3]. Specifically, the fermion parity of a topological JJ changes when the superconducting phase of the left superconductor ϕ of the JJ winds adiabatically by $\delta\phi = 2\pi$ [2,3]. Such a change in fermion parity of the JJ may be detected from the resulting 4π-periodic component in the current-phase relation of the topological JJ [3,36]. This is referred to as the fractional Josephson effect and can be detected using the fractional ac Josephson effect (FAJE).

The FAJE involves applying a finite dc voltage V across the junction so that the superconducting phase across the junction varies in time as $\phi(t) = \Omega_J t$ [37]. Here, $\Omega_J = V$ is the Josephson frequency, where we have set $\hbar = 1$ and the charge of the Cooper pair $2e = 1$. The 4π-periodic current-phase relation characteristic of a topological JJ results in a current that has a component at half the Josephson frequency, i.e., at $\omega = \Omega_J/2$ instead of $\omega = \Omega_J$ characteristic of conventional JJs [3,11,12,36,38,39]. In principle, the resulting ac current may be detected by a measurement of the radiation emitted from the junction [40,41]. Alternatively, the fractional Josephson effect can also be detected by measuring the size of the voltage steps, known as Shapiro steps [42,43]. For topological JJs, these voltage steps have been numerically found to be $\delta V = 2\Omega_J$, which is double the voltage steps for the conventional JJs [44,45].

Interestingly, evidence for both the FAJE [41] and doubled Shapiro steps [42,43,46] have been seen in TSs that are expected to support MZMs. However, there is evidence that such signatures might appear in nontopological systems as well. For example, both the signatures seem to also appear in the TS experiments when the devices are not in the topological parameter regime [41,43,46,47]. One possible spurious source of FAJE is the period-doubling transition seen in certain JJ systems [48]. In addition, the FAJE and doubled Shapiro steps are known (both experimentally [40] and theoretically [49,50]) to arise from Landau-Zener (LZ) processes in certain ranges of frequency. Avoiding such LZ processes might require particularly low frequencies in low-noise systems with multiple MZMs [51]. While the LZ process is known to potentially lead to FAJE [40,49], there have not been any generic nontopological scenarios presented in the literature so far.

In this Rapid Communication, we start by discussing a generic model of a resonant impurity coupled to a JJ [shown in Fig. 1(a)], which has a weakly avoided crossing in the energy spectrum as a function of phase [see Fig. 1(b)]. The present scenario requires only the coexistence of a highly transparent channel in a JJ [as seen in recent measurements of Andreev bound state (ABS) spectra [52]] and a weakly coupled impurity bound state. Such a coexistence can be found in a multichannel semiconductor-based JJ with a spatially varying density, as is the case of all of the recent experiments [41–43,46]. We use a scattering-matrix approach to show that this relatively generic situation can lead to an FAJE over a frequency range of a factor of a few even in the absence of any TS. In order to distinguish between this nontopological scenario from TS, it is important to be able to go to ultralow MHz frequencies in the FAJE measurements. Shapiro steps provide the setup where such a large range of frequencies spanning three orders of magnitudes (MHz–GHz) are possible [53]. In the second part of this Rapid Communication, we provide a rigorous framework connecting Shapiro steps to TS where we show that the low-frequency doubled Shapiro steps are guaranteed to appear in the overdamped driven measurements of topological JJs.

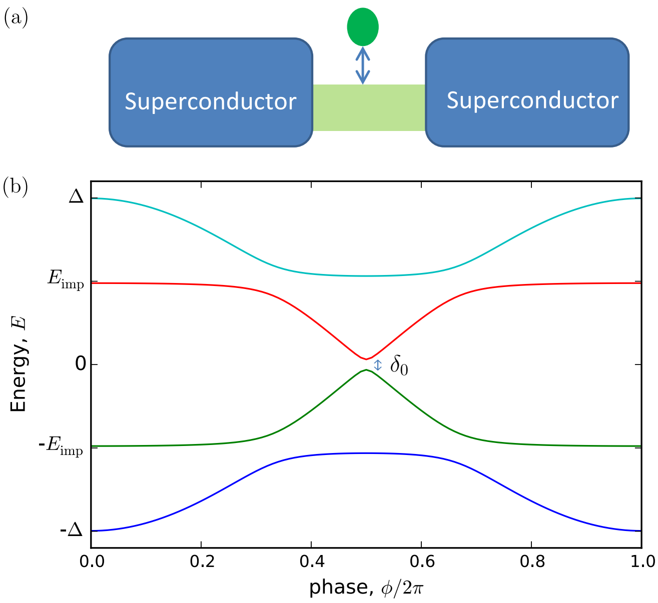

FIG. 1. (a) JJ configuration showing FAJE consists of a high transparency channel connecting two superconductors. The channel is tunnel coupled to an impurity bound state (shown as a disk adjoining wire). (b) Computed Andreev bound state spectrum for the setup in (a) shows a weakly avoided crossing at $E = 0$ and a gap to higher-energy states generated by a larger avoided crossing with the flat impurity bound state. The weakly avoided crossing can lead to an FAJE at finite voltages.

Let us first understand how an FAJE can occur in a nontopological setup such as the setup in Fig. 1(a). For simplicity, we consider the superconductors to be s wave with a highly transparent normal channel in between, together with a subgap impurity bound state. The highly transparent channel supports Andreev bound states (ABSs) in the junction that approach zero energy [see Fig. 1(b)] when the phase ϕ crosses $\phi = \pi$ [54]. Applying a finite voltage V across the junction causes the superconducting phase ϕ to vary in time as $\phi(t) = Vt$. This leads to the possibility of LZ processes exciting Cooper pairs across the superconducting gap. In general, these Cooper pairs are transported across the entire superconducting gap via multiple Andreev reflections [55,56], ultimately leading to a dissipative but otherwise conventional ac Josephson effect [55]. This situation is modified when the junction is tunnel coupled to impurity bound states. As shown in Fig. 1(b), the ABS spectrum of the JJ varies with phase ϕ where it crosses the relatively flat impurity bound state with energy E_{imp} at pairs of points. At such crossings, the junction exchanges a Cooper pair with the flat impurity state. When $\phi = \pi$, the ABS loses a Cooper pair to the condensate through a LZ process across the zero-energy gap δ_0. As the ABS energy approaches the second avoided crossing with the impurity bound state at energy E_{imp}, the ABS restores its Cooper pair at the expense of leaving the impurity bound state empty. Thus, the impurity bound state electron occupancy is flipped via the LZ process as the phase varies over a period of $\phi = 0$ to $\phi = 2\pi$ which is restored during the next 2π cycle. Therefore, while the spectrum of the junction is 2π periodic, the occupation of the impurity bound state is 4π periodic. Since the total energy E which includes the spectrum and occupation of the ABS and impurity bound states determines the supercurrent $I(\phi)$ by $I(\phi) \sim \partial_\phi E(\phi)$, $I(\phi)$ would also be 4π periodic with the phase ϕ. This manifests as a peak in the radiation spectrum from the current at a frequency of $\omega = \Omega_J/2$ instead of the usual Josephson frequency $\omega = \Omega_J$ peak.

While the qualitative argument above suggests the possibility of an FAJE occurring in nontopological semiconductor systems, it assumes the zero-energy LZ processes to be perfect and all other LZ processes to be completely avoided. In the following, we perform a completely unbiased quantitative analysis of the FAJE for the JJ shown in Fig. 1. To begin with, we note that at any finite voltage V, the occupation of an ABS fluctuates due to excitations out of the bulk gap (via multiple Andreev reflections). The quasiparticle fluctuations ensure that the system equilibrates to the grand canonical ensemble (with no conserved fermion parity) such that the expectation value of the current is 2π periodic as in the conventional system [57]. Thus, strictly speaking, the FAJE at any finite voltage is subject to random fluctuations and can only appear in the noise spectrum of the current [58,59]. To assess the range of voltages over which the JJ shown in Fig. 1(a) exhibits an FAJE, we compute the noise spectrum of the current

$$P(\omega) = \int d\tau e^{i\omega\tau}[\langle I(t)I(t+\tau)\rangle - \langle I(t)\rangle\langle I(t+\tau)\rangle], \quad (1)$$

where $\langle\cdots\rangle$ denotes the averaging over time t. The current [55] and its noise spectrum [58,59] can be computed by considering the scattering of quasiparticles between the superconducting leads, which are at different voltages. This approach has the advantage of including the contribution of not only the low-energy ABSs but also all bound and scattering states in the junction. We have expanded this formalism to general superconductor-normal-superconductor junctions [60]. Our general framework can be easily implemented with Kwant [61] which supplies the normal-superconductor scattering matrices. The resulting power spectrum $P(\omega)$ is plotted against the frequency scaled by the Josephson frequency, i.e., ω/Ω_J in Fig. 2 for various voltages for the system depicted in Fig. 1(a) with the spectrum shown in Fig. 1(b). The power spectrum at high voltages is quite broad, which becomes narrower at lower frequency and develops peaks in the vicinity of $\omega/\Omega_J = 1/2$ before splitting off to different values. The high-frequency spectrum is also several orders smaller in magnitude, which is expected in the adiabatic limit when fluctuations in the ABS occupation are small. While some of the peaks appear to move away from the ideal fractional value and come back, this might be difficult to resolve at a high level of broadening arising from nearby energy states and circuit-noise induced broadening.

The spurious FAJE peaks in Fig. 2 resulting from the LZ mechanism appear over a frequency range narrower compared to the parametrically large frequency range (i.e., $\Gamma,\delta \leqslant \omega \leqslant \Delta$) of the FAJE in a high-quality TS [58,59,62,63]. Here, Δ is the induced superconducting gap, which is a relatively large frequency ($\sim$GHz), and Γ and δ are respectively the quasiparticle poisoning rate and the MZM overlap that become vanishingly small ($\lesssim$MHz) in high-quality TSs.

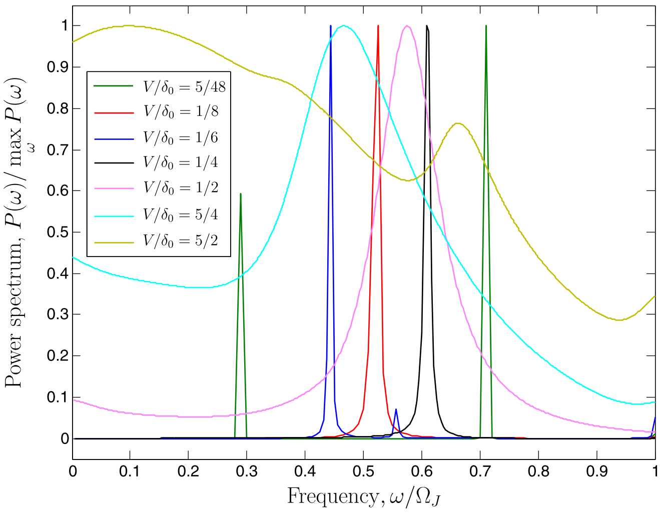

FIG. 2. Power radiated $P(\omega)$ as a function of frequency ω/Ω_J for different ratios of the applied voltage V relative to the zero-energy gap δ_0. The power spectrum $P(\omega)$ shows a fractional ac Josephson peak at $\omega = \Omega_J/2$ for a range of values of V/δ_0. The peak broadens out at higher voltages and shifts towards a more conventional peak at $\omega = \Omega_J$ at lower frequency (while becoming smaller). $P(\omega)$ has been rescaled so that all peaks are clearly visible.

It is clear from Fig. 2 that distinguishing a bona fide TS from an LZ-type mechanism induced by resonant bound states requires low-frequency ($\lesssim 50$ MHz) measurements of high-quality TS devices with $\Delta \gg \delta,\Gamma$. The FAJE which involves measuring small oscillating currents is difficult to perform for low frequencies because such small oscillating currents are typically measured using on-chip detectors [40,64] that are suited to measure relatively high frequencies ($\sim$GHz). On the other hand, the Shapiro stexp [37], which is a variant of the FAJE, has been demonstrated over a large range of frequencies from several MHz to GHz [53]. While this makes the Shapiro step promising for the detection of TSs, a rigorous proof establishing the doubled Shapiro step as a signature of TS is still missing from the literature. Below, we demonstrate analytically that the low-frequency doubled Shapiro steps can be used as a reliable signature of TS.

We begin by considering the Shapiro step experiment where a JJ shunted with a resistance R is biased with a time-varying current $I_{\rm bias}(t) = I_{\rm dc} + I_{\rm ac}\cos(\Omega_J t)$, with $I_{\rm dc}$ and $I_{\rm ac}$ being dc and ac bias currents, respectively. For the following analysis, we make a *key assumption* that we are working in the limit of low-frequency Ω_J so that the Josephson current $I_J(\phi(t))$ can be taken to be in equilibrium, apart from the conserved local fermion parity. The assumption of being at sufficiently low frequency can only be justified by studying the Shapiro steps over a few orders of magnitude in frequency (from $\Delta \sim$ GHz to $\delta,\Gamma \sim$ MHz). Using this assumption and the result of Bloch [57], we can establish that $I_J(\phi)$ for any nontopological system must be 2π periodic and thus rule out any nontopological FAJE such as those from the LZ mechanism.

Furthermore, assuming that the shunt resistance R is small enough to allow the JJ to be overdamped, the equation of motion for $\phi(t)$ for the resistively shunted JJ takes the standard form [37]

$$\frac{d\phi}{dt} = R[I_{\rm bias}(t) - I_J(\phi(t))]. \qquad (2)$$

For illustration purposes, we will choose a simple case of $I_J(\phi) = I_0\cos(2\pi\phi) + I_{\rm top}\cos(\pi\phi)$, where I_0 and $I_{\rm top}$ are the 2π- and 4π-periodic components of the critical current of the adiabatic current-phase relation, respectively. However, our results generally hold and do not depend on this parameter choice, as is proven by the analytic arguments in Ref. [65]. The dc voltage V across the JJ is calculated by considering the average change of the phase

$$V = \lim_{t\to\infty} \frac{\phi(t) - \phi(0)}{t}, \qquad (3)$$

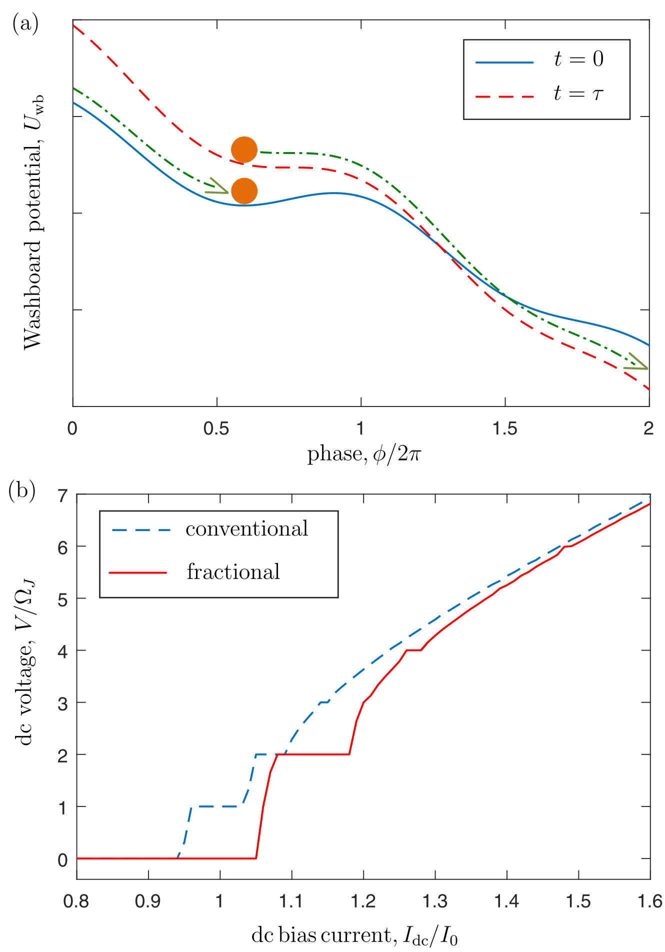

FIG. 3. (a) Schematic of a phase particle (orange disk) on a tilted washboard potential that describes the phase dynamics in an overdamped JJ. As the bias current increases from $t = 0$ to $t = \tau$, the phase particle is released from the local minimum and traverses the trajectory along the green dashed-dotted arrow, and stops when the current bias is back to its value at $t = 0$ and the phase particle has traveled by 4π (for the TS case shown here). This corresponds to a voltage step of $2\Omega_J$. (b) Shapiro step calculated numerically for a putative fractional Josephson system shows doubled Shapiro steps (see also Ref. [44]) as opposed to a conventional system with all integer Shapiro steps for an overdamped JJ. Here, $I_{\rm ac} = 0.1I_0$, $R = 25$, $I_{\rm top} = 0.15I_0$ (for fractional), and $I_{\rm top} = 0$ (for conventional).

where the limit is computed by choosing a sufficiently long simulation time for Eq. (2).

We will now show that overdamped JJs constructed out of TSs are generically characterized by a doubled Shapiro step in the strongly overdamped and low-frequency limit (i.e., $\Omega_J/I_J R \ll 1$). The dynamics of $\phi(t)$ described by Eq. (2) can be understood simply by an analogy of a "phase particle" rolling down a washboard potential according to the equation $\dot{\phi}(t) = -\partial_\phi U_{\rm wb}(\phi,t)$, where the washboard potential is written as $U_{\rm wb} = -R[I_{\rm bias}(t)\phi - \int d\phi I_J(\phi)]$. As seen in Fig. 3(a), because of the ac drive, the potential $U_{\rm wb}(\phi,t)$ varies in time with local minima at each cycle when $\phi(t) = \phi_0$ such that

$$I_{\rm bias}(t) - I_J(\phi_0) = 0. \tag{4}$$

In the adiabatic limit (i.e., $\Omega_J/I_J R \ll 1$), one can show that the phase particle approaches the minimum of the washboard potential exponentially in time once every period of the drive. This leads to a well-defined voltage that appears as a sharp plateau in the Shapiro steps [65].

Let us for now assume that [65] the phase particle approaches a minimum of $U_{\rm wb}$ during the time interval when such exists. In the conventional case of a 2π-periodic function I_J, this can occur once in a 2π period provided the critical current $I_{J,\max} > (I_{\rm dc} - I_{\rm ac})$. This will certainly occur if $I_{\rm dc}$ is small enough. In addition, if $I_{\rm dc} > (I_{J,\max} - I_{\rm ac})$, then there will be a range of time when $U_{\rm wb}$ has no minimum and the adiabatic solution breaks down. In this case, $\phi(t)$ will wind by a multiple of 2π and collapse to ϕ_0 after a winding of $2\pi n$. The result is that an integer voltage appears across the JJ. In the case of a topological JJ, the current-phase relation $I_J(\phi)$ has a 4π-periodic component and one can define two critical currents $I_{J,\max}$ and $I'_{J,\max}$, one associated with the range $\phi \in [4n\pi,(4n+2)\pi]$ and the other in the range $\phi \in [(4n-2)\pi,4n\pi]$. In our simple model $I_{J,\max},I'_{J,\max} = I_0 \pm I_{\rm top}$. As in the conventional case, the dc bias current must satisfy $I_{\rm dc} > (I_{J,\max} - I_{\rm ac})$ (assuming $I_{J,\max} > I'_{J,\max}$) to exit the zero-voltage state even in the TS case. On the other hand, if $2I_{\rm ac} < (I_{J,\max} - I'_{J,\max})$, then $I_{\rm dc} > I'_{J,\max} + I_{\rm ac}$ so that the phase particle cannot stop at one half of the minima. This leads to a doubled voltage step for the topological case, as seen from the numerical solution of Eq. (2) [see Fig. 3(b)].

In summary, we have shown that while the FAJE can be viewed as a smoking gun for the TS with MZMs, a detailed study of the frequency dependence of the FAJE is necessary before concluding a system to have realized the TS. We have shown this by considering a generic model of a high transparency channel in a JJ coupled weakly to a resonant impurity. We find this model to show an FAJE quite generically in semiconductor-based JJs, similar to the TS case with MZMs. Nevertheless, TSs are expected to show FAJE over a parameterically larger range of frequency. We argue that the current-phase relation over such a range of frequency, particularly at the low-frequency end, is better studied by considering the Shapiro step experiment. We present a way of understanding the Shapiro step experiment in terms of the tilted washboard potential that guarantees that the necessary and sufficient condition for the existence of doubled Shapiro steps in the low-frequency limit is that the JJ is formed from a TS. Thus, low-frequency Shapiro steps which have been demonstrated in conventional systems can serve as a smoking gun for MZMs.

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[65] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.95.060501, Sec. II, where we prove analytically that the period of the Shapiro step is generically doubled for the topological case in the adiabatic limit.