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Momentum of superconducting electrons and the explanation of the Meissner effect

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Momentum and energy conservation are fundamental tenets of physics, which valid physical theories have to satisfy. In the reversible transformation between superconducting and normal phases in the presence of a magnetic field, the mechanical momentum of the supercurrent has to be transferred to the body as a whole and vice versa, the kinetic energy of the supercurrent stays in the electronic degrees of freedom, and no energy is dissipated nor entropy is generated in the process. We argue on general grounds that to explain these processes it is necessary that the electromagnetic field mediates the transfer of momentum between electrons and the body as a whole, and this requires that when the phase boundary between normal and superconducting phases is displaced, a flow and counterflow of charge occurs in a direction perpendicular to the phase boundary. This flow and counterflow does not occur according to the conventional BCS-London theory of superconductivity, therefore we argue that within BCS-London theory the Meissner transition is a "forbidden transition." Furthermore, to explain the phase transformation in a way that is consistent with the experimental observations, requires that (i) the wave function *and* charge distribution of superconducting electrons near the phase boundary extend into the normal phase, and (ii) that the charge carriers in the normal state have holelike character. The conventional theory of superconductivity does not have these physical elements, the theory of hole superconductivity does.

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I. INTRODUCTION

Experiments [1,2] and theory [3] show that under ideal conditions the superconductor to normal transition in the presence of a magnetic field is a reversible phase transformation [4] between equilibrium states of matter that occurs without energy dissipation and without increase in the entropy of the universe. In this paper, we argue that the conventional BCS-London theory of superconductivity [5,6] cannot explain how mechanical momentum is conserved in this transition, and for this reason BCS-London theory as it stands is not a viable theory of superconductivity for any superconductor. In other words, BCS theory predicts that the Meissner transition is a "forbidden transition" [7], in contradiction with experiment [1,2]. Instead, we point out that the alternative theory of hole superconductivity [8] explains how the transition occurs in a reversible way conserving mechanical momentum. The key issue of reversibility and how it is addressed experimentally is discussed in Appendix B.

We restrict ourselves to nonrelativistic electrons, which is sufficient for most solids. In the absence of electric current, the average mechanical momentum of electrons at any point in space is zero. In the presence of an electric current, the mechanical momentum density of electrons at position $\vec{r}$ is [9,10]

$$\vec{\mathcal{P}}(\vec{r}) = \frac{m_e}{e}\vec{J}(\vec{r}), \qquad (1)$$

where $\vec{J}(\vec{r})$ is the current density at position $\vec{r}$, m_e is the *bare* electron mass, and e (<0) is the electron charge. Consider a cylinder of radius R and height h in a uniform magnetic field $\vec{H}$ parallel to its axis pointing in the $\hat{z}$ direction, hanging from a thread of negligible torsion coefficient. Assume the material is a type-I superconductor with a thermodynamic critical field H_c and London penetration depth λ_L, initially in the normal state, and the body is at rest. When it is cooled into the superconducting state, the magnetic field is expelled from the interior [1] (assuming $H < H_c$) through the development of a surface current

$$I = \frac{c}{4\pi} h H. \qquad (2)$$

I flows within a London penetration depth of the surface, so the current density is

$$\vec{J} = -\frac{c}{4\pi\lambda_L} H\hat{\theta} \qquad (3)$$

as follows from Ampere's law $\vec{\nabla} \times \vec{H} = (4\pi/c)\vec{J}$ and the requirement that $\vec{B} = 0$ inside the superconductor. Therefore the electrons acquired a nonzero momentum. The momentum density of the supercurrent is, from Eqs. (1) and (3),

$$\vec{\mathcal{P}} = -\frac{m_e c}{4\pi\lambda_L e} H\hat{\theta} \qquad (4)$$

in a volume $2\pi R\lambda_L h$, hence the total angular momentum of the supercurrent is

$$\vec{L}_e = -\frac{m_e c}{2e} h R^2 H\hat{z}. \qquad (5)$$

Note that L_e is proportional to the *bare* electron mass [10].

L_e is a macroscopic angular momentum carried by the supercurrent. For example, for $R = 1$ cm, $h = 5$ cm, and $H = 200$ G, $L_e = 2.84$ mg mm^2/s. From angular momentum conservation, we conclude that the body as a whole must rotate carrying equal and opposite angular momentum, since the total angular momentum before the system was cooled was zero and no angular momentum was imparted to the system as a whole upon cooling. Measurement of the body's angular momentum was never done this way, but instead by applying a magnetic field to an already superconducting body, which will develop a screening current with angular momentum equal to those given by Eqs. (3) and (5). Indeed, the body is found to rotate with angular momentum given by Eq. (5) with opposite sign [11–13]. This is called the "gyromagnetic effect."

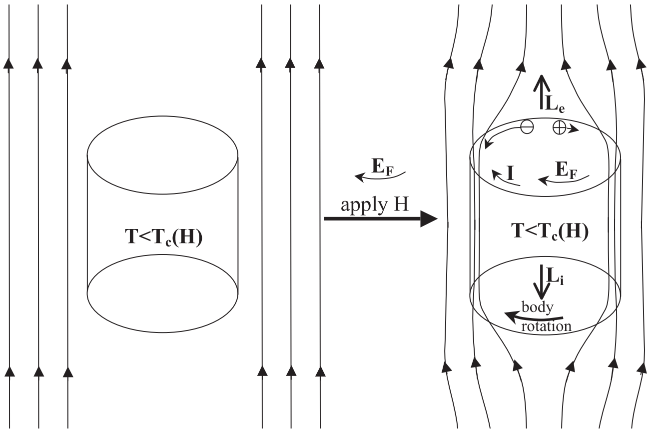

FIG. 1. Process I: a magnetic field is applied to a superconductor at rest. The body acquires angular momentum $\vec{L}_i$ antiparallel to the applied magnetic field. The supercurrent acquires angular momentum $\vec{L}_e = -\vec{L}_i$ parallel to the magnetic field. E_F is the Faraday electric field that exists during the process, which is clockwise as seen from the top, in the same direction as the body rotation.

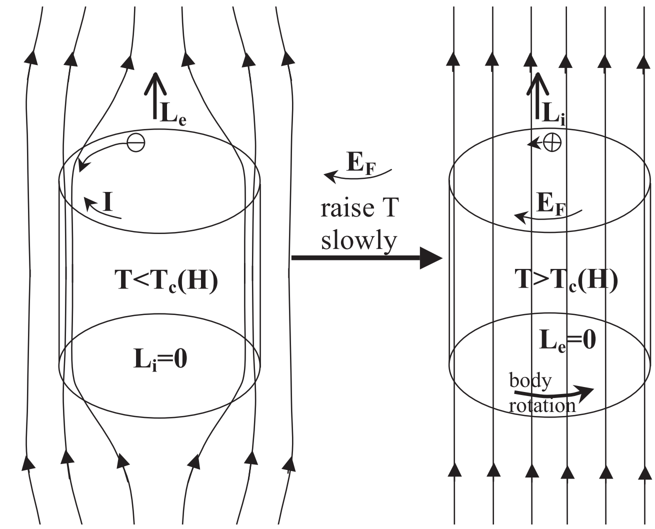

FIG. 2. Process II: a superconductor at rest in a magnetic field turns normal. The body acquires angular momentum $\vec{L}_i$ parallel to the applied magnetic field, which equals the angular momentum $\vec{L}_e$ initially carried by the supercurrent. E_F is clockwise as in Fig. 1, body rotation is counterclockwise.

Thus we cannot doubt that supercurrents carry mechanical momentum and angular momentum. Momentum conservation is a universal law of physics, hence when the state of the system changes so that the supercurrent changes, the change in the angular momentum of the supercurrent must be accounted for. Quantitatively, this momentum is certainly non-negligible. Consider that a typical current density in superconductors is of order 10^8 A/cm^2, much higher than current densities in normal metals. It is well known that for normal state current densities of order 10^6 A/cm^2 one begins to see significant electromigration effects [14], where the momentum of the conduction electrons is transferred to individual ions causing actual displacement of the ions. Such effects are not seen in superconductors. Moreover, no Joule heat is dissipated in the superconductor to normal transition in a magnetic field [2,15–18], indicating that there are no irreversible collision processes that also transfer momentum to the body as in normal conduction. Therefore superconductors need to have a way to transfer the large momentum of the supercurrent to the body as a whole that is different from the way normal electrons do it. A theory of superconductivity that cannot describe this momentum transfer process cannot account for momentum conservation and hence cannot be a valid theory of superconductivity.

II. WHAT NEEDS TO BE EXPLAINED

Consider three different processes in which a superconducting cylinder hanging from a thread will acquire angular momentum, shown in Figs. 1–3. We assume, consistent with the conventional theory of superconductivity and with experiment, that all processes are reversible [4].

Process I. The body is at rest at temperature $T < T_c$, and a magnetic field $H < H_c(T)$ is applied (Fig. 1). A clockwise supercurrent develops to prevent the magnetic field from penetrating its interior, and the body starts to rotate in the clockwise direction (as seen from the direction where the magnetic field is pointing).

Process II. The body is at rest in a magnetic field H and has a clockwise supercurrent preventing the magnetic field from entering its interior. Electrons in the supercurrent are moving counterclockwise. The temperature is raised to slightly above $T_c(H)$, the body enters the normal state, the supercurrent stops and the body starts rotating in counterclockwise direction (Fig. 2).

Process III. The body is at rest in a uniform magnetic field and initially at temperature $T > T_c(H)$. The temperature is lowered, the body enters the superconducting state and expels the magnetic field from its interior and starts rotating in clockwise direction (Fig. 3), while electrons in the generated supercurrent move in counterclockwise direction.

All these processes conserve total mechanical angular momentum (of the electrons in the supercurrent plus the ions in the body). Since there are no electric fields in the initial and final states, there is no momentum in the electromagnetic field. We also assume that the processes are slow enough that no momentum is carried away by electromagnetic radiation. We will argue that only process I can be explained by the conventional theory of superconductivity. Note that only in process I is the direction of the Faraday electric field that develops in the process, E_F, parallel to the motion of the ions, in processes II and III it is antiparallel. In the following, we discuss these three processes.

A. Process I

As the magnetic field is applied, an azimuthal Faraday electric field develops in the region within λ_L of the surface of the cylinder in clockwise direction, given by

$$E_F = \frac{\lambda_L}{c}\frac{\partial H}{\partial t} \tag{6}$$

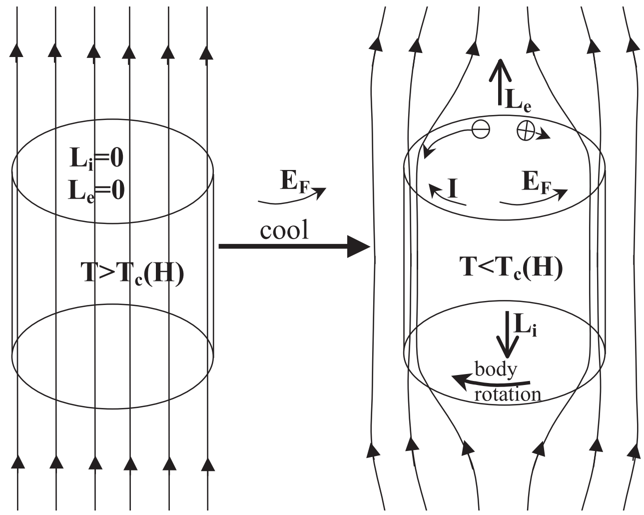

FIG. 3. Process III: a normal metal in the presence of a magnetic field is cooled into the superconducting state. The body acquires angular momentum $\vec{L}_i$ antiparallel to the applied magnetic field, and a supercurrent develops with angular momentum $\vec{L}_e = -\vec{L}_i$ parallel to the applied magnetic field. E_F is counterclockwise, body rotation is clockwise.

assuming the magnetic field penetrates a distance λ_L. We have for the velocity of a Bloch electron

$$\vec{v}_k = \frac{1}{\hbar}\frac{\partial \epsilon_k}{\partial \vec{k}}, \tag{7}$$

where ϵ_k is the band energy. Within semiclassical transport theory the equation of motion is

$$\frac{d\vec{v}_k}{dt} = \frac{1}{\hbar^2}\frac{\partial^2 \epsilon_k}{\partial \vec{k}\partial \vec{k}}\frac{d(\hbar \vec{k})}{dt} = \frac{1}{\hbar^2}\frac{\partial^2 \epsilon_k}{\partial \vec{k}\partial \vec{k}}(e\vec{E}_F). \tag{8}$$

Assuming an isotropic band and defining

$$\frac{1}{m_k^*} = \frac{1}{\hbar^2}\frac{\partial^2 \epsilon_k}{\partial \vec{k}\partial \vec{k}} \tag{9}$$

the equation of motion is

$$\frac{d\vec{v}_k}{dt} = \frac{1}{m_k^*}(e\vec{E}_F) = \frac{1}{m_k^*}\frac{e\lambda_L}{c}\frac{\partial H}{\partial t} \tag{10}$$

and the change in velocity of the electron when the magnetic field increases from 0 to H is

$$\Delta v_k = \frac{1}{m_k^*}\frac{e\lambda_L}{c}H. \tag{11}$$

The change in electronic momentum is $m_e \Delta v_k$, so the total change in electronic momentum is

$$P_e \equiv \sum_{k\ \mathrm{occ}} m_e \Delta v_k = \sum_{k\ \mathrm{occ}} \frac{m_e}{m_k^*}\frac{e\lambda_L}{c}H, \tag{12}$$

where the sum over k here and in what follows is over the *occupied* states in the band. The current density that develops is

$$J = \frac{1}{V}\sum_{k\ \mathrm{occ}} e\Delta v_k = \frac{1}{V}\sum_{k\ \mathrm{occ}} \frac{1}{m_k^*}\frac{e^2\lambda_L}{c}H \tag{13}$$

and using Eq. (3) yields for the penetration depth

$$\frac{1}{\lambda_L^2} = \frac{4\pi e^2}{c^2}\left(\frac{1}{V}\sum_{k\ \mathrm{occ}}\frac{1}{m_k^*}\right). \tag{14}$$

Using the expression for the current density Eq. (13) to replace the sum over k in Eq. (12) and the expression for the current density Eq. (3) yields for Eq. (12)

$$P_e = V\frac{m_e c}{4\pi\lambda_L e}H \tag{15}$$

with $V = 2\pi R\lambda_L h$, in agreement with Eq. (4), and yields Eq. (5) for the total angular momentum acquired by the electrons.

Now the equation of motion for a Bloch electron is

$$m_e\frac{d\vec{v}_k}{dt} = e\vec{E}_F + \vec{F}_{\mathrm{latt}}^k, \tag{16}$$

where $\vec{F}_{\mathrm{latt}}^k$ is the force exerted by the ionic lattice on the electron of wave vector k. Using Eqs. (8) and (9), we obtain

$$\vec{F}_{\mathrm{latt}}^k = \left(\frac{m_e}{m_k^*} - 1\right)e\vec{E}_F \tag{17}$$

and the total force exerted by the lattice on the electrons is

$$\vec{F}_{\mathrm{latt}} = \sum_{k\ \mathrm{occ}}\left(\frac{m_e}{m_k^*} - 1\right)e\vec{E}_F. \tag{18}$$

By Newton's third law, the total force exerted by the electrons on the lattice is then

$$\vec{F}_{\text{on-latt}} = -\vec{F}_{\mathrm{latt}} = -\sum_{k\ \mathrm{occ}}\left(\frac{m_e}{m_k^*} - 1\right)e\vec{E}_F. \tag{19}$$

The Faraday electric field also exerts a force on the positive ions. Assuming charge neutrality we have the same number of positive ions (charges) as negative electrons in the band, and the total force exerted on the ions (labeled by i) is

$$\sum_i \vec{F}_i = \sum_i m_i\frac{d\vec{v}_i}{dt} = \sum_i |e|\vec{E} + \vec{F}_{\text{on-latt}} \tag{20}$$

and using Eq. (19),

$$\sum_i \vec{F}_i = \sum_i m_i\frac{d\vec{v}_i}{dt} = -\sum_{k\ \mathrm{occ}}\frac{m_e}{m_k^*}e\vec{E}_F, \tag{21}$$

yielding for the total change in ionic momentum

$$P_i \equiv \sum_i m_i\Delta v_i = -\sum_{k\ \mathrm{occ}}\frac{m_e}{m_k^*}\frac{e\lambda_L}{c}H = -P_e, \tag{22}$$

hence the total angular momentum acquired by the ions is

$$L_i = \frac{m_e c}{2|e|}hR^2H = -L_e. \tag{23}$$

Thus, for a charge neutral system, the electrons and ions acquire equal and opposite momenta and angular momenta, as one would expect. The way the ions acquire momentum and angular momentum is partly from the Faraday field itself and partly from the force exerted by the electrons on the ions, as seen from Eq. (20). Irrespective of this, the total angular momentum acquired by the body (and the electrons) is independent of m^* and hence of the interactions between electrons and ions, as seen from Eq. (23). The interactions between electrons and ions only enter in determining the magnitude of the London penetration depth as seen from Eq. (14).

Note that this contradicts the conclusions of Frenkel and Rudnitsky [19], who argued that the fact that gyromagnetic experiments [11–13] agree with Eq. (23) with m_e rather than m^* is by itself evidence that electrons carrying the supercurrent are completely "free" from interactions with the lattice. Later in this paper, we will argue that Frenkel and Rudnitsky's conclusion still was correct, albeit for different reasons.

B. Process II

In process II (Fig. 2), the angular momentum of the supercurrent L_e given by Eq. (5) in direction parallel to the applied field has to be transferred in its entirety to the body as a whole when the system goes normal. In other words, the angular momentum of the electrons has to go from L_e to zero and that of the ions from 0 to L_e. The angular momentum of electrons and ions will change due to (i) electromagnetic forces, and (ii) interaction between electrons and ions. Let us examine them in turn.

1. Electromagnetic forces

As the magnetic field lines enter the body, a Faraday electric field pointing clockwise is generated throughout the interior of the cylinder (Fig. 2), that tries to prevent the magnetic field from entering (Lenz's law). This electric field imparts momentum to electrons in counterclockwise direction and to ions in clockwise direction. Thus this momentum transfer is in a direction exactly opposite to what is needed to reach the final state, where the counterclockwise electron current has stopped and the ions rotate in counterclockwise direction.

Can there be a magnetic Lorentz force in the azimuthal direction? It can result from radial motion of charge. Since the ionic charge cannot undergo radial motion, a magnetic Lorentz force cannot be the source of angular momentum for the ions. For the electrons there could in principle be radial motion, however, there is no such motion within the conventional theory of superconductivity.

2. Electron-ion forces

The Coulomb interaction between electrons and ions can transfer momentum between the two subsystems. Initially, the momentum of the supercurrent is carried by electrons bound in Cooper pairs. As the system becomes normal, Cooper pairs unbind and become normal quasiparticles, and the supercurrent stops. Within the conventional theory this process has been discussed by Eilenberger [20] using the time-dependent Ginzburg-Landau (TDGL) formalism. A term in the current density describes the current carried by normal electrons stemming from the momentum transferred to the normal electron fluid when the superfluid electron density decreases. Eilenberger states that "this momentum then decays with the transport relaxation time τ." However, such decay would necessarily lead to Joule heat dissipation and hence irreversibility, therefore this approach cannot be correct [4]. More generally, any approach that assumes that the momentum of the Cooper pair is transferred to normal quasiparticles cannot be correct since in normal metallic transport, decay of electric current is necessarily associated with thermodynamic irreversibility [21] and even electromigration for high current densities.

As already recognized by Keesom [16], *"it is essential that the persistent currents have been annihilated before the material gets resistance, so that no Joule-heat is developed."* The annihilation of the supercurrent has to be accompanied by transfer of the supercurrent momentum to the body as a whole in order to satisfy momentum conservation, with no energy transfer and no energy dissipation. In its 60 years of existence, the conventional theory of superconductivity has offered no clue as to how this happens.

One may speculate that transfer of momentum from the supercurrent to the body may occur through phonon emission or scattering by impurities. However, these are not *reversible* processes: in the reverse transformation from normal to superconducting, the body would not be able to transfer its momentum to the supercurrent by reversing the time arrow in these processes. For further discussion of the reversibility issue, see Appendix B.

C. Process III

Process III, shown in Fig. 3, is even more puzzling than process II. Here one has to explain not only how ions acquire momentum opposite to the direction of the force exerted by the Faraday electric field on ions, but also how electrons acquire momentum in direction opposite to the direction of the force exerted by the Faraday electric field on electrons, all without energy dissipation. Within the conventional theory, the Eilenberger formalism can be applied to describe how electrons acquire their momentum, but no mechanism exists for the body to acquire a compensating momentum in the opposite direction.

In summary, we have pointed out that no valid explanation exists in the literature of conventional superconductivity for how momentum is conserved in the processes shown in Figs. 2 and 3. We argue that any explanation of the momentum transfer between electrons and the body as a whole that involves collisions between electrons and ions, or impurities, or defects, or phonons, is necessarily a source of irreversibility, which is not observed [2], hence is invalid [22,23]. The only other way we know to transfer the momentum between the supercurrent and the body is mediated through the electromagnetic field, as discussed in the next section.

III. MOMENTUM TRANSFER MEDIATED BY THE ELECTROMAGNETIC FIELD

A key aspect of process II is that the momentum of the supercurrent gets transferred to the body, but its kinetic energy is not: the kinetic energy of the supercurrent remains in the electronic degrees of freedom, where it is used to pay the price of the condensation energy in rendering the superconducting electrons normal [4,24].

In most physical interactions, momentum transfer is accompanied by energy transfer. An exception is when magnetic fields are involved. A charge moving in a magnetic field will change its momentum but not its kinetic energy: the magnetic field does not do work on moving charges since the magnetic Lorentz force $\vec{v} \times \vec{H}$ is perpendicular to the particle's velocity $\vec{v}$. The momentum change of the particle is compensated by momentum change of the electromagnetic field. The momentum density of the electromagnetic field is given by

$$\vec{\mathcal{P}}_{\rm em}(\vec{r}) = \frac{1}{4\pi c}\vec{E} \times \vec{H} \tag{24}$$

with $\vec{E}, \vec{H}$ electric and magnetic fields.

Thus we argue that the process of transfer of momentum of the supercurrent to the body without energy transfer must involve the electromagnetic field in an essential way. It is natural to conclude that the transfer has to happen in two steps: the first step would transfer the momentum of the supercurrent to the electromagnetic field, and the second step would transfer the momentum from the electromagnetic field to the body as a whole. Even though both processes may occur concurrently, it is useful to think of them as separate processes.

At first sight, Eq. (24) does not appear to help, since the electric and magnetic fields at play in Figs. 2 and 3 are azimuthal and in the z direction, respectively, resulting in an electromagnetic momentum $\vec{\mathcal{P}}_{\rm em}$ in the radial direction. However,n the momentum of the supercurrent and the body are in the azimuthal direction. This then suggests that in processes II and III, *an electric field in the radial direction exists*. An electric field in the radial direction and a magnetic field in the z direction will give an azimuthal $\vec{\mathcal{P}}_{\rm em}$.

Consider process II, the annihilation of the supercurrent when the system goes normal. The momentum of the supercurrent, carried by negative electrons, is in counterclockwise direction (Fig. 2). Assume that in the process of the supercurrent disappearing an electric field $\vec{E}$ pointing radially *inward* is created, thus creating a *counterclockwise* electromagnetic field momentum according to Eq. (24). This could be a "storage box" for the momentum of the supercurrent. In a subsequent step, this momentum of the electromagnetic field would be transferred to the body in a separate process.

What we have just described would occur if the process of inward motion of the N-S phase boundary would also involve inward motion of negative charge, creating a transitory inward-pointing electric field, followed by inward motion of positive charge to retrieve the momentum stored in the field and pass it on to the body. We show the steps in this process in Fig. 4, where the counterclockwise and inward directions in Fig. 2 corresponds to the leftward and downward directions in Fig. 4, respectively.

In the process shown in Fig. 4, the momentum initially carried by the negative charge is transferred to the positive charge. There is no assumption on the masses of negative and positive charges, they could be the same or different. If the mass of the positive charge is much larger than that of the negative charge, its speed and kinetic energy will be much smaller.

In exactly the same (reversed) fashion process III can then be explained, as shown in Fig. 5, namely: if the outward motion of the phase boundary is associated with *outward* motion of negative charge, this would create a transitory radially *outgoing* electric field and hence a clockwise electromagnetic field momentum that would compensate the counterclockwise mechanical momentum acquired by the outward-moving electron due to the Lorentz force. In a subsequent step, positive charge would move outward and the clockwise momentum of the electromagnetic field would be transferred to the positive charge through the Lorentz force. The end result is negative and positive charges moving with the same momentum in opposite directions, as shown in the rightmost panel of Fig. 5.

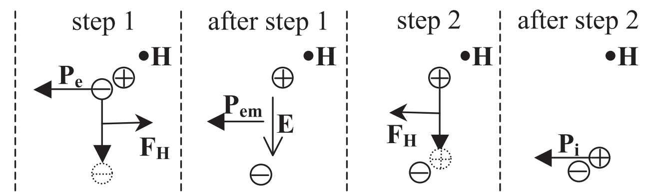

FIG. 4. Illustration of momentum transfer from a negative to a positive charge through the electromagnetic field, corresponding to process II, stopping of a supercurrent. Left and down directions correspond to counterclockwise and radially inward directions in Fig. 2. Magnetic field H points out of the paper. Initially, the negative charge has momentum P_e pointing to the left, the positive charge is at rest. In step 1, the negative charge moves down, the Lorentz force F_H imparts momentum to the right cancelling P_e. After step 1, the two charges are at rest, and an electric field E exists giving rise to momentum of the electromagnetic field $P_{\rm em} = P_e$. The mechanical momentum of the negative charge resides now in the electromagnetic field. In step 2, the positive charge moves down and the Lorentz force F_H imparts momentum to it to the left, which is being transferred out of $P_{\rm em}$. After step 2, the positive charge carries the momentum $P_i = P_e$ originally carried by the negative charge, and the momentum of the electromagnetic field is zero again since $E = 0$.

A problem with these explanations is of course that in a solid, positive ions cannot move radially inward nor outward. We will show in the next sections how superconductors get around this problem, through the remarkable properties of *holes*. Specifically, in Figs. 4 and 5, the negative charges correspond to *superconducting electrons*, and the positive charges correspond to *normal holes*. We will show that radially moving normal holes transfer azimuthal momentum to the body without energy dissipation.

IV. HOW SUPERCURRENT CARRIERS ACQUIRE AND LOSE THEIR MOMENTUM WITHOUT ENERGY DISSIPATION

The radial motion of negative carriers hypothesized in Figs. 4 and 5 is in the same direction as the motion of

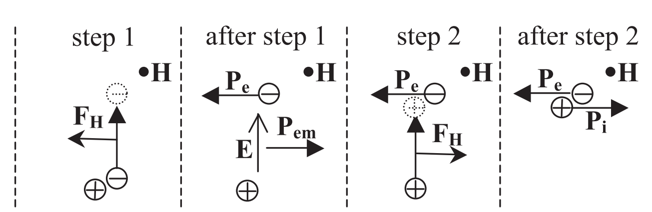

FIG. 5. Similar to Fig. 4 for process III, the Meissner effect. Left and up directions correspond to counterclockwise and radially outward directions in Fig. 3. Magnetic field H points out of the paper.

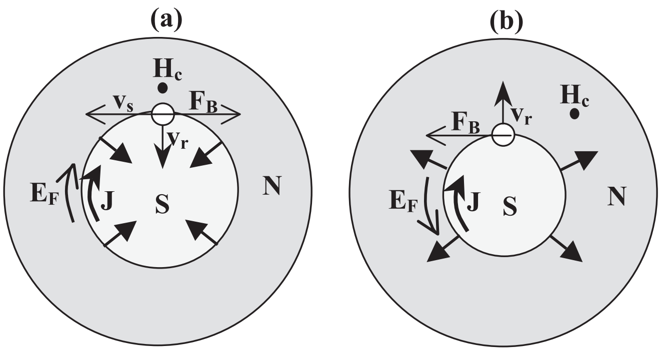

FIG. 6. The normal region is shaded in grey. The magnetic field H_c points out of the paper and the Meissner current J flows clockwise, corresponding to counterclockwise motion of electrons. (a) When the phase boundary moves inward, the magnetic Lorentz force on a superelectron moving radially inward as it becomes normal is clockwise, thus slowing the superelectron down as it becomes normal. (b) When the phase boundary moves outward, the magnetic Lorentz force on a normal electron moving radially outward as it becomes superconducting is counterclockwise, in the direction of motion of superelectrons in the Meissner current. In both cases the Faraday field E_F imparts a force in opposite direction.

the phase boundary in the respective situations, as shown in Fig. 6. The momentum parallel to the phase boundary acquired by a negative charge moving a distance Δx in direction $\hat{n}$ perpendicular to the phase boundary with radial speed v_r due to the Lorentz force imparted by the magnetic field is

$$\Delta \vec{p}_{\parallel} = \int \frac{e}{c}\vec{v}_r \times \vec{H} dt = \frac{e\Delta x}{c}\hat{n} \times H. \tag{25}$$

We assume that the momentum imparted by the Faraday electric field (in the opposite direcction) is much smaller and hence can be ignored. We will justify this assumption in a later section.

The speed of electrons at the normal-superconductor phase boundary in an applied magnetic field H is, according to Eq. (11)

$$v_s = \frac{e\lambda_L}{m_e c} H \tag{26}$$

provided we can replace m_k^* by the bare electron mass m_e in Eq. (11). We have recently argued [9] that BCS theory itself is inconsistent unless the dynamics of electrons in the supercurrent is governed by the bare mass m_e rather than the effective mass, and we will assume hereafter that this is the case. Another argument for this will be given in Sec. VI. A typical value for the superfluid velocity Eq. (26) for $\lambda_L = 400$ Å and $H = 500$ G is $v_s = 35\,225$ cm/s.

The momentum of the electron is $m_e v_s$, hence Eqs. (25) and (26) indicate that electrons making the transition from normal to superconducting or from superconducting to normal advance in the direction of the phase boundary motion a distance λ_L. For process II, this motion brings the velocity of the electron in the supercurrent from v_s to zero and stores its momentum in the electromagnetic field as shown in Fig. 4. For process III, this motion gives to the electron the momentum needed to carry the supercurrent and stores momentum of opposite sign in the electromagnetic field as shown in Fig. 5. Thus this accounts for the transfer of momentum from electrons to the electromagnetic field without energy dissipation, "step 1" in Figs. 4 and 5. Next, we need to understand how this momentum gets transferred back to the body, i.e., the processes denoted "step 2" in Figs. 4 and 5, through a "backflow process," which is necessary to preserve local charge neutrality.

V. HOW MOMENTUM IS TRANSFERRED TO THE BODY WITHOUT ENERGY DISSIPATION

After step 1 in Figs. 4 and 5 the momentum is stored in the electromagnetic field and needs to be retrieved and transferred to the body in step 2. This is achieved through the motion of normal *holes* in direction perpendicular to the phase boundary.

Consider the two Hall bars shown in Fig. 7. They are identical except one has negative and the other positive Hall coefficient R_H. The Amperian force on the bar is given by

$$\vec{F}_{\rm Amp} = \frac{I}{c}\vec{L} \times \vec{H}, \tag{27}$$

where $L = |\vec{L}|$ is the length of the sample and the vector $\vec{L}$ points in the direction of the flow of current I. The Amperian force is of course independent of the sign of the Hall coefficient.

For the bar in Fig. 7(a) the Hall coefficient R_H is negative, the carriers are electrons. The current density is given by

$$\vec{J}_x = -nev\hat{x} = J_x\hat{x} \tag{28}$$

flowing in the positive $\hat{x}$ direction ($e < 0$), where v is the magnitude of the drift velocity and n is the density of electron carriers. An electric field pointing to the right (negative $\hat{y}$ direction) exists, given by

$$\vec{E}_y = -\frac{v}{c}H\hat{y} = -E_y\hat{y} \tag{29}$$

that equals the magnetic Lorentz force pointing to the left, so that the forces on electrons in the y direction are balanced. Assuming the system is charge neutral, for every conduction electron there is a positive charge $|e|$ belonging to an ion that is not moving. The force on this ionic charge is

$$\vec{F}_{\rm ion} = |e|\vec{E}_y = eE_y\hat{y}, \tag{30}$$

pointing to the right. The total force on all the ions is

$$\vec{F}_{\rm ion,tot} = nAL|e|\vec{E}_y = nALe\frac{v}{c}H\hat{y} = -\frac{J_x A}{c}LH\hat{y}, \tag{31}$$

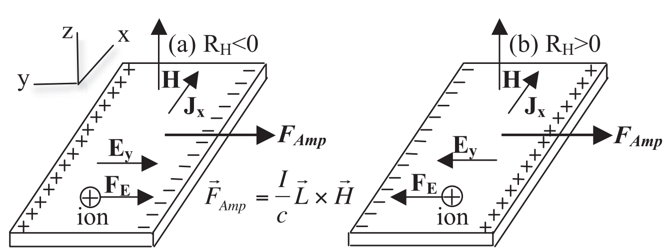

FIG. 7. Hall effect in metal bars with negative and positive Hall coefficients R_H. The Amperian force on the body is the same independent of the sign of the Hall coefficient. However, the physical interpretation is very different, as discussed in the text.

where A is the cross-sectional area of the sample, so the total current is $I = J_x A$. Hence in this case,

$$\vec{F}_{\text{ion,tot}} = \vec{F}_{\text{Amp}}. \tag{32}$$

For the electrons, electric, and magnetic forces are balanced and the electrons move along the $\hat{x}$ direction, hence no other force in the $\hat{x}$ direction is acting on electrons. The Amperian force results from the action of the Hall electric field E_y on the ions.

The situation is different for the Hall bar with $R_H > 0$ shown in Fig. 7(b). Here the Hall electric field is of opposite sign to the previous case,

$$\vec{E}_y = \frac{v}{c} H\hat{y} = E_y\hat{y} \tag{33}$$

pointing in the positive y direction. The current flows in the x direction, hence the net force in the y direction on current carriers has to be zero. The force on the ions from the electric field is now

$$\vec{F}_{\text{ion}} = |e|\vec{E}_y = |e|E_y\hat{y} \tag{34}$$

pointing *to the left*, i.e., in opposite direction to the Amperian force. How does the Amperian force come about?

The answer is, the electrons flowing in the x direction exert a force on the ions, given by

$$\vec{F}_{\text{e-i}} = 2eE_y\hat{y}, \tag{35}$$

so that the total force on the ion is

$$\vec{F}_{\text{ion,tot}} = \vec{F}_{\text{ion}} + \vec{F}_{\text{e-i}} = eE_y\hat{y} \tag{36}$$

just as in Eq. (30). The total force on the ions is again given by Eq. (32), the Amperian force.

The reason the electrons exert a force on the ions is that the ions exert a force on the electrons that are moving carrying the current. According to the semiclassical equations of motion for Bloch electrons the motion of electrons in solids results from the combined action of the external force and the force exerted by the ions on the electrons. A detailed analysis is given in Appendix A.

This then implies that in a Hall bar with a positive Hall coefficient where the drift velocity of current carriers (holes) is $\vec{v}_d$ in the direction of current flow, for each hole that moves a distance d, it takes a time interval $\Delta t = d/v_d$ and the momentum transferred in that time from the electrons to the ions is

$$\Delta\vec{P}_{\text{ion}} = -2\frac{e}{c}\vec{v}_d \times \vec{H}\Delta t = 2\frac{ed}{c}\hat{v}_d \times \vec{H}. \tag{37}$$

This momentum transfer from electrons to ions occurs without any irreversible scattering processes, and can only occur when the carriers are holes.

We can now understand how the momentum transfers between the supercurrent and the body shown in Figs. 4 and 5 occur in the context of the superconductor to normal transition and normal to superconductor transition, i.e., processes II and III. We will describe it for the Meissner effect, process III, Fig. 5. In the cylindrical geometry, the process is shown in Fig. 8.

We assume the superconductor-normal phase boundary is moving radially outward at speed $\dot{r}_0$. An azimuthal electric field pointing counterclockwise is generated at and near the phase boundary [25]:

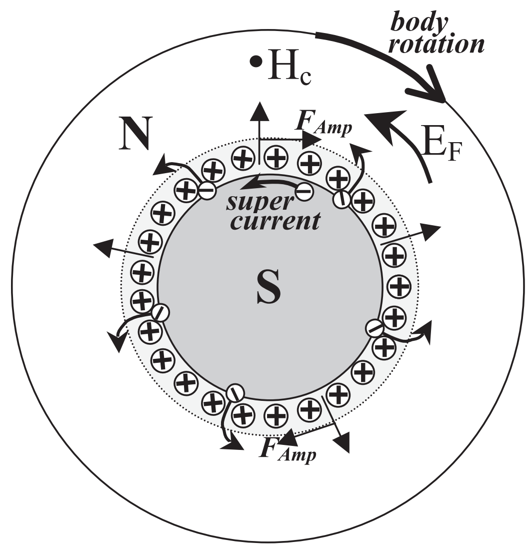

FIG. 8. Expansion of the superconducting phase (grey region) in a magnetic field pointing out of the paper. Electrons acquire counterclockwise momentum as they thrust outward through the action of the magnetic Lorentz force. A hole current flows outward in a boundary layer of thickness λ_L (light grey ring). The Amperian force on this current F_{Amp} transfers momentum to the body as a whole in clockwise direction, just like for the Hall bar in Fig. 7(b).

$$\vec{E}_F = \frac{\dot{r}_0}{c} H_c\hat{\theta}. \tag{38}$$

(1) Step 1. As the phase boundary advances into the normal region, electrons that were in the normal state become superconducting. We assume that the electron becoming superconducting thrusts radially outward (up in Fig. 5) at a high speed v_r a distance λ_L. In the process, it acquires through the action of the magnetic Lorentz force an azimuthal momentum in counterclockwise direction (to the left in Fig. 5):

$$\Delta\vec{p}_e = -\frac{e\lambda_L}{c} H_c\hat{\theta} + e\frac{\dot{r}_0}{c} H\Delta t\hat{\theta}. \tag{39}$$

The second term in Eq. (39) arises from the action of the Faraday field (38). Δt is the time it takes the electron to move a distance λ_L at speed v_r, $\Delta t = \lambda_L/v_r$. Under the condition

$$\dot{r}_0 \ll v_r, \tag{40}$$

the second term in Eq. (39) is much smaller than the first term and we assume it can be neglected, so that Eq. (39) gives the required momentum of the electron in the supercurrent, Eq. (26).

(2) After step 1. The outward motion of electrons in step 1 creates a radially outward electric field in a boundary layer of thickness λ_L in the normal region, and stores azimuthal momentum in the electromagnetic field, as shown in the second panel in Fig. 5. The radial electric field drives a radial outflow of current in this boundary layer of thickness λ_L (Fig. 8, light grey ring) that moves at the same speed as the phase boundary motion.

(3) Step 2. We assume that the normal current is carried by hole carriers. Normal current flows radially outward in the

boundary layer and exerts a force on the body. This force is the Amperian force $\vec{F}_{\rm Amp}$ shown in Figs. 7(b) and 8. It transfers the momentum stored in the electromagnetic field to the body as a whole, without energy dissipation, so the body acquires rotational velocity in the clockwise direction.

(4) After step 2. The momentum acquired by the electron going superconducting in the counterclockwise direction is exactly compensated by the momentum transferred to the body in the clockwise direction.

The fact that the momenta are exactly compensated does not need proof, it follows from the fact that the backflow propagation of the holes is exactly radial because of the balance of forces: the total azimuthal force acting on the outflowing hole with speed $\dot{r}_0$ is the sum of the clockwise magnetic Lorentz force and counterclockwise Faraday force, which equals zero:

$$\vec{F}_{\rm hole} = -|e|\frac{\dot{r}_0}{c}H_c\hat{\theta} + |e|\vec{E}_F = 0. \tag{41}$$

Nevertheless, let us verify that the momentum conservation holds. The momentum acquired by an electron going superconducting and thrusting radially outward a distance λ_L is

$$\Delta\vec{p}_e = -\frac{e\lambda_L}{c}H_c\hat{\theta} \tag{42}$$

neglecting the second term in Eq. (39) under the assumption that $v_r \gg \dot{r}_0$. The "backflow" normal holes move at speed $\dot{r}_0$ in the $+\hat{r}$ direction and traverse the boundary layer of thickness λ_L in time $\Delta t = \lambda_L/\dot{r}_0$. The net force per carrier exerted on the lattice during that time is given by Eq. (36), where E_y is the Faraday field E_F and the $\hat{y}$ direction is the $\hat{\theta}$ direction

$$\vec{F}_{\rm ion,tot} = e\vec{E}_F = e\frac{\dot{r}_0}{c}H_c\hat{\theta}, \tag{43}$$

which is the same as the Amperian force in Fig. 7(b). Hence the net momentum transferred to the ions per electron going superconducting is

$$\Delta\vec{p}_i = \vec{F}_{\rm ion,tot}\Delta t = \frac{e\lambda_L}{c}H_c\hat{\theta} \tag{44}$$

equal and opposite to Eq. (42), as expected.

Exactly the same steps in reverse explain how as the superconducting region shrinks the mechanical momentum of an electron in the supercurrent that becomes normal is transferred to the body through a radially inward flow of holes in a boundary layer of thickness λ_L.

Returning to the case of the expanding superconducting phase in Fig. 8, we also need to consider the effect of the Faraday field in the superconducting region within λ_L of the phase boundary, where the supercurrent flows. Its effect on the electrons in the supercurrent is to slow them down (force acts in clockwise direction), so that they eventually come to a stop when the boundary has moved beyond a distance λ_L, as discussed in Ref. [25]. Its effect on the body is to impart momentum in counterclockwise direction, which partially compensates the momentum transfer, Eq. (44), resulting in a net transfer of momentum which generates the body's rotation, as discussed in the next section.

VI. MACROSCOPIC TORQUE

Let us now analyze how the macroscopic rotation of the body comes about. The Amperian force per unit volume exerted on a radially outgoing current J_r in the presence of magnetic field H is

$$\vec{F}_{\rm Amp} = -\frac{H}{c}J_r\hat{\theta}, \tag{45}$$

where $\hat{\theta}$ is positive in counterclockwise direction. The radial hole current is given by

$$\vec{J}_r = n_s|e|\dot{r}_0\hat{r}, \tag{46}$$

and this current occupies a boundary layer of thickness λ_L, with volume $V = 2\pi r_0\lambda_L h$, with h the height of the cylinder. Hence the torque exerted by the Amperian force on the boundary layer of thickness λ_L flowing outward with speed $\dot{r}_0$ is

$$\vec{\tau}_1 = \frac{H_c}{c}2\pi r_0^2\lambda_L h n_s e\dot{r}_0\hat{z} \tag{47}$$

pointing in the $-\hat{z}$ direction, i.e., opposite to the direction of the magnetic field.

There is also a countertorque due to the clockwise force exerted by the Faraday electric field E_F on the ions in the superconducting region within distance λ_L of the superconductor-normal phase boundary, where supercurrent flows. The Faraday field in that region is given by [25]

$$\vec{E}_F(r) = \frac{H_c}{c}\dot{r}_0 e^{(r-r_0)/\lambda_L}\hat{\theta}, \tag{48}$$

and it exerts a torque

$$\vec{\tau}_2 = -2\pi n_s eh\int_0^{r_0}E_F(r)r^2dr\hat{z} \tag{49}$$

on the body. Doing the integral and assuming $r_0 \gg \lambda_L$ yields

$$\vec{\tau}_2 = -\frac{H_c}{c}2\pi n_s eh\lambda_L\left(r_0^2 - 2r_0\lambda_L + 2\lambda_L^2\right)\hat{\theta}, \tag{50}$$

so that the net torque on the body is (neglecting the higher order term proportional to λ_L^3)

$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 = \frac{H_c}{c}4\pi\lambda_L^2 h n_s e r_0\dot{r}_0\hat{z}. \tag{51}$$

By conservation of momentum, the ionic angular momentum is minus the electronic angular momentum, Eq. (5),

$$\vec{L}_i = -\vec{L}_e = \frac{m_e c}{2e}hr_0^2H_c\hat{z} \tag{52}$$

and the associated torque is

$$\vec{\tau}_i = \frac{d\vec{L}_i}{dt} = \frac{m_e c}{e}hr_0\dot{r}_0H_c\hat{z}. \tag{53}$$

Equating Eq. (53) to the net torque exerted on the body, Eq. (51), we find

$$\frac{1}{\lambda_L^2} = \frac{4\pi n_s e^2}{m_e c^2}. \tag{54}$$

This is the well-known expression for the London penetration depth [5]. On the other hand, we find from our formula for

Bloch electrons, Eq. (14), for either a band close to empty or close to full,

$$\frac{1}{\lambda_L^2} = \frac{4\pi n_s e^2}{m^* c^2}, \tag{55}$$

where n_s is

$$n_s = \frac{1}{V} \sum_{k\ \mathrm{occ}} 1 \tag{56a}$$

for a band close to empty, or

$$n_s = \frac{1}{V} \sum_{k\ \mathrm{unocc}} 1 \tag{56b}$$

for a band close to full, and $m^* = m_k^*$ at the bottom of the band for an almost empty band, or $m^* = -m_k^*$ at the top of the band for an almost full band.

In deriving the expression (55) for the London penetration depth, we assumed that superconducting carriers respond to the induced Faraday field as if they were Bloch electrons with effective mass m_k^*, Eq. (9). However, to satisfy momentum conservation, we found here that the London penetration depth has to be given by Eq. (54) with the *bare* electron mass m_e. The implication of this is inescapable: our original assumption, Eq. (10), leading to Eq. (55) was incorrect. Unlike normal Bloch electrons, superconducting carriers respond to an external field with their bare electron mass, in other words they are completely "undressed" from the electron-ion interaction. We recently reached this same conclusion through a completely different path, by examining inconsistencies within conventional BCS-London theory [9].

In summary, the macroscopic rotation of the body when the superconducting region expands results from the torque exerted by the radially outgoing hole current, Eq. (46), on the body in the clockwise direction exceeding the countertorque (50) exerted by the Faraday electric field on the ions in the counterclockwise direction in the region where the supercurrent flows by the amount given by the net torque (51). For a shrinking superconducting region all the signs are simply reversed.

VII. NEW PHYSICS OF SUPERCONDUCTIVITY

In the previous sections, we have described a plausible way to explain the momentum transfer between the electronic degrees of freedom and the body as a whole in a reversible way in processes II and III. We do not know any other possible way to do this, and no other way has been proposed in the literature. Next, let us consider what is required of a microscopic theory of superconductivity to allow this to occur. We argue that the following are *necessary conditions*. (i) The wave function *and charge distribution* of superconducting electrons close to the phase boundary extend into the normal state. (ii) The charge carriers in the normal state that are condensing to give rise to the supercurrent in the superconducting state are *holes*.

Requirement (i) follows from the fact that we assumed in the previous sections that when electrons go from normal to superconducting they "thrust" into the normal region a finite distance λ_L, thereby acquiring the momentum needed for the supercurrent through the magnetic Lorentz force. Within BCS theory, it is assumed that the superconducting order parameter does leak into the normal region, leading, e.g., to Josephson effects and proximity effects, however, BCS theory does not predict that the order parameter has any *charge* associated with it. This is because charge has to have a *sign* (negative or positive), and BCS theory is intrinsically electron-hole symmetric, so the order parameter is not associated with either negative or positive charge. Therefore BCS theory does not satisfy this requirement. BCS theory also does not satisfy requirement (ii), since within BCS the normal state carriers may be electronlike or holelike.

Therefore we conclude that BCS theory does not have the physical elements required to explain the reversible momentum transfer between electrons and the body that takes place in the superconductor-normal and normal-superconductor transitions in a magnetic field. Instead, the theory of hole superconductivity [8] does have those physical elements, as discussed in earlier papers [22,25,26] and recounted briefly in what follows.

(i) Within the theory of hole superconductivity, electrons in the condensate reside in mesoscopic orbits of radius $2\lambda_L$ [27], while they reside in microscopic orbits of radius k_F^{-1} in the normal state. Thus, when electrons go from normal to superconducting they expand their orbits to radius $2\lambda_L$, and since they are at the normal-superconductor phase boundary this is associated with negative charge leaking into the normal region. The azimuthal velocity acquired by expanding the orbit to radius $2\lambda_L$ is given by Eq. (26), the same as in a linear thrust over length λ_L in a direction perpendicular to the phase boundary [28]. (ii) Within the theory of hole superconductivity, as discussed in numerous papers and for numerous reasons [8], the normal state carriers are necessarily holes.

The essential physics of the Meissner effect within the theory of hole superconductivity is orbit expansion driven by lowering of the quantum kinetic energy [29,30]. Instead, in conventional BCS theory, superconductivity is driven by lowering of the potential energy. We have argued that no theory that explains superconductivity as driven by the potential rather than the kinetic energy can explain the Meissner effect [30].

VIII. MOMENTUM IN THE ELECTROMAGNETIC FIELD

Within our theory, the superfluid charge density is slightly inhomogeneous, since the orbit expansion leads to higher negative charge density within the London penetration depth of the surface, as shown schematically in Fig. 9. The excess charge density is given by [31]

$$\rho_- = e n_s \frac{\hbar}{4 m_e \lambda_L c}, \tag{57}$$

which gives rise to an outward pointing electric field in the interior of superconductors, that attains its maximum value E_m near the surface, given by

$$E_m = -\frac{\hbar c}{4 e \lambda_L^2}, \tag{58}$$

so that $E_m = -4\pi\rho_- \lambda_L$ [31]. Therefore there is electromagnetic momentum in the region within λ_L of the surface where both electric and magnetic fields are present, according to Eq. (24). The total electromagnetic angular momentum in that

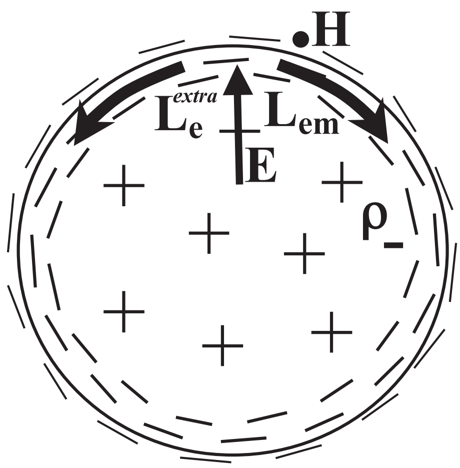

FIG. 9. Schematic view of superconducting region. Excess negative charge density ρ_- resides within a London penetration depth of the phase boundary. A radial electric field exists in the interior. The extra mechanical momentum $L_e^{\rm extra}$ carried by the excess charge density ρ_- is compensated by momentum $L_{\rm em}$ in the electromagnetic field.

region, of volume $2\pi r_0\lambda_L h$, is

$$\vec{L}_{\rm em} = -\frac{1}{4\pi c}E_m H_c r_0(2\pi r_0\lambda_L h)\hat{z}. \tag{59}$$

On the other hand, the extra mass density ρ_-/e carries mechanical angular momentum, given by [using Eqs. (57) and (5)]

$$\vec{L}_e^{\rm extra} = \frac{\hbar}{4m_e\lambda_L c}\vec{L}_e = -\frac{\hbar}{4m_e\lambda_L c}\frac{m_e c}{2e}hr_0^2 H_c\hat{z}, \tag{60}$$

so that

$$\vec{L}_{\rm em} = -\vec{L}_e^{\rm extra} \tag{61}$$

as required for momentum conservation.

It is interesting to note that for any value of ρ_- Eq. (61) would hold, provided that $E_m = -4\pi\rho_-\lambda_L$, which is the condition for the "surface charge density" $\sigma = \lambda_L\rho_-$ to screen the internal field E_m so that it does not leak out of the superconductor: the angular momentum of the mass density ρ_-/e moving at a speed given by Eq. (26) is exactly compensated by the angular momentum stored in the electromagnetic field.

In summary, the total electronic mechanical angular momentum of superconducting electrons in a magnetic field is compensated by the angular momentum of the body plus a small contribution ($\sim 1/10^6$) of electromagnetic field momentum:

$$\vec{L}_e^{\rm tot} = \left(1 + \frac{\rho_-}{en_s}\right)\vec{L}_e = -(\vec{L}_{\rm body} + \vec{L}_{\rm em}), \tag{62}$$

which completely accounts for momentum conservation and the mechanisms responsible for it.

IX. DISCUSSION

In this paper, we have argued that the only way that momentum can be transferred between the supercurrent and the body as a whole in a reversible way in processes where the normal-superconductor phase boundary moves is through mediation of the electromagnetic field, which necessitates flow of charge in a direction perpendicular to the phase boundary, and necessitates hole carriers in the normal state. Any alternative way to transfer the momentum between electrons and the body, i.e., scattering by impurities or phonons, would be an irreversible process incompatible both with experiment and with the established principles of superconductivity [4].

We have furthermore argued that the conventional BCS-London theory does not have the necessary physical elements to describe these processes in a reversible fashion. At the very least, it is a fact that no such description exists in the scientific literature.

An important aspect of our explanation is that it only works if it is assumed that the mechanical momentum of an electron in the supercurrent is

$$\vec{p}_e = -\frac{e\lambda_L}{c}H\hat{\theta} \tag{63a}$$

rather than

$$\vec{p}_e = -\frac{m_e}{m*}\frac{e\lambda_L}{c}H\hat{\theta} \tag{63b}$$

as predicted by the conventional theory [9]. The momentum formula (63b) works to explain momentum conservation in process I, but cannot explain how momentum is conserved in processes II and III. The reason is, the explanation of how momentum is transferred to the body discussed in Sec. V involves momentum transfer perpendicular to the motion, for which m^* does not play a role: the momentum transferred to the ions, Eq. (44), does not depend on m^*. We have discussed elsewhere [9] other reasons for why the correct momentum expression has to be Eq. (63a) rather than Eq. (63b).

BCS advocates argue that because at low temperatures the superconducting state with the magnetic field excluded has lower free energy than the normal state with the magnetic field inside, the system will somehow "find its way" to the lower free energy state and expel the magnetic field. They do not feel compelled to explain in the scientific literature *how*, within the confines of BCS theory or even within time-dependent Ginzburg Landau theory, the process occurs respecting momentum conservation and reversibility. We argue that such a stance is unacceptable. One might say that it is equivalent to saying that because an electron-positron pair has the same energy as a single 1.022-MeV photon, a theory predicting that the former will decay into the latter has a claim to validity. It does not, because such a process with a single photon would violate momentum conservation, two photons are needed. Hence the theory with only one photon cannot be a valid theory, no matter what other valid predictions it makes.

We argue that within BCS theory the Meissner transition is a "forbidden" transition [7], since the transition cannot take place respecting momentum conservation if only the supercurrent changes its momentum. As in other forbidden transitions in physics, it is necessary to ascertain how long it will take the system to get around the selection rules originating in conservation laws by using higher order processes. For example, consider gamma decay of excited atomic nuclei. Changes in the nuclear angular momentum by more than one unit cannot occur by emission of a single photon because this would violate angular momentum conservation. Changes by more than 1 unit can occur, but each additional unit of spin change inhibits the decay rate by about five orders of magnitude. For the highest known spin change of 8 units, the decay rate is suppressed by a factor 10^{35} and takes 10^{15} years instead of 10^{-12} seconds. Similarly, we believe any route to explain the Meissner effect within BCS theory satisfying conservation laws and reversibility is highly "forbidden" and would take time beyond the age of the universe for macroscopic systems. We would like to challenge BCS advocates to show that this is not so, by explaining the mechanism by which momentum is transferred between electrons and the body in a reversible fashion.

In contrast, the theory of hole superconductivity does have the physical elements necessary to explain these processes [8]. In summary, those physical elements that are not part of BCS theory are the following: (i) normal carriers are necessarily holes; (ii) when a system goes superconducting, not only the occupation of Bloch states near the Fermi energy changes as predicted by BCS, keeping the individual Bloch states unaltered; instead, the electronic wave function expands, and the highly dressed normal carrier becomes an undressed carrier with an extended wave function that does not "see" the short-wavelength ionic potential [32,33]; (iii) as a consequence of (ii), supercarriers respond to external fields according to the bare electron mass [9] rather than the effective mass as predicted by BCS, and (iv) as electrons become superconducting, negative charge extends beyond the normal-superconductor boundary into the normal region.

Because these issues are basic and fundamental to the understanding of superconductivity, we argue that it is imperative to resolve them. Physicists should stop using conventional BCS- London theory to describe real superconductors unless or until it can be shown that the theory does not violate momentum conservation.

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APPENDIX A: DERIVATION OF THE HALL COEFFICIENT AND HALL FORCE ON THE LATTICE

We consider the Hall effect in the geometry of Fig. 7. The Hall coefficient is defined as

$$R_H = \frac{E_y}{J_x H} \tag{A1}$$

with $\vec{H} = H\hat{z}$ the applied magnetic field, $\vec{J}_x = J_x\hat{x}$ the current density, and $\vec{E}_y = E_y\hat{y}$ the Hall field. The external force on an electron of wave vector k in a direction perpendicular to the current ($\hat{y}$ direction) is

$$F^k_{\text{ext}} = eE_y - \frac{e}{c}v_k H \tag{A2}$$

with

$$v_k = \frac{1}{\hbar}\frac{\partial \epsilon_k}{\partial k}. \tag{A3}$$

We assume an isotropic band with energy ϵ_k and omit vector labels on the wave vectors. The total force on an electron of wave vector k in a direction perpendicular to the current is

$$F^k_{\text{tot}} = m_e\frac{dv_k}{dt} = \frac{m_e}{m^*_k}\frac{d}{dt}(\hbar k) = \frac{m_e}{m^*_k}\left(eE_y - \frac{e}{c}v_k H\right) \tag{A4}$$

according to the semiclassical equation of motion, with

$$\frac{1}{m^*_k} = \frac{1}{\hbar^2}\frac{\partial^2 \epsilon_k}{dkdk} \tag{A5}$$

the effective mass tensor. On the other hand, we can write the force on the electron of wave vector k as the sum of the external force and the force exerted by the lattice:

$$F^k_{\text{tot}} = F^k_{\text{ext}} + F^k_{\text{latt}} = eE_y - \frac{e}{c}v_k H + F^k_{\text{latt}}. \tag{A6}$$

The total force on carriers per unit volume in a direction perpendicular to the current is, from integrating Eq. (A4) over the occupied states,

$$F_{\text{tot}} \equiv \int_{\text{occ}} \frac{d^3k}{4\pi^3}F^k_{\text{tot}} = m_e e\int_{\text{occ}}\frac{d^3k}{4\pi^3}\frac{1}{m^*_k}\left(E_y - \frac{H}{c}v_k\right) \tag{A7}$$

and the total force per unit volume exerted by the lattice on electrons in the transverse direction is, from Eq. (A6),

$$F_{\text{latt}} \equiv \int_{\text{occ}}\frac{d^3k}{4\pi^3}F^k_{\text{latt}} = F_{\text{tot}} - e\int_{\text{occ}}\frac{d^3k}{4\pi^3}\left(E_y - \frac{H}{c}v_k\right). \tag{A8}$$

Next, we evaluate Eqs. (A7) and (A8) for the cases of almost empty and almost full bands.

1. Almost empty band

The number of carriers and current are given by

$$n_e = \int_{\text{occ}}\frac{d^3k}{4\pi^3}, \tag{A9a}$$

$$J_x = e\int_{\text{occ}}\frac{d^3k}{4\pi^3}v_k, \tag{A9b}$$

and we assume that for the occupied states near the bottom of the band,

$$\frac{1}{m^*_k} \sim \frac{1}{m^*} > 0, \tag{A10}$$

independent of k. Equation (A7) yields

$$F_{\text{tot}} = \frac{m_e}{m^*}\left(n_e eE_y - \frac{H}{c}J_x\right) \tag{A11}$$

and setting $F_{\text{tot}} = 0$ yields

$$E_y = \frac{J_x H}{n_e ec}, \tag{A12}$$

$$R_H = \frac{E_y}{J_x H} = \frac{1}{n_e ec}, \tag{A13}$$

and from Eq. (A8)

$$F_{\text{latt}} = -\left(n_e e E_y - \frac{J_x H}{c}\right) = 0 \tag{A14}$$

using Eq. (A12). Therefore, for this case, the Hall coefficient R_H is negative, the total force exerted by the lattice on the carriers is zero, and conversely the total force exerted by the carriers on the lattice is zero.

2. Almost full band

The number of carriers and current are given by

$$n_h = \int_{\text{unocc}} \frac{d^3k}{4\pi^3}, \tag{A15a}$$

$$J_x = e\int_{\text{occ}} \frac{d^3k}{4\pi^3} v_k = -e\int_{\text{unocc}} \frac{d^3k}{4\pi^3} v_k, \tag{A15b}$$

and we assume

$$\frac{1}{m_k^*} \sim -\frac{1}{m^*} < 0 \tag{A16}$$

independent of k, for the unoccupied states near the top of the band. We have then

$$\int_{\text{occ}} \frac{d^3k}{4\pi^3}\frac{1}{m_k^*} = -\int_{\text{unocc}} \frac{d^3k}{4\pi^3}\frac{1}{m_k^*} = \frac{n_h}{m^*}, \tag{A17}$$

$$e\int_{\text{occ}} \frac{d^3k}{4\pi^3}\frac{1}{m_k^*} v_k = -e\int_{\text{unocc}} \frac{d^3k}{4\pi^3}\frac{1}{m_k^*} v_k == -\frac{J_x}{m^*}, \tag{A18}$$

hence from Eq. (A7),

$$F_{\text{tot}} = \frac{m_e}{m^*}\left(n_h e E_y + \frac{H}{c} J_x\right) \tag{A19}$$

and setting $F_{\text{tot}} = 0$ yields

$$E_y = -\frac{J_x H}{n_h e c}, \tag{A20}$$

$$R_H = \frac{E_y}{J_x H} = -\frac{1}{n_h e c}. \tag{A21}$$

Therefore, in this case, the Hall coefficient R_H is positive. To find the force exerted by the lattice on electrons from Eq. (A8), we use that

$$\int_{\text{occ}} \frac{d^3k}{4\pi^3} = \int_{\text{zone}} \frac{d^3k}{4\pi^3} - \int_{\text{unocc}} \frac{d^3k}{4\pi^3} = \frac{2}{v} - n_h \tag{A22}$$

with v the volume of the unit cell, and use Eq. (A15b), and obtain

$$F_{\text{latt}} = -\left[eE_y\left(\frac{2}{v} - n_h\right) + \frac{H}{c} J_x\right] \tag{A23}$$

and using Eq. (A20)

$$F_{\text{latt}} = -\frac{2eE_y}{v}, \tag{A24}$$

which unlike Eq. (A14) is *not* zero. Hence the total force per unit volume exerted by the carriers on the lattice is

$$F_{\text{on-latt}} = \frac{2eE_y}{v}. \tag{A25}$$

Now the electric field E_y also exerts a force on the lattice. The compensating ionic charge density per unit volume is $|e|(2/v - n_h)$, hence the direct force of the electric field on the ions per unit volume is

$$F_{\text{on-latt}}^{E_y} = -eE_y\left(\frac{2}{v} - n_h\right) \tag{A26}$$

so that the "net" force on the lattice per unit volume is

$$F_{\text{on-latt}}^{\text{net}} = F_{\text{on-latt}} + F_{\text{on-latt}}^{E_y} = n_h e E_y, \tag{A27}$$

or, using Eq. (A20)

$$\vec{F}_{\text{on-latt}}^{\text{net}} = -\frac{H}{c} J_x \hat{y} \tag{A28}$$

in agreement with Eq. (31).

APPENDIX B: THE KEY ISSUE OF REVERSIBILITY

This paper rests on the assumption that under ideal conditions the transition between normal and superconducting states in the presence of a magnetic field is reversible, in other words, that it occurs without change in the entropy of the universe. In this appendix, we discuss the history of this issue, and the experimental and theoretical evidence in its favor as it relates to our work.

Until the year 1933, when the Meissner effect was discovered [1], it was generally believed that the transition from the superconducting to the normal state when a current flows in the superconductor was necessarily irreversible: when the system became normal, resistance would become nonzero, the current would decay through the usual collision processes that occur in the normal state, and Joule heat K, with K the kinetic energy of the supercurrent, would be dissipated in the process. As a consequence, the entropy of the universe would increase by an amount $\Delta S_{\text{irr}} = K/T$, with T the temperature. No measurements were done to verify this assumption, presumably because it was considered to be a self-evident truth [34].

The first hint that in fact this self-evident truth might *not* be true came from the experimental finding [35] of a relation between the difference in specific heats in the normal and superconducting states and the temperature derivative of the critical magnetic field at the critical temperature:

$$C_s(T_c) - C_n(T_c) = \frac{V}{4\pi} T \left(\frac{\partial H_c(T)}{\partial T}\right)_{T_c}^2 \tag{B1}$$

with V the sample volume. This is known as the "Rutgers relation." Already before the discovery of the Meissner effect, Gorter [36] showed that Eq. (B1) follows from the assumption that the magnetic field B is zero in the interior of superconductors *and* that the relation

$$\frac{dQ}{T} = dS \tag{B2}$$

holds for the two phases, where Q is the heat absorbed (released) in the transition and S the entropy of the phase. Equation (B2) is equivalent to saying that the transition is reversible, hence that *no* Joule heat is dissipated when the supercurrent stops. Equation (B1) is simply derived from the relation [37]

$$dF = -SdT - MdH \tag{B3}$$

for the free energies of the normal and superconducting phases assuming the magnetization $M = 0$ for the normal phase and assuming

$$B = H + 4\pi\frac{M}{V} = 0 \tag{B4}$$

for the superconducting phase. The above equations are valid for a long cylinder with a magnetic field in a direction parallel to the axis. Using $S = -\partial F/\partial T)_H$, it follows from (B3) and (B4) that

$$S_n(T) - S_s(T) = \frac{L(T)}{T} = -\frac{V}{4\pi}H_c(T)\frac{\partial H_c(T)}{\partial T} \tag{B5}$$

along the phase transition line in the H-T phase diagram. L is the latent heat of the transition. Equation (B5) is the analogous of the Clausius-Clapeyron equation relating pressure and temperature for the liquid-solid or liquid-gas transition. It assumes that the free energies of the coexisting phases are the same *and* that the transition is reversible. From $C = T\partial S/\partial T)_H$ and Eq. (B5), it follows that

$$C_s - C_n = \frac{V}{4\pi}T\left[\left(\frac{\partial H_c(T)}{\partial T}\right)^2 + H_c(T)\frac{\partial^2 H_c(T)}{\partial T^2}\right] \tag{B6}$$

along the coexistence curve. Equation (B6) reduces to Eq. (B1) at the particular point $T = T_c$, where the magnetic field and the latent heat are zero. All these relations follow from the fact that the relation between the difference in free energies in the normal and superconducting states at temperature $T < T_c$ and the critical field $H_c(T)$ at that temperature is

$$F_n(T) - F_s(T) = V\frac{H_c(T)^2}{8\pi} \tag{B7}$$

if the transition is reversible [36].

Note that Eq. (B7) also follows from BCS theory [5]. Therefore, within conventional BCS theory, it is assumed, just as this paper assumes, that the normal-superconductor transition in a magnetic field is a reversible phase transformation under ideal conditions.

These relations, together with London's electrodynamic equations, imply that the kinetic energy of the supercurrent is precisely given by the difference in the free energies of normal and superconducting states Eq. (B7). The supercurrent density is given by

$$\vec{J} = en_s\vec{v}_s \tag{B8}$$

with $\vec{v}_s$ the superfluid velocity. London's equation is

$$\vec{\nabla}\times\vec{J} = -\frac{c}{4\pi\lambda_L^2}\vec{H} \tag{B9}$$

with λ_L the London penetration depth. In a cylindrical geometry, Eq. (B9) implies

$$J = -\frac{c}{4\pi\lambda_L}H \tag{B10}$$

so from Eqs. (B7) and (B10),

$$F_n - F_s = \frac{2\pi\lambda_L^2}{c^2}J^2, \tag{B11}$$

and using the standard equation for the London penetration depth [5]

$$\frac{1}{\lambda_L^2} = \frac{4\pi n_s e^2}{m_e c^2}, \tag{B12}$$

it follows that

$$F_n - F_s = \frac{n_s}{2}m_e v_s^2 \equiv K. \tag{B13}$$

The right-hand side of Eq. (B13) is the kinetic energy density of the supercurrent. At the phase boundary between normal and superconducting phases, Eq. (B13) holds and this guarantees that there is phase equilibrium between the two phases [24]. Equation (B13) also implies that when there is a small displacement of the phase boundary whereby a region goes from S to N, or from N to S, the resulting change in the kinetic energy of the supercurrent is exactly compensated by the difference in the free energies of the two phases. This implies that there is zero Joule heat dissipated when the supercurrent stops.

After the Meissner effect was discovered, it would seem very natural to expect that the transition in the presence of a magnetic field was perfectly reversible for the following reason: if the kinetic energy of the supercurrent is stored rather than dissipated as Joule heat when the system becomes normal, it will be available to be converted again to kinetic energy of the supercurrent in the reverse transformation as the system becomes superconducting and expels the magnetic field by generation of the Meissner current. Nevertheless, despite the theoretical consistency and inherent beauty of the above considerations, it was thought necessary to check this expectation experimentally, presumably because it was considered counterintuitive that a supercurrent could stop without any dissipation when the system becomes normal. In the period 1934–1938, W. H. Keesom and coworkers did very extensive experimental work to check these predictions in a variety of ways [2,15–18]. All the results found were consistent with the nonexistence of irreversible heat dissipation under ideal conditions (e.g., pure samples, the transition proceeding slowly), and the thermodynamic relations discussed above were found to hold to high accuracy.

Specifically, in Ref. [15], Keesom and coworkers made calorimetric measurements along a "Gorter cycle" [3,36] for Tl: cooling below T_c in zero field, then applying a field of magnitude just below the threshold value, then heating across the transition in the presence of the field to above T_c, then switching off the field. They obtained the latent heat associated with the transition directly from the measurements, and applied the first and second laws of thermodynamics to the heat exchanged along the cycle. Allowing for the possibility of an irreversible increase in entropy σ in the S-N transition, they found $\sigma = 0$ within experimental error, in other words that

$$\oint\frac{dQ}{T} = 0 \tag{B14}$$

holds in going around the cycle, hence concluded that "no irreversible entropy change occurs." In Ref. [16], they repeated and confirmed these results to higher accuracy, stating that to be consistent with the experimental results "it is essential that the persistent currents have been annihilated before the material gets resistance, so that no Joule heat is developed." In Ref. [17], the atomic latent heat of *Sn* was measured and compared with the theoretical expression (B5) that assumes reversibility, finding that "the agreement between the observed values and the calculated ones is striking." In Ref. [18], both sides of Eq. (B6) were measured as well as of Eq. (B5) versus temperature, finding agreement "under the assumption that the transition from the superconductive to the nonsuperconductive state is a reversible one." In Ref. [2], the reverse transition (N-S) in a magnetic field was also examined and it was found that the latent heat in the N-S transition was equal to the one previously measured for the S-N transition, leading to the conclusion that "also in the transition from the normal into the superconductive state no irreversible increase in entropy takes place." Quantitatively, the authors concluded from the measurements that "the maximum limit of the irreversible increase in entropy comes to 1.6% at 3 K and of 1% at 2.6 K [2]."

These experiments established that *at most* a small fraction (1%–2%) of the latent heat measured could be associated with irreversible processes. Particularly at low temperatures, where the kinetic energy of the supercurrent becomes substantially larger than the latent heat, this implies that not more than a tiny fraction (<1%) of the supercurrent could stop through onset of resistance with dissipation of its kinetic energy as Joule heat [38].

Several years later, Mapother [39] again tested the relations Eqs. (B5) and (B6) for *Sn* and In. He stated "In this article we present the results of a careful comparison between magnetic and calorimetric data for the elements, Sn and In" and "It will be shown that the thermodynamic consistency between the two types of measurement is, in general, of the order of 1% and limited mainly by the precision of the calorimetric data." Thus he established that the relations (B5) and (B6) hold to better than 1%, confirming reversibility.

These experiments and the associated theory are extremely strong evidence that the normal-superconductor transition in the presence of a magnetic field is reversible. Note also that state of the art calorimetry [40] is now substantially more advanced than it was in the 1930's. Thus, rather than to 1% accuracy, it may now be possible to establish experimentally that not more than 0.1% or perhaps even not more than 0.01% of the kinetic energy of the supercurrent is dissipated in irreversible processes. This would imply that 99.99% of the supercurrent stops without the current carriers undergoing irreversible collisions. The question then would be, how is 99.99% of the mechanical momentum of the supercurrent transmitted to the body as a whole without irreversible collisions?

We argue in this paper that the explanation of this presents an insurmountable challenge to the conventional theory of superconductivity. The conventional theory offers no mechanism by which "the persistent currents have been annihilated before the material gets resistance, so that no Joule-heat is developed," as demanded by Keesom [16]. The only way the conventional theory has addressed this issue is by proposing that the momentum of the supercurrent is passed on to normal electrons when Cooper pairs dissociate, that then transfer it to the body through collisions [20], which would have to be perfectly elastic and in addition generate no entropy.

So the conventional theory has to explain both how normal electrons can inherit the momentum of the supercurrent but not the kinetic energy of the supercurrent (or at least not more than 0.01% of the kinetic energy of the supercurrent), and how the momentum of these normal electrons can subsequently be transferred to the body as a whole in a reversible way, without entropy generation. We believe it is impossible to do either without violating basic laws of physics, even if the transition occurs infinitely slowly. In any event, it certainly has not been done in the scientific literature to date.

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