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Microwave spectroscopic observation of a Wigner solid within the $\nu = 1/2$ fractional quantum Hall effect

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We have studied the microwave spectra of a wide quantum well for Landau level fillings, ν, just below $1/2$, under conditions where the $\nu = 1/2$ fractional quantum Hall effect (FQHE) is present. One resonance in the spectra exhibits intensity variations with ν in striking agreement with that expected for a pinning mode of a Wigner solid of quasiholes of this FQHE state. This resonance is also quite sensitive to asymmetrization of the growth-direction charge distribution in the quantum well by gate bias. Another resonance in the spectra is associated with a different bilayer Wigner solid that also exists at much lower ν than the $1/2$ FQHE, and that appears to coexist with the $1/2$ quasihole solid.

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The fractional quantum Hall effect (FQHE) states at half-integer Landau fillings (ν) have long been of great interest [1–8], since they have correlations that differ from those of the fundamental Laughlin states found at odd denominators. At $\nu = 1/2$ the FQHE has been observed in wide [1–5] or double quantum wells [8] and is ascribed to the two-component Halperin-Laughlin Ψ_{331} state [9,10]. Ψ_{331} excitations carry charge $\pm e/4$, like the carriers of $\nu = 5/2$ states which are of interest in quantum computation [7]. Furthermore, at least when they are sufficiently dilute, a quasiparticle or quasihole of Ψ_{331} [10] has unequal, opposite charge in the top and bottom layers and, hence, an up or down dipole moment. Here we report evidence for a Wigner solid (WS) of quasiholes of the $1/2$ fractional quantum Hall liquid, from quantitative study of the microwave spectra of a wide quantum well (WQW) at ν close to $1/2$.

WSs in quantum Hall systems can be classified into two types [11]. Here we take a type-I WS to be a ground state of the entire system, such as is found at the low-ν termination of the FQHE series [12–26]. The type-II WSs are formed of quasiparticles or quasiholes in the presence of a gapped state such as a filled Landau level at an integer quantum Hall effect (IQHE) plateau [25,27–29], or an FQHE liquid [30]. The solid that we report here is of type II. Crucially, the density of carriers in a type-II WS, but not a type-I WS, is dependent on the magnetic field, because the carrier density increases as flux is subtracted or added to the parent, gapped state.

In a WQW the $1/2$ FQHE state exists for electron density, n, within a certain range [1–3,31–33]. Experiments [31] showed that increasing n in a WQW causes its carriers to become more bilayerlike by reducing the first-excited subband energy and by increasing the interlayer distance and the intralayer Coulomb interaction, $e^2/4\pi\varepsilon\ell_B$, where $\ell_B = \sqrt{\hbar/eB}$ is the magnetic length. Transport studies further showed [1–3,31,34] that the $1/2$ FQHE state occurs when the system is sufficiently bilayerlike, but only when the inter- and intralayer Coulomb interaction energies are still comparable. As ν in a WQW is decreased from $1/2$ by increasing B, the FQHE gives way to an insulator [31], which extends to the lowest ν measured. That insulator is interpreted as a type-I WS pinned by disorder, and a recent theory [33] of WQWs identifies this WS as a pinned bilayer rectangular lattice of the first excited subband [35].

The microwave spectroscopic measurements reported in this paper are of WS pinning modes. The pinning mode is a well-known experimental signature of WSs [15,18,20,23,27] and corresponds to a small oscillation of the solid within the potential of the disorder that pins it. For ν just below $1/2$ we find a resonance that can be identified as the pinning mode of the type-II $1/2$ FQHE quasihole WS. The resonance intensity allows measurement of the participating carrier density by means of a sum rule. For the resonance interpreted as type II, we find that participating density measured this way depends linearly on $1/2 - \nu$, quantitatively just as expected for quasiholes of the $1/2$ FQHE. This resonance also is suppressed by a small imbalance between the carrier density in the top and bottom layers, consistent with its interpretation as due to a solid of quasiholes of the $1/2$ FQHE ground state. Along with the pinning mode of the type-II $1/2$ FQHE quasihole WS, we also find a resonance due to the type-I WS which extends to the lowest ν we measured. The amplitude of the pinning mode of the type-I WS increases monotonically as ν decreases below $1/2$, and the presence of two resonances in the spectra indicates the system is some type of mixture of two WS phases.

Measurements were performed on a GaAs/AlGaAs WQW of width $w = 80$ nm with an as-cooled density of $n = 1.1$ in units of 10^{11} cm^{-2}, which we use for density throughout the paper. Figure 1(a) shows a top-view schematic of our microwave sample setup [23,24,27,36] which uses a coplanar waveguide (CPW) lithographed onto the top surface of the sample. We calculate the diagonal conductivity as $\sigma_{xx}(f) = (s/lZ_0)\ln(t/t_0)$, where $s = 30\ \mu$m is the distance between the center conductor and ground plane, $l = 28$ mm is the length of the CPW, $Z_0 = 50\ \Omega$ is the characteristic impedance without the two-dimensional electron system, t is the transmitted signal amplitude, and t_0 is the normalizing amplitude taken at $\nu = 1/2$. The microwave measurements were carried out in the low-power limit, in which the measurement is not sensitive to the excitation power, at a bath temperature of $T \approx 50$ mK.

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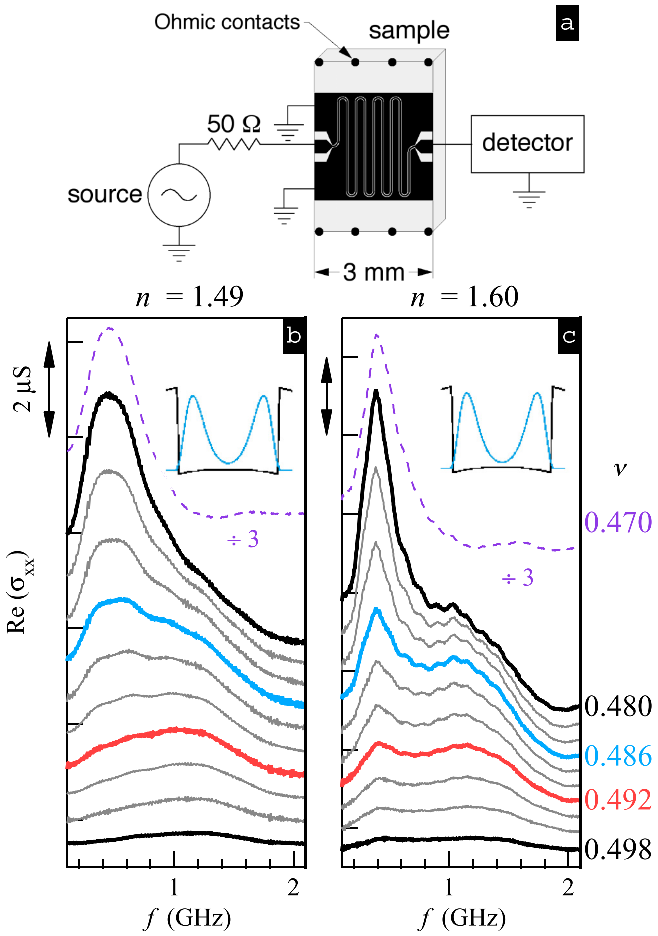

FIG. 1. (a) Schematic representation of the microwave setup. The gates are not shown. (b), (c) Microwave spectra, plotted as the real part of the conductivity Re(σ_{xx}) vs the frequency, f, for $n = 1.49$ and $n = 1.60$. The $\nu = 0.470$ trace is shown as a dashed line with values divided by 3. The weak bump near ~1.6 GHz is an artifact: the strong line at $\nu = 0.470$ has significant Im(σ_{xx}) at 1.6 GHz, which enables a standing wave between the transmission line and a reflection near the sample mounting. Traces from $\nu = 0.480$ to $\nu = 0.498$ are shown with 0.002 steps, and are successively offset by 0.5 μS. Insets: Simulation of the growth-direction charge distribution at the specified n, when the charge is symmetric about the well center.

As described elsewhere [3,4,28,31,35], we used bias voltage applied to front and back gates to control the carrier density and the symmetry of the growth-direction charge distribution. The back gate was in direct contact with the bottom of the sample and the front gate was deposited on a piece of glass that was etched to be spaced ~10 μm from the sample surface so as not to interfere with the microwave transmission line. Except as noted, the symmetric (balanced) growth-direction charge distribution was maintained by biasing the gates such that individually they would change the density by the same amount with equal and opposite electric fields. An asymmetric (imbalanced) distribution was obtained by first biasing one gate to get half the desired charge asymmetry, $\Delta n/2$. Then we biased the other gate with opposite polarity to maintain the same total density with applied electric fields in equal amount and the same direction.

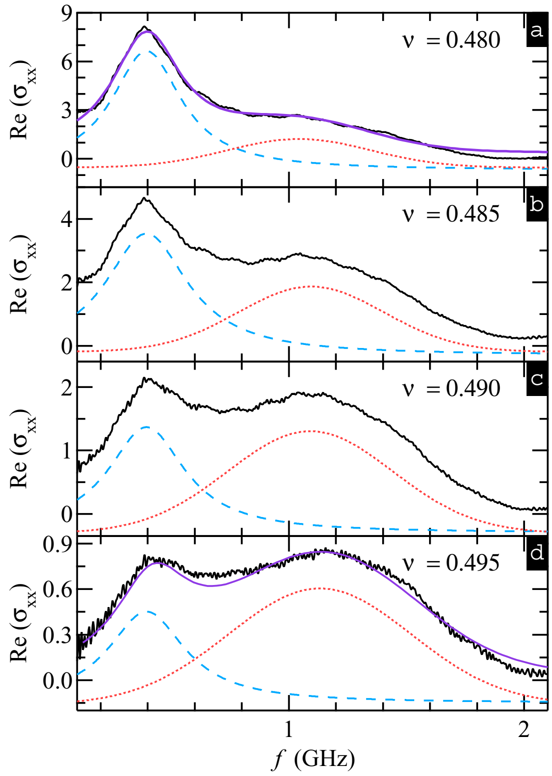

FIG. 2. (a)–(d) Re(σ_{xx}) vs f at several ν (solid traces) for $n = 1.60$. Each plot also contains the components of the individual peak fits for the type-I solid resonance (Lorentzian, dashed line) and the $\nu = 1/2$ FQHE quasihole solid (Gaussian, dotted line), which are vertically offset for clarity. f_{pk} for the type-I WS resonance is held constant and f_{pk} for the type-II WS resonance is allowed to vary for all fits. In (a) and (d) the fitting function is superimposed over the raw data to demonstrate the quality of the fit.

In Fig. 1(b) we plot Re(σ_{xx}) vs frequency, f, for several ν, at $n = 1.49$. At $\nu = 0.498$ we observe a resonance with peak frequency $f_{\text{pk}} \sim 1.2$ GHz. As ν decreases, this resonance has a nonmonotonic amplitude change and change in f_{pk}. An additional resonance is observed at lower ν with $f_{\text{pk}} \sim 0.45$ GHz. We ascribe this lower-f_{pk} resonance to a type-I WS, whose f_{pk} is consistent with that found in an earlier study [35]. In Fig. 1(c) at $n = 1.60$ we observe two well-defined resonances throughout the $0.480 \leqslant \nu < 0.5$ range. We interpret the higher-f_{pk} resonance as due to the type-II WS composed of quasiholes of the 1/2 FQHE, which coexists with the type-I bilayer electron solid.

To test our interpretation we compare the charge density obtained from the intensity of the type-II resonance to that expected for the type-II WS of quasiholes. We reproduce four spectra from Fig. 1(b) in Figs. 2(a)–2(d). The spectra are fitted to a sum of two single-peak functions. These fits are used to extract the two resonances over the entire ν range studied for $n = 1.60$ and $n = 1.49$, and to calculate the integrated intensities, $S = \int \text{Re}[\sigma_{xx}(f)]df$.

Pinning modes roughly obey a sum rule [37], whose predicted value for S/f_{pk} is $(S/f_{\text{pk}})^{\text{sr}} = \rho_c \pi/2B = e^2\pi\tilde{\nu}/2h$, where for a type-I WS, $\rho_c = ne$ and $\tilde{\nu} = \nu$, but for the type-II WS $\rho_c = ne|\nu^\star|/\nu$ and $\tilde{\nu} = \nu^\star = \nu - 1/2$. In

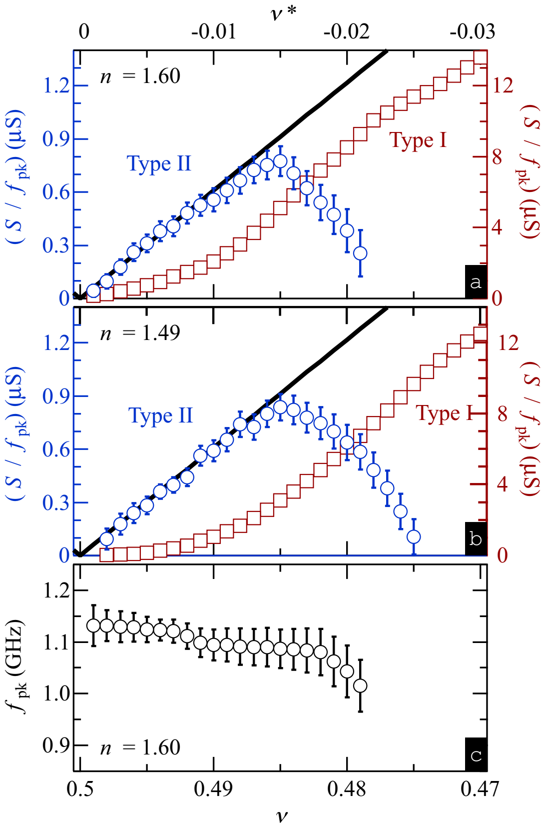

FIG. 3. (a), (b) Integrated spectrum divided by peak frequency, $S/f_{\rm pk}$, vs ν, for the resonance of type-II WS (circles, left axis), and the type-I WS (squares, right axis). The scales differ by a factor of 10. In each plot, the dark straight line is generated by the sum-rule formula $(S/f_{\rm pk})^{\rm sr}$ [37] (see text) without adjustable parameters, for full participation of quasiholes of the 1/2 FQHE, and refers to the left axis. (a) $n = 1.60$ and (b) $n = 1.49$. (c) $f_{\rm pk}$ for the type-II WS vs ν for $n = 1.60$; on the top axis $\nu^\star = 1/2 - \nu$ is shown. Error bars are estimated from using different fit functions and ranges.

Figs. 3(a) and 3(b) we plot the experimentally obtained $S/f_{\rm pk}$, $(S/f_{\rm pk})^{\rm expt}$, for the two resonances (symbols) and the calculated $(S/f_{\rm pk})^{\rm sr}$ (dark line). $(S/f_{\rm pk})^{\rm expt}$ of the type-II WS is in good agreement with $(S/f_{\rm pk})^{\rm sr}$, with no adjustable parameters, for $0.485 < \nu < 0.5$. For $\nu < 0.485$, $(S/f_{\rm pk})^{\rm expt}$ of the type-II WS resonance deviates from $(S/f_{\rm pk})^{\rm sr}$ and begins to decrease with increasing quasihole density. The good agreement between $(S/f_{\rm pk})^{\rm expt}$ and $(S/f_{\rm pk})^{\rm sr}$, achieved without any adjustable parameters, provides compelling evidence for the interpretation of the higher-$f_{\rm pk}$ resonance as due to a type-II WS made up of quasiholes.

The close agreement for the type-II WS of $(S/f_{\rm pk})^{\rm expt}$ and $(S/f_{\rm pk})^{\rm sr}$ implies that only a small percentage of carriers are available for the type-I WS. For the type-I WS resonance, we compare $(S/f_{\rm pk})^{\rm expt}$ to $(S/f_{\rm pk})^{\rm sr}$ to obtain the participation ratio $\eta \equiv (S/f_{\rm pk})^{\rm expt}/(S/f_{\rm pk})^{\rm sr}$. At $\nu = 0.485$, at which $(S/f_{\rm pk})^{\rm expt}$ begins to fall short of $(S/f_{\rm pk})^{\rm sr}$ for the type-II WS, we find $\eta \sim 10\%$ for $n = 1.49$ and $\eta \sim 17\%$ for $n = 1.60$. Since η of the type-I WS resonance continues to decrease as ν goes from 0.485 toward 1/2, near-full participation of the carriers in the type-II WS resonance is consistent with the observed type-I WS resonance intensity. As ν goes below 0.48 the type-I WS resonance amplitude increases as the electrons solidify into the type-I WS; by $\nu = 0.4$, $\eta \sim 100\%$ for the type-I WS [35].

Figure 3(c) shows the extracted $f_{\rm pk}$ vs ν for the type-II WS, for $n = 1.60$ ($n = 1.49$ data yield nearly identical values). $f_{\rm pk}$ decreases by ~5% for $0.5 > \nu > 0.485$ with larger decreases for $\nu < 0.485$. One would expect [27,30,38–40] that $f_{\rm pk}$ should decrease as $\nu^\star$ (and hence the quasihole density) increases due to the increase of the WS shear modulus. However, the local density within domains of the type-II solid can remain constant while the total proportion of the area occupied by these domains increases proportionally to $\nu^\star$. A similar situation occurred in the ν-driven solid-solid transition between the type-II WS with ν just above 4 and the bubble lattice [41]. In that case the transition range in which two pinning mode peaks were present showed a range of ν with weak change of $f_{\rm pk}$ even though the total density of carriers in the partially occupied Landau level was changing with ν. The overall picture of the system near $\nu = 1/2$ is then of a composite, with (1) domains of local $\nu = 1/2$ FQHE, (2) domains of type-II WS of quasiholes of that state, and (3) domains of a type-I WS.

In dc transport measurements there was a reentrant insulating range observed at ν slightly larger than $\nu = 1/2$ [31]. Our microwave measurements for $\nu > 1/2$ displayed no resonance; there was no observable pinning mode of the quasiparticles associated with $\nu = 1/2$ FQHE. A plausible explanation is that the quasiholes and quasiparticles have different interactions. Such a situation was predicted theoretically [11] near $\nu = 1/3$ for which the quasihole solid was calculated to have a higher melting temperature than the quasiparticle solid, though Ref. [30] found both quasihole and quasiparticle WS resonances of the 1/3 FQHE. Different from the situation for the 1/3 FQHE, pieces of the type-I solid are evidently present [42] in the range in which we observed the type-II solid. It is conceivable that the type-I pieces enhance the pinning of the type-II solid and enable the observation of its pinning mode.

In WQWs the 1/2 FQHE state has been shown to be highly sensitive to the symmetry of the growth-direction charge distribution [2,3,31]. To further test the association of the higher-$f_{\rm pk}$ resonance with the 1/2 FQHE, we investigated the role of charge asymmetry between the two layers.

Figure 4 shows $\mathrm{Re}(\sigma_{xx})$ vs f at $n = 1.49$ for two fixed ν. In Fig. 4(a) ($\nu = 0.490$) the trace for the symmetric case contains both resonances. We observe a loss of the higher-$f_{\rm pk}$, type-II WS resonance, and a concurrent enhancement of the type-I WS resonance near $f \simeq 0.45$ GHz on imbalancing the layer densities by $\Delta n = n_f - n_b/n = 2.9\%$, where n_f and n_b are respectively the front and back layer densities in the WQW. This result is more dramatic at $\nu = 0.498$, shown in Fig. 4(b). In that case the symmetric state shows only the higher-$f_{\rm pk}$ type-II WS resonance while the $\Delta n = 2.7\%$ case shows only the type-I resonance, but does not show the type-I WS. The loss of the type-II WS with this slight asymmetry of the charge distribution is explained if the 1/2 FQHE and its excitations are much more sensitive to the symmetry than the type-I WS. These Δn values are in reasonable agreement with Ref. [31], in which the 1/2 FQHE for $n = 12.9$ is suppressed by

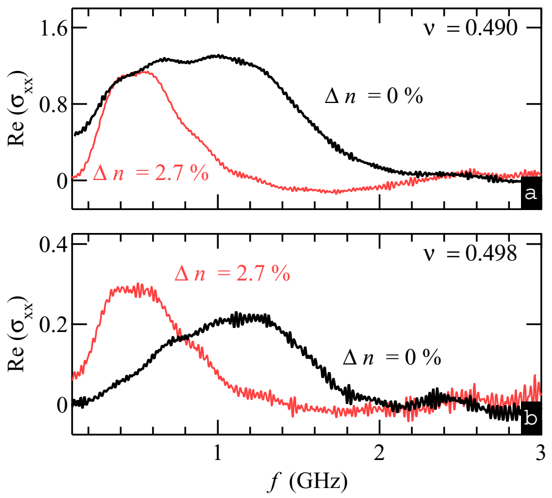

FIG. 4. Microwave spectra with asymmetric gating. Re(σ_{xx}) vs f at fixed ν for (a) $\nu = 0.490$ and (b) $\nu = 0.498$. Spectra are obtained at fixed total density, $n = 1.49$, and different charge configurations along the growth direction within the well, with Δn denoting the charge difference between the top and bottom layers, specified as a percentage of the total charge density n.

Δn of ~5.4%. Larger asymmetries of the well ($|\Delta n| \geqslant 10\%$) destroy the type-I WS in this ν range as well [31,35].

In summary, our study of a WQW provides strong evidence of a type-II WS of 1/2 FQHE quasiholes, which have unequal opposite-sign charges in the two layers. The solid appears to coexist along with the type-I solid characteristic of the WQW system at low ν.

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Data displayed in this paper are available by email request to engel@magnet.fsu.edu.

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[42] Reference [30] presents data on a resonance interpreted as a due to a type-II WS in the $1/3$ FQHE. When well developed, that resonance had $f_{\mathrm{pk}} \sim 1$ GHz. Located at about 2.4 GHz, in the wing of the pinning mode, is a much smaller shoulder, which was not addressed in Ref. [30]. The origin of that shoulder is unclear, because it cannot be identified with a known solid phase at nearby filling (as the lower-frequency resonance of the present paper can). At the small signal level of Ref. [30], the possibility exists that this small shoulder is a reflection, brought about by the change in input impedance of the transmission line, produced by the main resonance.